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Abstract

This paper analyses wage inequality and the welfare effects of changes in capital and labour income tax rates for different types of agents. To achieve this, we develop a model that allows for capital-skill complementarity given non-uniform distributions of asset holdings and labour skills. We find that capital tax reductions lead to the highest aggregate welfare gains but are skill-biased and thus increase inequality. However, our analysis also shows that the inequality effects of capital tax reductions are lower over the transition period compared with the long-run.

Key words: tax reforms, skill premium, inequality

JEL codes: E25, E62
Introduction

The literature on tax reforms in dynamic general equilibrium (DGE) models largely focuses on the aggregate welfare benefits and the distributional consequences of permanent reductions in capital tax rates. Studies within the representative agent framework suggest that tax reforms which reduce capital taxation will produce welfare gains for the society, even if the tax burden is concurrently shifted to labour (see e.g. Lucas, 1990, Cooley and Hansen, 1992 and, more recently, Angelopoulos et al., 2012, for the UK). The aggregate welfare benefits from tax reforms that reduce capital taxation can also be obtained in models with heterogeneous agents. However, these models make clear that such reforms can have large redistributive effects that will disadvantage different groups in the society (see e.g. Domeij and Heathcote, 2004, Greulich and Marcet, 2008, and Garcia-Milà et al., 2010).

This literature has considered different types of market incompleteness to demonstrate the distributional effects of tax reforms and, in particular, capital tax cuts. An important dimension in which agents differ, that is central to the analysis of capital tax reforms, is inequality in the distribution of assets or wealth. A common approach to modeling this type of heterogeneity is to assume that some agents do not have access to the capital markets, or more generally, that some agents depend more on labour relative to capital income (see e.g. Judd, 1985, Lansing, 1999, Krusell, 2002, and Garcia-Milà et al., 2010).

In such environments, agents whose capital income is significant can expect to gain after a capital tax cut. However, the total effects of a capital tax cut are not as clear for those agents who depend predominantly on labour income, usually termed as the workers. There are costs to workers from a capital tax cut, if this is accompanied by an increase in labour taxes. Nevertheless, there can also be benefits that take the form of increased labour productivity delivered by the increase in the capital stock. Therefore, to evaluate the distributional effects of capital tax cuts,
both the productive role of capital and its complementarity with labour need to be explicitly examined.

This complementarity between capital and labour becomes particularly important when the economic structure suggests a distribution in the skill supply of the labour force, in addition to the asset distribution, and even more so when these distributions are positively related. For instance, the PSID data (see e.g. Table 2 in Garcia-Milà et al., 2010) suggest that high wealth is positively related to higher wages, while evidence from the UK, discussed further below, suggests that skill acquisition, in the form of University education, is related to socioeconomic income group.

When the production structure exhibits capital-skill complementarities as suggested by e.g. Krusell et al. (2000), and Hornstein et al. (2005), capital-augmenting policies are expected to be skill-biased and thus increase the wage premium and inequality in the long-run. However, the dynamic effects on wage inequality in an environment where the distribution of assets is not uniform are more difficult to predict.

The importance of the transition period has also been highlighted in the welfare evaluation of tax reforms literature. This is because the benefits associated with the capital tax cut, in the form of higher labour productivity, tend to materialise for the worker many years after the reform (see e.g. Greulich and Marcet, 2008, and Garcia-Milà et al., 2010). However, when the production structure exhibits capital-skill complementarities, the timing of the effects on capital tax cuts on wage inequality will depend on the evolution of the complementarities of capital with the different types of labour over time.

With the above background in mind, this paper aims to analyse wage inequality and the welfare effects of changes in income tax rates for different types of agents, in a model that allows for capital-skill complementarity, given non-uniform distributions of asset holdings and labour skills. While the literature has allowed for joint inequality in asset holdings and labour
productivity in evaluating capital taxes (see e.g. Domeij and Heathcote, 2004, Conesa et al., 2009, and Garcia-Milà et al., 2010), our main interest is on the importance of the capital-skill complementarity under such joint distributions. This allows us to examine the post-reform evolution of wage inequality that is driven by an endogenous skill premium, and evaluate its contribution in determining the overall inequality effects of a given tax reform.

We focus on capital tax cuts, but also consider the effects of cutting labour taxes. To isolate the effects of changes in each tax rate on all agents, we consider changes in tax rates that are not revenue-neutral. Instead, given its current policy relevance, we consider tax reforms consistent with a lower steady-state debt-to-GDP ratio. Moreover, to focus on the interaction of asset and skill heterogeneity with a production structure that allows for different capital-labour complementarities, we also abstract from other sources of heterogeneity that have already attracted a lot of interest in the literature (e.g. stochastic or unobservable ability).

