Correction to 'Evaluation of normalisation methods for uniaxial bias extension tests on engineering fabrics'

Philip Harrison[1]* and Frank Härtel [2]

[1] University of Glasgow, Systems Power and Energy, School of Engineering, James Watt Building South, Glasgow G12 8QQ, UK

Tel.: +44 14 13 30 43 18. E-mail address: Philip.Harrison@glasgow.ac.uk

[2] Airbus DS GmbH, Claude-Dornier-Str., 88094 Immenstaad, Germany, frank.haertel@airbus.com

* Corresponding author

Abstract

This short communication is intended to correct certain erroneous conclusions drawn in the recently published paper by Härtel and Harrison (2014) [1]. The investigation of [1] was intended to evaluate the performance of normalisation methods for the uniaxial bias extension test. Predictions of two published theories for rate-independent behaviour were examined. New conclusions drawn in light of the error found in [1] are presented and corrected results are provided.

Keywords

A. Fabrics/textiles; B. Mechanical properties; D. Mechanical testing; E. Forming

1. Introduction

The aim of the current paper is to discuss an error and revise the conclusions of [1], in order to improve understanding on this topic. In [1] the performance of two independently published theories was evaluated. Three different hypothetical shear force versus shear angle behaviours were postulated and fed into the normalisation theories. The 'reasonableness' of the predictions was then assessed. It has since become apparent that the measure of accuracy used to decide which of the two theories was more accurate in [1] was incorrect. As a result, some of the conclusions of that investigation should be revised. The following correction is intended to be read in conjunction with the discussion of [1].

2. Evaluation Criterion

In [1], a criterion used to assess the accuracy of predictions of the theories was that;

(i) the relative contribution to the total force coming from Region B, relative to that produced by Region A (refer to Figure 1 of [1]), ***should not*** tend towards infinity as the shear angle in Region A approaches 90^{o}.

Assumption (i) misses the fact that purely mathematical shear force versus shear angle behaviours were used to evaluate the theories. Such mathematical functions are useful in testing the theoretical predictions, but it is a mistake to assume physically realistic behaviour when using such test functions. This realisation prompted an alternative mathematical analysis of statement (i), the analysis is presented in Section 2 and reveals that

(ii) the contribution to the total force coming from Region B, relative to that produced by Region A, ***should*** tend towards infinity as the shear angle in Region A approached 90^{o}.

This reversal in the evaluation criterion leads to a reversal of some of the main conclusions of [1]. In particular, it leads to the conclusion that instead of a problem with the normalisation theory of Method 2 of [1], there was instead a problem with the normalisation theory of Method 1 of [1]. Re-examination of the theoretical development of Method 1 revealed an error in the development of the final normalisation equation of Method 1 (originally presented in [2, 3]). The logic behind the theory is correct and all equations up to and including Eq (30) in [3] are without error. However, an algebraic mistake in deriving the final equation, Eq (31) of [3] has been found. In particular, the final normalisation equation given in [2, 3] as,

$$\psi(\theta) = c_A T_A = \frac{(\lambda-1)}{(2\lambda-3)} \frac{F_5 k_2}{L_5} - \frac{\psi(\theta/2)}{(2\lambda-3)} \left(\frac{1+cos\theta/2-sin\theta/2}{1+cos\theta-sin\theta} \right) \qquad \text{Eq (31) in [3]}$$

should have been written correctly as,

$$\psi(\theta) = c_A T_A = \frac{(\lambda-1)}{(2\lambda-3)} \frac{F_5 k_2}{L_5} - \frac{\psi(\theta/2)}{(2\lambda-3)} \frac{(sin\pi/4-sin\theta/4)}{(sin\pi/4-sin\theta/2)} \left(\frac{cos\theta/2}{cos\theta} \right) \qquad (1)$$

For definition of symbols in this equation see [1]. The error in Eq (31) of [3] was propagated into the investigation conducted in [1]. Eq (1) can now be written in a form suitable for comparison with Method 2 of [1] as,

$$F_{be}(\theta) = \frac{\sqrt{2}\cdot W}{(\lambda-1)} \left\{ \begin{matrix} F_{sh}(\theta)\cdot(2\lambda-3)\cdot cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right) + \\ F_{sh}(\theta/2)\frac{[sin(\pi/4-\theta/4)]}{[sin(\pi/4-\theta/2)]}\cdot cos\left(\frac{\pi}{4}-\frac{\theta}{4}\right) \end{matrix} \right\} \qquad (2)$$

it can be shown that this is exactly equivalent to Eq (6) in [1]. Consequently, the two normalisation methods (the corrected version of Method 1 given here and the unaltered version of Method 2 given in [1] and previously in e.g. [4, 5]), are equivalent and provide identical results. The consequence of this correction to the results of Figures 2, 3 and 5 in [1] are that the predictions of Method 1 are now exactly the same as those made by Method 2 in [1].

