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Large-scale, white-light, transformation optics using integral imaging

Stephen Oxburgh¹, Chris D White¹, Georgios Antoniou¹,
Ejovbokoghene Orife¹, Tim Sharpe² and Johannes Courtial¹

¹School of Physics & Astronomy, University of Glasgow, Glasgow G12 8QQ, UK

²Mackintosh Environmental Architecture Research Unit (MEARU), Glasgow School of Art, Glasgow G3 6RQ, UK

E-mail: johannes.courtial@glasgow.ac.uk

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Abstract

We describe a way to realise transformation-optical devices from structures of micro-structured sheets called generalised confocal lenslet arrays. The resulting devices should work for all visible light, and they should be relatively easy and cheap to (mass-)produce on the scale of metres, but they suffer from field-of-view limitations and significant transmission loss. Furthermore, the mapping between electromagnetic space and physical space is not through stigmatic imaging, but integral imaging. As an example application of this technology, we design and simulate an architectural window that cloaks insulation material with the aim of reducing heat loss.

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(Some figures may appear in colour only in the online journal)


1. Introduction

Transformation optics (TO) [1, 2] is an optical design paradigm, developed in the context of metamaterials [3], which appears to distort space when seen through a TO device. Standard TO devices are solid metamaterial [3] structures, which are difficult and costly to produce. The most famous application of TO is invisibility cloaking. In this case, the TO device—the cloak—is a (meta)material structure that aims to make a void inside the cloak appear infinitely small, thereby hiding the void and anything in it. At the same time, the cloak itself is designed to be invisible. This idea has caused great excitement, both in the scientific community and in the interested public.

Here we discuss, and study theoretically, an alternative way of realising TO, using microstructured light-transmitting sheets called generalised confocal lenslet arrays (GCLAs) [4]. Very general distortions of space can, at least in principle, be

realised using these ideas, including cloaking [5]. The sheets are pixellated, introducing discontinuities into transmitted wave fronts, apparently (but not actually) circumventing the limitations of ray optics due to theorems derived for continuous wave fronts. The GCLAs change the direction of transmitted light like the interface between different materials [6], and thus can be used to construct devices which are almost entirely empty. This should enable much larger and lighter devices to be built, thereby significantly extending the possible applications of TO.

In contrast to stigmatic imaging, in which all individual light rays that pass through the same point (the object point) before transmission through the imaging device afterwards intersect again in a single point (the image point), in *integral imaging* it is the axes of (in the simplest case cylindrical) bundles of light rays that intersect [7]. Such imaging results from passage through pixellated optical components in which each pixel redirects a small area of the wave front. Examples of such pixellated optical components are arrays of small telescopes, and the GCLAs we consider here are precisely such arrays of telescopes, each one formed by a pair of lenslets. Clearly, integral imaging is inferior to stigmatic

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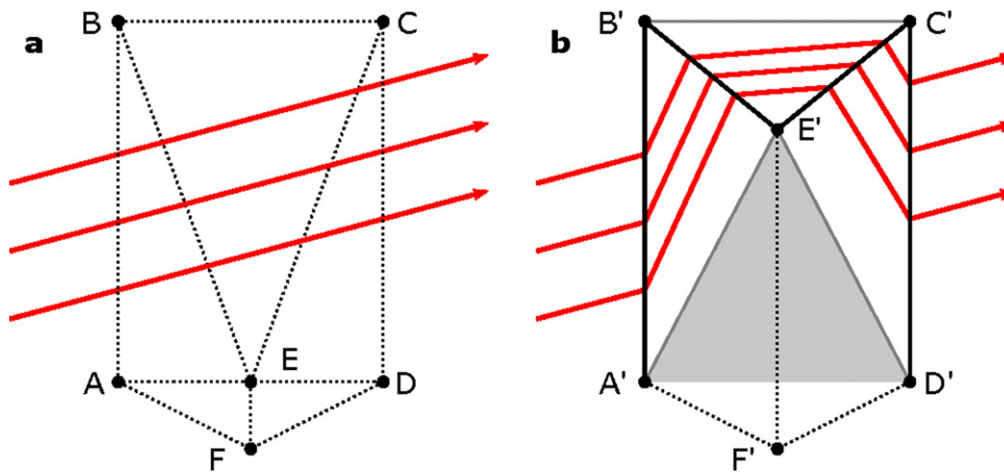


