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Deposited on: 23 January 2014
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Inquiry: An Interdisciplinary Journal of Philosophy
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/sinq20

To cite this article: Stephan Leuenberger , Inquiry (2013): Grounding and Necessity, Inquiry: An Interdisciplinary Journal of Philosophy, DOI:
10.1080/0020174X.2013.855654

To link to this article: http://dx.doi.org/10.1080/0020174X.2013.855654

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Grounding and Necessity

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(Received 14 December 2012; accepted 27 March 2013)

ABSTRACT  The elucidations and regimentations of grounding offered in the literature standardly take it to be a necessary connection. In particular, authors often assert, or at least assume, that if some facts ground another fact, then the obtaining of the former necessitates the latter; and moreover, that grounding is an internal relation, in the sense of being necessitated by the existence of the relata. In this article, I challenge the necessitarian orthodoxy about grounding by offering two prima facie counterexamples. First, some physical facts may ground a certain phenomenal fact without necessitating it; and they may co-exist with the latter without grounding it. Second, some instantiations of categorical properties may ground the instantiation of a dispositional one without necessitating it; and they may co-exist without grounding it. After arguing that these may be genuine counterexamples, I ask whether there are modal constraints on grounding that are not threatened by them. I propose two: that grounding supervenes on what facts there are, and that every grounded fact supervenes on what grounds there are. Finally, I attempt to provide a rigorous formulation of the latter supervenience claim and discuss some technical questions that arise if we allow descending grounding chains of transfinite length.

Not all facts are metaphysically brute. Rather, some hold in virtue of others, or because of others. Conversely, some facts ground other facts. The notion of grounding and its cognates can be used to articulate important philosophical theses. For example, physicalism is naturally taken to be the claim that all facts are grounded in some physical facts. More generally, there is a systematic connection between the notion of grounding and the more familiar notion of fundamentality: fundamental facts are those that are not grounded by other facts. Likewise, there is a systematic connection between grounding and explanation: by citing its

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grounds, one can, in principle, provide an explanation for a non-fundamental fact.

Recently, philosophers have attempted to clarify the notion of grounding, and to regiment its use. Among other things, they have examined candidate principles that articulate links between grounding and some of its other conceptual neighbours, besides fundamentality and explanation.

How does grounding relate to necessity, and to modality in general? Once upon a time, grounding would have been taken to be analysable in broadly modal terms, such as necessitation or supervenience. Such analyses have fallen out of favour. Still, it is tempting to think that grounding is a metaphysically necessary connection. After all, grounding is a species of metaphysical determination, and metaphysical determination—in contrast to nomic determination—leaves no room for contingency. In one version or other, the conclusion of this tempting line of thought seems to be widely accepted amongst theorists of grounding. For example, Fine writes that the relation of grounding ‘is like that of consequence in that a necessary connection must hold between the relata if the relation is to obtain’.1 Correia is more explicit about what its being a necessary connection is supposed to involve: ‘Necessarily, if the fact that A is grounded in some given facts, then it is impossible that the latter facts all exist but fail to ground the fact that A.’2 With varying levels of explicitness and commitment, Witmer et al., Audi, de Rosset, and Rosen, and Trogdon also endorse that view.3

Despite its popularity, the claim that grounding is a necessary connection seems to me mistaken. In my view, grounding is no more a necessary connection than causation is. My aim in this paper is to liberate it from the shackles of metaphysical necessity. If I were to succeed, this would be good news for those wishing to use grounding as a philosophical tool. For a close connection to metaphysical necessity would severely constrain its deployment. Put simply, it would not allow interesting grounding claims to be true. For example, it would rule out the truth of physicalism too easily—or so I shall argue.

Where does the tempting line of thought go wrong? Grounding is, of course, a species of metaphysical determination. To deny that would be to change the topic. But why should we accept that metaphysical determination

2Correia, Existential Dependence, 61.
3Witmer, ‘Intrinsicality without Naturalness’; Audi, ‘Clarification and Defense’; de Rosset ‘Getting Priority Straight’; Rosen, ‘Metaphysical Dependence’; Trogdon, ‘Grounding’. Schaffer, ‘Least Discerning’, 321, denies that grounding requires necessitation. He allows that a substance grounds its modes, even though it would be a category mistake to say that it necessitates them. His view is compatible with necessitarianism about grounding as a relation among facts. Trogdon, ‘Grounding: Explanation vs. Dependence’, diagnoses an ambiguity in our grounding talk and rejects necessitarianism on one of the disambiguations. He still accepts it for grounding in the sense of metaphysical explanation, the notion I am concerned with here.
is a matter of metaphysical necessity? Not because it is a species of determi-
nation—determination relations may be contingent, as we know from the
case of causation. Nor because it pertains to metaphysics—metaphysical
theses may well be contingent, as the recent debate about contingentism
has reminded us. Nor should we insist that grounding is a matter of
metaphysical necessity because it is both a determination relation and per-
tains to metaphysics—to do so would be to beg the question at issue.

In this paper, I shall presuppose that there is no ambiguity in our talk of
grounding: we have identified a single relation that we can theorize about. I
am indebted to previous writers on the subject who worked towards pinning
it down. Those who reject my presupposition can read my paper as arguing
that there is at least one salient relation of grounding that is only a con-
tingent connection.

The plan is as follows: In Section I, I formulate two widely accepted
principles linking grounding and necessity that I wish to contest. In
Sections II and III, I present what I take to be counterexamples to those
principles. I then consider the relationship between grounding and modality
more broadly, and ask whether grounding claims at least entail superveni-
ence claims. I formulate two supervenience-theoretic constraints on ground-
ing in Sections IV and V.

I. Candidate Linking Principles

Grounding claims come in a variety of logical forms. Those that I shall be
concerned with are of the form ‘B₁, B₂, ... ground A’, where ‘A’, ‘B₁’, etc.,
denote facts. So grounding claims are to be expressed by a predicate to be
flanked with a plural term on the left and a singular term on the right.

Other authors prefer to use a sentential operator, which can be read as
‘because’, to express the relevant claims. Typically, this operator is taken to
be variably polyadic. So a grounding claim may take the form ‘A because
B₁, B₂, ...’, where B₁, B₂, ... is a list of sentences or propositions. Such a list is
different from a conjunction: it does not unify its members into one entity.
Correia prefers the operator on the grounds of ontological neutrality: it
allows us to talk about grounding without being committed to facts. For my
purposes, this advantage is outweighed by an expressive limitation of the
operator. I am concerned with grounding claims that can be expressed by
philosophers, in English or in other natural languages. English is not rich
enough in its logical resources to provide, for every sentence in which the

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4For some metaphysical theses, such as physicalism, contingentism is arguably the prevalent
contingentism about composition, and Miller, ‘Defending Contingentism’, about the question
whether the parts are ontologically prior to the whole.

5Correia, ‘Grounding and Truth-Functions’.
predicate ‘grounds’ figures, an equivalent sentence using the operator. While English can denote infinite pluralities or classes Γ of facts, it cannot express the infinite list that corresponds to them. Moreover, some classes Γ may have members that English is unable to express.

