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ORIGINAL ARTICLE

De Jure and De Facto Validity in the Logic of Time and Modality

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What formulas are tense-logically valid depends on the structure of time, for example on whether it has a beginning. Logicians have investigated what formulas correspond to what physical hypotheses about time. Analogously, we can investigate what formulas of modal logic correspond to what metaphysical hypotheses about necessity. It is widely held that physical hypotheses about time may be contingent. If so, tense-logical validity may be contingent. In contrast, validity in modal logic is typically taken to be non-contingent, as reflected by the general acceptance of the so-called "rule of necessitation." But as has been argued by various authors in recent years, metaphysical hypotheses may likewise be contingent. If, in particular, hypotheses about the extent of possibility are contingent, we should expect modal-logical validity to be contingent too. Let "contingentism" be the view that everything that is not ruled out by logic is possible. I shall investigate what the right system of modal logic is, if contingentism is true. Given plausible assumptions, the system contains the McKinsey principle, and is thus not even contained in S5. It also contains simple and elegant iteration principles for the contingency operator: something is contingent if and only if it is contingently contingent.

Keywords contingentism; validity; tense logic; modal logic; McKinsey; S5

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Invariant truth-preservation

An argument preserves truth unless all its premises are true and its conclusion false. Truth-preservation is a minimal condition for logical validity. But while it is clear that it is not the whole story, it is unclear what needs to be added. One prominent idea involves necessity, and another invariance under substitution.

It is often said that a valid argument is a one that is necessarily truth-preserving. But while this may capture some concept of validity, it is not the one pertinent to logic. The argument from "Fred is green" to "Fred is coloured" is necessarily truth-preserving, presumably. But it is generally taken to be the task of metaphysics, rather than logic, to account for why it is a good argument.

On an another approach, an argument is valid if, roughly speaking, every argument of the same logical form is truth-preserving. The argument from "Fred is green" to "Fred is coloured" is ruled out by that criterion, as desired. It has the same form as

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the argument from the true "Fred is a frog" to the false "Fred is immaterial". So this approach is more promising. It requires clarification on three counts, though.[1]

First, we need to ask what the "form" of an argument is supposed to be. Typically, this question is answered by dividing the constituents of the premises and the conclusion into logical and non-logical ones. The logical form is then obtained by abstracting from the non-logical ones, in effect. But what counts as a logical constituent? For present purposes, the answer may be relative to a context of inquiry—propositional logic, predicate logic, tense logic, or modal logic, for example. In the case of propositional modal logic, the logical constituents may be taken to be negation, conjunction, disjunction, the material conditional and biconditional, the necessity operator, and the possibility operator. With the relativization explicit, we do not need to worry about whether the division of the vocabulary is arbitrary or principled.

Second, we need to specify the range of the quantifier in "every argument." Any collection of premises together with a conclusion counts as an argument. But what sort of entities are the premises and the conclusions? For present purposes, nothing hangs on it, as far as I can see. But for definiteness, I shall take premises and conclusions to be propositions. We could instead take them to be interpreted sentences, say of English. I shall use N to denote the class of propositions from which arguments are drawn. If the notion of logical validity under discussion is to have relevance for us, N needs to include every proposition that we might entertain, or express in the course of making an argument. Moreover, since the account appeals to the form of premises and conclusions, the propositions need to have some logical structure. I shall say more about this aspect in due course.

Third, we need to improve on the rough initial formulation—that an argument is valid if every argument of the same form is truth-preserving. Suppose that all propositions that are atomic (in the relevant context) are true, and their negations false. Now consider an argument with one atomic proposition as a premise and another one as the conclusion. In the stipulated situation, every argument of exactly that form will be truth-preserving. But we would not wish to count the argument as logically valid. We should require that every argument that has all the structure of the given one, and possibly more, is truth-preserving. This idea can be made precise using the concept of an N-substitution. Let $\neg$, $\wedge$, and $\Box$ stand for the propositional operators of negation, conjunction, and necessitation, respectively. I shall assume that N is closed under these operators. In the context of propositional modal logic, an N-substitution can then be defined as a mapping ρ from N to N such that $\rho(\neg A) = \neg(\rho A)$, $\rho(A \wedge B) = \rho A \wedge \rho B$, and $\rho(\Box A) = \Box(\rho A)$, and similarly for the other logical connectives. (Note that an N-substitution may map an atomic proposition to a logically complex one.) This function can then be straightforwardly extended from propositions to arguments.