We calibrate our model to the UK economy to assess the likely costs and benefits of tax reforms for the different agents. The UK is used to illustrate the quantitative analysis since the data suggest significant heterogeneities in both asset holdings and skills in the labour supply, which are also generally positively correlated. According to the Family Resources Survey in 2008-2009, 28% of households do not have any savings, 53% have savings up to £20,000 and 19% have savings above £20,000.\(^5\) Moreover, the Labour Force Survey of the Office for National Statistics,\(^6\) suggests that in 2003, 28% of the working population was employed in low-skill, semi-routine and routine occupations, whereas the remaining share worked in supervisory, technical, professional and managerial occupations. There is also support for associating skill with income group. For example, data from the Department for Education and Skills on the participation rates in higher education for different income groups show that the participation ratio was about three times higher in the 1990s for the three highest, relative to the three lowest groups.\(^7\) Finally, the tax structure in the UK stands in stark contrast with other European countries, by having
a very high capital to labour income tax ratio.\textsuperscript{8}

Our modeling permits us to capture key features of heterogeneity. Following the literature on credit constraints and income inequality (see e.g. Galor and Zeira, 1993, Benabou, 1996, and Aghion and Howitt, 2009), financial intermediation costs allow our model to generate heterogeneity in savings, which is consistent with the UK data. In addition, we use a constant elasticity of substitution (CES) specification for the production function, following e.g. Krusell et al. (2000), which assumes different degrees of complementarity with capital for skilled and unskilled labour. This allows our calibrated model to produce a wage premium that is in line with empirical studies.

Our analysis shows that while capital tax cuts lead to the highest aggregate welfare gains, the capital-skill complementarity amplifies inequality effects. The additional distributional effects that work through the structure of production and, more specifically, the complementarity between labour and capital materialise since capital tax reductions are skill-biased and thus increase the skill premium and wage inequality (see also e.g. He and Liu, 2008).

However, our analysis also shows that the inequality effects of capital tax reductions are lower over the transition period compared with the long-run. This is because the skill premium initially falls and then converges to the higher post-reform levels. This result is driven by the fact that, in general equilibrium, the complementarity between capital and labour inputs is higher in the short- than in the long-run. In particular, following the capital tax cut, the relative skill supply increases, as the agents that hold the capital stock find it optimal to increase labour income in order to increase investment in capital. The initial increase in relative skill supply decreases the skill premium. However, as the capital stock rises and the relative skill supply falls over time, the skill premium converges to a higher, post-reform level.

The rest of the paper is organised as follows. Section I presents the model which is calibrated and solved at the steady-state in Section II. Section III discusses the main results and Section
IV concludes.

\section{Model}

In this section we construct a closed-economy DGE model comprised of a representative capitalist and representative skilled and unskilled workers who all consume output in the product market and supply labour in the factor market in return for labour income. The first two income groups, subject to intermediation costs, allocate savings to physical capital and government bonds in return for capital income whereas unskilled workers do not save. The representative firm is owned by the capitalist who hires (skilled and unskilled) labour services and leases physical capital from the factor market for which it pays the competitive wage and interest rate, respectively. Finally, the government taxes economic activity, provides public spending and issues debt to balance its budget.

\subsection{Population composition}

The population size, $N$, is exogenous and constant. Among $N$, $N^c < N$ are identical capitalists, $N^s < N$ are identical skilled workers, and the rest, $N^u = N - N^c - N^s$, are identical unskilled workers. These three groups of agents are assumed to exogenously differ in their participation in asset markets and in the type of labour services they can offer. In other words, we do not allow for social mobility.\footnote{} Capitalists are indexed by the subscript $c = 1, 2, ..., N^c$, skilled workers by $s = 1, 2, ..., N^s$ and unskilled workers by $u = 1, 2, ..., N^u$. There are also $N^f$ firms, $f = 1, 2, ..., N^f$. We assume that the number of firms equals the number of capitalists, $N^c = N^f$, and that each capitalist owns one firm. It is useful, for what follows, to define $N^c/N = n^c$, $N^s/N = n^s$, $N^u/N = n^u = 1 - n^c - n^s$ and $N^f/N = n^f$. 
I.2 Firms

Each firm produces a single output, \( Y^f_t \), using physical capital, \( K^f_t \), and two distinct types of labour, unskilled, \( h^u_{t,t} \), and skilled, \( h^s_t \), where skilled labour is relatively more complementary to capital than unskilled labour. The production function is given by a constant returns to scale (CRS) technology assumed to take a constant elasticity of substitution (CES) specification following e.g. Krusell et al. (2000) and He (2012):

\[
Y^f_t = A \left\{ \mu \left( h^u_{t,t} \right)^{\alpha} + (1 - \mu) \left[ 1 - \mu \right] K^f_t \right\}^{\frac{1}{\gamma}}
\]

where \( A > 0 \) is constant productivity; \( 0 < \alpha, \nu < 1 \), are the parameters determining the factor elasticities, i.e. \( 1/(1 - \alpha) \) is the elasticity of substitution between capital and unskilled labour and between skilled and unskilled labour, whereas \( 1/(1 - \nu) \) is the elasticity of substitution between capital and skilled labour; and \( 0 < \mu, \rho < 1 \) are the factor share parameters. The above CES form allows us to capture the notion of capital-skill complementarity, which is considered to be a main driver of the skill premium and wage inequality (see e.g. Krusell et al., 2000, and Hornstein et al., 2005).