Figure 1. Pinch transformation. (a) A section of empty *electromagnetic space*, in which light-ray trajectories are straight lines, is divided into triangles. The point E marks the common corner of these triangles. (b) In *physical space*, this common corner is moved upwards to the position marked E', distorting the triangles and light-ray trajectories with them. The overall distortion is such that the rectangle ABCD is 'pinched' in the centre. The pinch transformation shown here is piecewise affine, which implies that, in the pieces, light rays travel in straight lines and change direction only at the boundaries of the pieces. As drawn, the transformation is also continuous, as the physical-space triangle A'F'DF is not empty (like in a cloak) but filled with the triangles A'F'E' and F'D'E'. In our proposed realisation of TO, these boundaries are realised by microstructured transparent sheets. For our proposed application of this transformation, we are interested only in light rays that pass through the lines A'B' and C'D'; the four surfaces shown by thick lines in (b) re-direct such light rays around the grey triangle, which is effectively cloaked. The transformation is drawn in 2D; it can easily be extended to 3D (see figure 3).

imaging, but many light-ray-direction changes cannot be realised ray by ray for all light fields, as this would result in outgoing phase fronts with non-zero curl (and therefore discontinuities at every point) [8, 9]. There is no such restriction for piecewise re-directing of light fields, in which those discontinuities get concentrated into the lines separating the pixels, resulting in generalised, but pixellated, refraction that can achieve more general, but integral, imaging. A famous example is the Gabor superlens [10], which can be shown to realise the most general one-to-one-and-onto mapping between object and image space any planar interface can perform [11]. (N.B. non-planar interfaces cannot perform non-trivial one-to-one-and-onto mapping between object and image space [12].) The GCLAs considered here can be understood as the homogeneous limit of a Gabor superlens; they can thus realise the most general one-to-one-and-onto mapping between object and image space any homogeneous interface can perform [13].

GCLAs have significant drawbacks such as diffractive blurring due to the small size of the pixels, a limited field of view, and only a fraction of the light undergoing the desired light-ray-direction change (the rest can be absorbed) [14, 15]. These drawbacks can be traded off against each other, and our current optical-engineering efforts [16] are aimed at finding suitable compromises. On the other hand, one of the attractive aspects of our proposal is that GCLAs can, at least in principle, be mass-manufactured cheaply and on a large scale, using existing technology. An example of sheets comprising pairs of microlens arrays being manufactured inexpensively on a large scale is Rowlux illusion film [17], a commercial sheet material that creates a depth illusion. The large-scale, large-volume, low-cost, manufacturability of the sheets that form our proposed TO devices should enable TO devices on

an architectural scale, using today's technology. As an example application, we discuss a potentially energy-saving architectural window capable of cloaking insulation material. This is one of the first everyday applications of the science of invisibility.

2. Transformation optics (TO)

TO works by moulding light-ray trajectories, as follows. Imagine a hypothetical empty space, referred to as *electromagnetic (EM) space*³, and a particular volume within it. Any light rays that travel through EM space do so along straight lines. Now imagine continuously distorting the inside of the volume, and the light-ray trajectories with it (such a continuous deformation is called a homeomorphism); outside the volume, space is not distorted and light-ray trajectories there are unaffected by the distortion. The resulting distorted space is called *physical space*. Figure 1 shows an example of an EM space and its corresponding physical space, with a few light-ray trajectories. TO's key realisation is that in a volume filled with suitable spatially-varying (meta)materials the light-ray trajectories are those in the physical space, but because observers outside the distorted volume would only see those parts of light rays which are unaffected by the distortion, this (physical-space) material structure has the appearance of the corresponding EM space.

In the case of the original invisibility cloaking proposal [2], a point in (empty) EM space is 'exploded' into a finite-size void in physical space. The appearance of this solid

³ Note that EM space can itself be curved, resulting in non-Euclidean TO [18].

structure is that of empty space; it is invisible, and the void and any object inside it is cloaked.

TO devices do not only distort light rays, but also light waves. The spatially varying materials that mould the light rays have the property that the optical path length of any curve through it is the same as that of the corresponding curve in EM space, and hence propagation of a light wave through a volume of physical space has precisely the same effect on the wave as propagation through the equivalent volume of EM space. The presence of an ideal TO cloak therefore has no effect on waves outside the cloak, not even a change in light phase.

Building a perfect macroscopic cloak for all visible light is at the very least hard, perhaps impossible. The obvious difficulties of manufacturing the required spatially varying three-dimensional nano-structures have led to cloaks in which the ideal material parameters were approximated to be more easily manufacturable, but ray-tracing simulations show that such simplifications happen at the cost of introducing visible imperfections [19–24]. There are more fundamental obstacles: on the scale of a few wavelengths, cloaks have been realised without approximating any material properties (e.g. [25]), but theoretical considerations have shown that significant increases in the size of the cloaked region require loss and bandwidth limitations to be much more tightly controlled [26–28].

A promising alternative is to design *ray-optical* cloaks that aim to mould light-ray trajectories without making any attempt to transform the phases accordingly, not even approximately. This approach has brought visible cloaking to a macroscopic scale [29–33].