We can then define an argument to be *invariantly truth-preserving* iff each one of its N-substitutions is truth-preserving. We shall also say that a proposition A is *invariantly true* iff each of its N-substitutions is true.

As I argue in the next section, invariant truth-preservation is one of at least two theoretically significant explications of logical validity.

De jure truth-preservation

Above, I briefly considered, and rejected, an account of logical validity as necessary truth-preservation. We may wonder whether it would be a good approach to combine the requirements of necessary and invariant truth-preservation. This idea can be cashed out by using one of the following two definientia: first, that it is necessary that the argument is invariantly truth-preserving; and second, that each N-substitution of the argument is necessarily truth-preserving. It will be assumed here that the two proposals pick out the same class of arguments.[2] I shall argue that one salient notion of validity does not require the necessity of invariant truth-preservation, and that another salient notion requires more.[3]

In tense logic, the validity of various principles depends on certain features of time—whether it has a beginning, whether it has an end, or whether it is cyclical. As John P. Burgess notes, "[t]he question which is the right system for tense logic is not one for the logician: the logician can indicate how this or that or the other system corresponds to this or that or the other theory of the nature of time, but which is the right theory of the nature of time is a question for the physicist" (Burgess 2008, p. 170). Burgess talks about the "nature of time," which suggests necessary features. But for all we know, the relevant features of time might be contingent. If time actually but not necessarily has an end, then an argument from $\neg G\bot$ to $FG\bot$ will be invariantly truth-preserving, but not necessarily so. ('G' can be read as "it will always be the case that," and 'F' as 'it will sometimes be the case that.") Still, it would be natural to consider it as tense-logically valid—whether or not physics is advanced enough to allow us to recognize it as such. We might think of this as a kind of, as it were, de facto tense-logical validity. (In the following section we shall see that something analogous arises also in the case of modal-logical validity.) So contingent invariant truth-preservation is enough for one salient notion of logical validity.

However, there is also a notion of logical validity which does not apply to the argument from $\neg G\bot$ to $FG\bot$, if time ends only contingently. Even if it invariantly preserves truth, it does not do so in virtue of its logical form. It is not invariantly truth-preserving as a matter of logic. Or, as I shall say: it is only de facto, and not de jure, truth-preserving.

If an argument is de jure truth-preserving, it is also necessarily truth-preserving. But the converse is arguably not the case: even invariant necessary truth-preservation does not entail de jure truth-preservation. Let *hyperdeterminism* (also known as *fatalism*) be the thesis that whatever is true is necessary. Hyperdeterminism may be true, or it may be false. But whatever truth-value it has, it presumably does not possess it as a matter of logic. Many of us find hyperdeterminism extremely implausible, and thus do not deem it true, let alone mandated by logic. But despite its implausibility, it is not ruled out by logic either. As David Kaplan (1995, p. 42) puts it, for a logician to rule it out "would be to meddle in metaphysics." The point to note is that if hyperdeterminism is true, it will be necessary that for any A, the argument from A to $\Box A$ is invariantly truth-preserving. But for some A, it will not be de jure truth-preserving. So necessary invariant truth-preservation need not be enough for one salient notion of logical validity. Arguably, we are interested in invariant de jure truth-preservation in many contexts of logical inquiry.

We may speculate that the two explications of validity—invariant truth-preservation and invariant de jure truth-preservation—correspond to two conceptions of logic. If we take logic to be a guide to safe expansion of our stock of beliefs, then we need not require more of a valid argument than invariant truth-preservation. For it will never recommend an inference from true premises to a false conclusion. If we take logic to be the study of certain relationships among certain abstract entities, namely propositions, then de jure truth-preservation seems to be more pertinent.[4]

De facto modal logic for contingentists

Burgess (2008, p. 170) notes that "the question which is the right system of [modal] logic would seem to be one not for the logician, but for the metaphysician." This is surely correct, if by "the right system" we mean one that codifies those arguments that are invariantly truth-preserving. For brevity, I shall use the term "de facto logic" for logic in so far as it is concerned with invariant truth-preservation.