Each firm acts competitively, taking prices and policy variables as given, and maximises profits:

\[
\Pi^f_t \equiv Y^f_t - r^k_t K^f_t - w_t h^s_t - w^u_{t,t} h^u_{t,t}
\]

subject to the technology constraint, (1); where \( w_t \) and \( w^u_{t,t} \) are, respectively, the wage rates of skilled and unskilled labour and \( r^k_t \) is the interest rate on capital.\(^{10} \) The different roles in the production function for skilled and unskilled labour imply that there will be a skill premium for the former, in the sense that the ratio of \( w_t \) to \( w^u_{t,t} \) will be larger than unity. We will calibrate the production function so that the implied factor input elasticities and the resulting wage premium
are in line with empirical studies.

I.3 Budget constraints of capitalists

The representative capitalist owns one firm and receives its profits. He also receives income from providing skilled labour services, $h_{c,t}$, to the labour market and income from interest on his accumulated stock of financial assets, in the form of capital, $K_{c,t}$, and government bonds, $B_{c,t}$.

The interest rate on government bonds is given by $r^b_t$. All these sources of income are taxed. In particular, financial asset and profit income are taxed at the constant rate $\tau^k$, while labour income is taxed at the constant rate $\tau^h$.

We assume that those agents holding assets need to pay intermediation or transaction premia due to imperfections in capital markets. For instance, these premia can represent the costs of gathering extra information relating to legal issues, asset-specific government regulations and intermediation fees. We follow Persson and Tabellini (1992) and Benigno (2009), and assume a quadratic cost function such that the capitalist incurs a cost of $\varphi^b_c K^2_{c,t}$ for holding physical capital and of $\varphi^b_c B^2_{c,t}$ for holding government bonds, where $\varphi^b_c, \varphi^k_c > 0$ measures the size of the transaction costs. The presence of this capital market imperfection and of the associated transaction costs help the model to capture a feature of realism. However, their main contribution here is that they will allow us, as we shall see below, to capture household heterogeneity in asset holdings.

The capitalist uses his income for consumption, $C_{c,t}$, investment in capital, $I_{c,t}$, and investment in government bonds, $D_{c,t}$. He also receives average (per agent) transfers from the government, $G_t (= G_t/N)$. Thus, his budget constraint is:

$$C_{c,t} + I_{c,t} + D_{c,t} = (1 - \tau^h) (r^k_{t} K_{c,t} + r^b_{t} B_{c,t}) +$$

$$+ (1 - \tau^h) \Pi^f_t + (1 - \tau^h) w_t h_{c,t} + G_t - \varphi^b_c B_{c,t}^2 - \varphi^k_c K_{c,t}^2$$

while the evolution equations of the stock of capital and government bonds, respectively, are
given by:

\[ K_{c,t+1} = (1 - \delta) K_{c,t} + I_{c,t} \] (4)

\[ B_{c,t+1} = B_{c,t} + D_{c,t} \] (5)

where \( 0 < \delta < 1 \) is the depreciation rate and \( K_{c,0}, B_{c,0} > 0 \) are given.

### I.4 Budget constraints of skilled workers

The problem of the skilled worker is similar to the capitalist’s, except that he pays different transaction costs, so that the capital market imperfections affect him to a greater extent. We assume that firm ownership gives an insider advantage in financial transactions to the capitalist (due, for instance, to past experience, socioeconomic background and networks) and thus the size of the transactions cost is lower for the capitalist. The idea that capital market imperfections can explain heterogeneity has been extensively examined in the income inequality literature (see e.g. Galor and Zeira, 1993, Benabou, 1996, and Aghion and Howitt, 2009). Most of these models assume, for simplicity, that the intermediation cost is either infinite for some agents (and thus these agents are effectively excluded from the financial market) or zero. In this paper, we examine the case of non-zero, finite intermediation costs for both capitalists and skilled workers where \( \varphi^b_{c} < \varphi^b_{s} \) and \( \varphi^h_{c} < \varphi^h_{s} \).

Accordingly, the budget constraints and the evolution equations for capital and government bonds for the \( s^{th} \) skilled worker are:

\[ C_{s,t} + I_{s,t} + D_{s,t} = (1 - \tau^h) (r^h_t K_{s,t} + r^h_t B_{s,t}) + (1 - \tau^h) w_s h_{s,t} + \Theta_t - \varphi^b_{s} B_{s,t}^2 - \varphi^k_{s} K_{s,t}^2 \] (6)

\[ I_{s,t} = K_{s,t+1} - (1 - \delta) K_{s,t} \] (7)
\[ D_{s,t} = B_{s,t+1} - B_{s,t}. \]  

I.5 Budget constraint of unskilled workers

Unskilled workers differ from capitalists and skilled workers in two important respects. First, capital market imperfections result in them being excluded from the financial markets as in the models of Benabou (1996), and Aghion and Howitt (2009). Second, we assume that exclusion from capital markets does not allow them to acquire the skills to provide skilled labour services, so that their labour effort differs, in nature, from the labour effort of the other two types of agents. Evidence from the UK, introduced later, suggests that skill acquisition, in the form of University education, is related to socioeconomic income group.

