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A statistical-process based approach for modelling beach profile variability

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ABSTRACT

This paper presents a methodology for modelling medium term (annual to decadal) cross shore beach profile change and erosion. The statistical-process based approach (SPA) presented here combines detailed statistical modelling of offshore storm climate with a process based morphodynamic model (XBeach), to assess, and quantify morphodynamic variability of cross shore beach profiles. Until now, the use of process based models has been limited to simulations at storm event timescales. This methodology therefore represents the first application of a fully process based model in longer term simulations, as such, the approach requires simulation of post-storm beach profile recovery as well as individual event impacts. Narrabeen Beach, NSW, Australia was used as a case study for application of the technique due to the availability of an extensive set of storm and beach profile data. The results presented here demonstrate that the methodology produces encouraging results for determining medium term beach profile variability and erosion.

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1. Introduction

Increasing awareness of the importance of medium to long term morphological change to coastal sustainability has led to a requirement for methodologies to support predictions over these time scales. Linked to this is the rising prevalence of process based morphodynamic modelling. These two factors have resulted in a requirement for the application of process based models to be extended, to allow the assessment of beach change beyond short term time scales. As this has yet to be achieved, this paper discusses how a statistical framework (Callaghan et al., 2008) can be combined with the process based coastal morphodynamic model XBeach (Roelvink et al., 2009), to form a statistical-process based approach for forecasting cross shore, storm driven, beach change at a medium term time scale.

Quantifying beach morphodynamic variability using a benchmark 1 in N year event has inherent limitations. Hawkes et al. (2002) show that, for a forcing system with multiple variates, such as storm events, the return period of the individual variates does not necessarily match those of the system response. One such reason for the difference is that, during beach erosion, the formation of a new equilibrium profile requires a finite time, meaning erosion is dependent on duration (Kriebel and Dean, 1993). A benchmark event is also unable to account for two (or more) storms occurring in quick succession and effectively merging into one erosive event. Should this occur, there is greater erosive impact on the beach than if separated by a time sufficient enough to allow for natural recovery (accretion). To improve the representation of the forcing conditions, Callaghan et al. (2008) developed a statistical framework for modelling extreme storm climate and beach erosion, known as the Full Temporal Simulation (FTS). This model combines the multivariate statistical modelling of individual storm events with a non homogeneous Poisson process for modelling event spacing which allows for the prediction of a time series of storm (erosion) events and calm (accretion) periods, leading to a more realistic quantification of beach erosion. They combined this model with the empirical storm erosion model of Kriebel and Dean (1993) to determine beach erosion. The usefulness of this model was also demonstrated by Ranasinghe et al. (2011a) who combined it with a simplified empirical model (Larson et al. 2004) to estimate dune erosion at Narrabeen Beach over a 110 year period incorporating sea level rise. Both approaches are limited as the post-storm recovery of the profile was estimated using an empirical technique, as the structural functions used to determine erosion are not capable of predicting post-storm beach recovery.

Process based techniques for the modelling of cross shore beach behaviour have existed for some time with numerous models available (UNIBEST-TC (Reniers et al., 1995), CROSSMOR2000 (van Rijn, 1996) and SBEACH (Larson and Kraus, 1989)), van Rijn et al. (2003) provide a detailed review of the capabilities of these models to predict cross shore profile change. This study involved storm and seasonal time scales with some models shown to produce good representation.
of different profile features at the seasonal timescale. However, the sustainability of process based models for simulating beach change beyond a storm timescale has yet to be demonstrated.

XBeach, as a tool for modelling coastal change, has been extensively validated against numerous flume experiments (1D) and some field case studies (2DH) (Roelvink et al., 2009). The model has then been successfully applied to simulate storm response of sandy beaches at Assateague Island, Maryland (Roelvink et al., 2009), Santa Rosa Island, Florida (McCall et al., 2010) and Ostend Beach, Belgium (Bolle et al., 2010). More recently, the use of XBeach has been extended to the modelling of gravel beach variability (de Alegria-Arzaburu et al., 2010; Jamal et al., 2010; Williams et al., 2012). Until now its use has been curtailed at the storm event timescale (hours to days). Although XBeach has been validated and used extensively for erosive conditions, it has not been successfully validated or used to simulate post-storm beach accretion and recovery.

The aim of this paper is to expand on the studies of Callaghan et al. (2008) and Ranasinghe et al. (2011a), to overcome the limitations of their models as a result of using empirical/data driven approaches to determine storm erosion/post-storm recovery. Here we will attempt to simulate medium term beach change using a fully process based approach, by combining the FTS approach with the XBeach model, hereafter known as the Statistical-Process based Approach (SPA). To the authors’ knowledge, this is the first attempt of calibrating XBeach to simulate beach recovery and, use a fully process based model to simulate beach change at medium term timescales. The SPA will provide useful insights into current capabilities of XBeach at medium term beach modelling.

2. Field site

2.1. Narrabeen Beach

Narrabeen Beach is located approximately 20 km north of Sydney, NSW, Australia (Fig. 1). It is a 3.6 km long embayed beach that experiences semi diurnal, microtidal conditions with a mean spring tidal range of 1.25 m (Short, 1984). The region is subjected to highly variable, moderate to high energy incoming wave conditions as the wave climate is driven by a number of cyclonic sources, with storms reaching the beach throughout the year (Short, 2006; Short and Trenaman, 1992). The beach predominantly exhibits an intermediate state, but has been shown to frequently change between all states (Wright and Short, 1984).

