STRUT-TIE METHOD OF DESIGN AND POSSIBLE SERVICEABILITY PROBLEMS: AN EXPLORATION BY NLFE

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Introduction: The strut and tie method presents a rational and consistent approach to the design of all parts in a reinforced concrete structure. With this approach, the load carrying mechanism of the structure is represented by approximating the compressive stress fields as struts, and tensile stress fields as ties. The stress in the struts and ties should not exceed the allowable compressive strength of the concrete or yield strength of the steel respectively. In the design of structures by this method there are two important issues to be addressed. The first issue is that of the visualization of an appropriate strut-tie model for a given structural system. In many structures there may be various load paths available and hence no unique strut-tie model exists. The second issue is that of validity of chosen models in relation to the serviceability and ultimate load characteristics of the resulting structure. It is important that the ductility of the structure should be maintained by insuring that crushing of concrete prior to yielding of steel is avoided at design loads. Since the strut and tie method involves a re-distribution of the stresses from the elastic pattern, it is necessary to determine the extent to which that re-distribution can be allowed for, while preserving the required performance from the structure. In this work, visualization of strut-tie models was carried out using elastic finite element analysis. The resulting stress fields were used to design structures which were analysed using an in-house non-linear finite element program and also physically tested in the laboratory.

Dimensioning: This involves sizing of the individual strut-tie components and ensuring safe load transfer at the areas where struts and ties intersect, i.e. the nodal regions. In order to avoid crushing of the concrete at design loads, the stresses in the concrete struts must be limited to αf_{cd}, where f_{cd} is the design uni-axial compression strength and α is a factor less than or equal to one depending upon the strut conditions. The strength of the concrete strut is affected by many factors such as the level of confinement, presence of transverse tensile stresses, and the multi-axial state of stress. Transverse compression such as in the case of confinement provided by transverse reinforcement is favourable to strength while transverse tension is detrimental to strength. Various values for α have been proposed by researchers (see Schlaich et.al [1]). The critical areas in the strut-tie models often occur at the nodes where three or more struts or ties meet. Where ties meet at a node, it is assumed that transfer of the tie forces takes place by bond, causing compression in nodes, as shown in node 1 of figure 4. The bearing capacity of the nodal zones must be assessed and the model adjusted if found to be inadequate.

Visualization: As a means of automatically visualizing the strut/tie models, elastic finite element analysis is used in combination with an evolutionary 'optimization' process. In this process, developed by Xie & Steven [2], low stressed parts of the structure are 'removed' by allocating a negligible stiffness to the associated element. The criterion for element removal, in this case, involves comparing the average Von Mises stress within an element with that present in the whole structure. If the average Von Mises stress is less than a certain percentage of the maximum, known as the

rejection ratio (rr), then that element is 'removed'. This process is repeated several times using increased values of (rr) until a steady state is reached, i.e. no more elements are removed or the structure becomes unstable. This procedure has the effect of automatically isolating the main stress paths within a structure and hence is able to outline the form of the strut and tie model. Hence, the visualization process need only be carried out until the main load paths become clear. Using this method, structures such as deep beams, corbels, and frame corners, where stress or geometrical discontinuities rule out the application of the Bernoulli theory, were designed in this study. With reference to practical detailing consideration, it is possible to guide the load paths to form a model which is more suitable for design.

Concrete Model: The main feature of the concrete model implemented in the finite element program are as follows:

- Compression: up to peak stress, the uni-axial stress (σ) strain (ε) follows the model defined by Liu et al.[3]. For post peak stress behaviour, the softening branch of the stress-strain relationship follows the model described by Saenz[4] in which:
$$\sigma = \frac{E\varepsilon}{1+(\varepsilon/\varepsilon_0)^2}$$
where E is the elastic modulus and ε_0 is the strain corresponding to peak stress
- Tension: prior to reaching cracking strain ε_{cr}, the stress-strain relationship is linear.
- For this work, the smeared cracking model was used. Tension stiffening is modelled as a linear softening of the stress strain relationship in the post cracking range.
- Shear retention: After cracking, the ratio (β) of cracked to uncracked shear modulus is described as a function of the strain normal to the crack ε_n and is given by:
$$\beta = 0.4\frac{\varepsilon_{cr}}{\varepsilon_n}$$
- Failure criterion: Since a 2-D state of stress is considered, the failure criterion adopted was that of Kupfer-Hilsdorf [5], linearised in terms of the octahedral shear and normal stresses.