Thus, the budget constraint of the \( u \)th unskilled worker is:

\[ C_{u,t} = (1 - \tau^u) w_{u,t} h_{u,t} + G_t \]  

where \( 0 < \tau^u < 1 \) is the tax rate on unskilled labour, \( h_{u,t} \) is the labour supply and \( C_{u,t} \) is consumption.

I.6 Utility function and optimal choices of agents

Each type of household \( i = c, s, u \) maximises:

\[ \sum_{t=0}^{\infty} \beta^t u(C_{i,t}, h_{i,t}) \]  

where \( \beta \) is the subjective time preference, subject to the relevant budget constraints given above.
We use the instantaneous utility function:

$$u_{i,t} = (C_{i,t}, h_{j,t}) = \left(\frac{(C_{i,t})^\gamma (1 - h_{i,t})^{1-\gamma}}{1 - \sigma}\right)^{1-\sigma}$$

where $0 < \gamma < 1$ is the consumption weight in utility and $\sigma > 1$ is the coefficient of relative risk aversion.

To maximise discounted utility, the representative capitalist chooses $\{C_{c,t}, h_{c,t}, K_{c,t+1}, B_{c,t+1}\}_{t=0}^\infty$ subject to (3 – 5), the representative skilled worker chooses $\{C_{s,t}, h_{s,t}, K_{s,t+1}, B_{s,t+1}\}_{t=0}^\infty$ subject to (6 – 8) and the representative unskilled worker chooses $\{C_{u,t}, h_{u,t}\}_{t=0}^\infty$ subject to (9).

I.7 Government budget constraint

Following the literature on tax reforms (see e.g. Lucas, 1990, Cooley and Hansen, 1992, Giannitsarou, 2006, Garcia-Milà et al., 2010, and Angelopoulos et al., 2012), we do not model government spending. Instead, government expenditure takes the form of transfers to the private agents, $G_t$. To finance these, it taxes income from labour and financial assets and issues government bonds, $B_t$. The budget constraint of the government is thus given by:

$$G_t + (1 + r^h_t)B_t = B_{t+1} + N^c[\tau^k (r^k_t K_{c,t} + r^h_t B_{c,t}) + \tau^h w_t h_{c,t}] +$$

$$+ N^s[\tau^k (r^k_t K_{s,t} + r^h_t B_{s,t}) + \tau^h w_t h_{s,t}] + N^u[\tau^u w_u h_{u,t}] +$$

$$+ N^p[\tau^p w_p h_{p,t}].$$

(12)

I.8 Market-clearing conditions

The market clearing conditions for the capital, bond, skilled and unskilled labour and product markets respectively are:

$$N^f K^f_t = N^c K_{c,t} + N^s K_{s,t}$$

(13)
\[ B_t = N^c B_{c,t} + N^s B_{s,t} \]  
\[ N^f h_t^f = N^c h_{c,t} + N^s h_{s,t} \]  
\[ N^f h_{u,t}^f = N^u h_{u,t} \]  
\[ N^f Y_t^f = N^c C_{c,t} + N^s C_{s,t} + N^u C_{u,t} + N^c [K_{c,t+1} - (1 - \delta) K_{c,t}] + \]  
\[ + N^s [K_{s,t+1} - (1 - \delta) K_{s,t}] + N^c (\varphi^b c B_{c,t}^2 + \varphi^b c K_{c,t}^2) + \]  
\[ + N^s (\varphi^b s B_{s,t}^2 + \varphi^b s K_{s,t}^2) \]  

where (17) gives the aggregate resource constraint of the economy.

I.9 Decentralised competitive equilibrium

The decentralised competitive equilibrium (DCE) is defined when (i) households and firms optimise, taking prices and policy as given; (ii) all constraints are satisfied; and (iii) all markets clear. After the relevant substitutions, we summarise the DCE in the paths of the following variables: \( \{C_{c,t}, C_{s,t}, C_{u,t}, h_{c,t}, h_{s,t}, h_{u,t}, w_t, w_{u,t}, K_{c,t+1}, K_{s,t+1}, K_{u,t+1}, B_{c,t+1}, B_{s,t+1}, r^k_t, \tau^b_t, \tau^h_t, \tau^u_t \} \) given the remaining fiscal policy instruments, i.e. taxes \( 0 \leq \tau^k, \tau^h, \tau^u < 1 \) and government transfers, \( G_t \equiv G \), which are all held constant to focus on permanent changes in tax rates. We define the relevant aggregate, economy-wide quantities as, \( X_t \), for \( X_t = \{C_t, I_t, K_t, B_t, Y_t\} \).

II Calibration and steady-state

In Table 1, we next calibrate the structural parameters of the model so that its steady-state solution, reported in Table 2 below, reflects the main empirical characteristics of the UK economy.
The calibration also provides empirical justification for the key modelling decisions made above.