Most implementations of TO devices so far require physical space to be filled with suitable materials, and for this reason we call them *solid*. The material properties reflect the local distortion of the EM space. There are also TO devices in which this is not the case (e.g. [32, 33]), which enables large parts of the device to be empty. Such devices have very limited functionality: in the case of [32], which still requires a large part of the device to be filled with material, empty regions cause the device to work only for a limited range of directions; in the case of [33], which is a simple cloaking device comprising four lenses, and which is therefore mostly empty, the distortion is limited to that which can be achieved with lenses and the device works only for a small range of directions.

3. Generalised confocal lenslet arrays (GCLAs)

GCLAs [4] are micro-structured sheets comprising pairs of microlens arrays combined into arrays of micro-telescopes which change the direction of transmitted light rays according to generalised laws of refraction [34]. Only very few examples of GCLAs have been demonstrated experimentally to date [35, 36]. The micro-telescopes can be miniaturised until they can no longer be resolved from the intended viewing position; they are the *pixels* of a transparent sheet performing *pixellated generalised refraction*.

GCLAs perform generalised refraction so general that they are examples of sheets that can appear to—but which do not actually—create wave-optically forbidden light-ray fields [9]. Such windows open up possibilities not normally considered. Wave-optics implies that configurations of light rays that can form inside TO devices cannot exist in empty space, but only in materials; here, the capability of GCLAs to create equivalent configurations of bundles of light rays in empty space allows TO devices made from GCLAs to be mostly empty.

The generalised law of refraction for GCLAs includes that required for idealised thin lenses [13], and so inhomogeneous GCLAs can act like idealised thin lenses [11]. In this sense, the paraxial cloak described in [33] is related to our TO devices.

With the exception of the approximate pinch-transformation window shown in figure 3(d), only a subset of GCLAs (namely imaging GCLAs [13]) is relevant to this paper. The generalised law of refraction for this subset has been shown to be equivalent to the light-ray-direction change at the interface between materials with different optical metrics [6]. Provided the GCLAs are in the $z = 0$ plane of a Cartesian coordinate system, the relationship between the symmetric metric on one side, which has elements $g_{ij}(=g_{ji})$, and that on the other side, which has elements $h_{ij}(=h_{ji})$, can be written in the form [6]

$$\begin{aligned}
 h_{11} &= g_{11}, & h_{22} &= g_{22}, & h_{12} &= g_{12}, & (1) \\
 h_{13} &= \frac{g_{13} + g_{12}\delta_y + g_{11}\delta_x}{\eta}, & h_{23} &= \frac{g_{23} + g_{12}\delta_x + g_{22}\delta_y}{\eta}, & (2) \\
 h_{33} &= \frac{g_{33} + 2\delta_y g_{23} + \delta_y^2 g_{22} + 2\delta_x g_{13} + \delta_x^2 g_{11} + 2\delta_x \delta_y g_{12}}{\eta^2}, & (3)
 \end{aligned}$$

where δ_x , δ_y and η are parameters describing the GCLAs, specifically the offset between corresponding lenslets in the x direction, the offset in the y direction, and the focal-length ratio, respectively. This relationship provides a direct link to standard TO [2].

A significant limitation of GCLAs is their limited field of view, which depends on the detailed optical design of the micro-telescopes and which has so far only been studied for the simplest GCLAs [14, 15]. Closely related is the appearance of unwanted additional images corresponding to light rays entering through the first lens of one telescope and exiting through the second lens of another telescope [14]; such errant light rays can be absorbed, resulting in a loss in optical power upon transmission through GCLAs [15]. Other unwanted effects include slight scattering, both for fundamental reasons (the Fourier spectrum of discontinuities is wide, but affects only a small fraction of the light) and for practical reasons such as surface roughness and manufacturing imperfections; a ray offset on the scale of the size of the microlenses; and some blurring of the view due to diffraction caused by the small size of the pixels and imperfect imaging by the (singlet) microlenses, especially if the light makes large angles with the normal to the GCLAs. It is possible to

ameliorate the last two effects, in the case of the offset by minimising the diameter of the microlenses (whereby the benefits of miniaturising the microlenses have to be balanced against adverse diffraction effects), in the case of blurring with improved optical engineering.