If we eschew talk of facts, the obvious way to achieve the effect of an infinite list is to use a truth predicate. Instead of ‘Γ grounds α’, we would say ‘p because all members of Γ are true’—where Γ stands for a list of truth-bearers, rather than facts. However, this comes at a cost. For consider the popular claim that all semantic facts are grounded by non-semantic facts. The semantic fact that ‘snow is white’ is true, say, is grounded by the fact that snow is white, or perhaps by that fact together with the facts that ground that ‘snow is white’ means what it does.6 The above recipe would turn ‘For every semantic fact α, there is Γ containing only non-semantic facts such that Γ grounds α’ into ‘For every semantic p, there is Γ containing only non-semantic truths such that p because every member of Γ is true’. On the face of it, that latter claim does not capture the intended claim: it does not express a dependence of the semantic on the non-semantic. Since truth is a paradigmatically semantic concept, ‘every member of Γ is true’ arguably does not count as a non-semantic claim.

Admittedly, the argument just given can be challenged. On some deflationary views about truth, it is not really a semantic concept. But my aim here is not to adjudicate the question what the logical form of grounding claim is. I merely wish to give some motivation for my choice. My hope is that the substance of what I say carries over, mutatis mutandis, to the operator. But I shall not discuss the operator formulations further.

Instead of enumerating the grounds, I shall often use Greek capitals to stand for them, and say ‘Γ grounds A’ instead of B1, B2, ..., ground A. Or in symbols: Γ < A.7 Permutation and repetition do not affect the truth value of a grounding claim. So, in particular, if B, Γ < A is true, so are B, B, Γ < A and Γ, B < A. I shall treat such lists as if they were classes. Since I do not think that anything turns on this, I shall not justify that practice.

I shall use O(A) and O(Γ) to express that A obtains and that all members of A obtain, respectively. If ‘A’ does not denote anything with respect to a possible world w, or denotes something that does not obtain in w, then O(A) is false with respect to w; and likewise for O(Γ).

For anyone familiar with Hume’s discussion of causation, the following is a natural reading of the claim that grounding is a necessary connection:

6See, for example, Mulligan, ‘Two Dogmas of Truthmaking’, 53, and Mulligan, ‘Truth Predicate’, 586. It is a very interesting question what the relationship between this idea and the preference for the minimal fixed-point in Kripke-type theories of truth is, but it is not a question I can do justice to here.

7The symbol < is used in Fine, ‘Pure Logic of Ground’. In that paper, it stands for a sentential connective rather than a predicate, however.
Entailment Necessarily, if $\Gamma < A$ then $\Box(O(\Gamma) \rightarrow O(A))$.

Entailment is my main target in this paper. A secondary target is another reading of the claim that grounding is a necessary connection, this one concerning the modal status of grounding claims themselves. To a first approximation, this other reading says that any true grounding claim must themselves be necessarily true: necessarily, if $\Gamma < A$, then $\Box(\Gamma < A)$. However, that claim is not a worthy target. For it clashes with the widely held principle that what is grounding or grounded is the case:

Factivity Necessarily, if $\Gamma < A$, then $O(\Gamma) \land O(A)$.

Given Factivity, the necessity of true grounding claims entails that only necessarily obtaining facts can be among their relata.

However, grounding would lose a great deal of its theoretical interest if it could not hold among contingent relata. Since Factivity seems innocuous, we should reject the necessity of grounding claims. This move vindicates a version of contingentism about grounding already, albeit not a very exciting one. However, there is a more controversial contingency claim in the neighbourhood. The factivity of grounding is compatible with the principle that true grounding claims hold in all possible worlds in which both the grounding and the grounded facts obtain:

Internality Necessarily, if $\Gamma < A$ then $\Box(O(\Gamma) \land O(A) \rightarrow \Gamma < A)$.

The label for this claim is borrowed from the metaphysics of relations. In one of its many senses, ‘internal’ applies to a relation provided that whenever it holds among some things, it holds among them in all possible worlds in which each of them exists.

Entailment and Internality are logically independent claims about grounding, even given Factivity. Accordingly, I will argue for their falsity separately, in Sections II and III, respectively.

II. Against Entailment

What motivates the widespread acceptance of Entailment? Above, I suggested an argument based on the premise that grounding is a species of metaphysical determination. The argument, as formulated then, was

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8 It would not lose all its interest. We might still be able to use it to make fine-grained distinctions among necessary truths, the sort of distinctions that supervenience relations are unable to capture.

9 Correia, *Existential Dependence*, 61, in a passage quoted earlier, endorses a principle that entails (together with Factivity) both Internality and Entailment. Other authors do not explicitly discuss Internality, as far as I am aware.
unconvincing, but perhaps it can be improved. Gideon Rosen points us in an interesting direction:

The facts that ground a given fact collectively ensure that it obtains as a matter of metaphysical necessity. This is one respect in which the grounding relation, which is a relation of metaphysical determination, differs from causal and other merely nomic forms of determination. There is a difference between the materialist who holds that facts about phenomenal consciousness are grounded in (and hence necessitated by) the neurophysiological facts directly, and the dualist who think that facts about the brain cause or generate conscious states according to contingent causal laws.¹⁰

Rosen is not claiming to be offering an argument for Entailment in this passage. Rather, he is illustrating the claim. Nonetheless, we can ask whether he provides the ingredients for an argument.

Materialism, or physicalism, holds that phenomenal facts—facts about conscious experience—are grounded by neurophysiological facts. Some versions of dualism hold that phenomenal facts are caused or generated by neurophysiological facts according to contingent laws. Since physicalism is incompatible with any version of dualism, grounding is different from the pertinent relations of causation and generation. So far, this argument is unexceptionable. But can its conclusion be used to establish Entailment? We need to account for the difference between grounding on one hand and causation and generation on the other. One way to do this would be to accept Entailment and reject the corresponding claims about causation and generation. But there may well be other ways.¹¹ For example, one could argue that grounding is *synchronic*, while causation and generation are *diachronic*. Moreover, there might be differences in their modal status even if Entailment and its analogue for causation both fail—a variant of Entailment restricted to worlds without alien natural properties may be true for grounding but not for causation.¹² If that is correct, it seems to be enough to account for the difference between these views.¹³

It appears, then, that Rosen’s observation about the difference between physicalism and dualism cannot be parlayed into an argument for Entailment. I shall not examine further why particular authors accept the latter. Before proceeding to make my case against it, I would like point out that contingentists

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¹⁰Rosen, ‘Metaphysical Dependence’, 118.
¹¹Note that Rosen claims otherwise; as I said, the argument I am criticizing is not Rosen’s.
¹²Lewis, ‘New Work for a Theory’.
¹³For a more thorough discussion of whether the contingentist can account for the difference, see Skiles, ‘Getting Grounded’.
about grounding can draw encouragement from the history of the debate on causation. After all, an analogue of Entailment for causation used to be widely accepted, but is now generally recognized to be false: the principle that if some facts \( \Gamma \) cause \( A \), then their joint obtaining necessitates \( A \). Early theorists of causation tended to characterize causation as a necessary connection, but subsequent theorists managed to say something informative about it even after rejecting that supposedly characteristic feature.

In the rest of this section, I shall identify potential counterexamples to Entailment—classes of facts that plausibly ground other facts without strictly implying them.

