On the Market Consistent Valuation of Fish Farms: Using the Real Option Approach and Salmon Futures

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Abstract

We consider the optimal harvesting problem for a fish farmer in a model which accounts for stochastic prices featuring Schwartz (1997) two factor price dynamics. Unlike any other literature in this context, we take account of the existence of a newly established market in salmon futures, which determines risk premia and other relevant variables, that influence risk averse fish farmers in their harvesting decision. We consider the cases of single and infinite rotations. The value function of the harvesting problem determined in our arbitrage free setup constitutes the fair values of lease and ownership of the fish farm when correctly accounting for price risk. The data set used for

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this analysis contains a large set of futures contracts with different maturities traded at the Fish Pool market between 12/06/2006 and 22/03/2012. We assess the optimal strategy, harvesting time and value against two alternative setups. The first alternative involves simple strategies which lack managerial flexibility, the second alternative allows for managerial flexibility and risk aversion as modeled by a constant relative risk aversion utility function, but without access to the salmon futures market. In both cases, the loss in project value can be very significant, and in the second case is only negligible for extremely low levels of risk aversion. In consequence, for a risk averse fish farmer, the presence of a salmon futures market as well as managerial flexibility are highly important.

Keywords: Agricultural Commodities, Aquaculture, Futures, Real Options, Risk Management

JEL Subject Classification: G13, Q20, Q22

Established in 2006 in Bergen (Norway) the Fish Pool is a derivatives market, where futures and options on fresh farmed salmon are traded in large quantities. Contract volumes traded at this market have grown to 102,295 tons, which is equivalent to 440 million Euro (4 billion Norwegian Krone), during 2013, continuing a strong upwards trend from previous years. Following its great success in the start-up phase the Oslo Stock Exchange acquired 71% of Fish Pool in December 2012 and currently owns 94.3%.

Bergfjord (2007), Dalton (2005), Bulte and Pennings (1997) as well as Gronvik (2009) provide good arguments for this trend. In short, markets for forwards and futures on fresh farmed salmon help companies which use this commodity in their production, for example, food processing companies. They can use these contracts to effectively hedge the price risk and plan ahead. The same contracts help producers, i.e. fish farmers, to reduce their (selling) price risk. An analysis of the welfare effects of futures markets in a rather general context is presented in Hirschleifer (1988). He discusses a two period model which includes consumers, processors, producers and speculators.

In this article we discuss how information reflected in the prices of contracts traded at a market such as Fish Pool can be used to compute fair (i.e. arbitrage free) prices for lease and ownership of fish farms. One of the most crucial elements in this process is to correctly account for risk premia persisting in the stochastically fluctuating salmon spot price and how these affect the harvesting decision of fish farmers. This will build upon key concepts from the derivatives pricing literature, including risk neutral valuation and hedging, which are motivated through the no-arbitrage principle.[1] The value function attached to the harvesting problem, obtained within a generally arbitrage free setting, will then provide the fair value of the salmon farm, either in terms of lease, when single rotation is considered, or in terms of ownership, when infinite rotation is considered. More specifically we are using the Schwartz (1997) two-factor approach to model the stochastic dynamics of the spot price.[2] This approach features a stochastic convenience yield and is considered to be a benchmark in the pricing of commodity futures. It generally provides a good fit to various shapes of the forward curve corresponding to the associated futures prices and can realistically describe classical conditions such as market backwardation or contango.[3]

The existing literature on the economics of salmon farming and aquaculture can broadly be classified into two categories. The first category focuses on models, where salmon prices are assumed to be deterministic. Representative examples are Bjorndal (1988), Arnason (1992), Cacho (1997) who also provides a good survey about general work that falls in this category, Yu and Leung (2006) as well as Guttormsen (2008). Some of these outputs emphasize additional important issues such as optimal feeding schedules or partial harvesting plans. The second category involves models where prices are assumed to follow a stochastic process. Forsberg and Guttormsen (2006) present a simplistic framework in discrete time, where the price process is specified without reflection on actual market prices. The harvesting decision is made on the basis of the agents subjective assessment on the distribution of prices without accounting for risk aversion. More sophisticated models have been developed in the related context of forestry management. Insley (2002) and Insley and Rollins (2003) present continuous time models, with general stochastic price dynamics, emphasizing the effect of mean reversion.[4] Here, the specification of the dynamic model is informed by historical data on timber prices (no derivatives). The harvesting decision is then based on the expected profits under the empirical distribution of prices. This approach ignores relevant risk premia that reflect the risk aversion of a representative agent. This shortcoming has been corrected in Chen, Insley and Wirjanto (2011) to which we will refer further below. In general, while adding the important feature of price uncertainty, the literature in the second category is still too disconnected from the existing financial literature on asset pricing. In contrast to this literature, our article is strongly linked to the Schwartz (1997) framework, which is considered to be a benchmark for commodity futures pricing. We estimate the parameters in our model on the basis of an extensive data-set of futures prices obtained from the Fish Pool market, covering the period from 12/06/2006 until 22/03/2012. Futures prices, as opposed to spot prices, allow us to determine the market measure that is used to price contingent claims. They are also far more abundant, spot prices are often only published irregularly and infrequently. By looking at the optimal stopping problem of an individual fish farmer, we then use real option theory to determine the monetary values for lease and ownership for a model fish farm. This analysis is undertaken under the market measure and hence reflects relevant risk premia and the risk aversion of a representative agent who is able to hedge risk exposure through salmon futures.[5] A related approach has been used to price forestry resources by Chen, Insley and Wirjanto (2011) based on lumber futures traded on the Chicago Mercantile Exchange. However these authors use a simplification of the Schwartz (1997) two-factor model, the so called "long-term model", which only features one stochastic state variable (a combination of spot price and stochastic convenience yield). This model leads to good approximations of the results that would be produced by the actual two-factor model if rotation periods are sufficiently long. In reality the rotation periods and harvesting cycles in salmon farming are however significantly shorter than for forestry resources, which is why we used the actual two-factor model from Schwartz (1997). We solve this more complex problem by appropriately adjusting the Longstaff and Schwartz (2001) least squares Monte Carlo approach, rather than using the long term approximation.

The methodology presented is applied to determine value of lease and ownership of a model fish farm. It is important to emphasize that this is for illustrative purposes, as some of the relevant costs are implicit or omitted. Our analysis has been guided by a number of practitioners from salmon farming businesses in Norway and Scotland and we would like to thank those involved for their contributions. Real option theory is applied in aquaculture management, but the financial models currently used do not seem to go beyond Black (1976), which is obtained from our set-up by fixing the convenience yield to a constant level. To further investigate the value of managerial flexibility, we assessed our harvesting strategy against simple ones, taking a similar line as in McDonald (2000). We show that our harvesting strategy adds approximately an extra 10% to the farm's value.

Finally, but perhaps most importantly, we look at the impact that the existence of a salmon futures market has on the harvesting decision of an individual fish farmer, depending on the level of risk aversion that this fish farmer exhibits. In order to do this, we assume that the fish farmer's preferences are modeled by a constant relative risk aversion (CRRA) utility function and that the fish farmer does not have access to the salmon futures market. We observe that the loss due to no access to the salmon futures market is only negligible for extremely low levels of risk aversion, but can be very substantial (more than 10%) for reasonable levels of risk aversion. We further observe that the average harvesting time is decreasing with the level of risk aversion, but can be higher or lower without access to the salmon futures market than it is with access. As such our conclusion is that the salmon futures market provides a highly valuable service for risk averse fish farmers. These results are also relevant and related to literature in the context of real options under risk aversion, which includes Hugonnier and Morellec (2013), Henderson (2007) and Ewald and Yang (2008).

The rest of the article is structured as follows. In the next section we will briefly review the Schwartz (1997) two-factor approach, while in the following section we summarize the results of our empirical estimation of the model. The optimal harvesting and rotation problem of an individual fish farmer and in consequence the valuation for lease and ownership of a model fish farm are discussed in detail in the penultimate section. The final section contains our main conclusions.