3. Mathematical Analysis of Power Generation in Test Specimen

The ratio between the power generated in Regions A and B of a uniaxial bias extension test is examined using an alternative approach (refer to Figure 1, repeated here from [3] for convenience). Considering the specific case when $\lambda = 2$ it is possible to determine the ratio of the power generated by Regions A and B (note that the power could be derived for arbitrary values of λ by appropriate modification of the area in Region A (see Figure 1), but for simplicity it is enough to restrict attention to $\lambda = 2$, the principle of the argument is the same). First consider the power generated by Region A, this can be written as,

$$P_A = L_A^2 h_o cos\theta\left[\tau_{ij}(\theta)D_{ji}(\theta)\right] \qquad (4)$$

where τ_{ij} is the stress associated with just shear deformation and neglects any contribution from stress acting along the fibre directions (the latter perform no work if inextensibility is assumed) and D_{ij} is the rate of deformation tensor and *L* is the side length of Region A. The term in square brackets in Eq (4) is the stress power per unit volume generated by the deformation while the term to the left of the square bracket in Eq (4) represents the current volume of material. In the principal system with zero fibre strain, if the thickness of the sheet remains constant with increasing shear angle, τ_{ij} and D_{ij} can be expressed as [6],

$$\tau_{ij} = \begin{bmatrix} \tau_{xx} & 0 \\ 0 & \tau_{yy} \end{bmatrix} \tag{5}$$

$$D_{ij} = \frac{\dot{\theta}}{2} \begin{bmatrix} \dfrac{1-\sin\theta}{\cos\theta} & 0 \\ 0 & \dfrac{-1-\sin\theta}{\cos\theta} \end{bmatrix} \tag{6}$$

and the stress can be directly related to the shear force per unit length using

$$\boldsymbol{\tau_{xx}} = \frac{\boldsymbol{F_s}}{\boldsymbol{h_o}} \left(\frac{\boldsymbol{\sin\theta - \cos^2\theta + 1}}{\boldsymbol{\cos\theta \cdot \sin\theta}} \right) \tag{7}$$

$$\tau_{yy} = -\frac{F_s}{h_o} \left(\frac{\sin\theta + \cos^2\theta - 1}{\cos\theta \cdot \sin\theta} \right) \tag{8}$$

Substituting Eq (7) and (8) into (5) and then (5) and (6) into Eq (4) produces

$$P_A = L^2 cos\theta F_s(\theta)\dot{\theta} \tag{9}$$

Using the usual assumption that the shear angle and the angular shear rate in Region B always remain half that in Region A, an equivalent expression can be found for Region B. The ratio of the power and therefore the ratio of the contributions to the axial force from Regions A and B, measured during a uniaxial bias extension test can then be written,

$$\frac{P_B}{P_A} = \frac{F_B}{F_A} = \frac{cos(\theta/2)}{2cos(\theta)} \frac{F_s(\theta/2)}{F_s(\theta)} \qquad (10)$$

The ratio between the two cos terms in Eq (10) naturally tends to infinity (due to the more rapidly decreasing area of Region A compared to Region B) and so the ratio of the power (and force) contributions from Region B and Region A, to the total measured force will also tend to infinity; the exact rate of this increase will be moderated by the form of the shear force curve, $F_s(\theta)$. This analysis confirms statement (ii) rather than statement (i).

4. Conclusions

The evaluation criterion used in [1] has been shown to be incorrect, leading to a reversal of some the conclusions stated in [1]. Namely, the difference between Methods 1 and 2 discussed in [1] should have suggested a problem with Method 1 not Method 2. The source of the problem was traced back to an algebraic error in the final step in the derivation of Eq (31) in [3]. The corrected version of this equation is given in Eq (1) of this communication and is expressed in terms of the measured axial force in Eq (2). The latter provides exactly the same solution as that given by the normalisation theory referred to as Method 2 in [1]. This finding provides strong mutual verification of the two normalisation theories; each theory was developed independently using different arguments, but ultimately produce identical results. Other conclusions in [1] remain valid.

References

[1] Härtel, F., and Harrison, P. Evaluation of normalisation methods for uniaxial bias extension tests on engineering fabrics. Composites Part A: Applied Science and Manufacturing, 2014, 67, 61-69

[2] Harrison P, Wiggers J, Long AC. Normalization of Shear Test Data for Rate Independent Compressible Fabrics, In: 10th International ESAFORM Conference on Materials Forming, 18–20th April, Zaragoza, Spain, 2007, 1011–1016

[3] Harrison P, Wiggers J, Long AC. Normalization of Shear Test Data for Rate-independent Compressible Fabrics. Journal of Composite Materials, 2008, 42, 2315-2344

[4] Cao J, Akkerman R, Boisse P, Chen J, Cheng HS, de Graaf EF, Gorczyca JL, Harrison P, Hivet G, Launay J, Lee W, Liu L, Lomov SV, Long AC, de Luycker E, Morestin F, Padvoiskis J, Peng XQ, Sherwood J, Stoilova TZ, Tao XM, Verpoest I, Willems A, Wiggers J, Yu TX, Zhu B. Characterization of mechanical behavior of woven fabrics: Experimental methods and benchmark results. Composites: Part A 2008; 39:1037–1053

[5] Hivet, G.; Duong, Ahn Vu, A contribution to the analysis of the intrinsic shear behaviour of fabrics, Journal of Composite Materials, 45 (6), 695-717, 2011

[6] Harrison, P, Yu, WR, Long, AC. Rate dependent modelling of the forming behaviour of viscous textile composites. Composites Part A, 2011, 42(11), 1719-1726