II.1 Population shares

We first wish to map out agent heterogeneity and thus distinguish the three types of households by their differing shares in the population, \( n^i \). According to the Family Resources Survey in 2008-2009, 28% of households do not have any savings, 53% have savings up to £20,000 and 19% have savings above £20,000. In light of this, since we assume that unskilled workers do not have savings, we set \( n^u \) equal to 30%. At the other end of the distribution, since we model capitalists as the income group with the highest share of savings and assets, we set \( n^c \) to 20% implying that \( n^s \) is 50%.

Other data providing an additional dimension by which unskilled workers differ from skilled workers and capitalists is that the former group offers a labour input that is lacking in skills. According to the Labour Force Survey of the Office for National Statistics, in 2003, 28% of the working population was employed in semi-routine and routine occupations, whereas the remaining share worked in supervisory, technical, professional and managerial occupations, which require an increasingly higher skilled labour input. Moreover, according to data from the Department for Education and Skills on the participation rates in higher education for different income groups, the participation ratio was about three times higher in the 1990s for the three highest, relative to the three lowest groups. Thus, there seems to be adequate support for associating skill with income group.

[Table 1 about here]

II.2 Productivity

We next turn to heterogeneity in productivity and returns to labour, which governs the choice of the relevant production parameters. Using the estimates in Krusell et al. (2000), we set
\( \nu = -0.495 \) and \( \alpha = 0.401 \) implying elasticities of substitution between capital and skilled labour and between capital (or skilled labour) and unskilled labour of about 0.67 and 1.67 respectively.\(^{13}\) Our calibration of \( \rho = 0.645, \mu = 0.275 \) and \( A = 1.65 \) allows us to obtain the labour share of income, the skill premium and the capital to output ratio consistent with the UK data.

**II.3 Savings**

Heterogeneity in savings is controlled for, as explained in the previous section, by the parameters that govern transaction costs in the financial markets. Following the models in e.g. Galor and Zeira (1993), Benabou (1996) and Aghion and Howitt (2009), we set these costs to infinity for the unskilled workers, which implies that these agents do not have any savings. As said above, about 28% of the UK households do not save. Regarding the households with positive savings, data from the Family Resources Survey of 2008-2009 suggest that households in the highest saving bracket have about five times higher savings than the other savers, on average. In terms of our model, this difference is applied to the representative capitalist and skilled worker by setting the ratio of transaction costs faced by each of these agents to be equal to 5. For simplicity, we set this cost in capital asset markets to be the same in the bond market. It is worth noting that we choose the level of the transaction costs parameter, so that in combination with an annual depreciation rate, \( \delta \), of 6%, the total ratio of capital to GDP in the steady-state is about 2 and that the transaction costs are about 1% of asset holdings. The latter is broadly consistent with the average difference between the lending and borrowing rates in the UK (see, e.g. World Development Indicators - WDI - database) over the past 30 years.

**II.4 Effective tax rates**

Effective average tax rates for capital and labour income are constructed by following the approach in Conesa et al. (2007). We use data from the National Accounts and the Public Sector,
Taxation and Market Regulation databases (available from OECD Statistics), to obtain the data for 1970-2005. The average capital tax rate over the time period is about $k = 0.44$, while the average labour income rate is around 0.27. Using data from Social Trends 38, Office for National Statistics, we are able to approximate the progressivity of the UK income tax system at about 1.6.\textsuperscript{14} A ratio of $\tau^h / \tau^u = 1.6$, together with the requirement that the weighted average of the two tax rates equals the effective labour income tax rate, would imply that $\tau^h = 0.304$ and $\tau^u = 0.19$. However, the progressivity of income taxation probably overestimates the progressivity of labour income taxation, which is our interest here. This is because, in light of the data discussed, we would expect the higher income brackets to have more capital income compared to the lower income brackets. On the other hand, the lower the progressivity ratio, the higher the implied value of $\tau^u$. We thus use a progressivity ratio of $\tau^h / \tau^u = 1.5$ for the calibration, which guarantees that $\tau^u$ is equal to the base income tax rate. Accordingly, we approximate the lower tax rate, $\tau^u$, at 20%, and the higher labour income tax rate, $\tau^h$, at 30%.

II.5 Parameters common to all agents

We next approximate the rate of time preference, $\beta$, so that $1/\beta$ is equal to 1 plus the ex-post real interest rate, where we use real interest rate data from OECD Main Economic Indicators, from 1970-2005. This gives a value of 0.976 for $\beta$. Following Kydland (1995), we set $\gamma$, the weight given to consumption relative to leisure in the utility function, equal to the average value of work versus leisure time, which is obtained using the data on hours worked from the OECD Economic Outlook database, from 1970-2005.\textsuperscript{15} We also use a common value from the literature for the intertemporal elasticity of consumption, $1/\sigma = 0.5$ or $\sigma = 2$.

