Entropy generation in thermal systems with solid structures - a concise review

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Abstract

Analysis of thermal systems on the basis of the second law of thermodynamics has recently gained considerable attention. This is, in part, due to the fact that this approach along with the powerful tools of entropy generation and exergy destruction provides a unique method for the analysis of a variety of systems encountered in science and engineering. Further, in recent years there has been a surge of interest in the thermal analysis of conductive media which include solid structures. In this work, the recent advances in the second law analyses of these systems are reviewed with an emphasis on the theoretical and modelling aspects. The effects of including solid components on the entropy generation within different thermal systems are first discussed. The mathematical methods used in this branch of thermodynamics are, then, reviewed. This is followed by the conclusions regarding the existing challenges and opportunities for further research.

Keywords: Entropy generation; Heat transfer; Irreversibilities; Solid material.

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Nomenclature

a_{sf}	interfacial area per unit volume of porous media, m^{-1}	T_s	temperature of the solid phase of the porous medium, K
c_p	specific heat at constant pressure, $\mathrm{J\cdot Kg^{-1}\cdot K^{-1}}$	t	time, s
h_{sf}	fluid-to-solid heat transfer coefficient, $\mathrm{W\cdot m^{-2}\cdot K^{-1}}$	u_f	velocity of the fluid in the porous medium, $\mathrm{m\cdot s^{-1}}$
J	current density, $\mathrm{amp\cdot m^{-2}}$	X	dimensionless axial distance
k	thermal conductivity of solid material, $\mathrm{W\cdot m^{-1}\cdot K^{-1}}$	x	axial distance, m
k_{ef}	effective thermal conductivity of the fluid $\left(\varepsilon k_f\right)$, $\mathrm{W\cdot m^{-1}\cdot K^{-1}}$	y	vertical distance, m
k_{es}	effective thermal conductivity of the solid $\left(\left(1-\varepsilon\right)k_s\right)$, $\mathrm{W\cdot m^{-1}\cdot K^{-1}}$	**Greek symbols**	
N_s'''	dimensionless local entropy generation rate	σ	electric conductivity, $\Omega^{-1}\cdot\mathrm{m}^{-1}$
Q	Dimensionless volumetric internal heat generation rate	ε	porosity
$\dot{q}$	Volumetric internal heat generation rate, $\mathrm{W\cdot m^{-3}}$	κ	permeability, m^2
$\dot{S}_f'''$	local entropy generation rate within the fluid phase of the porous medium, $\mathrm{W\cdot m^{-3}\cdot K^{-1}}$	μ_f	fluid viscosity, $\mathrm{Kg\cdot m^{-1}\cdot s^{-1}}$
$\dot{S}_s'''$	local entropy generation rate within the solid phase of the porous medium, $\mathrm{W\cdot m^{-3}\cdot K^{-1}}$	μ_{eff}	effective viscosity of porous medium, $\mathrm{Kg\cdot m^{-1}\cdot s^{-1}}$
T	temperature, K	θ	dimensionless temperature
T_f	temperature of the fluid phase of the porous medium, K	ρ	fluid density, $\mathrm{Kg\cdot m^{-3}}$

1. Introduction

Multidisciplinary efforts to achieve better control of the heat transfer rates in thermal systems are currently of significant academic and industrial interest [1]. This mainly stems from the rapidly growing concerns about energy efficiency in a wide range applications spanning from industrial to domestic appliances [2]. Further, the introduction of micro and bio-heat transfer and energy technologies has set new challenges for the thermal energy optimization [3][4][5]. Fundamentally, heat transfer has close ties with the first law of thermodynamics and this is, effectively, the only thermodynamic principle used in conventional heat transfer analysis. The second law of thermodynamics provides a measure of entropy generation rate, or irreversibility, within a system or process, and as a result, impacts the efficiency of the heat transfer process. Over the last few decades, there has been an increasing awareness about the influence of irreversibility on energy interactions [6]. This has led to the formulation of exergy analysis and the definition of exergetic efficiencies as an addition to the conventional energy efficiency approach [2][6][7]. Although initially developed for thermo-mechanical processes, the concept of exergy was extended to chemical systems as well [8], and it is now being applied to the problems in ecology, environment and economy [9][7]. Central to the conduction of exergy analysis is the calculation of entropy generation, which then determines the rate of exergy destruction [10][6]. This evaluates the level of degradation of energy and therefore introduces the concept of energy quality [11]. The first law analysis, however, does not recognize the variations in energy quality and only conducts an energy accounting [12]. This difference is the principal reason for the veracity of second law analyses when compared to those, which are solely on the basis of the first law of thermodynamics.

Application of second law analysis in thermal engineering, provides the possibility of optimizing a given system or process on the basis of the energy quality, which is very different from a first law analysis [13]. For example, it is well documented that the heat transfer in heat exchangers can be enhanced using various profiles of extended surfaces [14,15], or by optimizing the volume flow rate or the heat transfer coefficient [16]. However, design of a specific fin on the basis of the minimum entropy generation [17] results in different configurations compared to those obtained by the classical optimization methods [16]. This is also true when one is applying the concept of entropy generation to optimize the temperature field in electronic devices [18]. These types of thermodynamic analyses, i.e., the entropy generation and exergy analyses of a system, have been interestingly further extended to exergetic analysis of the human body [19][20]. Even for the human body, the obtained results from the energetic and the exergetic analyses were quite different to each other [19], which leads researchers to re-consider thermal systems by using the perspective of the second law of thermodynamics. This has been extended further to the consideration of the second law of thermodynamics over the first law for mechanical analysis in solid structures [21][22]. For instance, Baneshi et al. [21] used maximum entropy generation criterion to calculate the worst case scenario for mechanical systems under thermomechanical loads.

In reality, all thermal processes include some level of irreversibility, primarily due to the existence of temperature gradients. This renders an exergetic efficiency loss and results in reducing in the energy quality. In heat transfer processes, entropy generation has been reported in conduction [23][24][25][26], convection [27–29], radiation [30][31], and/or any combination of these modes of heat transfer [32][33]. Further, there are other sources of irreversibilities such as viscous dissipation [27–29] and magnetic fields [34]. Initiated by the seminal works of Bejan [35,36], several studies have been conducted on the minimization of entropy generation [37–41] [10][12] [42]. To date, there has been a significant focus on exergetic processes in forced and free convection [37–41] [10][12][42]. However, much less attention has been paid to the entropy generation in heat conductive media [43–45]. Given that solid structures are found in, nearly all thermal systems, this field has major potentials to grow, and is still far from maturity. Perhaps more importantly, after the introduction of complex solid components into thermo-mechanical systems, such as composite and multilayer structures, the first and second law analyses of conductive systems has gained considerable attention [46][47][48]. Hence, obtaining further familiarity with the process of entropy generation in solid components and their modelling is clearly of interest.

The current investigation presents a survey of the literature on the progress and challenges of the evaluation of entropy generation in primarily conductive systems. These include purely conductive problems and also those convective-conductive problems in which conductive process have dominating roles. The review covers the physical aspects of entropy generation in these problems and also explains the mathematical tools that can be utilized for the analysis and modelling. It ultimately identifies the ongoing challenges and the areas of interest for future research.

2. Entropy generation in thermal systems with conductive parts

The energy equation for a three-dimensional object that experiences internal heat generation/consumption is governed by the following formula.

$$\nabla\left(k\nabla T\right)+\dot{q}=0 \tag{1}$$

Calculation of the temperature field through this structure is the first step in obtaining the entropy generation rate. Depending on the nature of the thermal conductivity, the internal heat source and the boundary conditions, the temperature field can be found analytically or through numerical simulations. Regardless of the method of heat transfer analysis, the second law analysis can be conducted on this system to provide a ready-to-use relationship for determining the rate of entropy generation. Using the Classius-Duhem statement of the second law, the entropy balance equation may be written as [49][6]:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V s\,\mathrm{d}V=-\oint_A \frac{1}{T}\vec{q}\cdot\vec{n}\,\mathrm{d}A+\int_V \frac{1}{T}\dot{q}\,\mathrm{d}V+\int_V \dot{S}'''\,\mathrm{d}V\ . \tag{2}$$

Considering steady-state and applying the divergence theorem, the surface integral on the right hand side of the integral can be written in terms of the volume integral. That is,

$$\oint_A \frac{1}{T}\vec{q}\cdot\vec{n}\,\mathrm{d}A=\int_V \nabla\cdot\left(\frac{\vec{q}}{T}\right)\mathrm{d}V\ . \tag{3}$$

Using Eq. (3) to replace the surface integral with the right-hand side of Eq. (2), we obtain

$$-\int_V \nabla\cdot\left(\frac{\vec{q}}{T}\right)\mathrm{d}V+\int_V \frac{1}{T}\dot{q}\,\mathrm{d}V+\int_V \dot{S}'''\,\mathrm{d}V=0\,. \tag{4}$$

Since each integral in Eq. (4) is a volume integral, it follows that

$$\dot{S}'''=\nabla\cdot\left(\frac{\vec{q}}{T}\right)-\frac{\dot{q}}{T}\,. \tag{5}$$

By consideration of a one-dimensional solid wall of thickness *L*, made of a material with thermal conductivity *k(T)* as illustrated in Fig. 1, the energy Eq. (1) can be simplified to [50][49]

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[k(T)\frac{\mathrm{d}T}{\mathrm{d}x}\right]+\dot{q}=0. \tag{6}$$

Assuming one-dimensional heat transfer, which holds in the current configuration, and incorporating energy, Eq. (6), into Eq. (5), the following equation is obtained for the local entropy generation rate.

$$\dot{S}'''_{gen}=\frac{k(T)}{T^2}\left(\frac{\mathrm{d}T}{\mathrm{d}x}\right)^2. \tag{7}$$

Equation (7) is one of the fundamental equations used in entropy related analyses in solid media. It is imperative to note that as demonstrated by Aziz and Khan [49], the internal heat generation in solid media does not explicitly appear in the entropy generation equation.