The Schwartz (1997) Two-Factor Framework

Let us denote with $P(t)$ the salmon spot price at time t. This can be identified with the Fish Pool Index against which future contracts are settled at the Fish Pool market. In the Schwartz (1997) two-factor framework the dynamics of $P(t)$ is given by

$$dP(t) = (\mu - \delta(t))P(t)dt + \sigma_1 P(t)dZ_1(t) \tag{1}$$

$$d\delta(t) = \kappa(\alpha - \delta(t))dt + \sigma_2 dZ_2(t), \tag{2}$$

with constants μ, κ, α, σ_1 and σ_2 under the real world probability $\mathbb{P}$. The two Brownian motions $Z_1(t)$ and $Z_2(t)$ are assumed to be correlated, i.e.

$$dZ_1(t)dZ_2(t) = \rho dt. \tag{3}$$

The process $\delta(t)$ represents the stochastic convenience yield and can be recognized as a mean reverting Ornstein Uhlenbeck process, where α represents the mean reversion level and $\kappa > 0$ the mean reversion speed. It reflects the benefits and costs that an agent receives when holding the salmon, such as liquidity and storage/maintenance costs. The price dynamics (1) has an implicit mean reversion feature. If $\rho > 0$, then the instantaneous correlation between $P(t)$ and $\delta(t)$ is positive. Hence $P(t)$ is likely to be large when $\delta(t)$ is large and in this case $\delta(t)$ is likely to be larger than μ. The drift term in (1) will then push $P(t)$ downwards. The opposite happens if $P(t)$ is small, pushing $P(t)$ upwards. If in fact one chooses $\delta(t) = \kappa \ln(P(t))$, one obtains the dynamics of a geometric Ornstein-Uhlenbeck process in (1), and $\delta(t)$ defined in this way satisfies (2) with $\rho = 1$. In this case we obtain the so called Schwartz (1997) one-factor model.

A forward contract in this context is an agreement established at a time $s < T$ to deliver or receive the salmon at time T for a price K, which is specified at time s. In financial terms, the payoff at time of maturity T of such a forward contract is

$$H = P(T) - K. \tag{4}$$

We assume here and in the following that the interest rate r is constant. The value K that makes this contract have a value of zero under a no-arbitrage assumption is then given by

$$F_P(s,T) = \mathbb{E}_{\mathbb{Q}}\left(P(T)|\mathcal{F}_s\right). \tag{5}$$

This is called the forward price at time s. The symbol $\mathcal{F}_s$ denotes the information available at time s and we denote in the following with $\mathbb{F} = (\mathcal{F}_s)$ the associated filtration which represents the information flow.[6] The expectation in (5) is taken with respect to the pricing measure $\mathbb{Q}$, which takes into account a market price of convenience yield risk λ, i.e.

$$
\begin{aligned}
(6) \qquad dP(t) &= (r - \delta(t))P(t)dt + \sigma_1 P(t) d\tilde{Z}_1(t) \\
(7) \qquad d\delta(t) &= (\kappa(\alpha - \delta(t)) - \lambda)dt + \sigma_2 d\tilde{Z}_2(t),
\end{aligned}
$$

with $\tilde{Z}_1(t)$ and $\tilde{Z}_2(t)$ Brownian motions under $\mathbb{Q}$ and $d\tilde{Z}_1(t)d\tilde{Z}_2(t) = \rho dt$.[7] As the interest rate is assumed to be constant, we do not need to distinguish between forwards and futures.[8]

For convenience, we can always assume that current time is normalized to 0 and that the time of maturity T is relative to this, hence the same as the time to maturity. Since our model is Markovian, we can then denote the futures price in (5) as $F(P, \delta, T)$ depending on current spot price, level of convenience yield and time to maturity T. With this notation, Schwartz (1997) refers to Jamshidian and Fein (1990) and Bjerksund (1991) for an explicit expression for (5):

$$
\begin{aligned}
(8) \qquad F(P, \delta, T) &= P \cdot \exp\left(-\delta \cdot \left(\frac{1 - e^{-\kappa T}}{\kappa}\right) + A(T)\right) \\
(9) \qquad A(T) &= \left(r - \alpha + \frac{\lambda}{\kappa} + \frac{1}{2}\frac{\sigma_2^2}{\kappa^2} - \frac{\sigma_1 \sigma_2 \rho}{\kappa}\right) T + \frac{1}{4}\sigma_2^2 \left(\frac{1 - e^{-2\kappa T}}{\kappa^3}\right) \\
&\quad + \left(\alpha\kappa - \lambda + \sigma_1\sigma_2\rho - \frac{\sigma_2^2}{\kappa}\right)\left(\frac{1 - e^{-\kappa T}}{\kappa^2}\right).
\end{aligned}
$$

Note, that the futures price (8) has a log-normal distribution, which makes the analytical pricing of options in this framework possible. On the other hand note that at least one of the state variables, the convenience yield $\delta(t)$ is unobservable. In fact Schwartz (1997) assumes in general that both the commodity price $P(t)$ and the convenience yield $\delta(t)$ are unobservable, and only the future prices (8) are observable. In order to estimate the model, Schwartz (1997) then applies Kalman filtering techniques.[9]

Data and Empirical Estimates

Our data set consists of 1496 daily observations of futures prices on Fish Pool ASA from 12/06/2006 to 22/03/2012. For the whole sample period, complete data on the first 29 futures contracts sorted by different maturities are available. We use a similar notation as in Schwartz (1997) and denote with F1 the contract closest to maturity (with average maturity of 0.040 year) counting up to F29 which represents the contract farthest to maturity (with average maturity of 2.382 years).[10] Contracts in Panel A, Panel B, Panel C and Panel D are chosen as proxies for short-term, medium-term, long-term and mixed-term futures contracts respectively.[11] In each panel, five contracts (i.e., N=5) are used for the estimation. More precisely, Panel A contains F1, F3, F5, F7 and F9; Panel B contains F12, F14, F16, F18, F20; Panel C contains F24, F25, F26, F28 and F29 and Panel D contains F1, F7, F14, F20, F25. Note that Panel D is a combination of two short-term contracts, two medium-term contracts and one long-term contract and hence inhibits some form of pooling contracts with varying maturities. Note, that the pooling of too may contracts in one panel would lead to numerical problems in the Kalman filter. Table 1 describes the data features.[12]

[Table 1 about here.]

In this article we use an approach proposed by Schwartz (1997) to estimate the parameters in the model by use of the Kalman filter. The estimates are shown in table 2, and the root-mean-square deviation (RMSE) and the mean-absolute error (MAE) for each panel are shown in table 3. The risk-free rate r (3.03%) is chosen as the average Norwegian interest rate over the sample period. It can be observed that for each panel, all coefficients are significant at the 1% level; the correlation coefficient ρ is large; the speed of mean-reversion of the convenience yield κ, the expected return on the spot commodity μ, the mean-level of convenience yield α and the market price of convenience yield risk λ are all positive and reasonable. The volatility of the spot price σ_1 is relatively stable compared to the volatility of the convenience yield σ_2. Besides, it is also worth to note that the expected return on the spot commodity μ increases while the speed of mean-reversion κ decreases as the term of the contracts increases. The parameters obtained from Panel D seem to reflect and moderate the corresponding parameters from panels A, B and C. This is intuitive as Panel D is a mixture of contracts from these panels. According to table 3, the estimates are generally good. In order to asses the estimates against movements in the interest rates, we have run additional tests for a number of sub-panels, each corresponding to a different interest rate regime. We refer to part A of the online supplement of the article for this. In addition we have assessed the parameter estimates against possible seasonal effects and found that seasonality only marginally affects the fish farm valuation problem discussed in the later sections. These results are available in part B of the online supplement.

[Table 2 about here.]

[Table 3 about here.]

The term structures (real and model generated) reveal that both contango and backwardation are present in the market at different times – see part D of the online supplement for a specific comparison. In general, the model makes a good prediction for the short-term panel (filtered spot is near closest to maturity future and model generated forward curves match the shape of the actual forward curves) but finds it more difficult to capture the shapes of the forward curves corresponding to longer-term panels, where the actual term structure appears to be rather unconventional.[13] A more detailed analysis of these data, which also includes an implementation of the Schwartz three-factor model, featuring a stochastic interest rate, is presented in Ewald et al. (2015).

Optimal Harvesting Decision for An Individual Fish Farmer and Valuation of the Fish Farm

In this section we discuss the problem of optimal fish farming in the context of the previous sections. We consider a model fish farm, whose manager can decide when to harvest the fish. In this context and in the following we use the expressions manager and fish farmer synonymously. Both, the case of a single rotation as well as the case of sequential harvesting (infinite number of rotations), will be investigated. We assume that the manager of the fish farm acts rationally and chooses the harvesting time(s) in order to maximize benefits. The so determined value corresponds to the value of a lease (single rotation) respectively ownership (infinite number of rotations) of the fish farm. The methodology applied in this section is usually referred to as the real option approach and shares similarities with the valuation of financial option type derivatives, specifically those of American type, see Dixit and Pindyck (1994) for an overview.

Essentially the fish farmer's problem at each point in time is to decide whether it is better to postpone harvest, let the biomass in the pond grow and hope for beneficial movements in the salmon spot price while paying the costs for feeding and maintenance, or harvest the fish and cash in the revenue from selling on the spot market while paying a one time cost for harvesting. In the case of an infinite number of rotations, the fish farmer will after harvest be able to start a new harvesting cycle.