Steel Material Model: For plane stress applications, reinforcing bars are modelled using embedded elements and full bond is assumed between concrete and steel. Only axial loads are carried by the bars and hence shear resistance from dowel action is ignored. An elastic-perfectly plastic stress-strain idealization is adopted.

Application: As an example of application, the design and subsequent analysis of a symmetrical, two-sided corbel (C3A) is presented. Physical testing of this model was also carried out in the laboratory. The results of the visualization process are detailed in the elastic principal stress plots of figures 1-3. The strut-tie model is composed of the horizontal tension tie at the top of the corbel combined with the diagonal compression strut. The basic form of this model is similar to that first proposed by Neidenhoff [6]. The effective strength of the concrete strut is dependent upon the state of stress present and in this case it is in uni-axial compression. Dimensioning of the strut in this case can be derived from the width of the bearing plate at which the load is applied. Due to the presence of the tie, stress at the nodal zone under the bearing plate is limited to $0.75f_{cd}$. In this case, f_{cd} is taken as the uni-axial cube crushing

strength f_{cu} of the concrete. In this model, the critical zone with regard to crushing of the concrete occurs in the compression diagonal towards the column. The concrete in this zone, is in a state of bi-axial stress, and the level of stress concentration depends upon the geometry of the strut. Figure 4 gives details on determining the maximum stress σ_{max} at this point, where *t* is the thickness of the corbel.

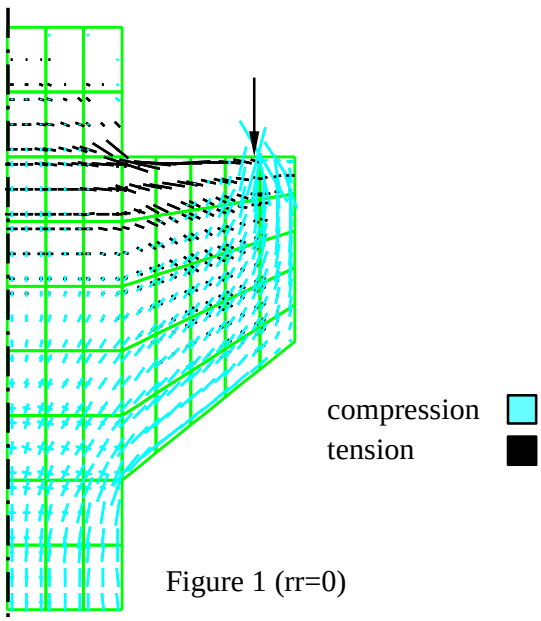

Figure 1 (rr=0)

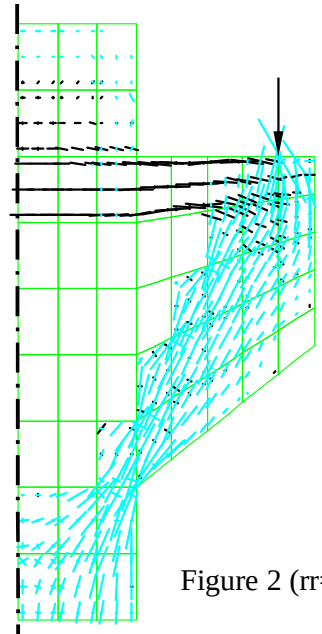

Figure 2 (rr=35%)