In the pioneering work of Ibanez et al. [51] it was shown that entropy generation in a cooling process could be optimized by controlling the convective cooling parameters. The entropy generation in a process of heat conduction is a function of the temperature field [23,50]. It, therefore, depends upon the thermo-physical properties of the medium, internal energy generation, and the thermal conditions imposed on the boundaries of the medium. Following Ref. [51], a number of studies calculated the classical entropy generation in pure conduction processes or minimized that in the processes within the pure conductive environments. Strub et al. [52] analyzed entropy generation in a wall with a purely sinusoidal temperature boundary condition on one side and a constant temperature condition on the other side. Later, Al-Qahtani and Yilbas [53] used the concept of entropy generation to analyze the one-dimensional transient heat conduction during laser pulse heating. Recently, Ali et al. [54] extended this work to the pure analytical treatment of energetic and entropic analyses of a cylinder due to laser short-pulse irradiation. It was shown that the general behaviour of the entropy generation was similar to the temperature field. However, when a parameter varies, the variation on the temperature and entropy generation fields may not be similar [54]. Aziz and Khan [17] studied heat transfer and entropy generation analyses of convective pin fins with convective heating at the base and convective heat loss at the tip. The fluid friction at the base, lateral surface and tip of the fin were also included in the calculations [17].

Subsequent to the introduction of entropy generation in solid media [51] a number of studies contributed to the advancement of this field, see for example [55][52][53]. These investigations focused on various conductive media such as homogeneous structures, functionally graded materials and composite geometries [23][50]. The next step was to include solid systems in convective media which resulted in the investigation of conductive-convective media. This group of problems is different from the conventional convection problems by the fact that the overall heat transfer in a conductive-convective system is strongly dependent upon the conduction of heat in the solid components. Entropy generation analysis, then, found its way into porous thermal systems [56][57][58][59], as an important class of conductive-convective systems.

In the following, the literature on these sub-categories is discussed.

2.1. Pure conductive media

One of the first articles on entropy generation in solid media was authored by Bisio in 1988 [60]. He analyzed entropy generation in one-dimensional transient conduction in a system with time-dependent thermal boundary conditions. Here, the authors assumed that the thermal conductivity of the slab varies with the local temperature as well as the coordinate along the direction of the heat conduction [60]. Many investigators followed Bisio's path and opted in favor of a thermodynamic analyses of conductive systems. The investigations of Ibanez et al. [51] and Kolenda et al. [55] were among these works. Ibanez et al. [51] considered entropy generation in a slab with uniform internal heat generation and different convective cooling conditions on the opposing faces of the slab. In this work, it was shown that the total entropy generation in the system can be minimized with the proper choice of the convective cooling parameters [51].

The work of Kolenda et al. [55] demonstrated that entropy generation in steady state conduction (one-dimensional or multidimensional) could be always minimized by placing heat sources in the conducting medium. Given this interesting conclusion, other researchers applied the second law of thermodynamics to conductive systems to analyze the entropy production within these structures [49][61][23]. These attempts led to the development of some contentions and debates. As an example, Aziz and Khan [49] conducted a theoretical analysis of entropy generation in heat generating solids and questioned the validity of some other entropy generation analyses of this problem. These authors mathematically demonstrated that the thermal energy sources do not explicitly appear in the entropy generation equation [49]. They therefore, concluded that the previous analyses, which considered internal heat generation as an extra source of irreversibility were incorrect [62][63]. In a subsequent study, Aziz and Khan [61] examined this new formulation of entropy generation [49] on all the three fundamental configurations, namely slabs, cylinders and spheres with constant boundary temperatures.

Later, Torabi and Aziz [23] investigated the generation of entropy in an asymmetrically cooled hollow cylinder with temperature-dependent thermal conductivity, internal heat generation and radiation effects on the outside surface. They calculated the local and total entropy generation rates for this geometry. These authors [23] demonstrated that with a proper choice of the convection parameters, the total entropy generation could be minimized (see Fig. 2). Torabi and Zhang [50] addressed entropy generation rate problems in the regular and functionally graded material slabs with temperature-dependent internal heat generation and convective-radiative boundary conditions. In addition to the effects of convection, this study included those due to radiation. Figure 3 shows that radiation could have similar effects to convection as in the previous studies [23]. This figure also indicates that the total entropy generation rate can be minimized by optimizing the radiation.

Given the increasing importance of the thermal analysis of composite multilayer structures this approach has recently gained increased significance. These types of structures are used extensively in manufacturing applications, largely due to their excellent mechanical and thermo-physical properties, when compared to those of conventional materials. Ka_ka and Yumruta_ [64] presented experimental and theoretical studies on the transient temperature and heat flow in multilayer walls and flat roofs. In addition, Amini-Manesh et al. [65] numerically demonstrated the importance of using composite materials as a substrate, which significantly enhances the flame stabilization in a reactive nano-film. Conduction heat transfer is also of significance in composite media, due to widespread applications of composite structures in the electronics industry [66][67][68]. It follows that predictions of the temperature and entropy generation within composite structures are important for many industrial applications. As a result, in recent years there have been a significant number of attempts to examine composite and multilayer media, from the second law perspective.

Torabi and Zhang [47] were among the first who investigated the generation of entropy in composite solid media. They considered one-dimensional steady conduction in a two-layer composite, hollow cylinder with temperature-dependent thermal conductivity and constant temperature (case one) or pure convective cooling on the internal and external surfaces (case two). Figure 4 shows the configuration of the double-layer hollow cylinder in their study [47]. These authors [47] further assumed perfect thermal contact between the two layers and varied the internal heat generation in each layer of the material. Substantial differences between the total entropy generation rate for the two cases were observed [47]. This implies that, in general, consideration of a constant thermal boundary condition in place of the actual convective boundary condition, cannot be adequately justified. Further, the current investigation indicates that in addition to the different values for the total entropy generation rate in the two investigated cases, the trend could sometimes be different. For example, by increasing the interface radius, the total entropy generation rate is increased in the first case. However, through a similar variation of the interface radius, the total entropy generation rate may decrease or increase in the second case [47]. More recently, by considering imperfect thermal contact resistance (TCR), Torabi et al. [48] extended the analyses of Ref. [47] to entropy generation analysis for the three fundamental double-layer solid structures previously investigated in Ref. [61]. It was observed that the existence of TCR could lead to an interesting phenomenon within the composite media, in which increasing the TCR reduces the total entropy generation rate. Hence, neglecting TCR in entropy generation simulation in multilayer materials can cause overestimation of the total entropy generation rate [48].

The investigations presented and discussed here, considered only a fraction of the general problem and there exists a handful of structures and conditions, which still need to be analyzed. For instance, the second law analyses of multilayer composite structures with more than two layers have yet to be conducted. Because the TCR has been shown to have a substantial influence upon the entropy generation in composite structures [48], the temperature jumps are speculated to play a similar role within the interface of conductive-convective systems. Further, velocity slip is expected to have a strong impact on the entropy generation rate within the system. Rigorous investigations of these effects remain as future research tasks.

The above discussion mainly has considered the steady-state processes. The number of transient analyses of entropy generation in pure conductive configurations is very limited. This is mainly due to the inherent difficulty of these analyses, which transform the ordinary differential equations into partial differential ones. A study of transient entropy generation analysis has been recently performed by Ali et al. [54]. This investigation opted in favour of transient entropy generation rate in a cylinder under laser-pulse heating [54]. Here, as mentioned in the introduction, a purely analytical solution was developed to obtain the temperature field. The calculated temperature field was then incorporated into the entropy generation formulation. However, due to the complexity of the derived mathematical formulation, the local entropy generation is difficult to be integrated over the entire geometry, and hence the total entropy generation has not been reported.

A summary of the recent investigated geometries and the employed solution methods, which will be discussed in section 3, has been provided in Table 1.

2.2. Conjugate heat transfer systems

The thermal aspects of the problem of conjugate heat transfer have been already reviewed [69][70]. As stated previously, entropy generation in convective systems has been the subject of intensive research for a relatively long time [42][71]. However, less attention has been paid to the media that contain both the conductive and convective constituencies. This is also true for conjugate conduction-radiation heat problems. Recently, this approach has been investigated by a number of researchers through the application of the second law to a number of convective-conductive or conductive-radiative media. Here, the process of entropy generation in these systems is considered, and a particular attention is paid on the role of the solid phase in this process.

Ibáñez and Cuevas [72] considered the fluid flow, heat transfer and entropy generation rates within a parallel wall microchannel, under a uniform transverse magnetic field. The magnetic field produced the Lorentz force and this was assumed to generate a fully developed flow [72]. The conjugate heat transfer problem in an MHD flow between two parallel solid walls of finite thicknesses, was analyzed from an optimization perspective. The principal objective in this investigation was to better understand the influences of the thermal conductivity of the fluid and channel wall, on the entropy generation [72]. This investigation demonstrated the existence of optimal values of these quantities, which result in the minimum entropy generation rates [72]. Later, these analyses were repeated for a viscous flow between parallel solid walls of finite thickness [45]. It was concluded that by the incorporation of wall dissipation in the evaluation of entropy generation in natural or forced convection, the channel wall thickness can be optimized [72][45]. This conclusion was also true for the optimization of the wall to fluid thermal conductivity ratio [72][45]. These findings are therefore of importance in the design of heat transferring devices [45][72].