In his decision whether to harvest now or postpone harvest, the fish farmer weighs up current benefits against expected future benefits. In the absence of the futures market discussed in the earlier sections, this expectation about future benefits would be based on the fish farmer's subjective beliefs, under which the price dynamics and convenience yield follow the dynamics (1) and (2). This is the standard approach taken in the aquaculture literature as well as most of the literature on forestry management, including Insley (2002) and Insley and Rollins (2003). However, this approach would miss out on two important facts. First, the decision maker is typically risk averse, and would evaluate future benefits through use of a concave utility function $U(\cdot)$. Second, the decision maker is able to hedge risk exposure through taking a dynamic position in the futures market. Assuming that there are sufficiently many futures contract, such that the underlying financial market is complete, the problem faced by the risk averse agent under the real world measure is equivalent to the problem a risk neutral agent faces under the now unique martingale measure. In fact the kernel of the martingale measure can be determined through marginal utilities, compare Pliska (1997) pages 43-44 and Schwartz (1997). As, the presence of the salmon futures allow the fish farmer to efficiently hegde the idiosyncratic risk in the salmon spot price through a dynamic portfolio of futures with differing maturities, by using the market (martingale) measure $\mathbb{Q}$ for the solution of the optimal stopping problem, we take account of the two facts highlighted above and in this way account for the relevant risk premia and risk aversion of a representative agent. In this way, the fish farmer follows a harvesting strategy which maximizes the financial value of the fish farm, which is an appropriate objective in the corporate setting that fish farms operate nowadays in the real world. This is in line with Schwartz (1997) for crude oil exploration, Chen, Insley and Wirjanto (2011) for lumber and many other studies.

The model discussed in this article is more complex than most of the models considered in the existing fish farming and aquaculture literature. Next to the two stochastic state variables, spot price and convenience yield, which we introduced in the previous sections, we now include a third state variable into our model which represents the biomass. For simplicity we assume that the average weight $w(t)$ of one individual fish during the harvesting cycle follows a deterministic dynamics represented by a von Bertalanffy's growth function, i.e.

$$w(t) = w_\infty \left(a - be^{-ct}\right)^3 . \tag{10}$$

Here w_∞ is the asymptotic weight. This growth function has been widely applied in the aquaculture literature. We assume that the total number of fish $n(t)$ at the fish farm unit during the harvesting cycle follows the dynamics

$$dn(t) = -m(t) \cdot n(t)dt, \tag{11}$$

where $m(t)$ denotes the mortality rate.[14] Note that salmon does not reproduce in the pens, and therefore the number of salmon in each pen has to decrease over time. The total biomass at the fish farm unit is then given as

$$X(t) = n(t)w(t). \tag{12}$$

Single-Rotation Fish Farming

Let us first consider the case of a single rotation. In this case the manager earns revenue from operating the fish farm but returns the fish farm to its owner when one harvesting cycle has been completed. The value determined in that way will correspond to the lease over the period of one harvesting cycle. We assume that to begin with, the fish farm is equipped with a fixed population of smolt[15] and hence initial release costs will not be explicitly accounted for in the single rotation problem. At the time of harvest, the fish farmer will make a profit of $P(t)X(t) - CH(t)$, where $P(t)X(t)$ constitutes the revenue and $CH(t)$ the harvesting costs. This potential profit needs to be evaluated against the option to defer harvest to a later time, and in the mean time pay for certain costs, e.g. feeding the fish. These costs are denoted as $CF(t)$. The optimal harvesting time is the stopping time τ, which is the solution of[16]

$$\max_{\tau} \mathbb{E}_{\mathbb{Q}} \left(e^{-r\tau} \left(P(\tau)X(\tau) - CH(\tau) \right) - \int_0^{\tau} e^{-rt} CF(t) dt \right). \tag{13}$$

It is not possible to obtain analytic solutions for an optimal stopping problem of such complexity. For this reason we revert to a numerical approach pioneered by Longstaff and Schwartz (2001) as well as Cortazar, Gravet and Urzua (2008). This approach is widely known as Longstaff-Schwartz or Least Square Monte Carlo approach.

The two state variables in (13) are $P(\cdot)$ and $\delta(\cdot)$.[17] To apply the optimal stopping rule derived in the following, the agent would substitute the filtered estimates for $P(\cdot)$ and $\delta(\cdot)$ for these variables. This approach has its theoretical foundation in the so called separation principle for optimal control under partial information which states that under appropriate conditions the problem of identifying the optimal decision problem with unobserved state variables can be decomposed into two parts, the filtering problem and the corresponding problem under observed state variables, compare for example Genotte (1986). A similar approach has been followed in Schwartz (1997).

Longstaff-Schwartz Approach

Following the Longstaff-Schwartz approach we proceed in steps as follows:

1. Path Simulation

We simulate a number M of paths over the time horizon T with time-discretization $\Delta t = \frac{T}{N-1}$ for the 2-factor model presented in the second section via the Euler-Maruyama scheme. Details are presented in part C of the online supplement.

2. Valuation Procedure

Similar as in the valuation and exercising of an American option, the fish farmer makes a decision by comparing the immediate harvesting value (VH) with the expected continuation value (VC) at each point in time. The harvesting value VH originates from sale revenue minus the harvesting cost (CH) while VC accounts for all possible discounted expected future rewards attached to waiting as well as costs for feeding (CF). Suppose the fish farmer makes decisions at K discrete points in time $0 < t_1 \leq t_2 \leq t_3 \cdots \leq t_K = T$. Let $x_{t_n} = [P_{t_n}, \delta_{t_n}]'$ denote the two combined stochastic state variables and as before X_{t_n} the biomass of fish based at the farm, while the σ-algebra $\mathcal{F}_{t_n}$ represents the information available at time t_n.[18] Then the optimal stopping time can be obtained from solving the following Bellman equation:

$$
\begin{aligned}
V(t_n, x_{t_n}) \;&=\; \max\{P_{t_n} X_{t_n} - CH_{t_n}, \\
&\quad -CF_{t_n}\Delta t + e^{-r\Delta t}\mathbb{E}_{\mathbb{Q}}[V(t_{n+1}, x_{t_n+1})|\mathcal{F}_{t_n}]\}
\end{aligned}
\tag{14}
$$

where $V(t,x)$ denotes the value function of the problem at time t and state $x = [P,\delta]'$.[19] Expressing the harvesting value VH and the continuation value VC as

$$VH(t_n, x_{t_n}) = P_{t_n} X_{t_n} - CH_{t_n} \tag{15}$$

$$VC(t_n, x_{t_n}) = -CF_{t_n}\Delta t + e^{-r\Delta t}\mathbb{E}_{\mathbb{Q}}[V(t_{n+1}, x_{t_n+1})|\mathcal{F}_{t_n}] \tag{16}$$

the procedure of determining the optimal harvesting time τ proceeds backwards from time T and harvesting occurs when

$$VC(\tau, x_\tau) < VH(\tau, x_\tau); \tag{17}$$

i.e. when the harvesting value is greater than the continuation value.

3. Estimation of the Continuation Value

At each point in time, the harvesting value VH can be readily obtained as a function of the state variables. However the expected continuation value VC is unknown, except at the terminal time T when $VC_T = 0$ as the real option has then expired and fish must be harvested.

However, no-arbitrage pricing dictates that the value of the un-exercised option at time t_n is equal to the sum of the expected remaining future cash flows until expiration, where the expectation is computed under the market measure $\mathbb{Q}$. Let C_{t_k} denote the cash-flow generated at time t_k, then the continuation value at time t_n is given as

$$VC(t_n, x_{t_n}) = \mathbb{E}_{\mathbb{Q}}\left[\sum_{k=n+1}^{\tau} e^{-r(t_k - t_n)} C_{t_k} \middle| \mathcal{F}_{t_n}\right], \tag{18}$$

with

$$C_{t_k} = \begin{cases} -CF_{t_k}\Delta t + VH_{t_k} & \text{if } \tau = t_k \\ -CF_{t_k}\Delta t & \text{otherwise.} \end{cases} \tag{19}$$

The Longstaff-Schwartz approach provides an easy and efficient way to estimate the expected continuation value. The unknown functional form of $VC(t_n, x_{t_n})$ in (18) can be expressed as a linear combination of a countable set of measurable basis functions L_j. In this article, we choose a class of quadratic functions for this purpose. The estimated continuation value at t_n for M simulated paths can then be calculated as,

$$\widehat{VC}_{t_n} = \hat{a}_1{x_1}^2 + \hat{a}_2{x_2}^2 + \hat{a}_3x_1 + \hat{a}_4x_2 + \hat{a}_5x_1x_2 + \hat{a}_6, \tag{20}$$

where the estimated coefficients $\hat{a}_j$ are obtained from regressing the discounted values of future cash flows introduced in (18), i.e., $\sum_{k=n+1}^{\tau} e^{-r(t_k-t_n)}C_{t_k}$, onto the basis functions for all simulated paths. Moreover Longstaff and Schwartz (2001) suggest that it is more efficient to only use in-the-money paths in the estimation, as the exercise decision is only relevant when the option is in the money. We follow this advice and use only paths with positive harvesting value to run the regression. The mechanics of this procedure are described in part C of the online supplement.