4. Ray-optical TO devices from GCLAs

In the TO devices we consider here, both EM space and physical space are divided into simplices (triangles in 2D, tetrahedra in 3D). A similar approach was taken in [32]; a 2D example is shown in figure 1. Each physical-space simplex is uniformly sheared and/or strained with respect to the corresponding EM-space simplex. Mathematically, such a uniform distortion is described by an affine mapping; the overall mapping from EM space to physical space is therefore piecewise affine. As neighbouring simplices in EM space are again neighbouring simplices in physical space, and as the affine mappings of neighbouring simplices map any point on the interface between the two simplices in EM space to the same point in physical space, the mapping is also continuous. Finally, the mapping is one-to-one, reflecting the fact that for each point in EM space there is one corresponding point in physical space. In the following we show that a structure comprising planar, homogeneous, surfaces that form the faces of the physical-space simplices and that refract according to a particular generalised law of refraction [13] which can be achieved with GCLAs [34] has the required properties.

As each physical-space simplex is a uniformly distorted version of the corresponding EM-space simplex, which represents a volume of homogeneous space, each physical-space simplex also represents a volume of homogeneous space. The light rays therefore travel in straight lines in each physical-space simplex, which we require as light rays travel in straight lines in the empty spaces between the sheets⁴. As the spaces on either side of each simplex face are homogeneous, the surfaces that represent the interface between these spaces also have to be homogeneous.

Note that two neighbouring simplices that are distorted in the same way require no light-ray-direction changing interface between them. Those combined simplices are not necessarily simplex-shaped; in the case of the pinch transformation shown in figure 3, for example, they are triangular prisms.

It is worth stressing that the transformations described here are continuous and piecewise affine. The pinch transformation in figure 1 is continuous as the space below the rectangle $A'B'C'D'$ is deformed such that it fills the triangle $A'D'E'$. In contrast, the original cloaking transformation [2] is not continuous as the entire edge of the central hole in physical space corresponds to the same point in EM space, and it

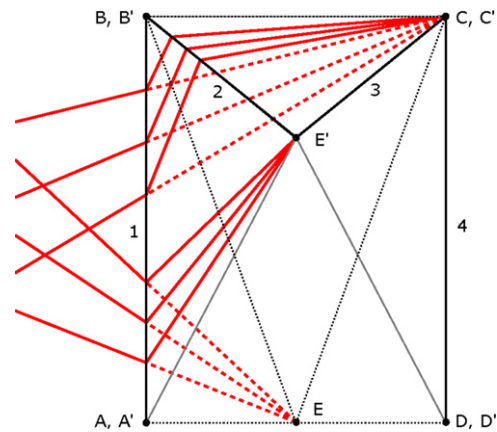


Figure 2. TO and imaging. The figure shows two light-ray bundles (solid red lines) incident (from the left) on a TO device realising the pinch transformation outlined in figure 1. The straight-line continuations (dashed red lines) of the rays before entering the device converge on two electromagnetic-space positions inside the device, C and E. The sheets re-direct the light rays such that the bundles converge at the corresponding physical-space positions, C' and E'.

is this discontinuity that leads to diverging material properties there [18].

In an effort to establish whether or not structures of GCLAs can form TO devices, we have previously established that the generalised law of refraction for GCLAs [34] is the same as that for the interface between spaces with different metrics [6] (see previous section). However, GCLAs could not realise *all* such metric interfaces, which poses the question ‘Are the metric interfaces that can be realised with GCLAs general enough to form TO devices?’ The answer comes from an unexpected—and unexpectedly simple—direction: imaging.

The mapping between EM space and physical space maps not only individual light-ray trajectories, but also *intersections* between light-ray trajectories, which means TO devices image between EM space and physical space. In our proposed realisation of TO devices, the imaging is performed by the surfaces that divide space (figure 2). The surfaces must image not only any point in the neighbouring empty segments, but anywhere in physical space; positions outside the adjacent segments are re-imaged by other surfaces.

Only planar surfaces can perform the required imaging [12], and in [13] we investigated the generalised law of refraction required by planar surfaces to perform the most general one-to-one imaging between the entire, homogeneous, object and image spaces⁵. Ideal GCLAs, comprising ideal thin lenses, change the direction of transmitted light precisely as required [13, 34]. In fact, they are the only known realisation of surfaces that change light-ray-direction in this way.

⁵ We do not necessarily require imaging between all of object space and all of image space here. However, the results in [13] are relevant here as imaging between subvolumes of EM space and physical space requires the same generalised law of refraction, but it does not need to be applicable over all possible incident and outgoing angles.

⁴ Note that there are also inhomogeneous spaces in which all light rays travel in straight lines; an example is the space resulting from the mapping performed by an idealised thin lens—another way in which our work is related to [33].

GCLAs image the plane of the GCLAs into itself, and they can image such that an arbitrary point outside this plane can be imaged to another arbitrary point outside this plane. The former means that the mapping between EM space and physical space is continuous⁶. The latter is used to calculate the parameters of the GCLAs (see appendix A).