In a recent study, Ibáñez et al. [73] extended the investigation of Refs. [45][72] to entropy generation analysis in a microchannel by considering the uniform heat flux boundary conditions and hydrodynamic slip between the channel walls and the fluid. This study illustrated the possibility of finding an optimum value of the heat flux, slip length, wall to fluid thermal conductivity ratio and Peclet number to minimize the global entropy generation rate. Further, as shown in Fig. 5, Ibáñez et al. [73] concluded that by fixing all other parameters and increasing the wall to fluid thermal conductivity ratio, the optimum rate of the global entropy generation increases.

Several other recent articles investigated the entropy generation in conjugate convective-conductive systems [74][75]. Torabi et al. [74] analyzed the first and second laws of thermodynamics within a cylindrical system. Nanofluid flow was considered between the inner and outer walls and these two solid structures were further included in the model. Temperature-dependent thermal conductivities, for solid materials, were assumed. Constant, but uneven, values of internal heat generations were incorporated into the energy equation of the solid parts. In a separate study, Torabi and Zhang [75] modeled temperature and entropy generation within a cylindrical system. Their model considers the inner solid cylinder, the MHD flow between the inner and outer cylinders and the outer hollow cylinder [75]. Thermal conductivities of the solid materials were assumed to be temperature-dependent while that of the fluid was considered constant [75]. These studies [74][75] demonstrated that due to the almost constant temperature within the inner solid layer of the rotating cylinder, this layer might not participate in the entropy generation. Nonetheless, the outer layer greatly affects the local and total entropy generation rates.

Currently, there is a shortage of information on the conjugate radiation-conduction heat transfer relationship and the associated entropy generation. To date, the only investigation in this area is the work of Makhanlall and Liu [32]. This investigation, included a second law analysis of a solid-liquid phase change systems involving both conduction and radiation [32], and assumed that both phases were semi-transparent and that the liquid motion does not occur within the system, as the material properties are identical for both phases. By calculating the entropy generation, the exergy destruction due to the conduction and radiation modes of heat transfer could be calculated [32]. Makhanlall and Liu [32] concluded that, although often neglected, radiation heat transfer plays an important role in the exergy destruction. Future research on the second law analysis of conductive-convective-radiative systems is expected to provide further insight into the influences of these combined modes of heat transfer upon the generation of entropy.

For the sake of completeness the above review has been tabulated in Table 2.

2.3. Porous media

It is well understood that porous solids can greatly facilitate heat transfer rates [76][77]. This is due to the dual influences of the solid medium on increasing the effective thermal conductivity of the composite solid-fluid medium, and also providing an extensive heat transfer surface area [77]. This has led to the wide application of porous materials in a variety of processes spanning from cooling technologies to bio-heat transfer [78]. However, the excellent heat transfer characteristics of porous media are associated with significant impedance of the flow. As a result, there have been considerable efforts on thermo-hydraulic optimisation of heat transfer processes in porous media, see for example Refs. [79][80][81][82]. A challenge before this approach is the multiplicity of possible optimisation criteria. For instance, optimisation can be on the basis of Nusselt numbers (defined at different locations across the system), temperature profiles or pressure drop. It is not immediately clear which of these is the most suitable criterion. Further, these features of the system are strongly interconnected and varying one would change the others [79]. This may highly obscure the optimisation process and even cause confusions. One possible remedy for this situation is the introduction of a single optimisation criterion, which includes all the pertinent thermal and hydrodynamic effects. The total entropy generation clearly provides such criterion. The irreversibilities encountered in heat transfer processes within porous media are, generally, due to the transfer of thermal energy across finite temperature difference and the viscous dissipation of the flow kinetic energy. The former, usually, occurs by conduction of heat within the solid matrix or convective and radiative heat exchanges between the solid and fluid phases [77]. The latter is due to the interactions occurring between the viscous flow and the solid matrix, and the extended contact surface area, between the moving fluid and the solid surface [76][83][84].

To model this situation, the method of representative elementary volume is often employed [76] and a system of equations that describe the transport of momentum and thermal energy are solved [76][77]. Depending upon the pore scale Reynolds number either of Darcy, Darcy-Brinkman or Darcy-Brinkman-Forchheimer equations are employed [76]. Consideration of local thermodynamic equilibrium divides the thermal energy transport into two major sub-categories. Local thermal equilibrium (LTE) greatly simplifies the problem and renders a single differential energy equation for the composite medium [77]. However, this approach ignores the generation of entropy as a result of heat exchanges between the solid and fluid phases. A more accurate approach releases the assumption of local equilibrium and establishes the local thermal non-equilibrium (LTNE) condition, and hence considers the irreversibility of local heat exchanges within the medium [78][85]. The generation of entropy due to the flow of fluid and heat in porous media has been investigated by a number of authors. In these works, usually, the problem of heat transfer is solved first and the resultant temperature and hydrodynamic fields are used to calculate the rates of entropy generation. The relative significance of heat transfer irreversibility (HHI) and fluid flow irreversibility (FFI) are measured by the so-called Bejan number defined as Be=HHI/(HHI+FFI) [10].Generation of entropy in porous media under natural convection has been studied most extensively. However, investigations of entropy generation under forced convection are less frequent. For conciseness reasons, the following discussion and review mostly concern the forced convection of fluid through a porous solid. The reader is referred to Ref. [71] for the survey of literature on entropy generation in free and mixed convection within porous media. Further, some very recent examples of investigations in this area could be found in [86][87][88][89]. Hooman and Ejlali [56] numerically solved the Darcy-Brinkman model and the LTE energy model for a fully filled porous pipe under developing condition. They calculated the rate of entropy generation and Bejan number for this configuration [56]. The effects of viscous dissipation on the temperature fields and entropy generation in a fully filled channel were investigated by Hooman and Haji Sheikh [90]. They conducted an extensive numerical study and demonstrated that viscous dissipation decreases the Nusselt number in the developing and developed regions of the flow conduit under isothermal wall heating [90]. In an attempt to provide optimization guides for the fully filled porous channels, Hooman et al. [57] analytically solved the Darcy-Brinkman model and the LTE energy equation. They considered three different boundary conditions on the external surface of the channel, and provided expressions for the Nusselt and Bejan numbers and rates of entropy generation. This analysis was later extended to the developing flows by considering a simple Darcian flow with the LTE condition and under constant temperature boundary condition [91]. A parametric study was conducted which demonstrated that entropy generation rate inversely correlates with the Peclet number and is directly related to Brinkman number [91]. The problem of entropy generation in a fully filled porous channel under fully developed condition was also considered by Mahmud and Fraser [92]. They considered constant but dissimilar wall temperatures and conducted analytical and numerical studies on the heat transfer and entropy generation characteristics of the system [92]. This investigation was later extended to unsteady cases by Kamish [93]. Further, through linearization of the governing equations, the unsteady heat transfer, fluid flow and entropy generation through a metal foam was investigated analytically by Mahmud et al. [94]. In a separate work, Mahmud and Fraser [95] calculated the rate of entropy generation during the natural convection of heat in a porous cavity under magneto-hydrodynamic effects. Hydro-magnetic effects were, also, considered in the problem of entropy generation in a channel fully filled with porous media and under mixed convection [96]. Entropy generation rates in partially filled conduits were calculated by Morosuk [97], who demonstrated that the maximum rate of entropy generation occurs on the porous-fluid interface. In this work, the behavior of entropy generation rate was mainly attributed to the hydrodynamic characteristics of the problem [97]. Entropy generation minimization has been used as a design criterion in porous heaters by Shakouhmand et al. [98]. They calculated the optimal porosity for a fully filled porous conduit and demonstrated that the optimum matrix porosity increases with the Reynolds number [98]. More recently, Mahdavi et al. [99] conducted a numerical study on the partially filled conduits and calculated the flow field, Nusselt number and entropy generation rates. They considered two configurations in which porous insert is either placed at the core of the pipe or is attached to the inner wall [99]. Their results highlighted the effects of the placement of porous material in the conduit on the heat transfer and entropy generation rate [99]. They also demonstrated that the thermal conductivity ratio can dominate the level of heat transfer enhancement [99].