Results

We apply the Longstaff-Schwartz approach presented as above to the fish farming problem, using our estimated parameters for the two-factor model in table 2 reflecting the values of futures contracts traded at the fish pool market during the sample period. A number of other parameters which are relevant to the fish farming process but can (and should) not be inferred explicitly from the salmon futures contracts are listed in table 4. These parameters include elements relevant to feeding costs, mortality and weight function [20] and have been obtained from Asche and Bjorndal (2011), pages 182 and 183. Plots of growth and biomass functions can be found in the part D of the online supplement. To calculate the feeding cost, a conversion ratio is used to measure the relationship between feeding quantity and growth/weight of the fish. We use the method of antithetic variates in order to improve the performance of the Longstaff-Schwartz method. In this article, $25,000$ paths and corresponding $25,000$ antithetic paths are simulated for 72 exercise points over 3 years. In other words, we assume that the fish farmer can make a decision about harvesting twice a month, which seems to be a good compromise between computational effort and the fact that in reality harvesting can take place at any day in the year.

[Table 4 about here.]

When should the fish farmer harvest? In the corresponding continuous time model where harvesting can occur at every instantaneous point in time, the optimal harvesting time can be characterized as the first time when $VC(t, x_t) = VH(t, x_t)$ with $x_t = [P_t, \delta_t]'$. This condition describes a two dimensional surface in the (t, P, δ) space, or equivalently for each time t a one-dimensional boundary which splits the (P, δ) space into two regions. In the first region it is optimal to postpone harvesting, while in the second region it is optimal to harvest. This boundary changes over time. In the partial differential equation formulation of the Bellman equation, the so called Hamilton-Jacobi-Bellman equation, this corresponds to the so called free boundary. The original work by Longstaff and Schwartz (2001) as well subsequent work is less conclusive as to how to obtain such a boundary. While under sufficient regularity assumption, in theory the free boundary is a smooth curve in the (P, δ) space which changes shape over time, the time discretization as well as the Monte Carlo/regression element in the Longstaff-Schwartz approach lead to a discrete set of combinations of $(P_m(t_n), \delta_m(t_n))$ where harvesting occurs. These combinations are affected by various estimation and discretization errors. In this article the free boundary/exercise threshold is obtained by non-linear least squares curve fitting for a class of functions, which we chose to be of the form $f(x) = ax^b + c$. This has been carried out for all scenarios introduced previously. Figure 1 shows the result for Panel A while figures for Panel B, Panel C and Panel D are included in part D of the online supplement. Blue spots in the figure represent combinations of P and δ where harvesting occurred. In consequence of the random nature of the problem as well as discretization error, these points do not all lie on the single fitted (thick red) line in the middle. For this reason we also present the boundaries of the 80% confidence intervals above and below the fitted line. These boundaries can also be interpreted as more or less conservative exercise thresholds.[21] The upward sloping concave shape of the curves is characteristic and has been observed in Schwartz (1997) (table XX and page 970) as well as Chen, Insley and Wirjanto (2011) figure 15. For smaller δ the threshold price which triggers harvesting is lower than for larger δ. As Schwartz (1997) page 970 explains, the intuition behind this is that when δ is low at current, because of the mean reversion feature it is likely to be higher in the next period. This will decrease the expected option value (which is decreasing in the convenience yield), and a lower P at current will suffice to make the harvesting payoff larger than the expected option value in the next period, which triggers harvesting. Further, from the dynamics of P, if delta is expected to go up, the growth rate of P is expected to decrease, and as the expected option value is also tied to the expected growth rate, this additionally contributes to decreasing the expected option value and hence lowering the threshold.

[Figure 1 about here.]

Table 5 shows for each panel, the lease value of the fish farm over one harvesting cycle and the average harvesting time, that is the average length of the harvesting cycle under the optimal harvesting rule along the different trajectories in the simulation. It can be observed that with the parameter settings obtained from the fish pool data as well as Asche and Bjorndal (2011), the average harvesting times are around 2 years, which is a realistic value. The lease values vary between 1.1142 million NOK (0.1186 million EUR) and 1.6467 million NOK (0.1752 million EUR)[22]

[Table 5 about here.]

The impact of the level of mean reversion in the price process on the harvesting decision in the forestry management context has been discussed in Insley (2002) and Insley and Rollins (2003). Specifically, Insely (2002) demonstrates that in the context of their one factor model, a lower level of mean reversion leads to later harvests. The situation in our two factor model is slightly more complex, as the mean reversion is only generated implicitly through the correlation between spot price and convenience yield, as explained previously. An analogue case can be made on the following basis though. By increasing the mean reversion speed κ in the convenience yield, the mean reversion feature in the spot price will be diminished as the convenience yield will become more and more like a constant convenience yield. In the extreme case, $\kappa = \infty$, the spot price will be a geometric Brownian motion which in average grows at the rate of $\mu - \alpha$, which is negative for Panels A, B and C, but positive for Panel D. Panels A and D show a significant larger estimate for κ than Panels B and C, hence the level of mean reversion in prices is less for Panels A and D, than it is for Panels B and D. The corresponding exercise boundaries in figure 1 and part D of the online supplement indicate that for the same level of convenience yield harvesting occurs at lower prices in Panels A and D than in Panels B and C. We may conclude from this that lower mean reversion in prices leads to earlier harvesting at lower prices. This finding is also supported by table 5, which shows average harvesting times for the four different panels. The result appears to be inverse to the result obtained in Insley (2002) however, this may be explained by the different sign in the asymptotic drift term, where in our case $\mu - \alpha$ tends to be negative, while in Insley (2002) and Insley and Rollins (2003) the drift term, at least in the geometric Brownian motion case, is positive. Overall, an exact comparison between the two models is very difficult as the mean reversion feature in our model is far more complex and depending on the combination of a number of parameters.

Infinite Rotations Fish Farming

We now consider the situation, when the fish farmer/manager initiates a new harvesting cycle, each time the previous harvesting cycle has been completed. This means, that at the time of harvest, the fish farmer not only receives revenue from selling the fish, but in addition obtains the value of the fish farm in its initial state, i.e. harvestable biomass zero, but with smolt released and current values for state variables P_t and δ_t. In addition to the harvesting costs, costs for releasing new smolts into the empty pen accrue at the end of the harvesting cycle. This enables the fish farmer to start a new harvesting cycle, and this procedure continues ad infinitum. As such this problem reflects ownership of the fish farm and its value will hence correspond to the value of ownership. Intuitively, the prospect of starting a new harvesting cycle after completing a previous harvesting cycle provides an incentive for the fish farmer to harvest earlier. It also presents an additional value, i.e. ownership costs more than a lease. For this reason we expect the average harvesting time to decrease and the value to increase. This is confirmed by our technical analysis. Further details can be found in part C of the online supplement.

Results

Once the value function V_0 has been obtained, we can obtain other relevant results, such as the average harvesting time and thresholds for the infinite rotation fish farming problem. In this part of the article, we consider Panel B over the whole sample period 12/06/2006-22/03/2012 as an example to illustrate the results for the infinite rotation case.[23] We adopt the average 10-year Norwegian bond rate as the proxy of infinite interest rate, i.e., 3.93%, during the sample period and estimates are shown in table 6. This rate suites the time-frame of the problem best. Nevertheless, to get a sense about the robustness of our results, we also considered the other panels as well as three appropriate sub-periods corresponding to three different regimes in the Norwegian base rate. These results are summarized in the part A of the online supplement.

[Table 6 about here.]

For technical reasons (we rely on a finite set of grid points) we have to disqualify paths in the Monte-Carlo part of the Longstaff-Schwartz method, which leave the grid space. In our particular application we chose these limits to be $P_t \in [10, 100]$ and $\delta_t \in [-1.5, 1.5]$. In theory, this presents an alteration of the Schwartz (1997) two-factor model, however in reality, prices outside the grid space have not been observed since the fish pool market has been created in 2006 and would in fact be highly questionable. The same holds for the convenience yields. As such, we expect that this feature of our analysis actually leads to better and more realistic results. Taking both accuracy and efficiency into consideration, 7×7 grid points are chosen, equidistant in each state variable. The termination criterion reflects the average matrix norm (L^1-norm) and termination occurs when this norm falls below 1%. Overall, we observe good convergence of our scheme.