But even though the logician, qua logician, cannot settle what the right system of de facto modal logic is, there are pertinent logical questions to explore for her. We have seen that tense logicians have investigated what the right system of de facto tense logic is, given certain physical hypotheses. Analogously, we can investigate what the right system of de facto modal logic is, given certain metaphysical hypotheses. The pertinent hypotheses concern modality, of course, and in particular, the division of sentences or propositions into the necessary and the contingent ones.

Hyperdeterminism is one important metaphysical hypothesis about modality. The question what the right system of de facto logic is, given hyperdeterminism, is easy to answer: it is the system *TRIV*, with its characteristic schema $\phi \leftrightarrow \Box\phi$ (Hughes and Cresswell 1996, p. 65). But for rival metaphysical hypotheses, the answer is likely to be more complex. I shall now consider a view that is at the opposite end of the spectrum from hyperdeterminism: what I call "contingentism." While hyperdeterminism takes everything to be non-contingent, contingentism takes virtually everything to be contingent. Or, more precisely, it takes a proposition to be possible unless that proposition is ruled out by logic. Here, "logic" is understood in a very inclusive sense, not restricted to some special branch like tense logic or modal logic. So it would count as ruled out by logic that Fred is a married bachelor. Roughly, A is ruled out by logic if $\neg A$ is true in virtue of the nature of its constituents. In that case, I shall call $\neg A$ "de jure true"—where de jure truth is a degenerate case of de jure truth-preservation, introduced in the last section. Alternatively, one might call the negation of what is ruled out by logic "analytic" or "conceptually necessary." So contingentism claims that A is possible unless $\neg A$ is de jure true.[5]

In particular, contingentism takes typical metaphysical truths—that is, those that are not also de jure truths—to be contingent. Contingentism, in the specific way that I understand it, belongs to a broader genus of contingentist views that have received a fair bit of attention in recent years (Cameron 2007; Miller 2009; Parsons ; Rosen 2002, 2006). I shall not discuss whether the version just sketched is true. But by clarifying certain logical questions about it, I hope to increase the plausibility of its coherence.

To investigate what the right system of de facto modal logic for the contingentist is, I shall work in the usual language L of modal logic. The vocabulary of L consists of a denumerable supply of propositional constants plus the primitive connectives $\neg$, $\wedge$, and $\Box$, and the defined ones $\vee$, $\rightarrow$, $\leftrightarrow$, and $\Diamond$. The formation rules and definitions are standard. I shall use ϕ and ψ as metalinguistic variables ranging over L-sentences.

The language L is not fully interpreted: the propositional constants have not been assigned a meaning. So there can be no question of truth-preservation from L-premises to L-conclusions. I shall therefore relate sentences of L to propositions—members of the class N introduced in Section "Invariant truth-preservation."[6]

A function f, defined on L, is an *interpretation* if it maps each propositional constant of L to an element of N, and satisfies the conditions $f(\neg\phi) = \neg f\phi$, $f(\phi \wedge \psi) = f\phi \wedge f\psi$, $f(\Box\phi) = \Box f\phi$, and similarly for the other connectives.

Let a sentence ϕ of L be *de facto valid* iff for every interpretation f, $f\phi$ is invariantly true. This definiens is equivalent to the simpler condition that for every interpretation f, $f\phi$ is true. Since the identity function is a substitution, invariant truth entails truth. For the other direction, suppose that $f\phi$ is true for every interpretation f, and let g be some interpretation. We need to show that $g\phi$ is invariantly true. Consider any N-substitution ρ. Then g', defined by $g'(\psi) := \rho(g\psi)$, is easily verified to be an interpretation. So $g'(\phi) = \rho(g\phi)$ is true, and $g\phi$ has been shown to be invariantly true.

Let a sentence ϕ of L be *de jure* valid iff for every interpretation f, $f\phi$ is invariantly de jure true. By the same reasoning as in the last paragraph, the definiens can be shown to be equivalent to the simpler condition that for every interpretation f, $f\phi$ is de jure true. De jure logic, as opposed to de facto logic, is concerned with de jure validities.