Assumption of LTE is the common feature of all the preceding studies of entropy generation in porous media. Deviation from thermodynamic equilibrium implies higher levels of irreversibility. In such cases, entropy generation is not limited to those due to viscous dissipation and external heat transfer sources and, the internal heat exchange processes introduce an important mechanism of entropy generation. When LTNE model is employed for thermal analysis of a porous system, an extra energy equation is used in the analysis due to the heat exchanges between the fluid and solid phases of the porous medium [100]. This new equation, which is coupled with the energy equation of the fluid phase, provides information on the temperature of the porous solid structure of the system. The difference between the solid and fluid temperature and the resultant heat exchange between the two is a major source of entropy generation in the solid porous structure. Given this, it is surprising that there is currently a small number of studies on entropy generation in porous systems under the LTNE conditions. In a recent work, Buonomo et al. [58] conducted a study on porous filled micro channels through using LTNE model. They analytically investigated the hydrodynamic and thermal processes between two parallel plates filled with a porous medium [58]. Due to the microscale size of the channel and rarefaction effects of the gas flow under consideration, the first order velocity slip and temperature jump conditions at the fluid-solid interface were used [58]. This study provided analytical expressions for the velocity and temperature fields as well as the local and total entropy generation rates [58]. Torabi et al. [59] took an LTNE approach and investigated temperature distribution, Nusselt number, and local and total entropy generation rates within a channel partially filled with a porous material. The lower wall of the channel was exposed to a constant heat flux and the upper wall was assumed to be adiabatic. Viscous dissipation effects were incorporated into the energy equations and analytical solutions were developed for the velocity and temperature fields [59]. Further, Nusselt number and local and total entropy generation rates were evaluated and a temperature and Nusselt number bifurcation phenomena were observed [59]. Furthermore, for the first time, a bifurcation phenomenon regarding the entropy generation rate was reported by this work (Fig. 6) [59]. The bifurcation in some other heat transfer characteristics, such as heat flux or Nusselt number, is defined as a sudden jump from a positive value to a negative value or vice versa [59][101][102]. However, since the entropy production is a positive quantity, by entropy bifurcation we mean that the value of entropy generation features a sudden jump [59]. Torabi et al. [100] considered the exothermic or endothermic feature of physical or physicochemical processes for entropy generation analysis of a channel partially filled with porous materials. Here, both local and total entropy generation rates have been investigated and illustrated. It was shown that, for a specific set of parameters, a certain thickness for the porous medium can be found such that the Nusselt number or the total entropy generation of the system can be optimized. Trevizoli and Barbosa [103] used the second law of thermodynamics to optimize a regenerator. One-dimensional Brinkman–Forchheimer equation was used to describe the fluid flow in the porous matrix and LTNE equation model was employed to determine the temperatures in the fluid and solid phases [103]. By using finite volume method, cycle-average entropy generation due to axial heat conduction, fluid friction and interstitial heat transfer were calculated. Variation of the total entropy generation versus particle diameter and aspect ratio showed the second law optimization is dependent upon other parameters of the system [103]. In a series of recent works, Ting et al. [104][105] analytically examined the thermal characteristics and entropic behavior of a fully-filled porous channel saturated with nanofluid. Their investigated configuration was subject to fully developed flow and asymmetric thermal boundary conditions [104], and could also include internal heat generation within the solid phase [105]. These authors applied LTNE between the nanofluid and the solid structure and demonstrated that the addition of nanoparticles reduces the local temperature difference between these two phases [105]. It was shown that the existence of internal heat sources in the solid phase majorly affects the system irreversibility. In particular, this study showed that the internal heat generation in the solid phase could have destructive influences upon the second law performance of the system [105]. Table 3 provides a summary of the recent investigation on LTNE analysis of the entropy generation in solid porous structures experiencing forced convection of heat.

In general, the analysis of entropy generation under non-local equilibrium in porous solids is still in its early stages. The previous heat transfer investigations [106][107][108][109] can be extended to the second law analyses. In particular, the bifurcation of entropy generation rate, deserves further attention as currently there is only a small number of studies of this phenomenon [58][59][100].

2.4. Thermoelectric systems

The need for electrical power is constantly growing across the world. The economic and environmental concerns associated with the conventional energy sources for conversion to electric power generation has led scientist to new concepts such as thermoelectric systems [110]. An example of such devices is a thermoelectric cooler which works as a reversed heat engine operating between the two heat reservoirs [25]. These devices which are in a solid state have better efficiency compared with the conventional systems for low capacity power applications [111]. Importantly, their solid structure offers advantages such as noiseless operation and low-cost maintenance [24][110]. Due to the solid structure of these systems, Fourier heat transfer model is considered for the thermal analyses [112][113][114]. In general, the unsteady energy balance in thermoelectric devices is written as [113]:

$$\rho c \frac{\partial T}{\partial t} = \nabla\left(k \nabla T\right) + \frac{J^2}{\sigma} - \beta \vec{J} \cdot \nabla T \tag{8}$$

The first, second, and third terms of the right hand side of Eq. (8) describe the heat conduction, Joule heating, and Thomson effect, respectively. However, when it comes to micro and nanoscales, more appropriate models such as Cattaneo-Vernotte are required [26]. Therefore, Eq. (8) may change to the following energy equation for the small scale devices under electrothermal effects [26].

$$\tau \frac{\partial^2 T}{\partial t^2} + \rho c \frac{\partial T}{\partial t} = \nabla\left(k \nabla T\right) + \frac{J^2}{\sigma} - \beta \vec{J} \cdot \nabla T \tag{9}$$

where the first term on the left hand side of Eq. (9) accounts for the structure and size of the solid structure on the heat transfer equation. As mentioned above, another two main terms, which are Joule heating and Thomson effect, appear in the energy equations [112][113][114]. In a recently published article [115], yet another heat generation term called Peltier effect has been discussed through the formulations. The thermal analysis of these systems has been well investigated throughout the literature; see Refs. [116][114] for a review of this topic. However, thermodynamic analysis of these solid systems is rather limited [24][25] [26][114]. In all of these analyses [24][25] [26][114] the considered geometry has been treated one-dimensional fin-type [117], to eliminate multi-dimensional partial differential equations. These analyses can be performed for steady state [24][25] or transient [26] processes. Similar to that discussed in the earlier subsections, when entropy generation in solid thermoelectric systems is considered, first the temperature field is obtained and consequently the thermal field is digested into the entropy formulation. Expectedly, that the entropy generation formulation consists of new terms which account for entropy generation due to Joule heating and Thomson effects [114]. Kaushik and Manikandan [24] provided some information regarding the geometry of the system for exergy analysis. With the exception of the cited articles there is no further thermodynamic optimization of thermoelectric solid systems. Hence, this topic needs further investigation and attention. Moreover, the transient energy analyses and consequently entropic analyses of nanoscales thermoelectric systems can be further analysed by using dual-phase-lag heat transfer models [118][119]. The most important contributions about thermodynamic analyses of thermoelectric systems have been tabulated in Table 4.

3. Analysis methods

In general, theoretical heat transfer modeling encounters one or more challenging partial differential equations, which could be coupled with each other. To solve these set of equations, the first step is to make reasonable simplifying assumptions. For instance, in solving energy equations within solid media unidirectional heat transfer is often considered to convert partial differential equation (PDE) into an ordinary differential equation (ODE) [120]. Let us consider a three dimensional wall (Fig. 7). The steady state energy equation without internal energy sources for this wall is given by [121]

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[k\,\frac{\mathrm{d}T}{\mathrm{d}x}\right]+\frac{\mathrm{d}}{\mathrm{d}y}\left[k\,\frac{\mathrm{d}T}{\mathrm{d}y}\right]+\frac{\mathrm{d}}{\mathrm{d}z}\left[k\,\frac{\mathrm{d}T}{\mathrm{d}z}\right]=0 \tag{10}$$

If the four surfaces which are perpendicular to y and z axes are assumed to be well insulated, the heat merely transfers through x direction. Then Eq. (7) reduces to the following ODE [122]:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[k\,\frac{\mathrm{d}T}{\mathrm{d}x}\right]=0 \tag{11}$$

Although due to some nonlinearities involved within the problem (say k=k(T)), the obtained ordinary differential equation may not be readily solvable, they take less expenses to be tackled.

Following this step, some mathematical manipulations may be necessary to decouple the equations. For example, the conduction energy equation in solid phase of porous media can be coupled with fluid phase energy equation in through convection terms. This occurs during the thermal analyses of porous systems under LTNE model (so-called two-energy equation model). Therefore, the following coupled equations may be considered [83][59]:

$$\rho c_p u_f \frac{\partial T_f}{\partial x}=k_{ef}\frac{\partial^2 T_f}{\partial y^2}+h_{sf}a_{sf}\left(T_s-T_f\right)+\frac{\mu_f}{\kappa}u_f^2+\mu_{eff}\left(\frac{\partial u_f}{\partial y}\right)^2 \tag{12a}$$

$$0=k_{es}\frac{\partial^2 T_s}{\partial y^2}-h_{sf}a_{sf}\left(T_s-T_f\right) \tag{12b}$$

where Eq. (12a) represents the energy equation in fluid phase and Eq. (12b) governs the heat flow through the solid phase of the porous system. As can be clearly seen in Eqs. (12a) and (12b), are coupled with each other through the term $h_{sf}a_{sf}\left(T_s-T_f\right)$ which counts for the convective heat transfer between the two phases. By increasing the order of derivatives, these coupled equations are decoupled to yield a new set of differential equations. Interested readers may refer to the recently published papers on heat transfer and entropy generation in porous media using LTNE model [123][124][102][59]. In some cases this step helps providing exact analytical solution for the problem [83][59]. However, under more involved situations, application of numerical methods is unavoidable [32][125][48]. Numerical techniques, such as Newton-Raphson, bisection, secant or fixed-point iteration methods, can be applied to find the coefficients of the solution [50][48]. They can be, further, applied to the original energy equation together with boundary conditions [32][125]. In the latter case, finite volume or finite difference methods could be employed. Consequently, the obtained temperature distributions are digested into the entropy formulations. If the system is a purely conductive one, the provided entropy equation in Section 2 is used. Regarding systems with other heat transfer modes, the appropriate formulation for the entropy generation has to be considered. For example, if a porous system under LTNE model is investigated, the entropy formulas may be written as:

$$\dot{S}'''_f = \frac{k_{ef}}{T_f^2}\left[\left(\frac{\partial T_f}{\partial x}\right)^2 + \left(\frac{\partial T_f}{\partial y}\right)^2\right] + \frac{h_{sf}a_{sf}\left(T_s - T_f\right)^2}{T_s T_f} + \frac{\mu_f}{\kappa T_f}u_f^2 + \frac{\mu_{eff}}{T_f}\left(\frac{\partial u_f}{\partial y}\right)^2 \tag{13a}$$

$$\dot{S}'''_s = \frac{k_{es}}{T_s^2}\left[\left(\frac{\partial T_s}{\partial x}\right)^2 + \left(\frac{\partial T_s}{\partial y}\right)^2\right] + \frac{h_{sf}a_{sf}\left(T_s - T_f\right)^2}{T_s T_f} \tag{13b}$$

where Eq. (13a) and Eq. (13b) give local entropy generation rate in solid and fluid phases of the porous medium, respectively. A more comprehensive discussion can be found in Refs. [59][83][104][100][58].