Figure 2 below shows the plot of the value function V_0 as a function of the two-state variables P and δ. This function represents the value for ownership of the fish-farm. It can be clearly observed that the convenience yield has a negative impact on the value of the fish farm, while the salmon spot price obviously has a positive impact. The former can be explained as follows: As discussed previously, ownership of the fish-farm has similar characteristics as holding an option contract on the commodity, but not the commodity itself. As the convenience yield benefits the commodity holder but not the option holder, an increase in the convenience yield will decrease the value of the fish-farm.[24]

[Figure 2 about here.]

Figure 3 shows the thresholds (free boundary for harvesting) for the infinite rotation problem. These have been obtained by appropriately adjusting the methods from the previous section. Compared to single-rotation fish farming, the average harvesting time in the infinite rotation fish farming problem reduces significantly from 2.4251 years to 2.1396 years. As indicated earlier, this is expected, as the prospect of starting a new harvesting cycle provides an incentive for earlier harvesting. The value for ownership of the fish-farm based on the estimates obtained from panel B has been computed as 20.6410 million NOK (2.1962 million EUR) while the lease value is 1.2324 million NOK (0.1311 million EUR). The value for ownership is about 17 times larger than the lease value, which is realistic as well. [25]

[Figure 3 about here.]

Is It Worth It ? The Value of Managerial Flexibility.

McDonald (2000) assessed in a general context optimal exercise rules obtained from a real option approach against simple rules of thumbs, including exercise at an optimal predetermined time. In this way he captured the value of managerial flexibility as well as the usefulness of the real option approach. In this section, we do a similar comparison within the context of the model presented here, which is of higher complexity than the one considered in McDonald (2000). As such we assess the real option approach discussed in the previous sections against a scenario where the manager sets a fixed harvesting date at the beginning of the harvesting cycle, disregarding any information updates over the harvesting cycle. The manager aims at setting this harvesting date optimally. The valuation problem corresponding to this setup is similar to the valuation of a a European option. Taking Panel A as an illustration, with $P_0 = 40.4$ and $\delta_0 = 0$, adopting the real option policy, the true lease value is computed as 1.5124 million NOK (*approx*. 0.1609 million EUR). Table 7 shows the values, fractions of optimal value covered and absolute difference between real option policy and fixed date strategies for various harvesting dates. We apply the calculation to each panel, including the case of infinite rotation, and find that these suboptimal policies can only cover up to a maximum of 90% of the optimal value. In consequence following any of these suboptimal strategies, the fish farm would voluntarily give up 10% of its value, when additional computational costs attached to the real option approach are only marginally higher than those attached to the fixed date strategies. As such we think, yes, the real option approach for the valuation of fish farms is worth it.[26]

[Table 7 about here.]

Risk Aversion: What is the Impact of Having a Salmon Futures Market ?

As we indicated earlier, in the presence of a complete salmon futures market, the fish farmer is able to efficiently hedge idiosyncratic risk in the salmon spot price. Individual risk preferences are aggregated in the market measure and its pricing kernel is in fact determined by marginal utilities. The specific level of risk aversion of one particular fish farmer who uses salmon futures appropriately hence has no effect on this fish farmer's harvesting decision, the optimal harvesting time is independent of the level of risk aversion. But what if the fish farmer does not use the futures market, or what if there were no salmon futures market ? In this section, we assume that the fish farmer does not have access to the salmon futures market and hence is unable to hedge price risk. We further assume that the fish farmer is risk averse and that preferences are characterized by a CRRA utility function $U(x) = \frac{x^{1-\gamma}}{1-\gamma}$, where γ represents the level of risk aversion.

Real options have been studied in the context of risk aversion with no or partial spanning in Hugonnier and Morellec (2013), Henderson (2007) and Ewald and Yang (2008). Hugonnier and Morellec (2013) assume that the project generates an instantaneous cash flow given by a geometric Brownian motion. They show that under the assumption of CRRA, the investment threshold is increasing with the level of risk aversion and the level of volatility. Investment is henceforth delayed and the difference in project value between firm- and utility maximizing policies can reach up to 20% for reasonable parameter values. A disadvantage of the the modelling framework in Hugonnier and Morellec (2013) is that it cannot cope with negative cash flows, at least for the CRRA case. Additionally, the assumption that instantaneous cash flows follow a geometric Brownian motion is limiting, given that the presence of mean reversion can significantly alter investment behavior. Henderson (2007) uses a different setup, where the payoff of the investment project is given by a geometric Brownian motion (which is similar to Hugonnier and Morellec (2013)), but utility is of exponential type. In addition it is assumed that a partial spanning asset exist. However, the case of no spanning asset is included by setting the relevant correlation $\rho = 0$. In this setup Henderson (2007) observes that increased volatility can in fact speed up investment behavior. Extending the setup in Henderson (2007) and including a mean reversion feature in the project's payoff, Ewald and Yang (2008) in fact demonstrate that the investment threshold can be decreasing with the level of risk aversion (with and without mean reversion). There is hence no clear indication as to how the level of risk aversion would in general affect investment behavior.

Let us now assume that in the context of the single-rotation fish farming model discussed earlier, the fish farmer's preferences are given by a CRRA utility function and that the fish farmer does not have access to the salmon futures market. To account for negative cash flows prior to harvest, e.g. feeding costs, we assume that these are made from bank loans which are redeemed at the time of harvest, when profits are made. This is a necessary assumption as CRRA utility $U(x) = \frac{x^{1-\gamma}}{1-\gamma}$ is not defined for negative values of x, but it is also a very realistic assumption. We only consider the single rotation case. The previous analysis to obtain the optimal harvesting time is then repeated, but under the real world measure (which reflects the fish farmer's subjective believes) and computing utilities for both harvesting and continuation value. We perform this analysis for varying levels of risk aversion γ and observe that for all panels the averaging harvesting time is decreasing with the level of risk aversion. Table 8 shows this for Panel D as an example. In this particular case the average harvesting time ranges from 2.5232 years with $\gamma = 0$ to 1.1979 years with $\gamma = 50$. The average harvesting time for the same panel with salmon futures is

2.2347 years (compare table 5). Perhaps more important than the average harvesting time is the relative loss in value, defined as in Hugonnier and Morellec (2013) as the relative difference in project value between firm- and utility maximizing policies. Table 8 shows that the relative loss in value can be significant, but crucially depends on the level of risk aversion. For very low risk aversion $\gamma \in [0, 1]$, the losses are only around 1.5%, but start to become more noticeable at $\gamma = 2.6$ where losses exceed 5% and become very large for high level of risk aversion at $\gamma = 8$, where they exceed 20% and reach a similar level as those reported in Hugonnier and Morellec (2013). The corresponding results for Panels A, B and C are similar and contained in the part F of the online supplement. In consequence, the salmon futures market provides a valuable service, in particular to those fish farmers which exhibit a high degree of risk aversion.

[Table 8 about here.]

Conclusions

In this article we presented a methodological approach, which can be used to determine the values of lease or ownership of a fish farm in a way which is consistent with market data obtained from the fish pool market, a recently established exchange in Bergen (Norway), where futures on fresh farmed salmon are traded. Our approach correctly accounts for risk premia due to stochastically fluctuating prices. Specifically, we considered the optimal harvesting problem for a fish farmer in a model where the price dynamics is determined by a Schwartz (1997) two-factor model. We looked at both cases of single and infinite rotations. The arbitrage-free value of lease and ownership of the fish farm have then been obtained from the value function of the harvesting problem with single and infinite rotation respectively. The data set used for this analysis contains a large set of futures contracts with different maturities traded at the Fish Pool market between 12/06/2006 and 22/03/2012. In the calibration of our model we adopted the Kalman filtering approach, while our numerical approach to solve the optimal stopping problem embedded in the harvesting decision of the fish farmer made use of the Least Square Monte Carlo and function iteration methods. We found this approach to be numerically stable and obtained very realistic results for a model fish farm.

We assessed the optimal strategy, harvesting time and value against the alternatives where the fish farmer has either no managerial flexibility or no access to the salmon futures market but exhibits risk aversion as modeled by a CRRA utility function. We observed that in both cases, the loss in project value can be very significant, and in the second case is only negligible for extremely low levels of risk aversion. As such we have established that the presence of a salmon future market as well as managerial flexibility are of high importance to risk-averse fish farmers.