The beach sediments are quartz and carbonate sands with median diameter ($D_{50}$) ranging from 0.25 to 0.50 mm (Wright and Short, 1984). The morphodynamic variability has been regularly and extensively monitored during the last few decades with beach profiles being surveyed at 5 locations (Fig. 1) along the beach by the Coastal Studies Unit, University of Sydney (Short and Trembanis, 2004). The beach profiles surveyed at profile 4, where long term longshore transport effects are minimal (Harley et al., 2011a; Ranasinghe et al., 2004), are used in the present study. Fig. 2 highlights the variability in profile 4 during the recording period.

2.2. Offshore wave data

Wave data collected between 1981 and 2005, offshore of Botany Bay (Fig. 1) at a water depth of 85 m, using a waverider buoy have
been used in this study. During the recording period the means \( H_s \) and \( T_s \) were approximately 1.5 m and 10 s respectively, with the overall wave climate being highly variable. Fig. 3 shows the plot of daily \( H_s \) during 1982 to provide an insight into typical wave conditions in the region. More information on the NSW wave climate can be found in Short and Trenaman (1992), Lord and Kulmar (2000), Kulmar et al. (2005), and Harley et al. (2010).

3. Statistical modelling

Statistical modelling of the storm climate at Narrabeen Beach follows the FTS procedure developed by Callaghan et al. (2008). An overview of the procedure used in the present study is given here as the approach adopted slightly differs from that of Callaghan et al. (2008). Here, pairs of peak significant wave height \( (H_{s,max}) \) of the storm events and the corresponding period \( (T_{s,max}) \) have been fitted rather than pairs of \( H_s \) and \( T_s \). The modified FTS therefore requires \( H_{s,max}, T_{s,max} \), Storm duration \( (D) \) of individual storms and spacing \( (S) \) between storms only.

Based on the analysis of the incident wave climate and erosion in the Narrabeen region, a correlation between large wave heights and erosion was evident. This led to the recording of storm events in the region being based around a wave threshold of 3.0 m (Kulmar et al., 2005), as this was shown to best correlate with observed erosion. Previous modelling studies at Narrabeen Beach (Callaghan et al., 2008; Harley et al., 2009; Ranasinghe et al., 2011a) have defined storm events when wave height exceeds this threshold. The study of Harley et al. (2009) used this threshold to model the response of Narrabeen Beach to storm events and found that using the 3.0 m value successfully captured the observed erosion.

The storms are extracted by clustering the wave data using the threshold wave height of 3.0 m; a criterion of 24 h between events to ensure independence; and minimum storm duration of 1 h. Following this approach, 539 storm events were identified for the 25 year wave record. Fig. 4(a) and (b) shows the details of actual storm events \( D \) vs. \( H_{s,max} \) and \( T_{s,max} \) vs. \( H_{s,max} \) respectively.

In order to determine a synthetic storm wave climate at Narrabeen Beach, the FTS procedure fits the Generalised Pareto Distribution (GPD), logistics distribution and a 3 parameter lognormal distribution to the storm events identified, following the procedure outlined by Coles (2001) and Callaghan et al. (2008). A Monte Carlo (MC) simulation using a Gibbs sampling technique (Geman and Geman, 1984) and Box–Muller method (Box and Muller, 1958) is then employed to generate a random synthetic time series of erosion and accretion periods, with parameter values attributed to \( H_{s,max}, T_{s,max} \) and \( D \) of storm events. For a full description of the FTS see Callaghan et al. (2008) and references therein. The procedure is summarised here for clarity:

1. Identify meteorologically independent storm events.
2. Fit the GPD to \( H_{s,max} \) and \( D \).
3. Fit the dependency (logistics) distribution between \( H_{s,max} \) and \( D \).
4. Fit the 3-parameter lognormal distribution to \( T_{s,max} \).
5. Fit a non-homogeneous Poisson process to \( S \).
6. Simulate the storm climate using the fitted distributions including storm spacing.

The number of random storm events required is dependent on the final use of the synthetic time series. This number has to be large enough to provide accurate estimation of the maximum return level of interest. Generation of more events than required will result in unnecessary computational time. According to Hawkes (2000), the maximum return period of interest requires a MC simulation size equal to the product of ten, the average number of events per year \( (N_y) \) and the return period \( (RP) \) (i.e. MC size = \( 10 \times N_y \times RP \)).

In order to demonstrate the SPA methodology presented in this paper, the maximum return period of interest was taken as 1 year. For 539 events over the 25 year period, \( N_y = 21.56 \); and with a maximum return period of 1 year this led to a random time series of 216 storm events (corresponding to 10 years) being generated. Fig. 4(c) and (d) gives plots of the randomly generated \( D \) vs. \( H_{s,max} \), \( T_{s,max} \) vs. \( H_{s,max} \) respectively. Comparison between the measured (Fig. 4(a) and (b)) and randomly generated events shows good correlation.

4. XBeach model

XBeach (Roelvink et al., 2009, 2010) is a 2DH coastal morphodynamic model developed to simulate dune erosion due to hurricane impacts based on the regimes outlined by Sallenger (2000). The model is based on the nonlinear shallow water equations and resolves nearshore hydrodynamics by employing a 2DH description of wave groups and infragravity motions. Wave group forcing is derived from a time varying wave action balance equation, which subsequently drives the infragravity motions and longshore and cross shore currents. The Eulerian flow velocities \( (u^E) \) determined by the model governing equations are used to force the sediment transport module.