Here, we briefly discuss the mathematical method used for the analysis of entropy generation in solid systems.

3.1. Exact analytical methods

It is often possible to find exact analytical solutions for the linear ordinary differential equations [14][121]. By taking this approach, along with the associated simplifying assumptions, the model may lack some realistic features. However, exact analytical solutions help developing a deep understanding of the behavior of the system under investigation. Importantly, the analytical results can be, further, used for validation of numerical tools. Analytical approach has been taken in a number of investigations of entropy generation in solid media [126][49][61], entropy generation in conjugate convection-conduction systems [72][45], and even for heat transfer and entropy generation within convective fins [17]. It should be noted that some energy equations for very fundamental geometries, such as the following discussed cooled homogeneous slab, which contain temperature-dependent parameters may be also solvable with exact analytical methods. An example of this can be seen in the interesting work of Aziz and Khan [61]. However, when dealing with the systems containing radiation effects [32][48], numerical methods are necessary to obtain the temperature distribution of the system and predict the entropy generation rate within the system.

To make the given descriptions clearer, an exemplary problem is solved analytically in this section. We consider a homogeneous slab made of a material with temperature-dependent thermal conductivity, which is cooled asymmetrically. The slab generates heat with a constant rate. Figure 8 shows the configuration of the problem. Considering these assumptions, the one-dimensional non-linear energy differential equation within the slab can be expressed as:

$$k\,\frac{\mathrm{d}}{\mathrm{d}x}\left[\left(1 + a\left(T - T_0\right)\right)\frac{\mathrm{d}T}{\mathrm{d}x}\right] + \dot{q} = 0 \quad 0 < x \le L \;, \tag{14}$$

with the following boundary conditions

$$\left.\frac{\mathrm{d}T}{\mathrm{d}x}\right|_{x=0} = 0 \tag{15a}$$

$$T\big|_{x=L} = T_c \tag{15b}$$

These equations can be easily non-dimensionalized using the dimensionless parameters described in the literature [50][47]. Hence, the nondimensional form of the equations are written as

$$\frac{\mathrm{d}}{\mathrm{d}X}\left[\left(1+\alpha\left(\theta-1\right)\right)\frac{\mathrm{d}\theta}{\mathrm{d}X}\right]+Q=0 \quad 0<X\le 1 \tag{16}$$

$$\left.\frac{\mathrm{d}\theta}{\mathrm{d}X}\right|_{X=0}=0 \tag{17a}$$

$$\theta\big|_{X=1}=\theta_c \tag{17b}$$

Although the main energy equation for this problem is nonlinear, it can be still solved using exact analytical methods. In this case, the energy Eq. (16) is a nonlinear second order ODE, which can be easily solved through a change of variable ($v=\frac{\mathrm{d}\theta}{\mathrm{d}X}$) and, therefore, transforming the original equation into a first order ODE, see Ref. [127]. Employing Maple 14, using the available boundary conditions and after some manipulation, the following relation can be used to describe the temperature of the slab:

$$\theta(X)=\frac{\alpha-1+\sqrt{\alpha^2-2\alpha+1-X^2\alpha Q+2\alpha\left(\frac{Q}{2}-\alpha\theta_c+\theta_c+\frac{\alpha\theta_c^2}{2}\right)}}{\alpha} \tag{18}$$

This formula can be then incorporated into the entropy generation relationship for the solid structure [50][47]. For the present one-dimensional problem, the volumetric entropy generation Eq. (5) can be written as:

$$\dot{S}'''=\frac{d}{dx}\left(\frac{q''_x}{T}\right)-\frac{1}{T}\dot{q} \tag{19}$$

Using Fourier's law, for a slab with regular material Eq. (19) becomes

$$\dot{S}'''=\frac{d}{dx}\left(-\frac{1}{T}k(T)\frac{dT}{dx}\right)-\frac{1}{T}\dot{q} \tag{20}$$

By incorporating the energy equation, i.e., Eq. (14) in Eq. (20), the latter reduces to

$$\dot{S}'''=\frac{k(T)}{T^2}\left(\frac{dT}{dx}\right)^2 \tag{21}$$

With the introduction of dimensionless variables, the dimensionless local volumetric entropy generation rate N_s for the slab with homogeneous material is given by

$$N_s = \frac{\left(1+\beta\left(\theta-\kappa\right)\right)\left(\frac{d\theta}{dX}\right)^2}{\theta^2} \tag{22}$$

Here, it should be noted that, as discussed by Aziz and Khan [49], the internal heat generation will not appear in the entropy generation formulation. In fact, the effects of internal heat generation on the temperature formulation influence the entropy generation rate in an indirect way. Due to the analytic nature of this method, appropriate parametric values can be used to determine the temperature and local entropy generation rate as depicted in Fig. 9.

This analysis can be further extended to double-layer composite walls by incorporating another energy equation for the second wall and related boundary conditions at the interface of the two walls. The two walls, depending on their size and surface condition, may or may not be in perfect thermal contact with each other. This has been discussed thoroughly in a series of recent publication by Torabi and co-workers [128][47][48]. For example, if the imperfect thermal contact is valid for the interface of the two walls, the following boundary conditions, in addition to the outer layers boundary conditions should be employed [48]:

$$T_1(x=x_2)-T_2(x=x_2) = -R_c\, k_1\left(1+a_1\left(T_1-T_0\right)\right)\left.\frac{\mathrm{d}T_1}{\mathrm{d}x}\right|_{x=x_2} \tag{23a}$$

$$k_1\left(1+a_1\left(T_1-T_0\right)\right)\left.\frac{\mathrm{d}T_1}{\mathrm{d}x}\right|_{x=x_2} = k_2\left(1+a_2\left(T_2-T_0\right)\right)\left.\frac{\mathrm{d}T_2}{\mathrm{d}x}\right|_{x=x_2} \tag{23b}$$

Where the right hand side of Eq. (23a) introduces a temperature jump into the system. For more information regarding the TCR and its effects on the temperature field of conductive systems, readers are encouraged to consult with the recently published articles in this regard [129][130][131].

3.2. Approximate analytical techniques

In conduction problems the governing equation becomes nonlinear if the thermal conductivity of the material is temperature-dependent [23]. Similarly, the convection term becomes nonlinear due to the power law dependence of heat transfer coefficient on the temperature difference. The radiation term contains two nonlinearities, one due to the Stefan–Boltzmann law of radiation, and the second due to the fact that the surface emissivity is a function of temperature. Other nonlinear terms arise in the differential equation of the material by introducing temperature-dependent internal heat generation [132][128]. Under most situations, even with one nonlinear term, the ODE describing the energy balance does not admit an exact analytical solution. The following energy equations have some of the above-mentioned characteristics and cannot be treated analytically [50][23].

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[k\left[1+b\left(T-T_0\right)\right]\frac{\mathrm{d}T}{\mathrm{d}x}\right]+\dot{q}\left(1+\alpha T\right)=0 \tag{24}$$

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left[k\left[1+b\left(T-T_0\right)\right]\frac{\mathrm{d}T}{\mathrm{d}r}\right]+\dot{q}\left(1+\alpha T\right)=0 \tag{25}$$

Consequently, a number of non-exact solution procedures can be used to address these types of problems. These methods which deliver a relation for the temperature distribution and, accordingly for the entropy generation within the material, are regarded as the approximate analytical techniques. They transfer the original nonlinear differential equation into an infinite number of simpler, analytically solvable differential equations. Since these tend to sum up a very large numbers of solutions, they are regarded as approximate solutions. When a reasonable number of transformed differential equations are solved and their results are added together, further differential equations are neglected. This imposes a truncated error in the results, which is an inherent characteristic of these techniques [133].

One of the powerful approximate methods, which has been used in heat transfer problems [134] and entropy generation analyses of solid media [23][135][128][50], is the differential transformation method (DTM). DTM is based upon Taylor series expansion, and is able to deliver highly accurate results in various challenging situations such as problems with convective-radiative boundary heat loss [23][135][50] and highly nonlinear internal heat generation [128]. Using such approximate methods can provide relations for the temperature and entropy generation along with a physical insight into the effective parameters. This is typically superior to the traditional numerical methods under many situations. This method transfers each term of the differential equation into a new term which is used in the series [23]. Afterwards, using the differential inverse operator, the final solution for the energy equation of the structure can be obtained. For example, consider the energy equation for the homogenous cooled wall in Ref. [50]. The energy equation is as follows:

$$\frac{\mathrm{d}}{\mathrm{d}X}\left[(1+\beta(\theta-\gamma))\frac{\mathrm{d}\theta}{\mathrm{d}X}\right]+Q(1+\alpha\theta)=0 \tag{26}$$

Using the DTM technique and taking the differential transform of Eq. (26), the above equation is transformed to:

$$\begin{aligned}&\left(1-\beta\cdot\gamma\right)\left(k+2\right)\left(k+1\right)\Theta\left(k+2\right)+\beta\left(\sum_{l=0}^{k}\Theta\left(l\right)\left(k+2-l\right)\left(k+1-l\right)\Theta\left(k+2-l\right)\right)\\&+\beta\left(\sum_{l=0}^{k}\left(l+1\right)\Theta\left(l+1\right)\left(k+1-l\right)\Theta\left(k+1-l\right)\right)+Q\cdot\alpha\cdot\Theta\left(k\right)+Q\cdot\delta\left(k\right)=0\end{aligned} \tag{27}$$

Now by varying parameter k form zero, the values of $\Theta\left(k\right)$ are determined. Then, by the following equation which is called differential inverse transform function is used to obtain the final energy equation.

$$\theta(X)=\sum_{k=0}^{n}X^k\Theta(k) \tag{28}$$

A more detailed discussion of this technique can be found in Refs. [136][137].