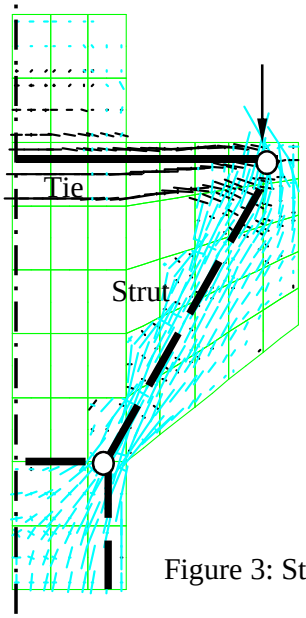

Figure 3: Strut & Tie model

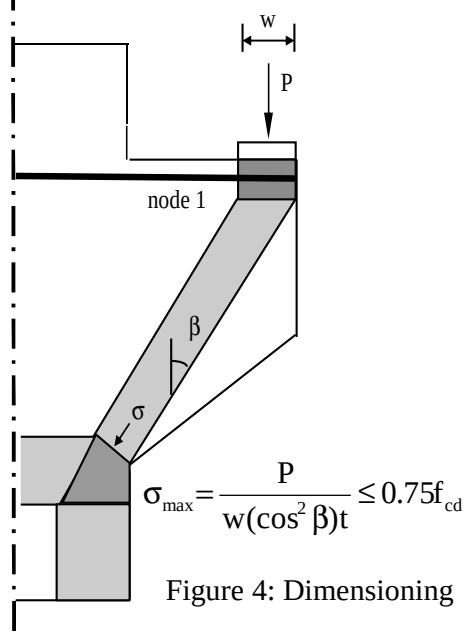

Figure 4: Dimensioning

The numerical concrete stresses occurring along the diagonal strut are given in figure 5. From this it can be seen that crushing of the concrete occurs at point B, at a load of around 130% of the design load. In the physical model, failure was initiated by crushing of the concrete in this zone at a load of around 135% of the design load. Yielding of the main steel in the top corbel-column junction, occurred while the crushing in the compression zone was initiated. The numerical and experimental steel strains in the main reinforcement are shown in figure 6 and display a relatively good correlation.

From this, it can be seen that the strut-tie model used in this case resulted in a reasonable lower-bound design solution. Full details of the test series are given in Cunningham [7].

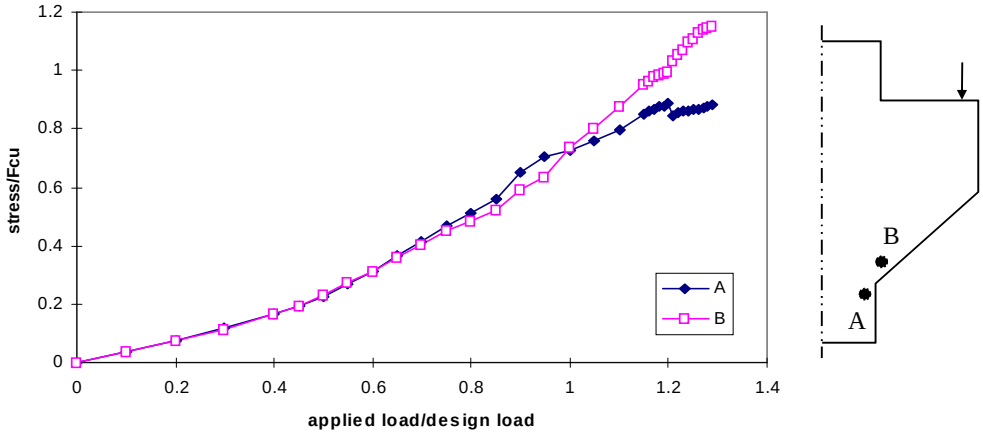

Figure 5: Corbel 3A; Numerical compressive stresses

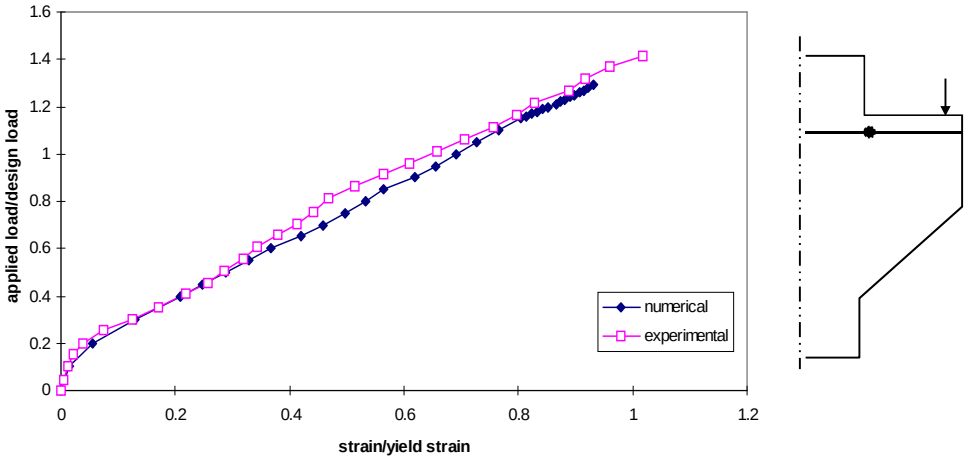

Figure 6: Corbel 3A; Strains in main reinforcement

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