The Lagrangian flow velocities \( (u^L) \) are determined from the shallow water equations and account for the wave induced mass flux and the subsequent return flows (Eqs. (1) to (3)). They are related to \( u^E \) by the Stokes drift \( (u^S) \) (Eqs. (4) and (5)).

\[
\frac{\partial u^L}{\partial t} + u^L \frac{\partial u^L}{\partial x} + v^L \frac{\partial u^L}{\partial y} - f v^L - v_h \left( \frac{\partial^2 u^L}{\partial x^2} + \frac{\partial^2 u^L}{\partial y^2} \right) - \frac{\tau_{sx}}{\rho h} - \frac{\tau_{tx}}{\rho h} - g \frac{\partial h}{\rho h} + F_x
\]
\[ \frac{\partial v^l}{\partial t} + u \frac{\partial v^l}{\partial x} + v \frac{\partial v^l}{\partial y} - f^l v^l - \nu_h \left( \frac{\partial^2 v^l}{\partial x^2} + \frac{\partial^2 v^l}{\partial y^2} \right) = \frac{\tau_{xy}}{\rho h} \frac{\tau_{by}}{\rho h} - \frac{g}{\rho h} \frac{\partial \eta}{\partial y} + \frac{F_y}{\rho h} \]

(2)

\[ \frac{\partial \eta}{\partial t} + \frac{\partial u^l}{\partial x} + \frac{\partial v^l}{\partial y} = 0 \]

(3)

\[ u^f = u^l - u^d \text{ and } v^f = v^l - v^d \]

(4)

\[ u^d = E_w \cos \theta \frac{\partial h}{\partial x} \text{ and } v^d = E_w \sin \theta \frac{\partial h}{\partial y} \]

(5)

where \( \tau \) is the bed shear stress; \( \eta \) is the water level; \( F \) is the wave-induced radiation stress; \( \nu_h \) is the horizontal viscosity; and \( f \) is the Coriolis coefficient.

The sediment transport module uses a depth averaged advection-diffusion equation (Galappatti and Vreugdenhil, 1985) to determine sediment concentration \( C_s \), using an equilibrium concentration \( C_{eq} \) as a source term (Eq. (6)). The sediment transport gradients \( q_x \) and \( q_y \) are determined from \( u^f, C_s \) and the diffusion coefficient \( (D_h) \) (Eqs. (7) and (8)).

\[ \frac{\partial h C_s^l}{\partial t} + \frac{\partial h C_s u^f}{\partial x} + \frac{\partial h C_s v^f}{\partial y} + \frac{\partial}{\partial x} \left( D_h \frac{\partial C_s}{\partial x} \right) + \frac{\partial}{\partial y} \left( D_h \frac{\partial C_s}{\partial y} \right) = \frac{h C_{eq} - h C_s}{T_s} \]

(6)

\[ q_x(x, y, t) = \left[ \frac{\partial h C_s u^f}{\partial x} + \frac{\partial}{\partial x} \left( D_h \frac{\partial C_s}{\partial x} \right) \right] \]

(7)

\[ q_y(x, y, t) = \left[ \frac{\partial h C_s v^f}{\partial y} + \frac{\partial}{\partial y} \left( D_h \frac{\partial C_s}{\partial y} \right) \right] \]

(8)

\[ \frac{\partial q_x}{\partial t} + \text{ morfac } \frac{\partial q_x}{\partial x} + \text{ morfac } \frac{\partial q_y}{\partial y} = 0 \]

(9)

\[ \frac{\partial q_y}{\partial t} + \text{ morfac } \frac{\partial q_x}{\partial x} + \text{ morfac } \frac{\partial q_y}{\partial y} = 0 \]

(10)

\[ C_{eq} \] is determined from, either, the Soulsby–van Rijn formula (Soulsby, 1997) or the van Thiel–van Rijn formulae (van Rijn, 2007a, 2007b; van Thiel de Vries, 2009) with the change in bed level computed from the sediment transport gradients \( (q_x \text{ and } q_y) \) and avalanching mechanism when a critical bed slope is exceeded (Eqs. (9) and (10)). For storm simulations the Soulsby–van Rijn (SVR) transport equation was used to determine \( C_{eq} \) (Eqs. (11) to (14)) with the van Thiel–van Rijn (vTvR) used for the recovery simulations (Eqs. (15) to (17)).

\[ C_{eq} = \frac{A_1}{\tau} \left( \left( \frac{|u|^f |u|^d}{\rho h} \right) - u^d \right) \]

(11)

\[ A_1 = A_{ib} + A_{is} \]

(12)

\[ A_{ib} = \frac{0.005 h (D_{50}/h)^{1.2}}{[s-1]g D_{50}} \]

(13)

\[ A_{is} = \frac{0.012 D_{50} D_{-0.6}}{[s-1]g D_{50}} \]

(14)

\[ C_{eq} = \frac{A_1}{\tau} \left( \sqrt{(u^f)^2 + 0.64 u^2_{rms} - u^d} \right) \]

(15)

\[ A_{ib} = \frac{0.015 h (d_{50}/h)^{1.2}}{[s-1]g d_{50}^{0.75}} \]

(16)

\[ A_{is} = \frac{0.012 d_{50} D_{-0.6}}{[s-1]g d_{50}} \]

(17)

Fig. 4. Determined storm events between 1981 and 2005. D vs. \( H_{s,max} \) (a) and \( T_{s,max} \) vs. \( H_{s,max} \) (b). Simulated random 10 year storm climate D vs. \( H_{s,max} \) (c) and \( T_{s,max} \) vs. \( H_{s,max} \) (d).
A detailed description of the XBeach model is given in Roelvink et al. (2009), Roelvink et al. (2010) and references therein.