When the convective aspects are incorporated into the system, the momentum and energy equations can be coupled [138][74][75]. In some cases, the velocity field can be directly incorporated into the energy equation [74]. More frequently, however, direct incorporation of the velocity into the energy equation result in an analytically unsolvable problem and therefore necessitates the use of numerical techniques [75][138]. To circumvent this barrier, a Tylor series of the velocity field is used. This makes the obtained temperature approximate, yet it has the advantage of producing an analytical relation for the temperature field. Recently, Mahian et al. [138] and Torabi and Zhang [75] used this procedure to obtain temperature distribution in convective media and accordingly incorporated these temperatures into entropy equations. The work of Mahian et al. [138] was performed in connection with purely convective systems, while Torabi and Zhang [75] investigated a conductive-convective system.

3.3. Numerical simulations

If the geometry is complicated and/or the energy equations, together with the boundary conditions involve strong nonlinearities, the use of numerical solution is mostly unavoidable. Further, numerical solutions are often used when energy and momentum equations are strongly coupled [99]. In these situations there are two main approaches to numerical solution of the equations. One method is to make use of commercial modelling software, customized for heat transfer and thermal systems. Two well-known of these are ANSYS Fluent and COMSOL Multiphysics. The other method is to develop a code in programing languages such as FORTRAN. One of the early investigation for the numerical entropy generation analyses of thermal systems, which involves heat conduction, has been done by Orhan et al. [139]. Here, finite control volume approach was used to predict the entropy generation rate for a phase-change process in a parallel plate channel. In a recently published work by Makhanlall and Liu [32] FLUENT 6.3 has been used to tackle the energy equations in a conductive-radiative environment. Under this setting, special treatment is needed to calculate the entropy generation rate within the system. This can be done by employing a feature of this software, called user defined functions. By adopting this approach, Makhanlall and Liu were able to provide local and total entropy generation and exergy destruction [32].

3.4. Combined analytical-numerical techniques

The last major approach for the solution of conduction dominated problems is to employ analytical and numerical techniques simultaneously. In some situations, the general solution of the problem can be found analytically. However, due to complex boundary conditions, such as radiation, in many cases, a particular solution cannot be developed analytically. Hence, it is necessary to apply a numerical method to calculate the constant coefficients for the particular solution. This approach has received considerable attention recently, as it presents a solution approach for a large number of problems [74][48][140]. To determine the coefficients a system of equations must be solved. Commercial mathematical packages such as Maple or Wolfram Mathematica may be used for this purpose. In a series of work [74][48][140], Maple's built-in fsolve command, which numerically approximates the roots of an algebraic function, was used in the second law analysis of various problems. This command employs numerical techniques, such as Newton-Raphson, bisection, secant and fixed-point iteration methods. Upon calculation of the coefficients, by incorporating them into the analytical formulation for the general solution, the temperature field and consequently the entropy generation rates can be readily calculated.

4. Conclusion

In recent years, entropy generation in solids has attracted considerable attention from the research community. The literature on entropy generation in thermal systems with solid structures was reviewed in the current work. The importance of the second law of thermodynamics in the analysis of these solid systems was presented and discussed. Following a concise mathematical discussion of the entropy generation in solid media, the literature pertaining to the entropy generation in pure conductive media, thermal systems with conjugate heat transfer and porous media, were reviewed. A relatively new technological manifestation of conduction dominated processes was detected in thermoelectric devices and the recent energy and exergy analysis in this field were reviewed. Subsequently, the review was extended to the solution method of the differential equations, required to address the most significant problems in this field. It was observed that there exists, significant potential for further research on the second law analyses of solid systems. Combined analytical-numerical and the numerical methods were identified as the techniques that offer the most attractive features for the foreseeable future of entropy generation modeling.

Moreover, authors would like to emphasize that as mentioned thorough the review, the entropy generation studies for purely radiative systems are very rare [30][31]. Since the primary heat transport modes in electronic systems are conduction and radiation [141], there is an urgent need for the consideration of entropy generation in radiative environments. As it has been stated in Bright and Zhang investigation [31], the general entropy generation equation for a radiative environment is similar to a conductive system. However, the intrinsic feature of radiative environment especially at micro and nanoscales, involves phonon transport equations which may hinders the solution of energy equations. This challenging and yet exciting task remains before the research community to be tackled.

Acknowledgements

This work was supported by the Hong Kong Research Grants Council (project no. CityU 11216815) and NSAF (grant no. U1330132).

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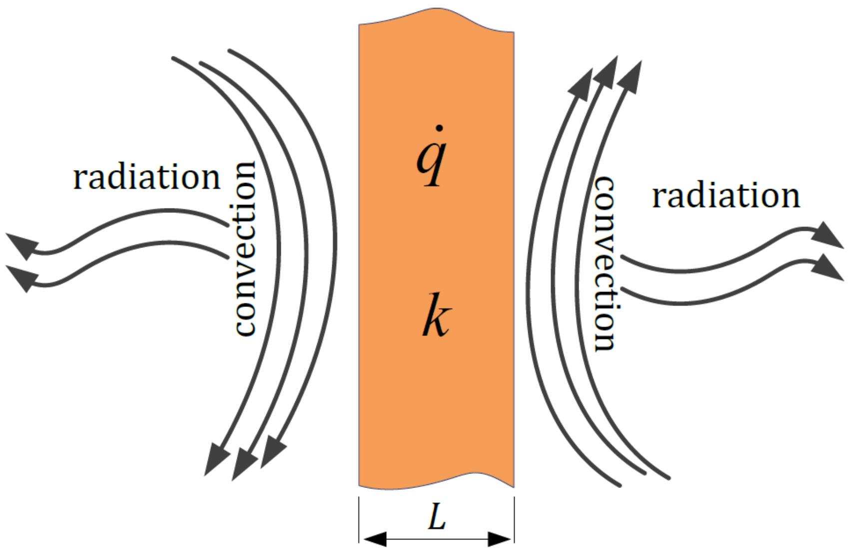

Fig. 1. Configuration of a solid wall with different modes of heat losses.

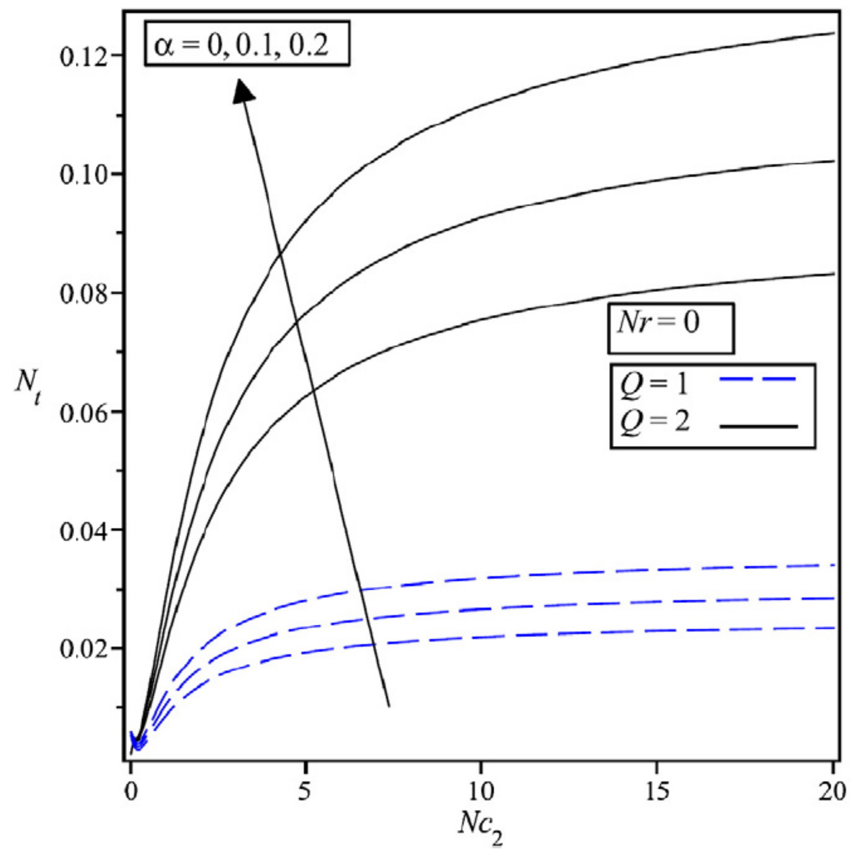

Fig. 2. Total entropy generation rate versus heat transfer rate in a hollow cylinder [23].