Our approach is of practical interest to companies in the fish farming business and can guide their decision process in the context of the acquisition of fish farm units. There are a number of ways how this study can be extended. One way is the inclusion of a stochastic mortality rate, possibly in a regime switching framework, where a high mortality regime corresponds to periods with disease outbreak such as fish lice or salmon anemia. It would be very interesting to understand how markets price the risk of disease outbreak and how this effects the valuation of fish farms. Stochastic growth as well as stochastic feed costs would be other interesting lines of research to pursue.

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Notes

[1] No-arbitrage, which is essentially an equilibrium condition, dictates that contingent claims (risky future pay-offs in cash) have to be valued as the discounted expectation under a so called risk neutral or martingale measure. Under this measure asset prices net of total cost of carry follow martingales. Such a measure is only unique if the underlying market is complete. In general the martingale measure that is used by market participants needs to be inferred from an empirical analysis of relevant derivatives prices. This measure is sometimes also referred to as the market measure or pricing measure. It aggregates the various risk premia that the different market participants attach to relevant assets and implicit drivers such as volatility or convenience yield. In this article a rich set of futures contracts is used to identify the market measure, and risk aversion of agents and the corresponding risk premia are accounted for correctly by pricing contingent claims as the discounted expectations under this measure.

[2]We assume this to be exogenously given and do not investigate the micro structure that explains price formation in the salmon market, as this would lead to a model too complex to be used effectively for our purposes. Important contributions to the aspect of price formation in the salmon market context have been made by Asche, Bremnes and Wessells (1999) and Asche, Bjorndal and Young (2001).

[3]Solibakke (2012) presents an approach using stochastic volatility to model the Fish Pool market. However, only front months contracts are considered and the term structure, which can only be obtained from contracts with longer maturities, is not accounted for. In fact, it is well known that stochastic volatility alone cannot produce realistic term structures. While Solibakke (2012) makes excellent contributions to the understanding of the dynamics of short term contracts at the Fish Pool market, our analysis, which looks at the valuation of lease and ownership of fish farms, looks further ahead into the future and requires information from contracts with longer maturities and the forward curves in particular. We recognize that stochastic volatility on top of stochastic convenience yield would be a desirable feature, but this would lead to a model too complex to handle efficiently.

[4]The models presented in Insley (2002) and Insley and Rollins (2003) are single-factor models and hence feature significantly reduced mathematical complexity as compared to the model discussed in our article.

[5]This is the only way to price the fish farm in a market consistent way, so as to not introduce

arbitrage. Application of the CAPM to price the fish farm is problematic from a number of aspects. As Dusak (1973), Carter, Rausser and Schmitz (1983) and Baxter, Conine and Tamarkin (1985) highlight, zero net-supply of futures contract (there is a long position for every short position) make it difficult to account for these assets in the market portfolio. Additionally, as Ewald and Salehi (2015) have demonstrated, correlation from returns in futures position with the returns of the market portfolio is close to zero. Further note that Bessembinder (1992) as well as Malkiel and Xu (2006) confirmed that idiosyncratic risk is priced in agricultural futures markets.

[6]More precisely, $\mathbb{F} = (\mathcal{F}_s)$ denotes the augmented and completed filtration generated by the Brownian motions $Z_1(s)$ and $Z_2(s)$.

[7] In fact we have $\tilde{Z}_1 = Z_1 + \frac{\mu - r}{\sigma_1} t$ and $\tilde{Z}_2 = Z_2 + \lambda t$. The market price for pure price risk is reflected in the expression $\frac{\mu - r}{\sigma_1}$. It is important to note that both sets of equations (1) and (2) as well as (6) and (7) are identical through this identification, but that the respective representations reveal the different statistical properties under the two different measures $\mathbb{P}$ and $\mathbb{Q}$.

[8]The difference between futures and forwards is that the former are exchange traded, while the latter are mostly traded over the counter (OTC). The exchange usually requires the agent to set up a margin account, the amount held reflecting price movements in the market, protecting buyer and seller from possible default of the other party. The mechanism of the margin account can in principle affect the futures price, but under the assumption of constant rates, it is well known that both futures and forward price coincide.

[9] Note that $F_p(s, T)$ in equation (5) as well as in equations (8) and (9) is observed under the real world measure and that empirical observations a priori only reveal properties of the distribution under the real world measure $\mathbb{P}$. The Kalman filter is therefore set up by using the state dynamics (1) and (2) and expressions (8) and (9) which are all under the measure $\mathbb{P}$. It is important to note though, that via (8) and (9) and endnote 6 it is straightforward to translate statistical properties under $\mathbb{P}$ into statistical properties under $\mathbb{Q}$. We refer to Schwartz (1997) for further details on the application of the Kalman filter in the context of futures markets.

[10]Contracts expire at the end of each months and over the course of the month time to maturity decreases before rollover at the end of the month. F1 is a contract with a notional one month maturity, but because of the time to maturity decreasing over the month, the average maturity is just about half a month, which is 0.040 years. In the same way, F2 is a contract which has an average maturity of one and a half month and so on.

[11]The Schwartz (1997) model is able to capture a variety of shapes for the forward curves, but not all. Different time horizons and combinations of contracts in the different panel emphasize different parts of the forward curve. The nature and in particular the time horizon of the problem motivate the choice of a specific panel.

[12]Panel D is a combination of contracts from the other panels and is therefore not displayed in table 1.

[13]The slightly odd looking actual term structure for longer dated salmon futures contracts is likely to be caused by the rather low trading volume of these contracts.

[14]At this point we may well assume that the mortlity rate $m(t)$ is stochastic. In fact this is assumed in Ewald et al. (2015) and it is shown there how a stochastic mortality rate feeds into the stochastic convenience yield, as it adds to the the cost of storage. In the examples discussed later we assume for simplicity that the mortality rate is constant deterministic.

[15]Infant salmon is commonly referred to as smolt.

[16]Harvesting of farmed salmon occurs all year round, as such there are no specific windows that would constrain the harvesting time. However, Norway imposes limits on maximum allowable biomass of 780 tons per license, compare fisheries.no (2014). Reaching the threshold of maximum allowable biomass can sometimes trigger early harvesting. We do not model this effect explicitly.

[17]Weight $w(\cdot)$ or bio-mass $X(\cdot)$ could in principle be interpreted as further state variables. However, in our modeling framework these variables are deterministic. The value function $V(t, P, \delta)$ as well as the optimal stopping rule depend implicitly on these variables through t, as can be seen in figure 1 for example. Taking account of these variables explicitly would only complicate our Monte Carlo based approach. This is a different matter in PDE based approaches, where including weight as an explicit state variable can be helpful, compare Insley and Rollins (2003). The matter would also be different, if we were to allow for stochastic weight growth or stochastic mortality, which we leave for future research.

[18]In the time discretized setup, we indicate time dependence via sub-indices, i.e. $P_{t_n} = P(t_n)$, which is common in the literature.

[19]Note that since the biomass $X(t)$ is deterministic, it does not need to be accounted for explicitly as a state variable, but will instead be reflected by the time dependency of the value function.

[20]The von Bertalanffy's growth function is derived from the polynomial function provided in the book via $w(t) = w_\infty \left(a - be^{-ct}\right)^3$, where w_∞=6, a=1.113, b=1.097, c=1.43.

[21]Alternative simple harvesting rules are discussed in a later section of this article.

[22]These are realistic values. A comparison with actual prices paid for the acquisition of fish farms is however difficult for the reason that some of the data are confidential. In December 2014, Marine Harvest acquired the assets of Acuinova, a former Chilean salmon farming company, for a total of 125 million USD. Included in this deal are a hatchery, smolt facility, 36 seawater licenses and primary and secondary processing facilities. The expected harvest volume of this unit lies above 15,000 metric tons in 2015, according to Seafoodsource (2014). The relationship between Fish Pool salmon futures and the share prices of Marine Harvest and The Scottish Salmon Company has been investigated in Ewald and Salehi (2015).

[23]Panel B represents the medium-term contracts and covers an appropriate mix of maturities, suited to the nature and time frame of the problem. Contracts in panel B are also among the most liquid contracts and hence the price information obtained from these contracts is likely to be the most reliable. Contracts with longer maturities than those present in panel B are far less liquid and hence less reliable for our purpose.

[24]This can also be observed for options on dividend paying equity in the classical Black-Scholes framework, where the continuously paid dividend replaces the convenience yield.

[25]Note that with the computed average harvesting time of 2.1396 years in the infinite rotation case, ownership entitles to roughly 17 harvesting cycles in 36 years, which considering discounting makes this value seem realistic as well.