5. Modelling storm induced erosion and post-storm recovery

The calibration and validation of XBeach for simulating storm induced beach erosion and post-storm recovery are presented below. The results from this section demonstrate the model setups required for use within the SPA.

5.1. Storm erosion modelling

Calibration and validation of XBeach were achieved by modelling a variety of observed storm events that occurred during the measurement period. It is essential that storm events, which accompany measured pre- and post-storm profiles, are used for calibration. It is also essential that only one storm occurred between the selected profiles. Due to the high occurrence frequency of storms in the Narrabeen region (one approximately every two weeks) and the fact that profile measurements had been carried out only once a month, storm events that satisfy both requirements are sparse. Analysing the 25 year storm and beach profile survey records, four storm events with varying $H_{s\text{max}}$ and $D$ were selected for calibration. The selected storms are shown in Table 1.

As the available wave data are non directional, wave approach was assumed to be orthogonal to the shoreline with one directional bin. Hourly $H_{s}$ values along with the energy spectrum were determined for each storm event, with the offshore wave boundary condition defined in XBeach each hour using the JONSWAP spectrum. Using 1 h duration JONSWAP spectra best encapsulate the actual storm profile.

To produce a computationally efficient setup and ensure that the processes are represented as accurately as possible, variable grid spacing was applied to the simulations. The grid spacing was defined as approximating a minimum short wave period ($T_{\text{min}}$) of 5 s, keeping at least 12 grid points per wave group length and assigning a minimum grid spacing of 2 m (the recommended minimum value). This resulted in 617 grid points across the model domain, with a maximum spacing of 13 m near the offshore boundary varying down to a minimum value of 2 m nearshore. For all simulations, an average $D_{50}$ value of 0.37 mm has been implemented (Wright and Short, 1984).

The accuracy of model simulations is assessed using a Brier Skill Score (BSS), which has also become a common practice within coastal numerical modelling (Pedrozo-Acuna et al., 2006; Roelvink et al., 2009; Sutherland et al., 2004; van Rijn et al., 2003; Williams et al., 2012). The BSS for comparing measured and simulated profiles is given in Eq. (18).

$$\text{BSS} = 1 - \left( \frac{\sum (x_p - x_m)^2}{\sum (x_b - x_m)^2} \right)$$

where $x_p$ is the predicted profile from XBeach; $x_m$ is the measured profile (post-storm) and $x_b$ is the initial (pre-storm) profile. The BSS classification given by van Rijn et al. (2003) states that BSS = 0, ‘bad’; 0–0.3, ‘poor’; 0.3–0.6, ‘reasonable/fair’; 0.6–0.8, ‘good’; and 0.8–1.0, ‘excellent’.

As the SVR formula was developed under bed and suspended load conditions, it is not strictly valid for high velocity, sheet flow situations such as those encountered in this application. To overcome this, XBeach allows for a threshold velocity condition to be enforced reducing the stirring velocity ($u^2_{\text{stirring}}$) during sheet flow situations (Eq. (16)). Under waves and currents, sheet flow situations occur when the Shields parameter ($\theta$) exceeds 0.8 (Sousbby, 1997) with a suggested limiting maximum value ($\theta_{\text{max}}$) of 0.8 to 1.0 (Roelvink et al., 2010). The $u^2_{\text{stirring}}$ limitation is enforced by defining $\theta_{\text{max}}$ (Eq. (17)). McCall et al. (2010) set $\theta_{\text{max}} = 1.0$ for modelling hurricane impact at Santa Rosa Island, FL, USA, and investigated the effects of a range of $\theta_{\text{max}}$ (0.8–1.2) on model results. As the mean $D_{50}$ at Narrabeen Beach (0.37 mm) is greater than that at Santa Rosa (0.20 mm), the influence of sheet flow will be less and the requirement for a limiting $\theta_{\text{max}}$ will be lower. However, as $\theta_{\text{max}}$ is necessary for using the SVR formula in high velocity situations, it was decided to use $\theta_{\text{max}} = 1.0$ in the present study.

$$u^2_{\text{stirring}} = \left\{ \begin{array}{ll} \frac{|u|}{\theta_{\text{max}}} & \text{if } \theta_{\text{max}} > \theta > \theta_{\text{max}} \\ \frac{0.018u_{\text{rms}}^2}{c_f} & \text{if } \theta \leq \theta_{\text{max}} \end{array} \right.$$  

(19) where $\theta$ is the Shields parameter; $\theta_{\text{max}}$ is the maximum Shields parameter (start of sheet flow); $\Delta$ is the relative density of sediment; $c_f$ is the flow friction coefficient; and $C_e$ is the drag coefficient.

Invoking $\theta_{\text{max}}$ reduces $u^2_{\text{stirring}}$, which in turn reduces $C_e$ and therefore limits the volume of sediment carried by the water column. This will reduce the sediment transport rates, and the overall erosion of the beach. In order to further improve model outputs, three parameters that influence model simulations (Chézy coefficient, $C$; permeability coefficient, $k$; and wet cell gradient prior to avalanching, $wetSP$) were varied in an attempt to calibrate the model against the measured post-storm profiles.