### II.i. Physical facts and phenomenal facts

Consider \( \Phi_\alpha \), the class of all actually obtaining physical facts. Among other things, \( \Phi_\alpha \) includes facts about all particle positions and velocities, and all field values at all spacetime points. The success of physical sciences in providing reductive explanations of various phenomena gives us some reason to think that physicalism is true, and hence also suggests that every non-physical fact is grounded by some (proper or improper) subclass of \( \Phi_\alpha \). In particular, phenomenal facts—such as \( \text{Red} \), the fact that I am having a red experience—are so grounded, if physicalism is true. The link between physicalism and grounding claims exploited here is implicit in the passage by Rosen quoted earlier: he takes it to be a commitment of physicalism that neurophysiological facts—here included among the physical facts—ground phenomenal facts. Presumably, then, there is some \( \Phi \subseteq \Phi_\alpha \) such that the antecedent of the instance of Entailment produced by \( \Phi \) and \( \text{Red} \) is true: \( \Phi \) grounds \( \text{Red} \). What about the consequent, the claim that \( O(\Phi) \) strictly implies \( O(\text{Red}) \)? Plausibly, it is possible that all facts in \( \Phi \) obtain but \( \text{Red} \) does not. After all, such a combination of facts is conceivable. It thus appears possible, and we have no particular reason to think that the appearance of possibility is misleading in this case.

This argument against Entailment can be formalized using the following three premises:

1. Possibly, physicalism is true and \( \text{Red} \) obtains as a non-fundamental fact.
2. Necessarily, there are no physical facts \( \Gamma \) such that \( \square(O(\Gamma) \rightarrow O(\text{Red})) \).
3. Necessarily, if physicalism is true, then for every non-fundamental fact \( A \) there are physical facts \( \Gamma \) such that \( \Gamma \prec A \).

From these premises, it follows (even in a very weak modal logic) that it is not necessary that for all \( \Gamma \), if \( \Gamma \prec \text{Red} \) then \( \square(O(\Gamma) \rightarrow O(\text{Red})) \), and hence that Entailment has a false instance.

\[ \text{14} \]Steven Nadler writes that ‘[o]ne of the central ingredients of the philosophical analysis of causation traditionally has been necessity. A causal relation is a necessary relation. . . . If \( a \) is the cause of \( b \), then \( a \), in some sense, necessitates \( b \)’. (Nadler, ‘Malebranche on Causation’, 113).
This argument against Entailment can be attacked in different ways. In the following, I shall defend it against four objections, the most pressing one—which may well have occurred to the reader—last.

The objection from dualism attacks premise (1). If physicalism is incompatible with the existence of phenomenal consciousness, it is not possible that the former is true and Red obtains, and there is no counterexample to Entailment. It thus seems that dualists have not been given any reason to reject Entailment.

The objection is correct as far as it goes. However, in the present dialectic, it does not go very far. Typically, theories of grounding are intended to be neutral on a wide range of metaphysical claims. They cannot be neutral throughout; after all, grounding is itself a topic in metaphysics. Whatever the exact extent of the required neutrality, it is clear that the truth or falsity of physicalism ought not to be settled by a theory of grounding. But my argument might still saddle proponents of Entailment with a commitment to the falsity of physicalism, for all the objection from dualism says. Therefore, the objection does little to ease the pressure on Entailment.

The objection from necessitarianism attacks premise (2). The counterexample to Entailment relies on the truth of a controversial possibility claim: that it is possible that $Φ$ and $\neg$Red are both true. In support of that possibility claim, I gestured at conceivability, and the appearance of possibility. Notoriously, it is hard to substantiate these claims, and flesh out a modal epistemology to back up one’s modal claim. Some philosophers are ready to reject our pre-theoretical possibility judgements. They may hold, for example, that metaphysical possibility is just logical compatibility with the laws of nature. Necessitarians of that stripe will be unimpressed by the considerations I adduced.

As with the objection from dualism, I dispute the dialectical significance of the point. I am addressing the question how grounding relates to modality. In doing that, I am assuming certain views about modality that I take to be fairly orthodox, and investigate how they relate to grounding, which is less well explored. It is clear that assuming necessitarianism makes Entailment a much weaker, more easily defensible claim. To see this, suppose that the most extreme version of necessitarianism, namely fatalism, is true, such that every truth is necessary. Then Factivity will already guarantee the truth of Entailment. But as it is normally introduced, Entailment is not supposed to be hostage to non-standard views about possibility.

The objection from physicalist commitments attacks premise (3). Above, I took evidence for physicalism to be ipso facto evidence for the thesis that all facts are grounded by some class of actual physical facts. This may be questioned: perhaps physicalism only requires that a given fact is grounded by some physical facts, or by $Φ_@$ plus a totality fact.

To a first approximation, a totality fact is a fact expressed by a ‘that’s all’-clause. Totality facts have been invoked as truthmakers for general or
universal facts, which are expressed by universal quantifications. In that context, a totality fact is one that rules out the existence of any individuals beyond those in a certain specified class. In the present context, a totality fact would be one that rules out that there are any fundamental facts beyond those in $\Phi_\Box$.

According to this objection, my argument above failed to motivate the claim that some class of physical facts grounds $Red$ by itself.

In response, I would like to pose a dilemma for the objector’s appeal to totality facts. As with properties, we can distinguish between sparse and abundant facts. Sparse facts are natural as opposed to disjunctive, relatively fundamental as opposed to derivative.

For the first horn of the dilemma, suppose that totality facts are sparse, and assume with the objector that $\Phi_\Box$ plus the totality fact grounds $Red$ but $\Phi_\Box$ alone does not. It then follows that the totality fact is not in $\Phi_\Box$ and is thus not a physical fact. Surely, grounding satisfies the following weak version of transitivity: if $\Gamma < A$, and $A, \Gamma < B$, then $\Gamma < B$. Letting $A, B,$ and $\Gamma$ be the totality fact, $Red$, and $\Phi_\Box$, respectively, it follows that $\Phi_\Box$ does not ground the totality fact. Hence there is a sparse non-physical fact that is not grounded by $\Phi_\Box$. This, I submit, is incompatible with physicalism. While the view may or may not require that every abundant fact is grounded by the physical facts, it surely requires that every sparse fact is so grounded.

On the second horn of the dilemma, suppose that totality facts are merely abundant. Then it is extremely implausible that they play an irreducible role in grounding claims. That is, if a fact is grounded at all, is grounded by a class that only includes sparse facts. Merely abundant facts do not essentially feature among the grounds of a given fact. To have a abundant facts play an ineliminable grounding role is to put the cart before the horse.

As we have seen, the objector claims that $\Phi_\Box$ together with the totality fact grounds $Red$ but $\Phi_\Box$ alone does not. By the principle just motivated, $Red$ is also grounded by a class that consists exclusively of sparse facts. Since $\Phi_\Box$ includes all physical facts, some sparse facts are not physical facts. Again, I submit that this is incompatible with physicalism. I thus conclude that the physicalist ought not to allow that $\Phi_\Box$ plus a totality fact grounds something that is not grounded by any subclass of $\Phi_\Box$ alone.

Last but not least, the objection from premise incompatibility attacks the conjunction of premises (1) and (2). This objection is the most pressing one.

If $\neg (O(\Phi_\Box) \rightarrow O(\text{Red}))$ is possible, then there is a possible world that is a physical duplicate of ours in which my doppelgänger does not have a red experience, and is thus a zombie or at least a partial zombie—he exhibits a

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15According to the strong transitivity principle for grounding (explicitly asserted by Rosen, ‘Metaphysical Dependence’, and a consequence of the system in Fine, ‘Pure Logic of Ground’), $\Gamma, \Delta < A$ whenever $B, \Delta < A$ and $\Gamma < B$. The weak version I am using follows by setting $\Gamma = \Delta$.
kind of superblindsight. But, the objection goes, if such creatures are possible, then physicalism is false, and it is not the case that phenomenal facts are grounded by physical ones. For physicalism stands and falls with the impossibility of zombies, of the full or the partial variety. After all, the so-called ‘Zombie Argument’ uses exactly such possibility claims to argue against physicalism.