Our question is what sentences of L are de facto valid, given contingentism? We may observe that at least de jure valid ones are, since de jure truth entails truth. This suggests that before we embark on the project of specifying the de facto valid sentences of L, we need to know what the de jure valid ones are. Clearly, all tautologies are de jure valid, since they are de jure true under every interpretation. Since B is de jure true whenever A and $A \rightarrow B$ are, the right system of de jure logic admits the rule of modus ponens: if $\vdash \phi \rightarrow \psi$ and $\vdash \phi$, then $\vdash \psi$ ('$\vdash \phi$' means that ϕ is de jure valid). Given the plausible principle that whenever A is de jure true, so is $\Box A$, the system also admits the rule of necessitation: if $\vdash \phi$, then $\vdash \Box\phi$. Further, it follows from the definition of de jure validity that it admits the rule of substitution: if $\vdash \phi$ and σ is an L-substitution, then $\vdash \sigma\phi$. (An *L-substitution* σ is a mapping from L to L such that $\sigma(\neg\phi) = \neg\sigma$, $\sigma(\phi \wedge \psi) = \sigma\phi \wedge \sigma\psi$, $\sigma\Box\phi = \Box\sigma\phi$, and similarly for the other connectives.) But there is an enormous range of systems that satisfy these constraints. What the right system of modal logic is has been debated for almost a 100 years, and at least some of the debate can be taken to be concerned, at least implicitly, with de jure logic for genuine necessity. No consensus has emerged. So we may worry that our project does not get off the ground.

Fortunately, the situation is better than it may seem. It turns out that many features of the general contingentist's de facto modal logic are robust across a large range of systems. In the following, I shall only make relatively uncontroversial assumptions about

the right system of de jure modal logic. First, that it is a *normal* modal logic, in the standard technical sense: besides satisfying the constraints derived in the last paragraph, it contains all instances of the schema **K** as axioms:

$$\mathbf{K} \quad \Box(\phi \to \psi) \to (\Box\phi \to \Box\psi)$$

Second, I shall assume that it contains all instances of the axiom schema **T**:

$$\mathbf{T} \quad \Box\phi \to \phi$$

Let S be the right system of de facto modal logic—that is, the set of L-sentences that are de facto valid. We are already in a position to establish a few results about S. I shall start with those that do not need the assumption of contingentism.

First, S admits the rule of modus ponens. For suppose that $\vdash_S \phi$ (i.e. that ϕ is in S) and $\vdash_S \phi \to \psi$. Then for every interpretation f, $f\phi$ and $f(\phi \to \psi) = f\phi \to f\psi$ are true. Hence $f\psi$ is true, and thus $\vdash_S \psi$.

Second, S admits the rule of substitution. Suppose that $\sigma\phi$ is not de facto valid. Then for some interpretation f, $f(\sigma\phi)$ is not true. Define an interpretation g by $g\psi := f(\sigma\psi)$. Since $g\phi = f(\sigma\phi)$ is not true, ϕ is not de facto valid.

These two results tell us that the class of de facto valid L-sentences is closed under logical consequence and under substitution. It thus deserves to be called a "logical system." There is no compelling case for requiring our de facto logic to be closed under necessitation, a point to which I will return.

Our next result depends on the assumption of contingentism. We show that every instance of the schema **4** is de facto valid:

$$\mathbf{4} \quad \Box\phi \to \Box\Box\phi$$

Let f be any interpretation, and suppose that $f(\Box\Box\phi) = \Box\Box f\phi$ is false. Then $\Box f\phi$ is not de jure true. Since whenever A is de jure true, so is $\Box A$, $f\phi$ is not de jure true. By contingentism, $\Box f\phi = f(\Box\phi)$ is false. So $f(\Box\phi) \to f(\Box\Box\phi) = f(\Box\phi \to \Box\Box\phi)$ is true, for any f, and hence **4** is de facto valid.

It is intuitively clear that there are no false instances of **4** if contingentism is true: on that view, every proposition that need not be necessary, as far as logic is concerned, already fails to be necessary in the actual world. There could not be a possible world with even fewer necessities, as it were.

If the principle **4** is already part of de jure modal logic, as the orthodox view has it, then it is a trivial and immediate consequence that **4** is de facto valid. What the above argument shows is that given contingentism, **4** will only have true instances even if it fails to be de jure valid.

Hugh Chandler (1976) and Nathan Salmon (1989) have suggested that **4** does have false instances. As we have just shown, the contingentist has to disagree. But Salmon, at least, is clear that he only takes himself to have provided a compelling argument against instances of **4** being logical truths—de jure truths, in our terminology: "[T]he mere logical possibility, as opposed to the truth, of the modal intuition is beyond all reasonable doubt" (p. 31). A contingentist need not deny this.