XBeach uses the Chézy bed friction relationship in the flow module. The relationship between flow friction coefficient ($C_f$) and $C$ (Eq. (21)) means a reduction in $C$ or an increase in $C_f$ will result in a decrease in erosion, by reducing flow velocities. The default values of $C_f$ and $C$ are set in XBeach as 0.003 and 55 respectively.

$$C_f = \frac{C}{C^2}$$

(21)

When $\theta_{\text{max}}$ is implemented, in addition to the decrease in flow velocities, reducing $C$ will reduce $u^2_{\text{stirring}}$ and $C_{eq}$. This further limits the volume of sediment that can be transported in the water column, reducing the sediment transport rates and the overall erosion. As

### Table 1

<table>
<thead>
<tr>
<th>Storm</th>
<th>Profile dates</th>
<th>Storm dates</th>
<th>$H_{\text{max}}$ date</th>
<th>$H_{\text{max}}$ (m)</th>
<th>$D$ (hrs)</th>
<th>$T_p$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Start – 31/05/83</td>
<td>Start – 04/06/83</td>
<td>04/06/83</td>
<td>3.89</td>
<td>77</td>
<td>12.4</td>
</tr>
<tr>
<td>4</td>
<td>Start – 14/10/94</td>
<td>Start – 20/10/94</td>
<td>20/10/94</td>
<td>5.33</td>
<td>22</td>
<td>9.85</td>
</tr>
</tbody>
</table>
Table 2

Results for XBeach calibration of storm events.

<table>
<thead>
<tr>
<th>Storm</th>
<th>BSS</th>
<th>Vol. Err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.91</td>
<td>+6%</td>
</tr>
<tr>
<td>2</td>
<td>0.52</td>
<td>+9%</td>
</tr>
<tr>
<td>3</td>
<td>0.81</td>
<td>−1%</td>
</tr>
<tr>
<td>4</td>
<td>0.78</td>
<td>−1%</td>
</tr>
</tbody>
</table>

sediments at Narrabeen Beach are medium to coarse sands it is possible that the C value may be less than the default value of 55. Sensitivity tests were carried out to determine appropriate C and γ values for the model setup. The XBeach model includes a basic groundwater module in order to simulate infiltration and exfiltration to and from the beach. This module utilises the principle of Darcy flow and includes vertical interaction between surface water and groundwater. The infiltration and exfiltration to and from the beach is defined by a flow velocity (w) and is positive from surface water to the groundwater. When the ground water level (zgw) is greater than the bed level (zb), exfiltration takes place and the volume of groundwater joins the surface water within the same numerical time step (Eq. (22)). Infiltration of surface water into the beach takes place when the groundwater level is less than the bed level and is determined using the Darcy flow relationship (Eq. (23)).

\[
\begin{align*}
\frac{w_i^j - z_{gw}^j}{\Delta t} & = k \frac{\nabla dz}{\Delta t} \text{por} \quad \text{(22)} \\
\frac{w}{\Delta z} & = -k_e \left( \frac{dp}{\Delta z} + 1 \right) \quad \text{(23)}
\end{align*}
\]

The k values tested were determined using the average grain diameter (0.37 mm), a kinematic viscosity at 20 °C and 35 ppt salinity (\(\nu = 1.05 \times 10^{-6}\); average conditions at Narrabeen Beach) and a range of porosities suggested for sandy beaches (Soulsby, 1997). All simulations have no geometric variation in k (i.e. \(k_e = k_y = k_z\)) and the groundwater level remained constant at MSL.

The parameters discussed above were tested systematically in order to determine the most appropriate model setup. The best setup was taken as the one that produced the highest average BSS and lowest average volumetric error across all storm events. Table 2 and Fig. 5 compares measured and simulated post-storm beach profiles for the individual storm events given in Table 1. The results in Table 1 show that the model setup has resulted in BSS ranging from 0.91 ('excellent') to 0.52 ('reasonable/fair'). These give an average BSS and volumetric error of 0.76 ('good') and +3% respectively. The parameters that produce these results are provided in Table 3. A C value of 45 gives a cf value of 0.005 corresponding to a rippled sandy bed (Soulsby, 1997) and can therefore be considered valid for Narrabeen Beach based on the studies of Short (1984) who shows that the bed across the entire nearshore region at Narrabeen Beach is extensively rippled during median wave conditions.

Storm 2 produces the lowest BSS. However, it can be seen from Fig. 5(b) that the measured post-storm profile after storm 2 contains a nearshore bar, which indicates that the profile has not been measured at the end of the storm but during an intermediate state of recovery following the storm event. The post-storm profile in this case was measured 14 days after the end of the storm event allowing sufficient time for this bar to form. If the profile measurement was taken closer to the end of the storm event the BSS may be considerably higher as the beach would be exhibiting a dissipative state with a wide planar surf zone as in storm 1. Although it is shown in Table 1 that the post-storm profile for storm 3 was measured 26 days after the event, as the profile is curtailed at approximately MSL (+0.02 m AHD) there is no evidence of nearshore morphology meaning the BSS is higher. Given that the average BSS for all storms is 0.76, which results in a 'good' BSS rating and an excellent average volumetric error of +3%, it can be concluded that the XBeach model has been effectively calibrated for modelling storm erosion at Narrabeen Beach.