5. Pinch-transformation window

In this section we provide an application for the pinch transformation of figure 1, namely a thermally insulating window. Windows are an important element of buildings, having both functional (sunlight and daylight, optimising solar gain, ventilation) and phenomenological (view, social connection, appearance) purposes. Glass and glazing is increasingly a key element of building facades, but in the context of climate change [37] there are increasing demands on the energy and carbon performance of buildings [38–40], which require improved thermal performance and reduced heat loss. Heat loss through windows is generally estimated to be around 10% of overall losses in housing, but has been shown to be as high as 40% for some building types [41]. However windows are currently the weak link in the thermal envelope. The performance requirements in current UK building standards [42, 43] are seven times lower for windows than for opaque elements (and in very-low-energy dwellings, the likely performance of even the very best available glazing (the U-value of triple glazed, argon-filled, windows with low-emissivity coatings is approx. $0.8 \text{ W m}^{-2} \text{ K}^{-1}$) will be eight times worse than the walls ($0.1 \text{ W m}^{-2} \text{ K}^{-1}$)). While standards for heat loss through walls can be easily increased through additional insulation, improving performance of windows has been much more challenging, with the focus on low emissivity coatings, multiple layers (double glazing currently being the norm, although now being superseded by triple glazing) and aerogel fills. There is therefore a clear need for a window which would significantly reduce heat loss while retaining the visual qualities of glazing. For our proposed window, the energy-saving merits also need to be evaluated, most likely against simpler windows that are also quadruple-glazed.

Our window design comprises a 3D version of the 2D pinch transformation (figure 1), realised with GCLAs, such that a wedge of insulating material in a standard (cuboid-shaped) window opening in a wall can be hidden. Ray-tracing simulations of our design are shown in figure 3(c), where the parameters have been calculated from imaging considerations (see figure 2). The simulation is idealised in a number of ways, neglecting, for example, field-of-view limitations and wave-optics effects such as diffraction (see appendix B for details).

The simulation shows that, within the limitations of our simulation, the concept of TO with GCLAs-type surfaces

works. Here, the wedge of opaque material is not visible, and in fact the view through the structure (used as a window) is the same as that through the wall opening without the wedge of opaque material and the surfaces.

However, the simulation also shows a significant reduction in brightness. This is due to the way in which the telescopes that make up GCLAs change the direction of transmitted light rays, namely by stretching or contracting the beam cross-section transversally, which in turn modifies the beam's Fourier spectrum, i.e. the directions of plane-wave components present in the beam. If the beam cross-section is stretched by a factor η such that $|\eta| > 1$, which happens on transmission through a telescope in which the lenses have the same size and shape and in a direction such that the lens with the shorter focal length is encountered first, only a fraction $1/|\eta|$ of the light incident is transmitted [15]. Inspection of the parameters of the pinch-transformation window shown in figure 3(c) reveals that for such a window to funnel light through a clear fraction f of the full window area results in an intensity transmission coefficient of f^2 .

The transmission coefficient can be increased from f^2 to f , at the expense of introducing an offset into the rays upon transmission through the window, by setting the horizontal beam expansion factor of all GCLAs in the window to 1. The GCLAs then do not consist of arrays of telescopes made from confocal lenses, but from confocal *cylindrical* lenses. The inner GCLAs are no longer imaging all of object space [13], not even integrally, and so the device is only an approximation to a TO device. The corresponding view is shown in figure 3(d). Nearby objects, such as the far edge of the window opening, are seen in a slightly shifted position, while more distant objects appear closer to their true position.

6. Discussion

In this paper, we have outlined a path towards TO devices that work for visible light on large size scales, and which can be manufactured using current fabrication methods. This opens up architectural uses, and we have presented an insulating window as an example.

It is worth stating that the lens cloak [33] could also be used to realise an energy-saving window that cloaks insulation material. This appears to be serendipitous, as the lens cloak was simply designed to be a combination of lenses that images every point back into itself, and this determines the mapping between EM space and physical space. It might well be possible to design TO devices from lenses that realise a given mapping between EM space and physical space (or at least a suitable approximation of this mapping), but at present it is not clear how to do this. In contrast, structures of GCLAs can, in principle, realise arbitrary piecewise affine mappings of a simplicial complex between EM space and physical space, and the simplicial approximation theorem [46] states that such mappings can approximate arbitrary continuous mappings. A larger number of simplices allows a better approximation, but because of the imperfections of GCLAs it

⁶ Conversely, a sheet that does not image the plane of the sheet into itself, which requires the sheet to offset light rays upon transmission, could in principle realise discontinuous mappings.