Given that we will evaluate policies that reduce the debt-to-GDP ratio below, we calibrate the share of government spending in GDP, $G/Y = 31\%$, to obtain a $B/Y$ ratio of 70% based on official forecasts for 2011-2013 (see e.g. the Pre-Budget Forecast, June 2010, Office for Budget
II.6 Steady-state

The steady-state solution of the model is given in Table 2 below in terms of the aggregate variables. The figures show that the capitalists consume in total 19.4% of total income (or about 23% of total consumption), skilled workers consume in total 44% of total income (or around 52% of total consumption) and unskilled workers consume in total 22.3% of total income (or approximately 26% of total consumption). In addition, the capitalists in total own about 67% of the capital and government bonds in the economy. As said above, the ratio of savings, $I_c/I_s$, and assets, $K_c/K_s$ and $B_c/B_s$, of the representative capitalist to the representative skilled worker, is equal to five. Note also that the net (after depreciation, tax and transaction costs) interest rates on capital and bonds, are given respectively by:

$$\tilde{r}_k = r^b(1 - \tau^k) - \delta - 2\phi_c^b \left( \frac{n_c}{n_c + n_s} \right) K_c - 2\phi_s^b \left( \frac{n_s}{n_c + n_s} \right) K_s$$  \hspace{1cm} (18)

$$\tilde{r}_b = r^b(1 - \tau^b) - 2\phi_c^b \left( \frac{n_c}{n_c + n_s} \right) B_c - 2\phi_s^b \left( \frac{n_s}{n_c + n_s} \right) B_s$$  \hspace{1cm} (19)

and are equal in the steady-state. Table 2 shows that these returns are 2.5%, which compares favourably with the 1970-2005 average UK real interest rate, equal to 2.1% in the WDI database.

[Table 2 about here]

It is next worth noting that the ratio of average hours worked by unskilled workers to the average hours worked by skilled labour in the model is 1.1, which is the same as in the UK Labour Force Survey (LFS) data. These work-time allocations imply Frisch (or $\lambda$-constant) labour supply elasticities of 3.97 for capitalists, 2.18 for skilled and 2.19 for unskilled workers, which are...
similar to values calibrated in macro models (see e.g. Browning et al., 1999, Chetty et al., 2011, and Keane and Rogerson, 2012, for a discussion regarding micro and macro elasticities). The value for the capitalists suggests that, as expected, this group is the relatively least dependent on labour income, and is consistent with the research in Low (2005) and Domeij and Floden (2006), which suggests that agents without full access to asset markets prefer increased work hours. Table 2 also shows that the labour’s share of income in the model, 
\[ \frac{w(N^c h_c + N^s h_s) + w_u N^u h_u}{Y} = 0.633 \]
is close to the value (i.e. 0.601) obtained from the OECD’s International Sectoral Database (ISDB) for 1970-2005.

Turning to the skill premium in the UK, Walker and Zhu (2008), estimate a college premium (defined as the log difference between the wage rate for skilled and unskilled labour) of about 18% for males and 28% for females, while Machin (1996), computes the ratio of wages between non-manual and manual jobs in manufacturing that ranges between 1.3 and 1.5, from 1970 to 1990. For the US, Hornstein et al. (2005) report a college premium, in terms of wage ratios, that ranges from about 1.47 in 1982 to 1.79 in 2000. The skill premium obtained predicted by our calibration is \( w/w_u = 1.37 \) or \( \ln(w/w_u) = 31\% \). Finally, note that in the steady-state \( C_c = 0.439, C_s = 0.402 \) and \( C_u = 0.331 \). Thus in terms of welfare, \( U \), higher consumption and lower work effort make the capitalists better off, followed by the skilled and unskilled workers, respectively. The weighted average measure of aggregate or Benthamite utility, \( U_a \), is also reported.\textsuperscript{19}

## III Tax reforms

In this section, we examine five different tax reforms that meet a debt-to-GDP target of 60% in the steady-state. The latter provides us with a common base for conducting the policy reforms.\textsuperscript{20} We start by changing the capital income tax rate, \( \tau^k \), holding all other rates constant. Next we examine changes in the labour income tax rates, first on skilled labour, \( \tau^h \), and second on
unskilled labour, $\tau^u$, each implying that the progressivity of labour income taxation has been altered. We then examine the case where the government changes the effective average labour tax rate, i.e. $\tau^h$ and $\tau^u$ move proportionately, so that the progressivity in the labour income taxation remains unaffected. Lastly, we evaluate the distributional effects of varying all tax rates proportionately.

For each tax reform considered, we first find the steady-state tax rate(s) required to obtain the target debt-to-GDP ratio and then impose these new tax rates on the pre-reform economy to obtain the dynamic paths post-reform. Following, for example, Giannitsarou (2006) and Ardagna (2007), to obtain the dynamic solution of the model and simulate the transition paths after the tax reform, we use a log-linear approximation to the DCE described above. We calculate post-reform welfare, conditional on the initial, pre-reform steady-state, for each type of agent and for the aggregate economy, using the discounted utility expression in (10). This is initialised from the pre-reform steady-state and thus includes the transition period from the pre- to the post-reform steady-state. In contrast, post-reform steady-state welfare excludes the transition path.