The hypothetical objector does not claim to know the falsity of either premise (1) or premise (2). Rather, she claims to know that the two premises are incompatible.

In response, I question the objector’s claim about the modal commitments of physicalism. Admittedly, the incompatibility of physicalism with the possibility of zombies is widely accepted. However, we need to heed a distinction that is often overlooked. In one sense, a zombie world is a physical duplicate of the actual world in which some actual phenomenal facts do not hold. In another sense, a zombie world needs to satisfy a further condition in addition to those just mentioned: it is a purely materialistic world, one in which there is nothing over and above the physical facts. I claim that the possibility of zombie worlds in the first sense is compatible with physicalism, and that the possibility of zombie worlds in the second sense is not needed for my putative counterexample to Entailment. The second claim is obvious: in describing the relevant possibility claim, I did not need to mention that there is nothing over and above the physical. The first claim, however, needs more argument.

Physicalism is often introduced with a creation metaphor: it is true if God was finished creating the world once He had put all the physical facts in place. Consider a variant of that metaphor. In the actual world, God had put all the physical facts in place by the end of day seven. This was enough to make it the case that \( \text{Red} \) obtains. God henceforth left the world alone. In world \( \text{wb} \), God on day eight ensured that in the region occupied by my brain, a non-physical fundamental property, to be called ‘chromaplasm’, is instantiated. Chromaplasm makes visual phenomenology disappear. In \( \text{wb} \), I do not have a red experience, i.e. \( \text{Red} \) does not hold. The presence of chromaplasm is a blocker of \( \text{Red} \) in \( \text{wb} \).

Clearly, \( \text{wb} \) is not materialistic, because of the presence of chromaplasm. The question, then, is whether in this scenario, there is indeed a subclass of \( \Phi_\text{a} \) that grounds \( \text{Red} \) in the actual world. Pace Hawthorne, it seems to me that it is. The mere possibility of \( \text{wb} \) ought to be compatible with physicalism. What God could have done after day seven ought not to bear on the

\[16\text{Notoriously, the ‘nothing over and above’ condition is hard to spell out without appeal to grounding or a cognate notion. But this difficulty need not concern us here.}\]

\[17\text{Hawthorne, ‘Blocking Definitions’, introduces the idea of such blockers.}\]

\[18\text{Hawthrone, ‘Blocking Definitions’.}\]

\[19\text{I address Hawthorne’s argument in Leuenberger, ‘Ceteris Absentibus Physicalism’, where the compatibility of physicalism with the possibility of blockers is defended in more detail.}\]
question whether our world—where, by hypothesis, He stopped after day seven—is physicalistic. Since in the actual world, God only created physical facts and then retired, physicalism is true. All the actual fundamental facts are physical, after all, and that seems to be sufficient condition for the truth of physicalism. Hence *Red* is grounded by some class of physical facts.\(^{20}\)

It seems to me that none of these objections to my argument are convincing. To conclude the discussion, I would like to emphasize that my conclusion is not merely that Entailment fails for some relation in the grounding family. That would hardly be news. For example, Fine introduces the notion of natural grounding,\(^{21}\) for which Entailment fails. But ‘grounding’, in my usage, does not refer to natural grounding. In the above argument, it was important for me to make the case that the actual world of that scenario was materialistic. If I had been concerned with natural grounding, I would not have bothered. For I take it that even dualism is compatible with the claim that every non-physical fact is naturally grounded by some physical facts. Consider a scenario where both mental and physical properties are metaphysically fundamental, and where there are fundamental psycho-physical laws that determine the distribution of mental properties as a function of the distribution of the physical ones. (Chalmers is sympathetic to a view of that kind.)\(^{22}\) Then any mental facts will presumably be naturally grounded by the physical facts. But I would certainly deny that the former are grounded—genuinely grounded, grounded *simpliciter*—by the latter.

**II.ii. Grounding dispositions**

Admittedly, not all the assumptions relied on in constructing my counter-example are uncontroversial. It will thus strengthen my case against Entailment if I can identify another falsifying instance. While still controversial, the assumptions required will be different. Even though the two counterexamples are structurally similar, they may convince different readers.

Rosen lists the following as a candidate grounding claims:

> The dispositions of a thing are always grounded in its categorical features . . . . A glass is fragile in virtue of the arrangement of the molecules that make it up, perhaps together with the laws of chemistry and physics.\(^{23}\)

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\(^{20}\)The physicalist has far greater resources to resist arguments for the conceivability of a zombie world in the second sense—a purely materialistic world without consciousness—than those for the conceivability of a zombie world in the first sense. However, a detailed consideration of conceivability arguments is beyond the scope of this article.


\(^{22}\)Chalmers, *The Conscious Mind*.

\(^{23}\)Rosen, ‘Metaphysical Dependence’, 110.
Suppose that such a ‘categoricalist’ view of dispositions is right. Let $g$ be a fragile glass, and $D_{b,s}g$ the fact that $g$ is disposed to break when struck. Categoricalism requires that $D_{b,s}g$ has some categorical basis $K(D_{b,s}g)$. We can think of that basis as consisting of facts. $K(D_{b,s}g)$ will be a subclass of the class $K_g$ of all instantiations of categorical properties by parts of $g$. More precisely, if $x$ is a part of $g$ and $F$ any categorical property, then either the fact that $x$ has $F$ or the fact that $x$ lacks $F$ is in $K_g$, and likewise for categorical relations. According to categoricalism, $K(D_{b,s}g)$ grounds $D_{b,s}g$.

Suppose further that the laws of nature involving the actual categorical properties are contingent: in some other possible worlds, $K(D_{b,s}g)$ is nomically related to different causal powers. Then some possible duplicate of the glass does not have the disposition. If the categorical basis grounds the disposition by itself, then this constitutes a counterexample to Entailment.

However, the grounding claim in question may not be plausible anyway. It is much more plausible that it is together with certain laws of nature that $K(D_{b,s}g)$ grounds the disposition—a variant that Rosen mentions as well. But that claim is vulnerable to a modified counterexample. Suppose that in world $w_5$, there is a fifth fundamental force, associated with an alien property whose bearers attract each other. While the corresponding law of $w_5$ is not a law of the actual world, all actual laws are also laws in $w_5$—such as the laws linking mass to the gravitational force, electric charge to the electromagnetic forces, colour charge to the strong nuclear force, as well as the laws relating forces to motion. In $w_5$, the glass $g$ is in many respects just like it is in the actual world. Its parts are exactly the same with respects to the bearers of actual fundamental forces. However, the parts instantiate the alien property in $w_5$, in such a way that it would not break if struck. So it seems that $K(D_{b,s}g)$ and $D_{b,s}g$ form another counterexample to Entailment.

Before considering objections, I shall set out the argument in three premises again.

(1′) Possibly, categoricalism is true and $D_{b,s}g$ obtains.

(2′) Necessarily, there are no categorical facts $\Gamma$ such that $\square(O(\Gamma) \rightarrow O(D_{b,s}g))$.