![Fig. 5](image-url)

Fig. 5. Comparison of measured and simulated storm profiles for a range of storms at Narrabeen Beach.
5.2. Post-storm recovery modelling

As with the storm erosion recovery calibration, it is essential to identify extended calm periods between two consecutive profile measurements, where no storm events had occurred, for calibration of the model for beach recovery. This led to two periods being selected from the measure data (Table 4). In Callaghan et al. (2008) a recovery time of 400 h is set to determine full recovery. This translates to approximately 17 days and is slightly higher than the average actual spacing between events (16 days) determined from the measured data. The fact that the periods chosen for calibration have durations (29 and 20 days) greater than the average spacing and the full recovery threshold of Callaghan et al. (2008), the transformation towards a reflective beach state is expected.

Simultaneous analysis of the wave climate during the calm periods between storms shows that they satisfy the calm wave criteria with all measured wave heights being below the 3.0 m threshold. Details of the wave conditions during the recovery periods (average $H_s$ and $T_s$), are provided in Table 4. Similar to storm erosion modelling, all beach recovery simulations were forced using the JONSWAP spectra that represent the measured wave conditions during the periods.

The sediment transport rate ($q_t$) in XBeach (Eq. (22)) is determined using a representative velocity ($u_{repr}$); the sum of the current flow velocity ($u^c$) and an advection velocity ($u^a$) from wave skewness and asymmetry (Eqs. (23) and (24)). A strong asymmetric wave motion leads to an increase in shear stress imparted on the bed (Walstra et al., 2007). This leads to greater sediment mobilisation, favouring onshore sediment flux. In addition, high crest velocities in the onshore direction, attributed to skewed waves in the shoaling zone, mobilise and transport more sediment than the wave troughs (directed offshore) further increasing net onshore transport of sediment (Grasso et al., 2011).

$$q_t = C_s u_{repr} - D_h \frac{\partial c}{\partial x} - 1.6 C_s \nu_{magp} \frac{\partial z}{\partial x}$$

(24)

where $C_s$ is the sediment concentration, $u_{repr}$ is the Eulerian transport velocity, $D_h$ is the sediment diffusion coefficient, $h$ is the water depth and $\nu_{magp}$ is the Lagrangian transport velocity.

$$u_{repr} = u^c + u^a$$

(25)

$$u^a = (faSk \times Sk - facAs \times As)u_{ms}$$

(26)

The factors applied to skewness ($faSk$) and asymmetry ($facAs$) determine the magnitude and direction of net sediment transport. The values selected for these factors therefore determine the predominant sediment transport direction. The permeability of the beach also plays a significant role in beach formation during the accretion phase (Jensen et al., 2009). For this reason, the groundwater flow module was activated for all post-storm recovery simulations with the permeability of the beach taken from the storm erosion tests ($0.0031 \text{ m/s}$).

In order to calibrate XBeach for post-storm recovery conditions at Narrabeen Beach, a set of sensitivity tests on $facAs$ and $facSk$ was carried out to determine suitable values.

To accurately replicate the upper beach berm formation during the recovery conditions it is important to consider the tidal variation at Narrabeen Beach. The tidal variation controls the maximum level of wave run up and thus the maximum level to which sediment can be transported onshore. Manly Hydraulics Laboratory (MHL) provided data of tidal variation for the Sydney region. These were averaged over 19 years (1990 to 2010) and are provided in Table 5. The sensitivity testing was carried out using simplified semidiurnal mean, spring and high tidal cycles. The high tidal cycle corresponds to a variation between High Spring Water Solstice and Indian Low Water Springs.

Unlike in storm conditions, it is less likely that sheet flow conditions will occur due to the smaller incident wave heights. For this reason the $\theta_{max}$ criterion was not implemented in the recovery simulations.

To make the recovery simulations, and the overall SPA methodology computationally feasible a morphological acceleration factor ($morfac$) (Ranasinghe et al., 2011b; Roelvink, 2006; Splinter et al., 2011; Vousdoukas et al., 2012) of ten was used for all recovery simulations.

The sensitivity testing was carried out systematically with the model setup producing the highest BSS without tidal variation and then, with tidal variation determined. As earlier, BSS and volumetric errors between measured and simulated profiles were used to assess model performance. Table 6 shows results from the sensitivity tests and Table 7 provides the parameter setup. Fig. 6 compares measured recovery profiles with simulated profiles, for both recovery periods. From these results it can be seen that there is little difference in the average BSS volumetric errors for each tidal range. All tidal variations give ‘good’ average BSS (0.74, 0.76 and 0.75) and volumetric errors (−4%).

The results provided in Table 6 are very encouraging and show that, by accounting for the processes that govern accretion, XBeach can be calibrated to produce ‘good’ predictions of post-storm beach accretion.

5.3. Simulation of annual beach change

In order to assess the validity of combining the XBeach storm erosion and post-storm recovery model setups, beach profile change that occurred from 25/08/81 and 16/08/82 (ca. one year) at Narrabeen Beach was simulated and compared with measured data (Fig. 7). Between the first and last profile measurements during this period, 19 storm events were recorded. All beach recovery simulations were implemented using a spring tidal variation.

To assess the accuracy of the procedure in predicting the annual time series, the relative mean absolute errors (RMAE) in the subaerial

Table 3

Calibrated parameters for modelling storm induced erosion.