We can use the de facto validity of **4** to show that the *rule of strict equivalence* is also admissible for the contingentist's de facto logic (so that S qualifies as a "traditionally strict classical" system in the terminology of Chellas and Segerberg (1996)):

$$\textbf{RSE} \quad \text{If} \quad \vdash_S \Box(\phi \to \psi), \quad \text{then} \vdash_S \Box(\Box\phi \to \Box\psi).$$

For suppose that $\vdash_S \Box(\phi \to \psi)$. By **4** and the rule of modus ponens, $\vdash_S \Box(\Box(\phi \to \psi))$. It is easily verified that $\Box(\Box(\phi \to \psi)) \to \Box(\Box\phi \to \Box\psi)$ is de jure, and hence de facto, valid. Using the rule of modus ponens again, $\vdash_S \Box(\Box\phi \to \Box\psi)$.

So far, I have only made minimal assumptions about de jure validity. To obtain further results, I now wish to exploit a more substantial assumption:

$$\textbf{CH} \quad \text{If} \quad \Diamond A \ \textsf{is} \ \textit{de jure true}, \quad \Diamond\neg A \ \text{is not de jure true.}$$

If A is of simple subject–predicate form, such as the proposition that Socrates is wise, then the resulting instance of **CH** is plausible: there is no logical or conceptual feature of that proposition that would be incompatible with the metaphysical hypothesis that Socrates is essentially wise, after all. To motivate the general claim, we may note, with Kaplan, that however implausible hyperdeterminism is, it is not the business of the logician to rule it out. But if **CH** were false for some A, it would be de jure true that both A and $\neg A$ are possible, and hence that hyperdeterminism is false. (**CH** is short for "coherence of hyperdeterminism.")

From **CH** and contingentism, it follows that for all A, $\Diamond\Box A$ or $\Diamond\Box\neg A$ is true. It is tempting to think that the following is de facto valid:

$$\phi \to \Diamond\Box\psi.$$

But this principle entails hyperdeterminism, for reasons familiar from Fitch's knowability paradox.[7] For suppose that hyperdeterminism is false. Then for some A, $A \wedge \neg\Box A$ is true. By the principle, $\Diamond\Box(A \wedge \neg\Box A)$ is true. But this is inconsistent, given our assumption about de jure modal logic.

However, CH and contingentism together entail that the so-called "McKinsey axiom" is de facto valid:

$$\textbf{McK} \quad \Box\Diamond\phi \to \Diamond\Box\phi.$$

For suppose that $f(\Diamond\Box\phi) = \Diamond\Box f\phi$ is false. By contingentism, $\neg\Box f\phi$, and hence $\Diamond\neg f\phi$, is de jure true. Given **CH**, $\Diamond f\phi$ is not de jure true. By contingentism, $\Box\Diamond f\phi = f(\Box\Diamond\phi)$ is false. Hence $f(\Box\Diamond\phi) \to f(\Diamond\Box\phi) = f(\Box\Diamond\phi \to \Diamond\Box\phi)$ is true. Since f was chosen arbitrarily, **McK** is de facto valid.

McK is not a thesis of S5. In fact, it fails in every "non-hyperdeterministic" world (that is, every world in which some instance of $\phi \to \Box\phi$ is false) in every Kripke model for that system. So the correct system of de facto modal logic is not contained in $S5$. Below, we will use a further assumption to rule out that the system extends $S5$, and will conclude that the two are not comparable.

We can illuminate the import of **McK**, and thus of **CH**, by stating some of its consequences in an extension of the language L that includes the contingency operator ∇, defined as follows:[8]

$$\mathbf{D}\nabla \quad \nabla\phi := \Diamond\phi \wedge \Diamond\neg\phi.$$

McK is equivalent to (1):

(1) $\Diamond \neg \nabla\phi$.