[26]Using different combinations of initial values for price and convenience yield, i.e. $P_0 \in (35, 40.4, 45)$ and $\delta_0 \in (-0.5, 0, 0.5)$, in addition to table 7, eight additional cases have been considered and the results are presented in part E of the online supplement. While absolute values such as the optimal value and suboptimal value do vary with different initial values, the fractions of value captured by the corresponding suboptimal policies are relatively stable, with the highest values all occurring at the 2nd year.

Table 1. Contracts Features, 12/06/2006 - 22/03/2012

Contract	Mean Price (Standard Deviation)	Mean Maturity (Standard Deviation)
	Panel A	
F1	30.55 (6.14) *NOK*	0.040 (0.024) *year*
F3	30.08 (5.42)	0.207 (0.024)
F5	29.71 (4.92)	0.374 (0.024)
F7	29.33 (4.59)	0.542 (0.024)
F9	28.97 (4.29)	0.709 (0.024)
	Panel B	
F12	28.71 (4.03) *NOK*	0.960 (0.024) *years*
F14	28.45 (3.81)	1.127 (0.024)
F16	28.23 (3.51)	1.295 (0.024)
F18	28.15 (3.40)	1.462 (0.024)
F20	28.07 (3.29)	1.629 (0.024)
	Panel C	
F24	27.67 (2.88) *NOK*	1.964 (0.024) *years*
F25	27.59 (2.77)	2.047 (0.024)
F26	27.53 (2.68)	2.131 (0.024)
F28	27.47 (2.56)	2.299 (0.025)
F29	27.42 (2.49)	2.382 (0.025)

Table 2. Estimation Results for Whole Sample, Avg. Rate 3.03%, 12/06/2006-22/03/2012

Parameter	Panel A F1, F3, F5, F7, F9 (Short Term)	Panel B F12, F14, F16, F18, F20 (Medium Term)	Panel C F24, F25, F26, F28, F29 (Long Term)	Panel D F1, F7, F14, F20, F25 (Mixed Term)
μ	0.364 (0.102)***	0.692 (0.078)***	0.818 (0.136)***	0.520 (0.108)***
κ	4.342 (0.110)***	1.092 (0.058)***	0.495 (0.045)***	1.664 (0.054)***
α	0.493 (0.126)***	1.034 (0.117)***	1.286 (0.178)***	0.460 (0.143)***
σ_1	0.236 (0.009)***	0.158 (0.001)***	0.219 (0.003)***	0.214 (0.011)***
σ_2	1.270 (0.062)***	0.221 (0.006)***	0.163 (0.008)***	0.448 (0.026)***
ρ	0.892 (0.011)***	0.803 (0.014)***	0.921 (0.005)***	0.806 (0.029)***
λ	1.799 (0.554)***	1.131 (0.172)***	0.630 (0.132)***	0.690 (0.230)***
Log-Likelihood	-17344.3	-22184.1	-24116.7	-15995

Note: Standard errors in parentheses. [***] significant at 1% level; [**] significant at 5% level; [*] significant at 10% level.

Table 3. RMSE and MAE of Log Prices

Panel A						
	F1	F3	F5	F7	F9	ALL
RMSE	0.0177	0.0269	0.0173	0.0140	0.0228	0.0203
MAE	0.0131	0.0208	0.0125	0.0098	0.0168	0.0146
Panel B						
	F12	F14	F16	F18	F20	ALL
RMSE	0.0097	0.0128	0.0116	0.0088	0.0094	0.0106
MAE	0.0072	0.0088	0.0078	0.0059	0.0064	0.0072
Panel C						
	F24	F25	F26	F28	F29	ALL
RMSE	0.0085	0.0085	0.0090	0.0061	0.0076	0.0080
MAE	0.0040	0.0035	0.0043	0.0033	0.0039	0.0038
Panel D						
	F1	F7	F14	F20	F25	ALL
RMSE	0.0198	0.0350	0.0220	0.0152	0.0265	0.0246
MAE	0.0150	0.0280	0.0158	0.0105	0.0167	0.0172

Table 4. Relevant Parameters for Fish Farming

Parameters	Value
Mortality Rate	10%
Conversion Rate	1.1
Number of Recruits	10000
Time Horizon (*years*)	3
Asymptotic Weight (*kg*)	6
Variable Harvesting Cost per kg (*NOK*)	3
Variable Feeding Cost per kg per year (*NOK*)	7

Table 5. Lease Value of Fish Farm and Harvesting Time

Contracts	Harvesting Time (years)	Pond Value (million)	
$Panel_A$	2.0715	1.5124 NOK	0.1609 EUR
$Panel_B$	2.4043	1.2220	0.1300
$Panel_C$	2.3550	1.1142	0.1186
$Panel_D$	2.2347	1.6467	0.1752

Note: Exchange rate used here is 1 $NOK = 0.1064$ EUR, http://www.xe.com/ [last access: 02/10/2015].

Table 6. Estimation Results for Panel B, Avg. Rate 3.93%, 12/06/2006-22/03/2012

Parameter	Medium-term Contract (F12, F14, F16, F18, F20)
μ	0.654 (0.103)***
κ	1.012 (0.096)***
α	1.135 (0.193)***
σ_1	0.153 (0.002)***
σ_2	0.206 (0.014)***
ρ	0.736 (0.037)***
λ	1.142 (0.293)***
Log-Likelihood	-22101.70

Note: Standard errors in parentheses. [***] significant at 1% level; [**] significant at 5% level; [*] significant at 10% level.

Table 7. Optimal Policy vs. Suboptimal Policy: Panel A with $(P_0 = 40.4, \delta_0 = 0)$

Fixed Harvesting Date (years)	Suboptimal Value (million)		Suboptimal Value/Optimal Value	Optimal Value - Suboptimal Value (million)	
1.0	0.8931 *NOK*	0.0950 *EUR*	59.05%	0.6194 *NOK*	0.0659 *EUR*
1.5	1.2291	0.1308	81.27%	0.2833	0.0302
2.0	1.3172	0.1402	87.09%	0.1952	0.0208
2.5	1.2647	0.1346	83.62%	0.2477	0.0264
3.0	1.1511	0.1225	76.11%	0.3613	0.0385

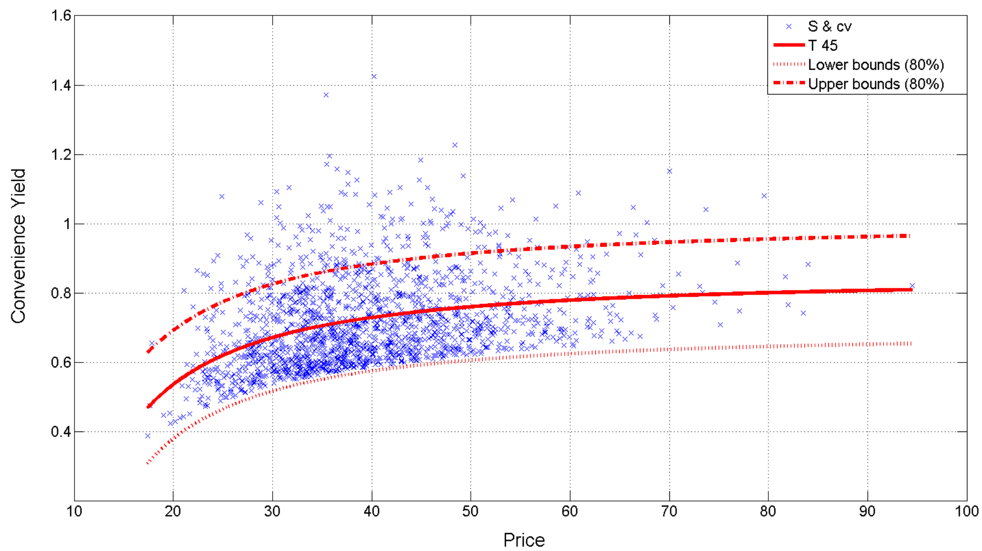
Note: Exchange rate used here is 1 $NOK = 0.1064\ EUR$, http://www.xe.com/ [last access: 02/10/2015].

Table 8. Lease Value of Fish Farm and Harvesting Time Under CRRA: Panel D

γ	Harvesting Time (years)	Pond Value (million)		Percentage Loss
0	2.5232	1.6199 *NOK*	0.1724 *EUR*	1.63%
0.1	2.5198	1.6207	0.1724	1.58%
0.3	2.5090	1.6220	0.1726	1.50%
0.5	2.4969	1.6224	0.1726	1.48%
0.9	2.4635	1.6208	0.1725	1.57%
1.1	2.4424	1.6179	0.1721	1.75%
2	2.3252	1.5885	0.1690	3.53%
5	2.0057	1.4510	0.1544	11.88%
8	1.7075	1.2558	0.1336	23.74%
18	1.5840	1.1675	0.1242	29.10%
33	1.4131	1.0538	0.1121	36.01%
50	1.1979	0.9559	0.1017	41.95%

Note: Percentage Loss = relative difference in project value between firm- and utility maximizing policies, which reflects the percentage loss due to not having access to the futures market at different levels of risk aversion. Exchange rate used here is 1 $NOK = 0.1064$ EUR, http://www.xe.com/ [last access: 02/10/2015].