<table>
<thead>
<tr>
<th>Parameter description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limiting Shields parameter ($\theta_{max}$)</td>
<td>1.0</td>
</tr>
<tr>
<td>Chézy coefficient (C)</td>
<td>45</td>
</tr>
<tr>
<td>Coeff. permeability ($k$)</td>
<td>0.0031 m/s</td>
</tr>
<tr>
<td>Wet cell max gradient (wetslp)</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Table 4

Details of recovery periods used for XBeach calibration.

<table>
<thead>
<tr>
<th>Recovery</th>
<th>Profile dates</th>
<th>$H_{mean}$ (m)</th>
<th>$D$ (days)</th>
<th>$T_s$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25/08/81–23/09/81</td>
<td>1.16</td>
<td>29</td>
<td>9.5</td>
</tr>
<tr>
<td>2</td>
<td>25/07/82–16/08/82</td>
<td>1.11</td>
<td>20</td>
<td>9.5</td>
</tr>
</tbody>
</table>

Table 5

Tidal variations for Sydney region.

<table>
<thead>
<tr>
<th>Tide</th>
<th>Low level (m)</th>
<th>High level (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>−0.484</td>
<td>0.524</td>
</tr>
<tr>
<td>Spring</td>
<td>−0.607</td>
<td>0.647</td>
</tr>
<tr>
<td>High</td>
<td>−0.856</td>
<td>0.995</td>
</tr>
</tbody>
</table>

Table 6

Results for XBeach calibration of recovery periods.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>0.84</td>
<td>−2%</td>
</tr>
<tr>
<td></td>
<td>Spring</td>
<td>0.86</td>
<td>−2%</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>0.89</td>
<td>−2%</td>
</tr>
<tr>
<td>2</td>
<td>Mean</td>
<td>0.63</td>
<td>−6%</td>
</tr>
<tr>
<td>2</td>
<td>Spring</td>
<td>0.65</td>
<td>−6%</td>
</tr>
<tr>
<td>2</td>
<td>High</td>
<td>0.61</td>
<td>−6%</td>
</tr>
</tbody>
</table>
beach volume, the position of the 0 m and 2 m beach contours and the profile envelopes were determined. The position of the beach contours is referenced seaward from the top of the dune (+10 m AHD). The RMAE calculations were carried out in line with van Rijn et al. (2003) and changed into percentages for analysis (Eq. (27)). The results are given in Table 7, Figs. 8(a), 9(a) and 10(a).

\[
\text{RMAE} = \frac{\left| x_p - x_m \right|}{x_m} \times 100\%
\]

These results show that the combined model reproduces the volumetric beach change and position of the 0 m and 2 m contours with RMAE of 7%, 13% and 13% respectively. The maximum, minimum and mean profile errors are 5%, 11% and 9% respectively.

The difference between simulated and measured annual profile change may be partly attributed to the simplified approach of using a predefined storm threshold and switching between erosion and accretion mechanisms according to this threshold. This does not allow for the bar dynamics and intermediate beach states, associated with the gradual recovery of the beach, to be captured. This may potentially imply the overestimation of beach volume and the position of the 0 m and 2 m contours evident in Figs. 8(a) and 9(a). However, as the aim of this paper is to demonstrate the first attempt of using a process model for medium term beach change, we adopted this simple approach.

Although a reasonable degree of success is achieved from combining model setups separately calibrated for individual erosion and recovery events, an additional recovery model setup was calibrated using the entire annual time series, to provide a comparison. This procedure will allow for a better representation of the predominant beach state, rather than focusing on a reflective state during the calm periods between storm events.

Calibration was conducted on the same parameters (facSk and facAs) to determine which combination produced the best representation of the overall annual variability. The results are provided alongside the individual event calibrations in Table 7, Figs. 8(b), 9(b) and 10(b).

These results show that, when calibrating the recovery model over an annual time period, the RMAE of the volumetric change and position of the 0 m and 2 m contours reduced to 4%, 9% and 13% respectively. Additionally, the maximum, minimum and mean profile errors change to 4%, 13% and 5% respectively. It is evident, that although the majority of the RMAEs reduce, there is an increase in the error associated with the minimum profile envelope. This increase is due to a greater level of recession now occurring. This highlights the concern raised previously that not including the reflective state in the recovery simulations will

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Table 7
Calibrated parameters for modelling post-storm recovery for individual events and over the annual period.

<table>
<thead>
<tr>
<th>Parameter description</th>
<th>Event value</th>
<th>Annual value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Limiting Shields parameter ((\theta_{\text{max}}))</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>Factor on skewness (facSk)</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Factor on asymmetry (facAs)</td>
<td>1.0</td>
<td>0.8</td>
</tr>
<tr>
<td>Coeff. permeability ((k))</td>
<td>0.0031 m/s</td>
<td>0.0031 m/s</td>
</tr>
<tr>
<td>Tidal variation</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Accel. Factor (morfac)</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

---

Fig. 6. Comparison of measured and simulated beach profiles of post-storm recovery.

Fig. 7. Wave measurements and storm events from 25/08/81 to 16/08/82.

Fig. 8. Comparison of simulated and measured beach volumes from 25/08/81 to 16/08/82. Individual event calibration (a) and annual series calibration (b).
limit the accretion and result in continual recession during longer simulations.