Since all instances of **T** are in S, we also have:

(2) $\nabla\phi \rightarrow \Diamond\nabla\phi$

Putting (1) and (2) together, and applying $\mathbf{D}\nabla$, we obtain:

$$\mathbf{FF} \quad \nabla\phi \rightarrow \nabla\nabla\phi$$

"**FF**" is short for "Fracanzani Formula," since to the best of my knowledge, Antonio Fracanzani of Vicenza (also referred to as "Frachantian" or "Frachantianus") was the first to advocate that principle, in a treatise from 1494/1495.[9]

Moreover, the Converse Fracanzani Formula holds even without the assumption of **CH**:

$$\mathbf{CFF} \quad \nabla\nabla\phi \rightarrow \nabla\phi$$

For we have $\vdash_S \nabla \quad \nabla A \rightarrow \Diamond\nabla A$, and $\vdash_S \Diamond\nabla A \rightarrow \Diamond\Diamond A$. From **4**, we have $\vdash_S \Diamond\Diamond\phi \rightarrow \Diamond\phi$. So $\vdash_S \nabla\nabla\phi \rightarrow \Diamond\phi$. By parallel reasoning, we establish $\vdash_S \nabla\nabla\phi \rightarrow \Diamond\neg\phi$.[10]

Putting **FF** and **CFF** together, and iterating them, we obtain the following for all $n \geq 1$ (where $\nabla^0\phi = \phi$ and $\nabla^{n+1} = \nabla\nabla^n\phi$):

(3)$\nabla\phi \leftrightarrow \nabla^{\mathrm{n}}\phi$

The contingentist thus obtains a simple and elegant principle for iterating the contingency operator.

We can obtain further results about S if we make the plausible assumption that **McK** is not de jure valid. Suppose that under interpretation f, it is not de jure true. Contingentism then entails that $\neg\Box(f\neg(\Box\Diamond\phi \rightarrow \Diamond\Box\phi)) = f(\neg\Box(\neg(\Box\Diamond\phi \rightarrow \Diamond\Box\phi))$ is true. If S is consistent, then the necessitation of **McK** is not de facto valid. Therefore, S does not admit the rule of necessitation, and is not a normal system of modal logic.[11] From a result of Scroggs (1951), it follows that S does not extend $S5$. The two systems are not comparable.

Concluding thoughts

Throughout, I have steadfastly ignored an elephant in the room: the question whether S, as specified, is consistent, or whether it includes every sentence of L. If contingentism is

incoherent, then we can produce an argument, for any ϕ, that if contingentism holds, ϕ is de facto valid. It can be argued that *S* is consistent if the right system of de jure logic is *T* or *S*4, but not if it is *B* or *S*5. However, it is beyond the scope of this paper to provide a more general answer, and a discussion of the assumptions on which this argument depends.

To conclude, I wish to add a further remark on the analogy of tense logic and modal logic. Validity in tense logic may be contingent, and I have urged that the same applies to validity in modal logic. I have thus explored the idea that tense logic and modal logic may be alike with respect to the modal status of their validities. But while this treats the modal and the temporal as alike in a certain respect, it arguably does not yet make them analogous. For what corresponds, in tense logic, to the rule of necessitation (which need not hold in systems of de facto modal logic) is the rule of temporal generalization. We might investigate what is invariantly true given certain hypotheses about what instant we are currently at—for example, that we are not at the first nor the last moment. For such de nunc validities, temporal generalization may fail. Once we study de nunc logic, the term "history of logic" may acquire a wholly new meaning.

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Notes

1 In some places, Quine advocated an account of that kind, broadly construed (e.g., in Quine 1940, p. 2).
2 The assumption—which only serves expository convenience here—follows from the conjunction of the Barcan Formula and the Converse Barcan Formula for propositional quantification.
3 I shall leave it open whether there is any distinctive theoretical role to play for the notion of necessary invariant truth-preservation.
4 See Harman (2002) for relevant discussion.
5 The converse, that if $\neg A$ is de jure true, A is not possible, should be acceptable to contingentists and non-contingentists alike.
6 On another, perhaps more familiar approach, we interpret propositional constants in a model, and ask about truth-preservation in every model in a certain class, instead of truth-preservation *simpliciter*. But in the context of our investigation, this alternative approach would appear to be less direct. On the face of it, hyperdeterminism and contingentism are hypotheses about what modal propositions are true, rather than about what model is intended (e.g., a possible worlds model where every world only accesses itself).
7 As remarked in passing by Williamson (2007, p. 280).
8 The symbol ∇ was introduced by Montgomery and Routley (1966). (In their systems, it is a primitive rather than a defined symbol.)

9 "Dico ... quod si alidquid est contingens, est illud esse contingens." From Frachantianus Vicentinus, "Questiones ... in Consequentiis Radulphi Strodi", quoted after Boh (1984, p. 504).

10 **CFF** is axiom S41a in Montgomery and Routley (1966).

11 Indeed, *S* is not even a *classical* system, in the technical sense.

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