(a)



(b)

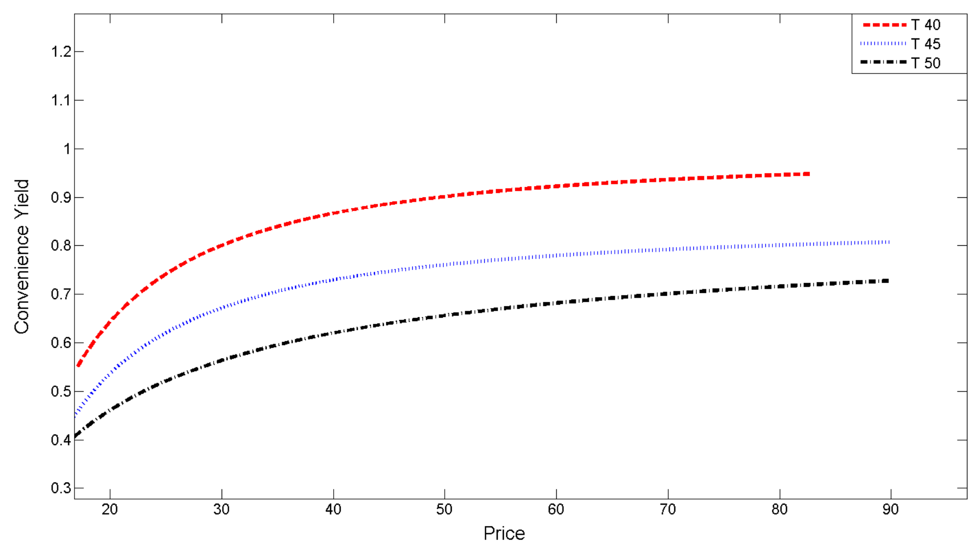


Figure 1. Threshold for Panel A: (a) threshold at one time; (b) threshold at different times

Note: S and cv denote spot price and convenience yield respectively. Blue spots in (a) represent combinations of S and cv where harvesting occurred at time point 45. The boundaries of 80% confidence intervals above and below the fitted (thick red) line are also presented and can be interpreted as more or less conservative exercise thresholds. Thresholds at time points 40, 45 and 50 are shown as red, blue and black line accordingly in (b).

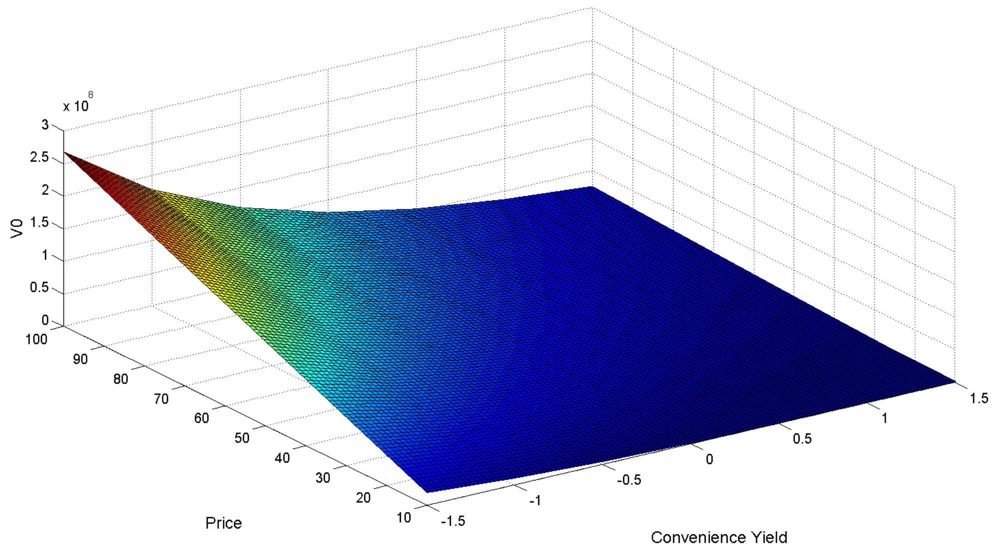
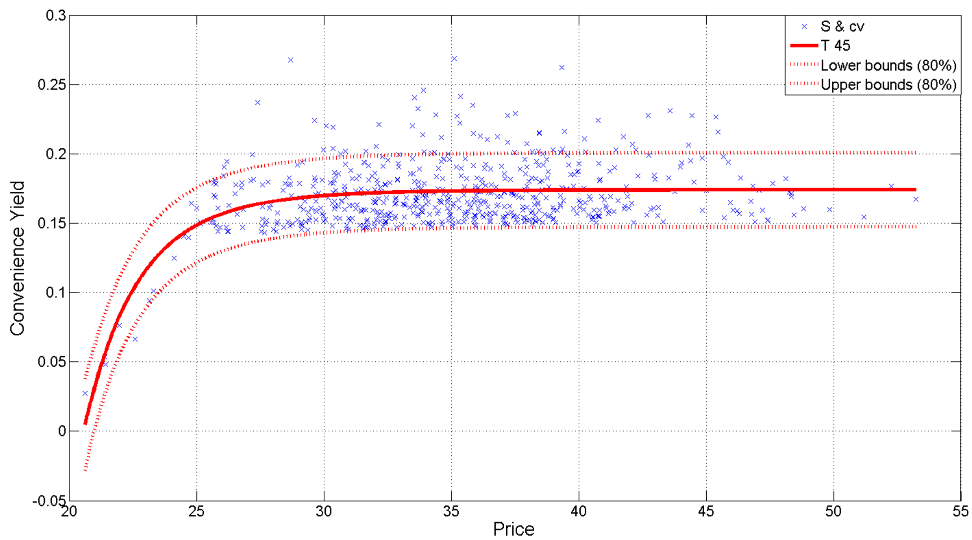


Figure 2. Value of ownership of the fish farm using parameters obtained from Panel B, 12/06/2006-22/03/2012

Note: The ownership value of the fish-farm V_0 is expressed as a function of the two-state variables, i.e. price and convenience yield. It can be clearly observed that the convenience yield has a negative impact on the value of the fish farm, while the salmon spot price has a positive impact.

(a)



(b)

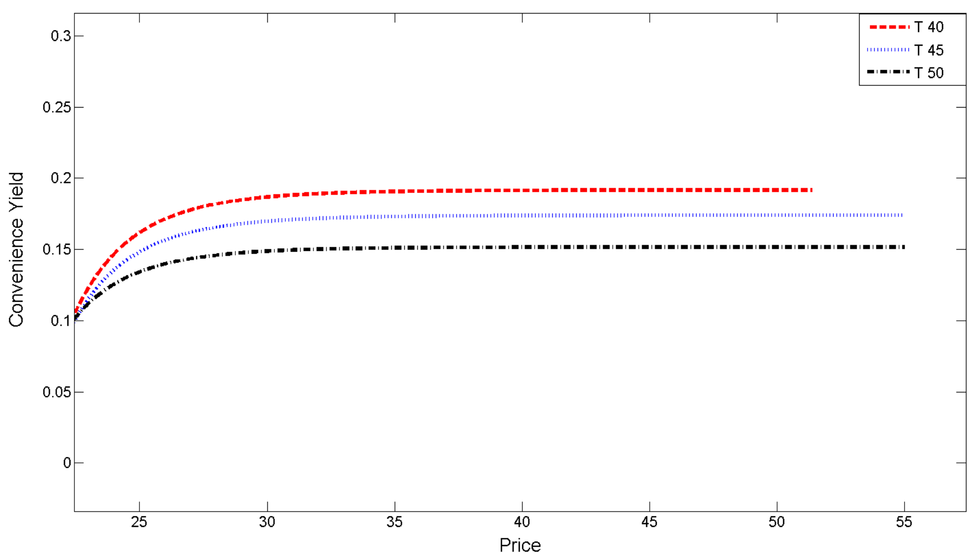


Figure 3. Infinite rotation fish farming, threshold for Panel B: (a) threshold at one time; (b) threshold at different times

Note: S and cv denote spot price and convenience yield respectively. Blue spots in (a) represent combinations of S and cv where harvesting occurred at time point 45. The boundaries of 80% confidence intervals above and below the fitted (thick red) line are also presented and can be interpreted as more or less conservative exercise thresholds. Thresholds at time points 40, 45 and 50 are shown as red, blue and black line accordingly in (b).