### III.1 Laffer curves in tax revenue and debt

Prior to undertaking the welfare analysis, it is first useful to demonstrate the general equilibrium effects of tax changes on factor returns and quantities by examining the impact of tax changes on the tax revenue from all tax bases. The relationship between the tax revenue from a particular tax base and the associated tax rate is, in general, given by a Laffer curve (see e.g. Schmitt-Grohé and Uribe, 1997, and Trabandt and Uhlig, 2012). In our model, changing a tax rate can lead to either increases or decreases in the tax revenue collected from this tax base, depending on whether the economy is on the upward or downward sloping part of the curve, respectively. In the CES production function with capital-skill complementarity that we employ, a tax rate
change will have spillover effects to the tax revenue collected from the other tax bases. For instance, an increase in the capital tax rate will decrease the capital supply, but will tend to increase or decrease the supply of unskilled labour, depending on whether the latter substitutes for or complements capital in production. Thus, the tax revenue collected from the tax base of unskilled labour income can either rise or fall after the increase in the capital tax.

As an illustration, we plot the Laffer curves associated with changes in $\tau^k$ in Figure 1. The $B/Y$ curve (lower-right panel) indicates that the target for the debt-to-GDP ratio can be obtained by either increasing or decreasing $\tau^k$ to 65.3% or 40.7%, respectively. In the remaining subplots in Figure 1, we normalise the current level of tax revenue and debt to 100, to depict the predicted changes brought about by changes in the tax rate. The relationship between tax revenue from assets and the capital tax rate (upper-right panel) shows that the economy is on the upward slopping part of this Laffer curve. Increasing $\tau^k$ increases the tax revenue collected from capital, while falls in $\tau^k$ decrease tax revenue from this source. However, the upper-left and upper-middle panels in the Figure suggest that decreases in $\tau^k$ crowd-in both skilled and unskilled labour and, accordingly, the tax revenue from these sources increases.

The UK economy appears to be near the peak of the total tax revenue and debt Laffer curves with respect to $\tau^k$. The results for the tax revenue Laffer curve are very similar to those reported in Trabandt and Uhlig (2012) for the UK. In particular, both models predict for this economy that the gain in tax revenue by increasing the capital tax to the point where the tax revenue is maximised is only a few percentage points.$^{22}$

[Figure 1 about here]

The results for the Laffer curves associated with the remaining tax instruments are, in general, similar and indicate that, also consistent with Trabandt and Uhlig (2012), the UK is closer to
the peak of the capital tax Laffer curve, compared to the labour taxes Laffer curves. They also imply that, similar to the analysis in Schmitt-Grohé and Uribe (1997), for a given level of debt, when a tax rate is the variable chosen to satisfy the government budget constraint, there can be two long-run solutions. In Table 3 we summarise the tax changes required to obtain steady-state equilibria that cohere with the target $B/Y$ ratio of 60%. Given the Laffer curves in tax revenue and debt discussed above, this target is consistent with both increases and decreases in tax rates. Table 3 suggests that reductions in each of the taxes individually or jointly are generally smaller than the respective increases.

III.2 Evaluation of tax reforms in the long-run

We next calculate the welfare for each agent at the steady-state of the equilibria in Table 3 and present, in Table 4, the welfare gains/losses relative to the pre-reform economy. To calculate these welfare changes, we follow Lucas (1990) and compute the percentage extra consumption that an individual would require so as to be equally well off between the two regimes. This is defined as:

$$\xi_i = \left( \frac{U_{i,ss}^{post}}{U_{i,ss}^{pre}} \right)^{\frac{1}{\alpha - \beta}} - 1$$

for each agent $i = c, s, u$, where $ss$ denotes welfare calculated in the steady-state.

The first observation regarding the results in Table 4 is that, as expected, welfare is always reduced for all agents for increases in tax rates. Therefore, we do not consider these equilibria further in the analysis which follows.
Regarding the fall in tax rates, the results in Table 4 show that there are different welfare effects on the agents. In general, tax cuts imply gains (or, at least, no losses) for all types of agents, with the exception of reductions in $\tau^u$ or $\tau^u, \tau^h$ combined and thus are not Pareto improving. The biggest welfare gains at the aggregate level are obtained for a capital tax cut. However, this is also the tax reform with the largest distributional effects, ranging from sizeable welfare gains for the agents that own capital and supply skilled labour, to near-zero welfare gains for unskilled workers. This trade-off between efficiency and equity is central to the analysis of capital tax reforms and is well-documented in the related literature (see e.g. Domeij and Heathcote, 2004, and Garcia-Milà et al., 2010). However, here it is obtained for a capital tax cut that is not followed by a labour tax increase.

The key to interpreting these results lies in the interaction of the asset and skill inequalities with the structure of production. As discussed above when analysing Figure 1, a fall in $\tau^h$ increases the capital stock and this raises the productivity of both types of labour, so that labour supply and labour income are increased. Therefore, workers also gain from a reduction in the capital tax. This positive productivity spillover effect is an important driver of the zero long-run optimal capital tax results in models that assume a relatively high complementarity between the labour input of the worker and capital stock (e.g., as in models using Cobb-Douglas production functions).