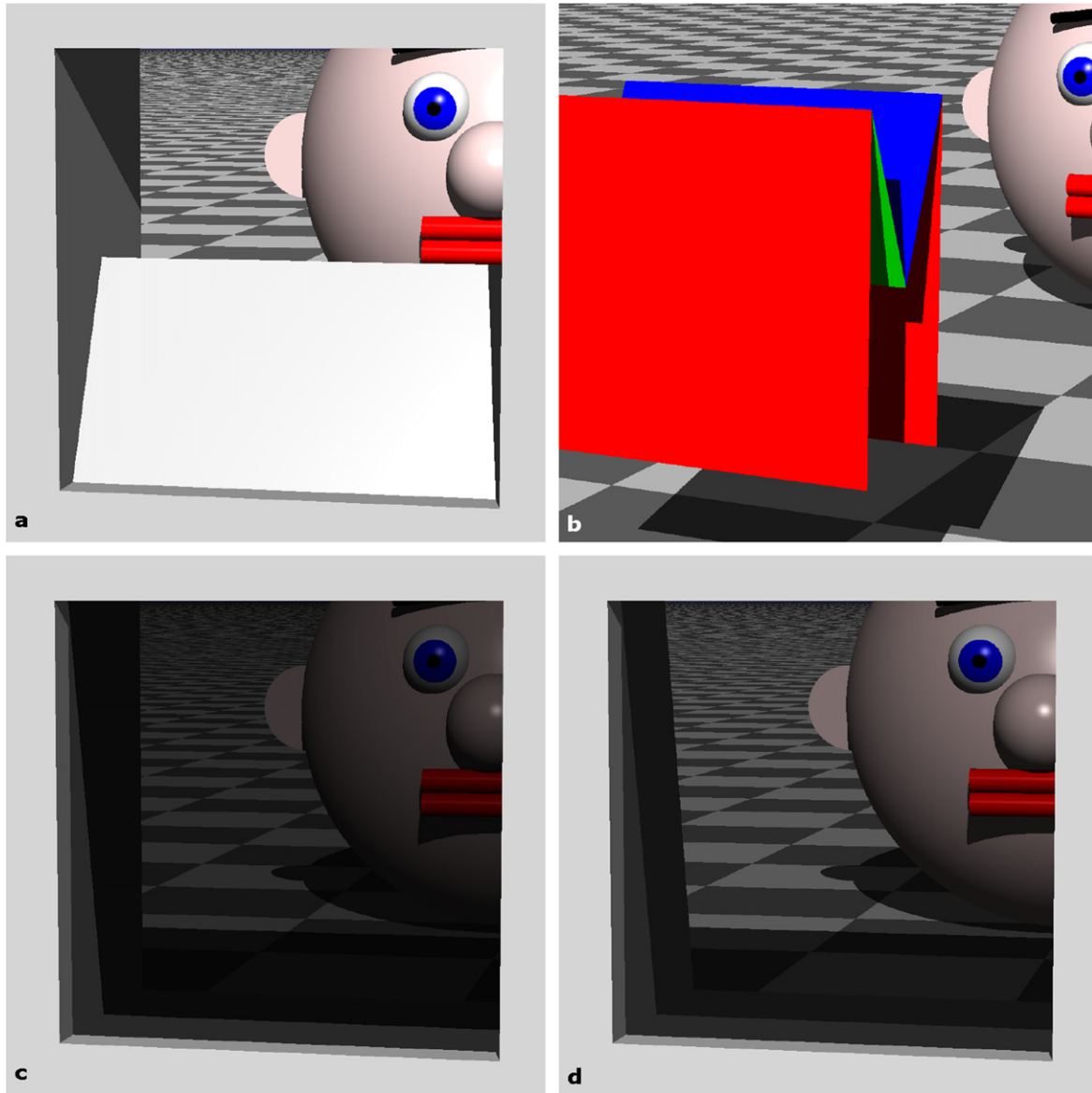


Figure 3. Ray-tracing simulation of pinch-transformation windows that cloak a wedge inside a standard, cuboid, window opening in a wall. The window opening with the uncloaked wedge is shown in (a). A fraction $f = 1/2$ of the window area is clear. The pinch-transformation window comprises four planar GCLAs-type surfaces, each shown as a coloured rectangle in (b). The view of the window opening in (a) with the pinch-transformation window present is shown in (c). The window redirects light rays around the wedge, such that they continue along their original straight-line trajectory behind the window. The wedge is invisible, but the window transmits only 25% ($=f^2$) of the intensity. The transmission coefficient can be increased to 50% ($=f$) by replacing all lenses in the GCLAs with horizontal *cylindrical* lenses of the same focal length, at the expense of introducing a slight offset into the transmitted light rays; the view through such an approximate pinch-transformation window is shown in (d). The simulation neglects field-of-view limitations and the pixellation of GCLAs. The images were created using the open-source, free, ray-tracer Dr TIM [44, 45]. More details on the simulation can be found in appendix B.

is normally advisable to minimise the number of surfaces light has to pass through.