5.4. Medium term erosion modelling

The verification of XBeach for single event and annual timescales justifies its use within the longer term SPA methodology. To demonstrate the SPA, beach variability from a random 10 year synthetic storm climate, derived from the FTS, was simulated and the results compared with measured data. The initial beach profile for model simulation was taken as the average profile determined from the beach profile surveys.

As Narrabeen Beach is subjected to a multidirectional wave climate, the inclusion of wave direction within a longer term simulation is important. Callaghan et al. (2008) showed that there is no correlation between $H_{\text{max}}$ and wave direction ($\theta$), meaning that $\theta$ can be assigned to the synthetic storms from empirical data. Therefore, to provide a more complete realisation of the Narrabeen storm climate, each random event was assigned an empirical direction from storm events recorded between 1992 and 2009 by Manly Hydraulics Laboratory.

From the rule of thumb approach of Hawkes (2000), a 10 year simulation should produce an accurate 1 year erosion return level. In an attempt to quantify this, the erosion return levels from the SPA were determined and compared to those from the measured data. The erosion return levels from the measured profile data were determined using the four methods described by Callaghan et al. (2008), i.e. (a) Block averaging procedure; (b) Consecutive volumes with no correction; (c) Consecutive volumes corrected for the number of storms; and (d) Consecutive volumes corrected for the number of effective storms. In line with the studies of Hoffman and Hibbert (1987); Callaghan et al. (2008) and Ranasinghe et al. (2011a), beach erosion volume above 2 m contour is used for comparison.

Fig. 11 compares return levels of measured and SPA simulated beach erosion. It should be noted that the divergence of the simulated erosion after the 1–2 year return period is a result of only 10 years of simulations being used to determine the return levels.

From Fig. 11 it is clear that, when using the event calibrated recovery model, beach erosion is consistently overestimated. The annually calibrated recovery model gives better results due to its ability to develop an intertidal berm. Harley et al. (2009) showed that the erosive impact of storm events at Narrabeen Beach partially depends on the state of the beach during the precedent calm period. When a reflective state occurs prior to storm events, erosion was shown to be greater as there is a larger volume of sediment available for erosion from the beach; and there is no nearshore bar system to dissipate incoming wave energy. Calibrating the recovery model over an annual time series results in less accretion during calm periods and therefore provides less erodible sediment for the subsequent storm event, explaining the reduction in erosion volumes evident in Fig. 11.

Even though the annually calibrated model gives better results, the model is still unable to reproduce the full range of states, due to the simple mechanism used in the approach to switch between erosive to...
accretive conditions. Recently, this issue was tackled by Davidson et al. (2013) who employ a disequilibrium technique to define periods of erosion and accretion. The basis of this approach uses a time varying dimensionless fall velocity (Gourlay, 1968; Wright et al., 1985) to distinguish between erosion and accretion depending on its relationship to the equilibrium state. Inclusion of this within their simplified model yielded a good estimation of the variability in the shoreline position at Narrabeen Beach over a six year period.

6. Conclusions and recommendations

This paper has presented the first attempt at establishing a methodology to allow the simulation of medium term beach change using a fully process based model.

The methodology presented here expands on those of Callaghan et al. (2008) and Ranasinghe et al. (2011a) by replacing the empirical structural functions, used by them, with a fully process based model. The SPA allows for the determination of antecedent beach profiles providing a more detailed continuous simulation of beach variability. The calibration and validation of the coastal morphodynamic model XBeach at Narrabeen Beach presented here, show the ability of the model to simulate beach recovery under calm conditions as well as storm induced beach erosion. Although Jamal et al. (2010) investigated the accretion of gravel beaches using their XBeach variant, their work is limited to a wave timescale only. The recovery simulations discussed in this paper therefore provide the first attempt at modelling sandy beach accretion at a timescale of days to weeks. The success achieved here is, therefore, not only useful in terms of developing the SPA, but also for longer term coastal morphodynamic simulations using process based models.

The combination of calibrated model setups, presented in Section 5, demonstrates the ability of XBeach to reproduce the behaviour of Narrabeen Beach at an annual time scale, with a reasonable degree of accuracy. However, the simplified storm threshold approach used to switch between storm erosion and post-storm recovery restricts the development of the intermediate state found at Narrabeen Beach. Better results were achieved by the calibration of the recovery model over the annual storm data rather than individual periods.

The use of the SPA methodology with the annually calibrated recovery model, to simulate a 10 year storm climate, was shown to produce erosion volume estimations comparable to those measured at Narrabeen Beach. The event calibrated model consistently overestimated erosion volumes. This highlights the major limitation of the SPA being the inability to simulate the full range of beach states. Even though calibrating the recovery model over an annual time series partially overcomes this limitation, this approach is still unable to reproduce the full range of beach states and rather, provides a compromise between intermediate and reflective states. The implementation of a more detailed mechanism to account for the gradual transition from erosive to accretive conditions, such as that of Davidson et al. (2013), will allow the development of intermediate beach states and significantly improve the credibility of the approach.

Although the SPA approach presented here is not without limitations, it does provide a valuable first step towards modelling beach change at medium term time scales using process based models. The methodology demonstrates the flexibility of XBeach for continuous simulations of beach change at annual time scales; and estimations of medium term erosion return levels using the SPA approach.

With some further studies to mitigate the limitations highlighted above, this type of modelling framework may become a very valuable decision making tool in future coastal management projects.

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