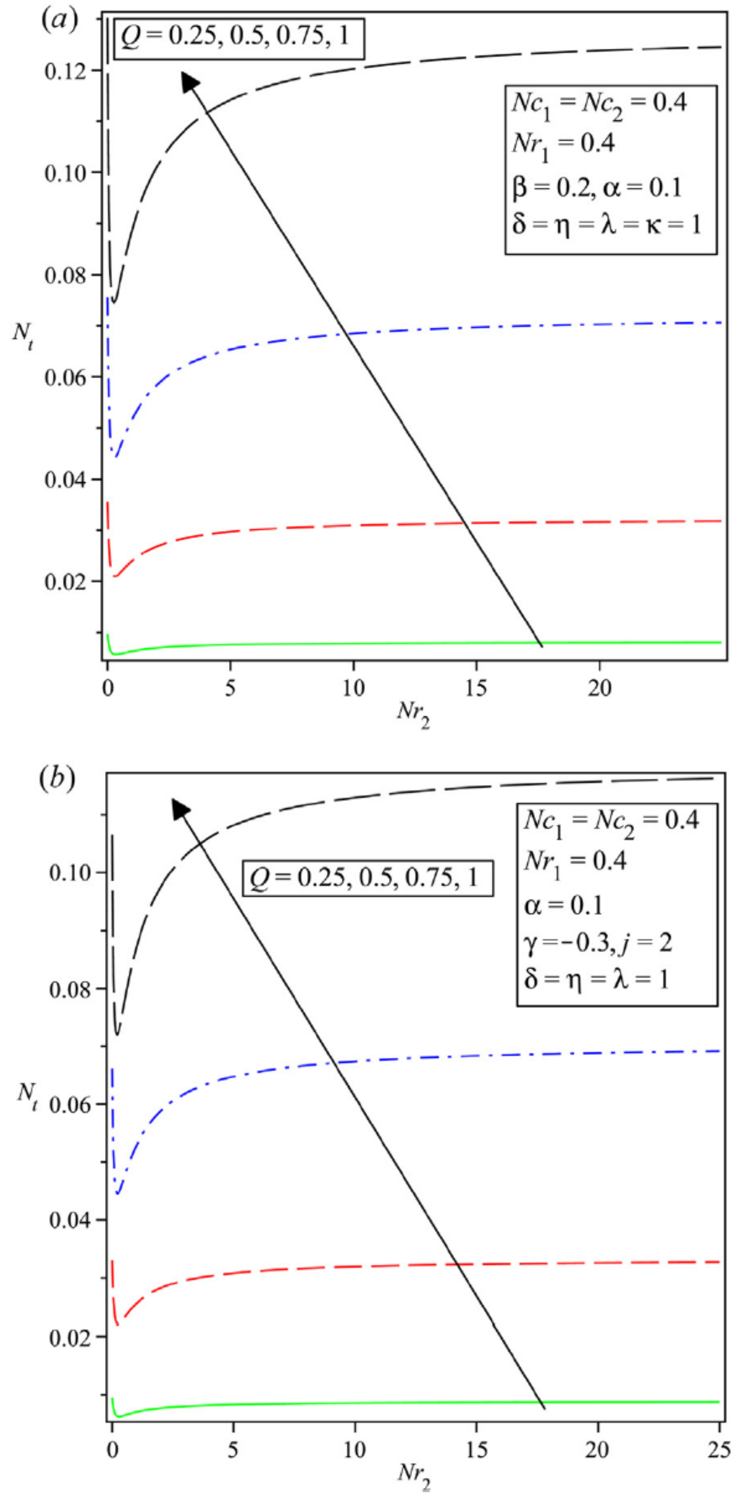

Fig. 3. Total entropy generation rate versus conduction-radiation parameter for (a) homogenous slab and (b) FGM slab [50].

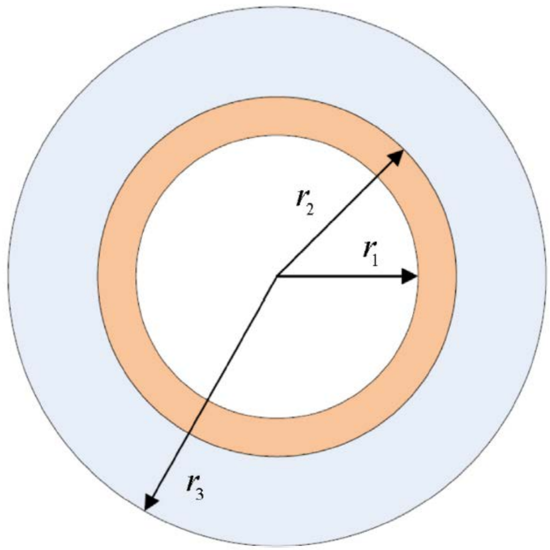

Fig. 4. Configuration of double-layer composite hollow cylinder studied by Torabi and Zhang [47].

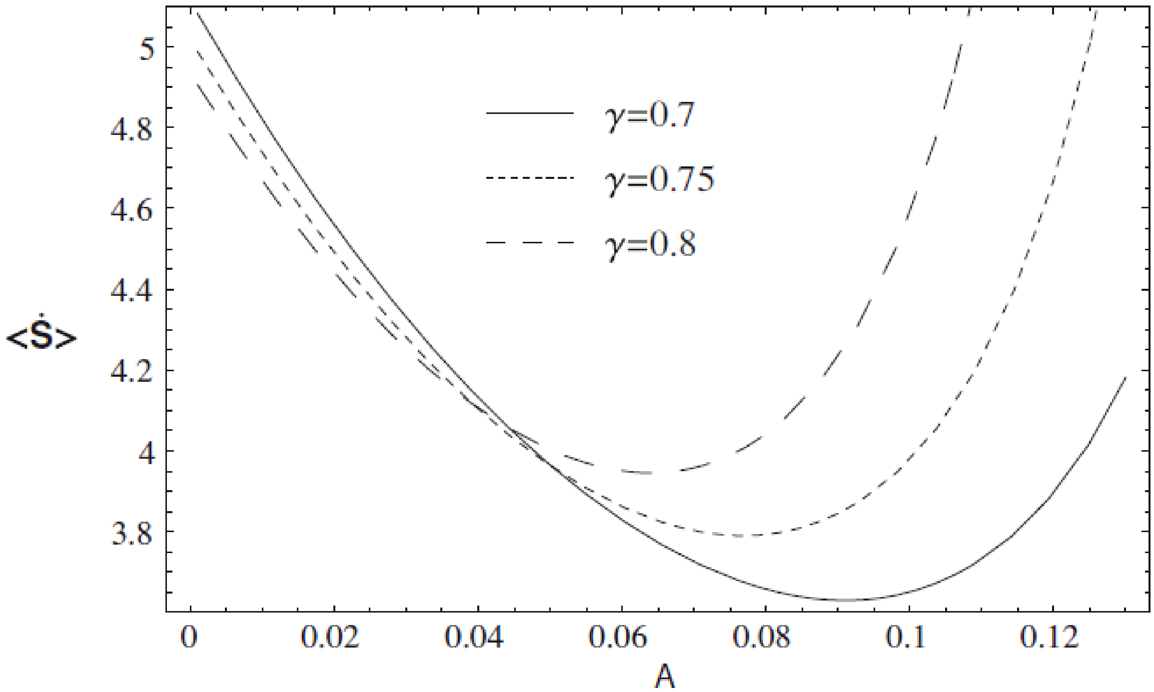

Fig. 5. Total entropy generation rate versus slip length in a microchannel studied by Ibáñez et al. [73].

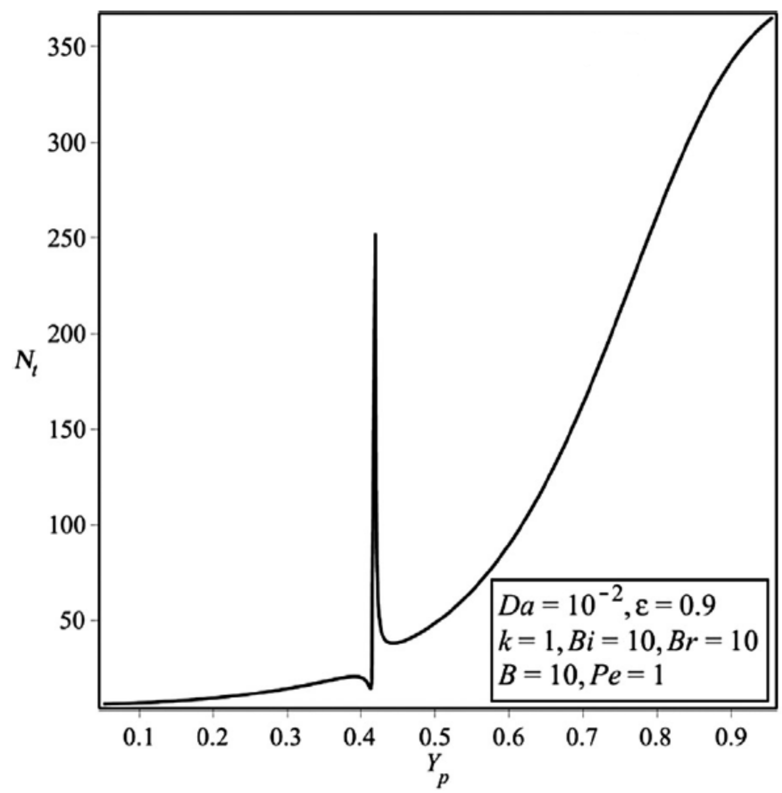

Fig. 6. Total entropy generation rate versus porous thickness in a partially porous channel using LTNE model [59].