(3′) Necessarily, if categoricalism is true, then for every dispositional fact $A$ there are categorical facts $\Gamma$ such that $\Gamma < A$.

This argument has a very similar structure to the one in the last section. Dialectically, it has different strengths and weaknesses, however. As far as I

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24 I am here assuming the widely held view that dispositions are intrinsic (see Lewis, ‘Finkish Dispositions’, 147–8, for example). McKitrick, ‘Case for Extrinsic’, argues that some dispositions are extrinsic, in the sense of differing among nomic duplicates. On such a view, dispositions are presumably not grounded by intrinsic facts alone. Still, as long as they have a (partly extrinsic) basis, I could run a modified version of my argument.
am aware, there is no body of literature asserting the incompatibility of (1') and (2'), so an analogue to the objection from incompatible premises is not particularly threatening. On the other hand, the commitments of categoricism are less easy to pin down than the commitments of physicalism, so the present argument is more vulnerable in that respect.

The objection from dispositionalism attacks premise (1'). The response, predictably, is similar as before: whether or not categoricalism about dispositions is true, it ought to be compatible with a theory of grounding.

The objection from masking attacks premise (2'). This objection starts from the well-known observation that an object may be disposed to break if struck even though it is not the case that it would break if it were struck. For the disposition may be finkish—prone to disappearing when the stimulus is present—or masked. Since in world $w_5$, striking does not in any way change the basis of the disposition, the falsity of the counterfactual cannot be due to finkishness. According to the objection, however, $g$ is still disposed to break if struck in $w_5$, except that that disposition is masked by the presence of the fifth fundamental force, in the way that the fragility of a vase may be masked by the presence of bubble wrap.

One way to resist this objection would be to argue that masks must be extrinsic to the bearer of the disposition. However, that claim has recently come under sustained pressure. So it is more promising to allow that some masks may be intrinsic, while insisting that the particular case I described is not one of them. If we ascribed to $g$ in $w_5$ the disposition to break if struck, then by analogy, we would need to ascribe to actual objects many dispositions that they do not have. Let $w_3$ a world in which some region duplicates the region occupied by my desk with respect to three fundamental forces, but not with respect to the strong interaction. Then the object occupying that region—whether it counts as a desk or not—is disposed to disintegrate if touched—or indeed, to disintegrate upon any stimulus whatsoever. After all, it is the strong interaction that keeps the protons in the atomic nuclei together, despite their electric repulsion. By reasoning analogous to the one deployed in the objection from masking, we would have to accept that my desk also has the disposition to disintegrate if touched—except that that disposition is masked by the presence of the strong interaction. But clearly, my desk is not disposed to disintegrate if touched, so something must be wrong with the reasoning.

It is difficult to characterize the distinction between cases where an object lacks a disposition and cases where that disposition is masked. One constraint, however, is that masking cases cannot be normal. That is, it cannot be the case that normally, if $x$ is disposed to give response $r$ to stimulus $s$, then that disposition is masked. As is familiar from other contexts, the notion of normalcy need not be cashed out in statistical terms. (According

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25For example in Clarke, 'Opposing Powers'.
to the promising proposal of Bird,\textsuperscript{26} normal interferers—which do not qualify as masks—are those that originate in a design feature or a natural function.) But however it is cashed out exactly, normalcy is a world-relative matter. The fact that it is not normal for the fifth force to operate in this world does not prevent its operation from being normal in \(w_5\). We can easily fill out the description of that world in such a way that the presence of that force is normal. So the objection from masking fails.

The objection from categorialist commitments attacks premise (3\textsuperscript{′}). According to this objection, a categorialist can adopt a more liberal understanding of categorical facts—they need not belong to \(K_g\) as I specified it. In particular, she can include facts involving categorical properties that do not exist in the actual world. Let \(F_5\) be the categorical property with which the fifth force of \(w_5\) is associated. Then the categorical facts also include, for each part \(x\) of \(g\), the fact that \(x\) does not have \(F_5\). Since in \(w_5\), some parts of \(g\) do have \(F_5\), \(w_5\) is not a world where the categorical grounds for \(D_{b,g}\) obtain. Hence the example is consistent with a strict implication between those grounds and the obtaining of the disposition.

I have two responses to this objection. The first is to dispute that the appeal to negative facts about \(F_5\) is compatible with the spirit of categorialism. Typically, categorialists look with suspicion on fundamental facts that involve, in some sense, non-actual worlds, or brute counterfactuals. They hold that facts about non-actual worlds or about what would happen if things were different do not qualify as explainers—only this-worldly facts do. But the fact that \(g\), or one of its parts, does not have \(F_5\) does not appear to be a this-worldly fact in the relevant sense. After all, one of its constituents is the alien property \(F_5\).

The second response concedes, for the sake of the argument, that non-instantiations of alien properties may qualify as categorical grounds, but urges that categorialism, if true, is itself only contingent. This assumption, while controversial, does not beg any questions against Entailment: categorical facts may strictly imply dispositional facts even though the latter can obtain in the absence of the former. If categorialism is only contingent, then the objector is not entitled to assume that it holds in \(w_5\), and that the fifth force is associated with a categorical property. It may be a fundamental dispositional property of the parts of \(g\) to be such as to satisfy a cluster of counterfactuals—roughly, to move in a certain way given a certain regime of other forces.\textsuperscript{27} Let \(F'_5\) be that property. Then clearly, negative facts about \(F'_5\) are not among the categorical facts in the world. For one of their constituents is a property that is not only non-actual, but also non-categorical.

\textsuperscript{26}Bird, ‘Can Dispositions Have Intrinsic Finks and Antidotes?’.
\textsuperscript{27}According to dispositional essentialism, actual fundamental properties are dispositional. For a development and defence of that view, see Bird, \textit{Nature’s Metaphysics}. For discussion of the possibility of such purely modal properties, see Manley, ‘Dispositionality’.
Hence \( w_5 \) is a world where all the actual categorical facts about \( g \) obtain, and the counterexample stands.

A variant of this second response is also effective against the suggestion that the categoricalist can appeal to certain quantified negative facts, such as the fact \( T_4 \) that there are no fundamental properties except \( F_1, F_2, F_3, \) and \( F_4 \). If there are possible non-categorical properties, then this fact is not itself a categorical fact. This follows from a plausible constraint: if the proposition that a given fact \( A \) obtains is not a proposition about the categorical subject-matter, then \( A \) is not a categorical fact. Following David Lewis,\(^{28}\) I am modelling subject-matters as partitions of the class of possible worlds, and take a proposition to be \textit{about} a subject-matter just in case its truth value is a function of what cell the actual world belongs to. In the case of the categorical subject-matter, the cells of the partition are maximal classes of worlds that are duplicates of each other in all categorical respects. The truth value of the proposition that \( T_4 \) obtains can vary within such a cell: it may be true in the actual world and false in a categorical duplicate with fundamental dispositional properties. So \( T_4 \) is not a categorical fact, and cannot be indispensable in grounding dispositional fact, if categoricalism is true.

Clearly, these counterexamples exhibit a general pattern: Entailment fails because in some possible worlds—blocker worlds—there are extra fundamental facts whose obtaining ensures the non-obtaining of certain facts that, in the actual world, are grounded in other facts.