The 3D pinch transformation is closely related to the transformation employed by the carpet cloak [47]. Both transformations are designed to make an indentation in one surface of the device appear planar when viewed through the device, with the aim of hiding the presence of the indentation and anything in it. In the case of the carpet cloak, the indented surface is additionally made to look like a planar mirror. It is tempting to think that this could also be achieved in our proposed windows by simply placing mirrors at the surfaces

corresponding to the lines $A'E'$ and $E'D'$, but theoretical considerations and ray-tracing simulations (not shown here) demonstrate that this is not generally the case.

Remarkably, many devices enabled by our proposed realisation of TO devices have no analog in traditional TO devices. An example is a ‘tardis window’, a stigmatic imaging device that scales all dimensions of the empty space seen through it by an equal factor. It is impossible to realise such a device, using TO or otherwise: due to a theorem about stigmatic imaging [48], this would require that the spatial region in the scaled space be filled with a material. However, it *is*

possible to realise an approximation of such a device using GCLAs, as the imaging by GCLAs is integral and therefore almost, but not quite, stigmatic. This might, for example, enable novel types of goggles for medical use.

One of the limitations is the finite field of view of the individual GCLAs, already mentioned above. Changing the geometry of the telescopelets, specifically making the GCLAs arrays of different parts of the lenses (not necessarily the centre), allows the centre of the field of view to be moved; in this way, it should be possible to align the fields of view of individual GCLAs such that whole devices work over some range of light angles. Whether or not this is sufficient for a particular application needs to be evaluated case by case.

It is worth noting that the value of the fraction f of the window area through which the pinch-transformation window funnels the light has a significant effect on the transmission coefficient of the window. Specifically, as f decreases, the transmission coefficient decreases, rapidly. The example in figure 3 is calculated for $f = 1/2$, resulting in intensity transmission coefficients $f^2 = 0.25$ and $f = 0.5$ for the non-approximate and approximate pinch-transformation window, respectively. For smaller fractions, e.g. $f = 1/4$, these transmission coefficients become 0.0625 and 0.25, respectively: the non-approximate pinch-transformation window then absorbs approx. 94% of the light. This high loss indicates a trade-off between the amount of insulation one is able to install, and the apparent brightness of the window. The possibility of improving the latter, e.g. through additional design modifications, deserves further investigation.

Detailed optical engineering, for which our proposal provides a significant motivation, will be able to improve GCLAs. Beyond such incremental improvements it is, perhaps, possible to develop radically improved alternatives to GCLAs. Obvious candidates for such sheets are very thin metamaterial structures ('metasurfaces') [49], but to date only much simpler laws of refraction have been achieved [50]. Ideally, those metasurfaces would not be pixellated, and thus perform stigmatic imaging instead of the integral imaging considered here. It is not known whether or not non-pixellated refraction according to the required more general laws of refraction is even possible; what is known is that it is impossible to refract certain incident light fields in this way as this would turn them into fields with a wave front that is discontinuous at all points and therefore wave-optically forbidden [8, 9].

It is worth noting that we calculate the parameters of our surfaces purely from simple imaging considerations, whereas the material parameters of other TO devices are usually calculated using differential geometry, the branch of mathematics dealing with curved spaces [51]. This beautiful but complicated mathematics still applies to our devices and makes them work, but does not need to be employed in the design of our devices.

Our ideas could perhaps be applied also to generalisations of TO devices, of which there are a number. Within optics, cloaking has been applied in space *and* time [52], and recently cloaking has been demonstrated in the diffuse limit [53]. Outside of optics, TO ideas have been applied to mould

heat flow ('transformation thermodynamics') [54, 55], acoustic waves [56], and earthquake waves ('seismic cloaking'), with the potential of directing those around critical areas as a form of earthquake protection [57–59]. When applied to seismic cloaking, for example, our ideas could potentially greatly simplify the protection of large areas by merely requiring the modification of thin volumes in the ground—two per GCLAs, each one being a phase hologram of a lens array for seismic waves; note that phase holograms can be much thinner than the wavelength they are designed for [50]—instead of modifying an entire volume. As our ideas are ray-optical in nature, one condition that would need to be satisfied for our ideas to be applicable is that the scale of the structure is much greater than the relevant wavelength. This can be the case for seismic cloaking, which deals with wavelengths of a few metres to 100s of metres [59] and typically very large structures.

We have argued here that structures of surfaces performing generalised refraction offer an interesting alternative to metamaterials in constructing TO devices, and we have suggested a viable realisation in terms of GCLAs, which are micro-structured transparent sheets which can, in principle, be manufactured inexpensively, using current technology, on the scale of tens of meters. In practice, our approach involves compromises, but in many ways these are complementary to the compromises involved in metamaterial TO. Our approach may therefore significantly extend the reach and practical application of TO.