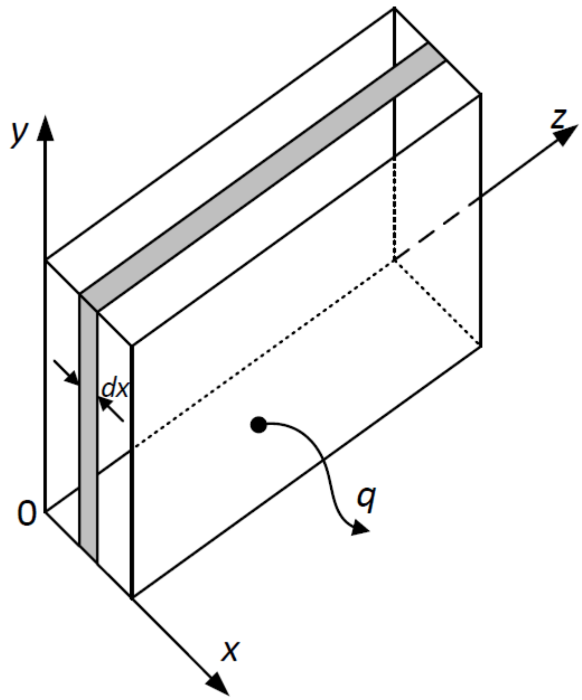

Fig. 7. Configuration of a three dimensional cooled wall.

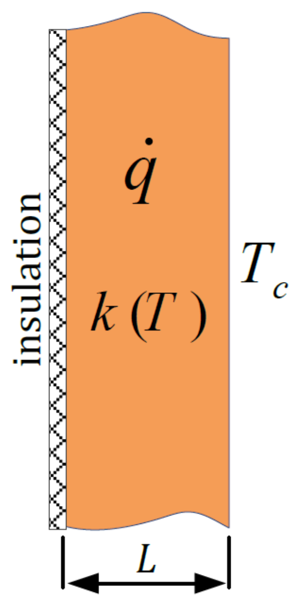

Fig. 8. Configuration of a cooled wall.

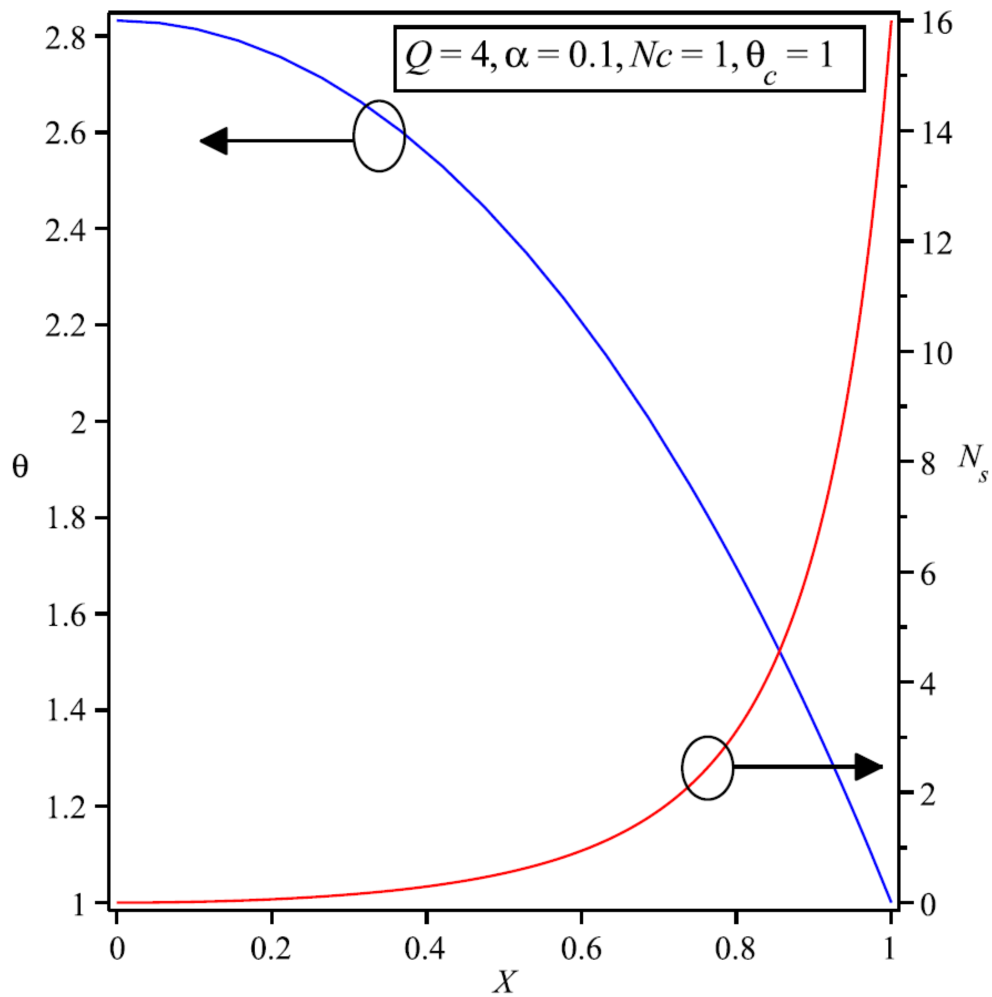

Fig. 9. Temperature distribution and local entropy generation rate for the cooled wall.

Table 1. Summary of the recent investigated pure conductive geometries

Authors	Geometry	Boundary condition	Solution method	Main contribution
Kolenda et al. [55]	One-dimensional or multi-dimensional walls	Constant temperature	Numerical	One of the pioneering works
Aziz and Khan [49]	Wall	Asymmetric convection	Exact analytical	Incorporation of internal heat source
Aziz and Khan [61]	Wall, hollow cylinder and sphere	Inner and outer constant temperatures	Exact analytical + numerical for some cases	Incorporation of internal heat source + new fundamental geometries
Torabi and Aziz [23]	Hollow cylinder	Inner convective, outer convective-radiative	Analytical DTM	Analytical solution with radiation boundary condition
Torabi and Zhang [50]	Walls	Convective-radiative at both sides	Analytical DTM	Incorporation of walls materials such as temperature-dependent or spatial-dependent thermal conductivities
Torabi and Zhang [47]	Double-layer hollow cylinder	Constant temperature for both inner and outer layers + convective cooling for both inner and outer layers (Two cases)	Combined analytical-numerical	Composite cylindrical structure
Torabi and Zhang [128]	Double-layer wall	Convective-radiative at both sides	Analytical DTM	Composite walls with radiation heat loss
Torabi et al. [48]	Double-layer wall, hollow cylinder and sphere	Inner convective, outer convective-radiative	Combined analytical-numerical	Incorporation of imperfect thermal contact for the interface boundary condition
Ali et al. [54]	Cylinder	Constant temperature	Combined analytical-numerical	Transient heat transfer and entropy generation analyses of a cylinder

Table 2. Summary of the recent investigations in conjugate thermal systems

Authors	**Geometry**	**Modes of heat transfer**	**Solution method**	**Main contribution**
Ibáñez and Cuevas [72]	Microchannel	Conduction and convection	Analytical	A pioneering work about conductive-convective systems
Ibáñez et al. [45]	Microchannel	Conduction and convection	Analytical	Optimization of walls' thickness
Ibáñez et al. [73]	Microchannel	Conduction and convection	Analytical	Incorporation of slip boundary
Torabi et al. [74]	Cylindrical system	Conduction and convection	Combined analytical-numerical	Consideration of copper–water nanofluid
Torabi and Zhang [75]	Cylindrical system	Conduction and convection	Combined analytical-numerical	Consideration of magnetohydrodynamic flow
Makhanlall and Liu [32]	Square cavity	Conduction and radiation	Numerical	A pioneering work about entropy generation in conductive-radiative systems
Jejurkar and Mishra [142]	Annular microcombustor	Conduction and radiation	Numerical	A pioneering work about entropy generation in microcombustors

Table 3. Summary of the recent investigations in forced convection in porous systems using LTNE model

Authors	**Geometry**	**Boundary condition**	**Fully or partially filled**	**Solution method**	**Main contribution**
Buonomo et al. [58]	Channel	Constant heat flux	Fully filled	Analytical	First entropy generation investigation for LTNE model
Torabi et al. [59]	Channel	Constant heat flux lower wall and adiabatic upper wall	Partially filled	Combined analytical-numerical	Entropy generation study for partially filled channel
Torabi et al. [100]	Channel	Constant heat flux	Partially filled	Combined analytical-numerical	Incorporation of internal heat sources
Trevizoli and Barbosa [103]	A simplified passive regenerator	Constant temperature and symmetry boundaries	Fully filled	Numerical	Incorporation of oscillatory flow using a time-dependent pressure function
Ting et al. [104]	Channel	Constant heat flux	Fully filled	Analytical	Addition of nanoparticles to the fluid phase
Ting et al. [105]	Channel	Constant heat flux	Fully filled	Analytical	Addition of nanoparticles to the fluid phase and heat generation in solid phase

Table 4. Summary of the recent entropy generation investigations in thermoelectric solid systems

Authors	**Geometry**	**Steady or transient**	**Solution method**	**Main contribution**
Chakraborty et al. [114]	Rectangular thermoelectric leg	Steady	Analytical	A pioneer work on the entropy analysis
Kaushik and Manikandan [24], Manikandan and Kaushik [25]	Annular thermoelectric leg	Steady	Analytical	Influence of Thomson effect and exergy analysis
Figueroa and Vázquez [26]	Rectangular thermoelectric leg	Transient	Numerical	Hyperbolic heat transfer analysis for small scale devices