There are quite a few more potential cases of grounding without necessitation. One could argue that facts involving intrinsic properties ground facts involving \textit{maximal} properties—properties such as being a rock that cannot be had by any proper part of one of its bearers—even though they do not strictly imply them.\(^{29}\) Drawing on Dancy,\(^{30}\) one could argue that some deontic facts fail to be strictly implied by their grounds.

Some of these further examples might be more convincing for some readers than those that I have chosen. However, I take it that the strategy for how to undermine Entailment has been sufficiently illustrated, and I will now move on to consider Internality.

III. Against Internality

Internality asserts that grounding facts are contingent only on the obtaining of the relata:

\textbf{Internality} Necessarily, if \( \Gamma < A \) then \( \Box(O(\Gamma) \land O(A)) \rightarrow \Gamma < A \).

\(^{28}\)Lewis, ‘Relevant Implication’.

\(^{29}\)Sider, ‘Maximality and Intrinsic’.

\(^{30}\)Dancy, \textit{Ethics without Principles}.
As I noted in Section I, Internality is logically independent from Entailment. For this reason, the arguments of the previous section do not establish the falsity of Internality. However, my proposed counterexamples to Entailment are also counterexamples to Internality—or so I shall argue.

Suppose again physicalism is true in the actual world, such that some class of physical facts $\Phi$ grounds Red. Let the world $w_{bb}$ be like $w_b$ in so far as it is a physical duplicate of the actual world in which the region corresponding to my brain is suffused with chromaplasm. But unlike in $w_b$, that same region is further suffused with antiplasm—a non-physical fundamental property that works as an antidote to chromaplasm. Since the presence of that property cancels out the effect of the blocker, Red does hold in $w_{bb}$. But intuitively, Red is not grounded by $\Phi$ alone in $w_{bb}$; the fact that the blocker-blocker antiplasm is instantiated plays a crucial role in an account of how Red is grounded in that world. If that intuitive verdict is right, $\Phi$ and Red yield a false instance of Internality: $\Phi$ grounds Red, but it is possible that $\Phi$ does not ground Red even though they both hold.

The argument that the second counterexample to Entailment above also produces a counterexample to Internality proceeds in a similar way. In the actual world, $K(D_b,sg)$ grounds the fact that the glass $g$ is disposed to break. In $w_5$, as we saw, some property associated with a fifth fundamental force stabilizes the glass. In $w_6$, all the fundamental facts of $w_5$ hold, but in addition, there is an alien property $G$ associated with a sixth fundamental force. The distribution of that force just cancels out the fifth fundamental force present in $w_5$ but not $@$. Hence $g$ is disposed to break in $w_6$. But intuitively, facts about the distribution of $G$ play an ineliminable role in an account of how $g$’s disposition is grounded in $w_6$. Since $K(D_b,sg)$ and the dispositional fact obtain in both $@$ and $w_6$, but stand in the relation of grounding only in $@$, they provide a counterexample to Internality.

The two counterexamples follow the same pattern: Internality fails because there is a possible world—a blocker-blocker world—where the grounds, the grounded fact, a blocker as well as a blocker-blocker obtains. As with Entailment, we could produce further potential counterexamples using this pattern.

**IV. Supervenience of Grounding**

So far, my aim has been negative: to show that certain putative principles linking grounding and modality fail. This may help to fuel a certain kind of scepticism about metaphysicians’ talk of grounding. Some philosophers feel that theorists of grounding have not done enough to pin down one concept, and hence failed to give legitimacy to such talk. The rejection of widely accepted modal constraints on grounding may play into their hands.

Friends of grounding may thus be interested in identifying modal constraints on grounding that are compatible with examples of the kind I
presented. In the remainder of the paper, I try to formulate principles that are in the spirit of Internality and Entailment, and yet do not fall foul of the possibility of blockers and blocker-blockers. The proposed principles do not directly involve the notion of necessity, but rather the notion of supervenience. They thus assert a link between grounding and modality broadly construed.

I shall start by formulating a weaker supervenience-theoretic variant of Internality. In the next section, I shall then attempt to do the same for Entailment.

Internality is equivalent to a supervenience claim: any pair of worlds that are indiscernible with respect to the obtaining of $\Gamma$ and $A$ are indiscernible also with respect to whether $\Gamma$ grounds $A$. Another, weaker supervenience claim results from expanding the supervenience base to include all facts, and not just those in $\Gamma$ and $A$. To make this precise, say that $w$ and $w'$ are factually indiscernible just in case every fact obtains in $w$ iff it obtains in $w'$. We then have:

**Supervenience of Grounding** If $w$ and $w'$ are factually indiscernible, then $\Gamma$ grounds $A$ in $w$ iff $\Gamma$ grounds $A$ in $w'$.

This thesis seems to capture at least part of the intuition that motivated Internality: that grounding links are not external relations among facts. When God had created all the facts, He was done creating the world. He did not have to add grounding links among the facts. Grounding relations follow from the identity of those facts, as it were.

Supervenience of Grounding is compatible with the counterexamples to Internality. The world $w_{bb}$ where $\Phi$ does not ground Red differs from the actual world in what facts obtain—chromaplasm is instantiated in $w_{bb}$ but not in the actual world, for example.

It might be thought to be trivially true that grounding supervenes on what facts obtain. For suppose that $\Gamma$ grounds $A$ in $w$ but not in $w'$. Then it would seem that there is a fact, namely, that $\Gamma$ grounds $A$, that obtains in $w$ but not in $w'$. Hence the two worlds are factually discernible. However, in my usage, not every truth corresponds to a fact. Facts are relata of the grounding relation, but instances of that relation do not in turn constitute facts—there is no fact that $\Gamma$ grounds $A$, even if $\Gamma$ does ground $A$.

The non-triviality of the above supervenience is thus hostage to a certain conception of facts. Fortunately, though, this concerns only its letter, not its

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31The equivalence holds under the assumption that all worlds are accessible to each other.
32Fine, ‘Varieties of Necessity’, 244–5, argues that an analogous supervenience principle for natural necessity fails. Presumably, the associated relation of natural grounding also fails to supervene. If so, Supervenience of Grounding articulates another respect in which grounding, on my conception, is different from natural grounding as construed by Fine.
If there are facts about grounding, then the idea behind Supervenience of Grounding needs to be expressed differently. A natural approach would be to take the supervenience base to consist of facts that do not involve grounding—‘non-aetiological facts’, as we might call them. Depending on the details of the theory, these might be the facts which do not have the relation of grounding among their constituents.

So Supervenience of Grounding is not trivial. Still, I expect it to be an uncontroversial constraint on grounding. It is hard to make sense of two worlds with exactly the same facts obtaining in them, but differing in the explanatory connections between those facts.

V. Supervenience of the Grounded on its Grounds

It remains for us to capture the idea that the grounded supervenes on its ground. One way to do is invokes the idea of fundamentality. Say that $A$ is fundamental in world $w$ iff $A$ obtains in $w$, and there is no $\Gamma$ that grounds $A$ in $w$. Say that $w$ and $w'$ are fundamentally indiscernible iff every $A$ that is fundamental in $w$ is fundamental in $w'$, and vice versa. Then the following has some plausibility:

**Supervenience on Fundamentals** If $w$ and $w'$ are fundamentally indiscernible, they are factually indiscernible.

That is, the bottom of the grounding hierarchy fixes all the facts above it. Thanks to Supervenience of Grounding, it will also follow that if the fundamental facts are the same in two worlds, exactly the same grounding claims will be true in both worlds.