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Appendix A. Calculation of the parameters of the GCLAs

To calculate the parameters of the GCLAs that form a TO device, we use the fact that each sheet has to be homogeneous and images, one-to-one, all object space into all image space and vice versa. Such sheets have recently been investigated theoretically [13], and for a sheet in the $z = 0$ plane the mapping between the object coordinates (x, y, z) and the image coordinates (x', y', z') was found to be of the form

$$x' = x - z\delta_x, \quad y' = y - z\delta_y, \quad z' = \eta z. \quad (\text{A.1})$$

The parameters δ_x , δ_y and η describe the sheet's generalised law of refraction, given by equations (15) and (16) in [13], which can be achieved with GCLAs. We can solve these equations for the parameters to obtain

$$\delta_x = \frac{x - x'}{z}, \quad \delta_y = \frac{y - y'}{z}, \quad \eta = \frac{z'}{z}. \quad (\text{A.2})$$

Note that a single pair of object and image coordinates with $z \neq 0$ completely determines all sheet parameters.

To calculate the parameters of the sheets, we identify suitable pairs of object and image positions for each sheet, and describe these in a Cartesian coordinate system such that the sheet itself lies in the $z = 0$ plane.

We use the fact that the sheets, individually or in combinations with other sheets, image the positions of vertices in EM space into the corresponding physical-space positions. For example, the outside sheet marked as '1' in figure 2 images the vertex E in EM space to the physical-space position E'. This determines the parameters of sheet 1. Sheet 1 also images the position of EM-space vertex position C to an intermediate position C₁, which sheet 2 images to the physical-space vertex position C' (figure 2). This determines the material parameters for sheet 2. The parameters for the remaining sheets can be calculated similarly.

Appendix B. Raytracing simulations of structures of idealised GCLAs

Our simulations are performed using our custom ray-tracing software Dr TIM [44, 45]. Dr TIM is open-source software [60], with the code for previous versions also available as supplementary information to [44, 45], which also contain information on simple source-code-modification tasks. Dr TIM is also freely available and interactive; a Java archive (JAR) of the software is available in the supplementary material, which also contains a section explaining how to repeat the simulations shown in this paper. The aim of this section is to explain the limitations of these simulations.

Following standard raytracing practice [61], Dr TIM creates visual images of a virtual scene by tracing the path of light rays backwards, starting from a virtual camera. Each backwards-traced light ray moves in the direction of a pixel of a virtual screen in front of the camera, then via the surfaces of any intersected scene objects towards a light source. The images created by Dr TIM are intended to be somewhat photo-realistic, but they are simplified in a number of ways. The most obvious simplification is the neglect of wave-optics effects such as diffraction, but the tracing of rays is also simplified.

The most substantial simplification that is relevant here is an idealisation of GCLAs, due to the fact that they can be implemented in different ways, which affects the optical properties. For example, the lenses can have different F numbers, which affects the ray offset, the field of view, and the size of aberrations; the lenses can have different designs, e.g. singlets or doublets, affecting chromatic aberrations and deviations from the law of refraction calculated in [34]; they can be arranged as hexagonal or square arrays (or in more unusual arrangements) with different shapes and amounts of 'dead area' around the lenses, which affects the size and shape of the field of view; there could be absorbing baffles between neighbouring telescopes, which determines what happens to light outside the field of view [14]; there could be field lenses inserted into each common focal plane, which again has an effect on the field of view and vignetting; and they could have different anti-reflection coatings. Clearly,

these limitations can have a very noticeable effect on the ray-tracing image, to the extent that they can affect the feasibility of a given device.

In order to avoid these numerous design choices, and for ease of implementation, but at the price of results which are unrealistic in some respects, we have chosen to simulate here idealised GCLAs which only suffer from a reduction in light intensity due to geometrical losses [15]. No other imperfections are simulated. Our idealised GCLAs redirect all transmitted light according to the law of refraction calculated in [34], without offsetting them. Note that the field-of-view limitations of simple CLAs without field lenses have previously been simulated [14] and calculated [15]. More realistic simulations of the view through individual GCLAs, which trace each ray through the detailed structure of GCLAs, can be found in [4].

Another noticeable simplification, standard practice in raytracing, concerns illumination. A perfect, non-absorbing, TO device should be invisible to the illumination light and therefore not throw any shadow. Dr TIM calculates only shadows due to objects blocking the direct line of sight to a point light source; it does not attempt to calculate the significantly more complex shadows due to devices that change light-ray trajectories. In the simulations shown in this paper, all sheets throw shadows.

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