To see what kind of possibility is ruled out by Supervenience on Fundamentals, suppose that physicalism is true, and that all facts are grounded by the fundamental physical facts. In particular, my having a red experience is grounded by that class. If Supervenience on Fundamentals holds, there is no possible world whose fundamental facts are exactly the same as in the actual world, but in which those facts ground a fact incompatible with my having a red experience—say my having a green experience. Surely, this is a scenario that we want to rule out.

Note that Supervenience on Fundamentals is compatible with my counterexamples to Entailment. The actual world and $w_b$ are not fundamentally indiscernible, since there are fundamental facts involving chromaplasm in $w_b$ that do not obtain in the actual world.

However, Supervenience on Fundamentals rules out too much in another respect. It is incompatible with the possibility of factually discernible worlds all of whose facts are ‘gunky’. In mereology, a gunky object is one all whose parts have proper parts. In the theory of grounding, a fact is gunky if it is not
fundamental, and no fact that belongs to any of its grounds is fundamental either. There are several models for such facts. If objects are gunky and facts about wholes are determined by facts about their proper parts, then facts about objects will be gunky. On a quasi-Leibnizian view, every property is conjunctively analysable, and so facts involving those properties will presumably be gunky.

If all facts in a world are gunky, then none are fundamental. Any two worlds with only gunky facts are thus vacuously fundamentally indiscernible, by the definition above.

But a general theory of grounding ought not to rule out the possibility of gunky facts. So Supervenience on Fundamentals needs to be modified to be compatible with that possibility. The idea to be retained is that the upper parts of the grounding chains supervene on the lower parts, whether the lower parts are fundamental or not. I propose to cash this out by another supervenience claim. Schematically, it reads as follows:

**Supervenience on Grounds** Any two possible worlds that are ground-indiscernible are factually indiscernible.

It remains to define a notion of ground-indiscernibility. The informal desiderata for the definition are that the resulting instance of Supervenience on Grounds is plausible, and that it captures the idea that the grounded depends on the grounds.

We can lay down some formal desiderata for the relation to be defined, and the instance of Supervenience on Grounds that it yields. First, ground-indiscernibility ought to be entailed by factual indiscernibility together with Supervenience of Grounding. After all, other kinds of differences between worlds, if there be such, are not relevant in this context. Second, ground-indiscernibility ought to be an equivalence relation. For given Supervenience of Grounding and the first desideratum, Supervenience on Grounds entails that worlds are ground-indiscernible if and only if they are factually indiscernible. Since factual indiscernibility is an equivalence relation, ground-indiscernibility must be one as well. Third, ground-indiscernibility ought to entail fundamental indiscernibility, but not vice versa, such that Supervenience on Grounds turns out to be logically weaker than Supervenience on Fundamentals. In particular, it ought to allow that two worlds with only gunky facts are factually indiscernible. Fourth, Supervenience on Grounds ought to be a consequence of Entailment, unlike Supervenience on Fundamentals. My interest here is in finding a plausible fall-back position from Entailment, and I do not wish to smuggle in constraints that do not

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33This characterization is adequate only if, as is standard, a strong transitivity principle for grounding is assumed.

34On this point, I am in agreement with Rosen, ‘Metaphysical Dependence’, 116. For a dissenting view, see Schaffer, ‘Monism’.
follow from the latter claim. The satisfaction of each of these four formal desiderata should be demonstrable from the definition alone, without any substantive assumptions about the space of possible worlds or facts.

The formal desiderata are all satisfied by the relation of factual indiscernibility. But plugging in this relation into Supervenience on Grounds produces a merely trivial claim, which is unable the capture the idea of dependence.

I shall now define a relation of ground-indiscernibility that satisfies both the informal and the formal desiderata. The intuitive idea is this. Suppose we pick a fact \( A \) in \( w \), then one of its grounds in \( w \), then a ground of that latter fact, and so on indefinitely. If whatever our choices, we always either stop with a fundamental fact that also obtains in \( w' \), or else eventually only pick facts that also obtain in \( w' \), then the ultimate grounds, as it were, of \( A \) in \( w \) are all present in \( w' \). If this holds for all facts of \( w \), and conversely for all facts of \( w' \), then \( w \) and \( w' \) are ground-indiscernible.

To make these ideas precise, I shall make use of the concept of a transfinite sequence.

I shall assume, in the following, that all descending grounding chains are set-sized. That is, even though I allow descending grounding chains that are infinitely long, I do not allow ones that are proper class long. Furthermore, I shall assume that even though there may be proper class many facts, the facts that belong to some ground of a given fact \( A \) form a set.\(^{35}\)

Let \( f \) be a sequence of facts, and \( \Gamma \) a class of facts. We say that \( \Gamma \) is an \( s \)-ground of \( f \) in world \( w \) iff there is a non-empty final segment \( f' \) of \( f \) such that \( \Gamma < B \) for all \( B \in f' \). This condition simplifies in the case in which \( f \) has a last element: \( \Gamma \) is an \( s \)-ground of \( f \) just in case \( \Gamma \) grounds that last element.

For a fact \( A \) of \( w \), an \( A \)-sequence \( f \) in \( w \) is a function \( f \) with \( f(0) = A \), and such that \( f(\alpha) \) belongs to some \( s \)-ground of \( f|\alpha \) whenever \( f \) is defined for an ordinal \( \alpha > 0 \). Here, \( f|\alpha \) is the restriction of \( f \) to the domain \( \alpha \), that is, the sequence consisting of all the predecessors of \( \alpha \) in \( f \). Clearly, there are \( A \)-sequences for every fact \( A \), for example the sequence \( \langle A \rangle \) defined only for the ordinal 0.

An \( A \)-sequence \( f \) is maximal in \( w \) iff there is no \( A \)-sequence \( f' \) of which it is a proper initial segment. That is, the maximal \( A \)-sequences are those that are not \( s \)-grounded. It can be shown that there are maximal \( A \)-sequences for every fact \( A \).\(^{36}\)

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\(^{35}\)On the supposition that there are worlds with descending grounding chains of proper class length, it is very plausible that there are factually indiscernible worlds that are ground-indiscernible according to the definition I am about to give. More work would be required to formulate a supervenience constraint on grounding that accommodates infinite descent of proper class length.

\(^{36}\)Sketch of proof: Given our assumptions, the \( A \)-sequences form a set \( S^A_w \), which is partially ordered by the relation that holds between \( f \) and \( g \) if \( f \) is an initial segment of \( g \). Let \( C \) be a chain in that partial order. Then it can be verified that \( \bigcup C \) is a supremum for \( C \) in \( S^A_w \). By Zorn’s Lemma, it follows that \( S^A_w \) has a maximal element.
The sequences just defined can now be used to define ground-indiscernibility. A \( w \)-sequence is an \( A \)-sequence in \( w \), for some \( A \). For an ordinal \( \beta \), a \( w \)-sequence \( f \) is \( \beta \)-convergent in \( w' \) iff \( f' \) with \( f'(\gamma) = \eta f(\beta + \gamma) \) is a maximal \( f(\beta) \)-sequence in \( w' \). Finally, we can complete the definition: \( w \) and \( w' \) are ground-indiscernible iff every maximal \( w \)-sequence is \( \beta \)-convergent in \( w' \), for some ordinal \( \beta \), and every maximal \( w' \)-sequence is \( \beta' \)-convergent in \( w \), for some ordinal \( \beta' \).

So-defined, ground-indiscernibility and Supervenience on Grounds satisfy all four formal desiderata. Pertinent proofs are to be found in the Appendix.

Importantly, Supervenience on Grounds is metaphysically plausible, although certainly non-trivial. It captures, at least partially, the defensible core of the idea that there is a necessary connection between grounds and what they ground.

Acknowledgements

Earlier versions of this paper benefitted from the feedback of audiences in Berlin, Stirling, Brisbane, and Canberra. I would also like to thank Alexander Skiles, Jeff Snapper, and two anonymous referees for written comments that significantly improved this paper. This work was supported by the Arts and Humanities Research Council (grant number AH/J004189/1).

References


Appendix: The formal desiderata

Proposition 1. Given Supervenience of Grounding, factually indiscernible worlds are ground-indiscernible.

Proof. Suppose that w and w’ are factually indiscernible. Let f be any maximal w-sequence. Since w and w’ are factually indiscernible, f(0) obtains in w’. By Supervenience of Grounding, Γ < A in w iff Γ < A in w’. Using this, it is easy to verify that f satisfies the conditions on a maximal f(0)-sequence in w’. Since f(γ) = f(0 + γ), f is 0-convergent in w’. Since f was chosen arbitrarily, we conclude that every maximal w-sequence is 0-convergent in w’, and vice versa, and hence that w and w’ are ground-indiscernible. □
Proposition 2. Ground-indiscernibility is an equivalence relation.

Proof. The symmetry of ground-indiscernibility follows immediately from its definition. Reflexivity holds because every \( w \)-sequence is \( 0 \)-convergent in \( w \). For transitivity, suppose that \( w \) and \( w' \), and \( w' \) and \( w'' \), are ground-indiscernible, respectively. Consider any maximal \( w \)-sequence \( f \). Since \( w \) and \( w' \) are ground-indiscernible, there is some \( \beta \) such that \( f' \) is \( \beta \)-convergent in \( w' \). That is, \( f''(\gamma) = f'(\beta + \gamma) \) is a maximal \( f(\beta') \)-sequence in \( w' \). Since \( w \) and \( w' \) are ground-indiscernible, there is some \( \beta' \) such that \( f'' \) is \( \beta' \)-convergent in \( w'' \). That is, \( f'''(\gamma) = f''(\beta' + \gamma) \) is an \( f(\beta') \)-sequence in \( w'' \). By the associativity of ordinal addition, \( f'''(\gamma) = f'(\beta + (\beta' + \gamma)) = f(\beta + \beta' + \gamma) \). So \( f \) is \( \beta + \beta' \)-convergent in \( w'' \). Since \( f \) was chosen arbitrarily, every \( w \)-sequence is \( \gamma \)-convergent in \( w'' \), for some \( \gamma \). The same argument could be given with the roles of \( w'' \) and \( w \) reversed. So \( w \) and \( w'' \) are ground-indiscernible, establishing transitivity.

Proposition 3. Supervenience on Fundamentals is strictly logically stronger than Supervenience on Grounds.

Proof. We first show that ground-indiscernibility entails fundamental indiscernibility. For suppose that \( w \) and \( w' \) are not fundamentally indiscernible. Without loss of generality, we can assume that some fact \( w \) is fundamental in \( w \) but not in \( w' \). Consider the unique \( A \)-sequence \( f \) in \( w \), whose domain is the ordinal 1. Clearly \( f \) is maximal. Since \( A \) is not fundamental in \( w' \), \( f \) is not a maximal \( w' \)-sequence. Hence \( f \) is not \( 0 \)-convergent in \( w' \); and since it stops after \( f(0) = A \), it fails to be \( \beta \)-convergent in \( w' \) for any ordinal \( \beta \). It follows that \( w \) and \( w' \) are not ground-indiscernible. However, the converse does not hold. Let world \( w \) consist of the denumerably many facts \( A_0, A_1, \ldots \), with \( A_{i+1} < A_i \), for all integers \( i \), and world \( w' \) of the denumerably many facts \( B_0, B_1, \ldots \), with \( B_{i+1} < B_i \) and \( A_i \neq B_j \) for all integers \( i \) and \( j \). Then \( w \) and \( w' \) are fundamentally indiscernible, since neither contains any fundamental facts. However, \( f \), defined on \( \omega \), with \( f(i) = A_i \), is a maximal \( w \)-sequence that fails to be \( \beta \)-convergent in \( w' \) for any ordinal \( \beta \).

Proposition 4. Entailment entails Supervenience on Grounds.

Proof. Assume that Entailment holds, and that \( w \) and \( w' \) fail to be factually indiscernible. Suppose, without loss of generality, that some fact of \( w \) does not obtain in \( w' \). We shall define a maximal \( w \)-sequence \( f \) such that \( f(\beta) \) does not obtain in \( w' \), for any \( \beta \). This sequence fails to be \( \beta \)-convergent in \( w' \) for any \( \beta \), and hence \( w \) and \( w' \) are not ground-indiscernible.

Let \( D^w_w \) be the class of facts that obtain in \( w \) but not in \( w' \). Fix \( A \in D^w_w \), and say \( \sigma \) is a \( D^w_w \)-sequence if it is either empty or an \( A \)-sequence in \( w \) all of whose elements are in \( D^w_w \). Define a function \( h \) on the class of \( D^w_w \)-sequences as follows: \( h(g) = \{ \langle g, A \rangle \} \) if \( g \) is not \( s \)-grounded in \( w \); otherwise \( h(g) = \{ \langle g, B \rangle : B \in D^w_w \text{ and } B \in \Gamma \}, \text{ for some } \Gamma \text{ that is an } s \text{-ground of } g \text{ in } w \}. \) Clearly, the members of \( \{ h(g) : g \text{ a } D^w_w \text{-sequence } \} \) are pairwise disjoint. We now show that all of them are non-empty. Case (i): There is no \( s \)-ground of \( g \) in \( w \). Then \( \langle g, A \rangle \in h(g) \). Case (ii): \( \Gamma \) is an \( s \)-ground of \( g \) in \( w \). Then there is some \( B \) in some final segment of \( g \) such that \( \Gamma < B \) in \( w \). By Entailment, \( B \) holds in every possible world where all members of \( \Gamma \) hold. Since \( B \in D^w_w \), some member \( C \) of \( \Gamma \) is also in \( D^w_w \). So \( \langle g, C \rangle \in h(g) \), and \( h(g) \) is therefore non-empty. By the Axiom of Choice, there is a class that contains exactly one representative from each member of \( \{ h(g) : g \text{ a } D^w_w \text{-sequence } \} \). Since this is a class of ordered pairs, we take it to be a function \( h' \) that maps a \( D^w_w \)-sequence to a member of \( D^w_w \).
We now use recursion to define a sequence by $f^\prime(0) = A$ and $f^\prime(a) = h'(f^\prime \upharpoonright a)$. As a class of ordinals, $s_A = \{ \alpha : f^\prime(\alpha) = A \text{ and } \alpha > 0 \}$ is either empty or has a least member $\gamma$. Let $f$ be $f'$ if $s_A$ is empty, and $f' \upharpoonright \gamma$ if it is not.

It is now straightforward to verify that $f$ is a maximal $A$-sequence in $w$, and that all elements of $f$ are in $D_w^\psi$. ■