However, consistent with Krusell et al. (2000), a higher capital stock benefits skilled more than unskilled labour, so that the wage premium increases to 32.9% (implying a wage ratio of 1.39) after the reform. Hence, in this model, capital-skill complementarities work to amplify the inequality implications of capital tax cuts. In contrast, reductions in $\tau^u$ or $\tau^u$ and $\tau^h$ result in increases in unskilled labour, which in turn increase skilled labour but crowd out capital, thus leading to lower capital income.\textsuperscript{25}
III.3 Tax cuts and the skill premium

The general message from the above analysis is that the complementarity and/or substitutability between factor inputs is important when assessing the effects of tax reforms. This finding is consistent with related research which has emphasised the importance of different patterns of production and sector- and factor-specific technical changes on inequality (see e.g. Hornstein et al., 2005, for a review). Here, the tax reform plays a similar role to factor-specific technological progress given the way it affects factor returns and productivity (see also e.g. He and Liu, 2008). By reducing $\tau^k$ or $\tau^h$ the government is effectively introducing a skill-biased change, while reductions in $\tau^u$ favour the unskilled.

He and Liu (2008) also evaluate the effect of capital tax cuts on the skill premium for a model that is calibrated to US data and conclude that the capital tax cuts will lead to modest increases in the skill premium. In particular, the elimination of the capital tax and its substitution with labour taxes, results in an increase of the skill premium of about 3.3% in their model.

In our model for the UK, the effects of the capital tax cut on this premium are bigger, since the skill premium rises by 1.8% for a small reduction in the capital tax, by 7.9%. Our model differs in two important ways. First, we allow for agents that differ in both capital ownership and skill supply, whereas He and Liu (2008) use a representative agent model. The higher concentration of capital that we assume, consistent with the British data, tends to increase the impact of a capital tax cut on the skill premium. In particular, given that the marginal propensity to save increases with income, the increase in the supply of capital after the capital tax cut is expected to be higher in a society characterised by higher concentration of wealth. Second, He and Liu (2008) allow for endogenous skill formation, so that, in their model a capital tax cut also leads to a larger rise in the relative skill supply, which acts to moderate the skill premium. In light of these findings, our long-run quantitative results can be interpreted as an upper bound on changes in
inequality. Nevertheless, for shorter horizons, the composition of skill in the population is more likely to remain unchanged.

### III.4 Skill premium and inequality during the transition

We next evaluate the aggregate and distributional effects of the above tax reforms for all periods after the reform. In particular, we assume that the economy in the current period is at its pre-reform steady-state, as summarised by Table 2, when we implement the required permanent tax reforms. These consist of permanent, unanticipated, debt-impacting reductions in the tax rates, which are changed in the current period in each experiment to their new steady-state values in Table 3, keeping in each case the remaining tax-spending policy instruments fixed. By expressing the pre-reform steady-state in percent deviations from the post-reform steady-state, we then simulate the response of the economy post-reform and calculate welfare including all periods after the reform. Thus, in contrast to the steady-state results reported in Table 4, the transition path is now taken into account. Table 5 shows the welfare gains/losses for each agent and tax reform considered.\(^{27}\)

[Table 5 about here]

We first compare welfare gains/costs in Table 5 to the corresponding steady-state values in Table 4. Consistent with the literature, the results indicate that the larger benefits in terms of aggregate welfare are obtained by capital tax cuts and that these are smaller, compared to the long-run.\(^{28}\) Moreover, the results show that the inequality effects after the capital tax cuts are also smaller relative to the steady-state. In particular, capital tax cuts result in smaller welfare gains relative to the long-run for capitalists and skilled workers, while the welfare figures are roughly the same for the unskilled workers. In other words, the inequality effects are dampened by the inclusion of the transition period.
To further investigate this result we focus again on the capital tax reduction. To this end, in Figure 2 we plot the transition paths of capital, labour input and consumption by agent, the relative supply of skilled labour, defined as $\frac{N^c h^c + N^s h^s}{N^u h^u}$, and the skill premium. The paths of consumption and hours are important as these will ultimately determine welfare for each agent. The pre-reform steady-state and the transition paths are expressed in percent deviations from the post-reform steady-state.\footnote{Figure 2 about here}

Figure 2 shows that a tax reform based on reducing the capital tax implies an increase in the capital stock as the economy gradually converges to the new equilibrium. The capital tax cut has created incentives for those agents who hold capital, i.e. capitalists and skilled workers, to increase their capital accumulation and thus increase investment. For this to be achieved, capitalists and skilled workers can temporarily decrease consumption, but they also can increase their income by increasing their labour supply. Therefore, in general equilibrium, the increase in the return to capital also increases labour supply for those agents who hold capital. For the capitalists, in particular, the labour supply initially increases above the new steady-state and then converges to it. As they become wealthier over time, given the higher capital stock, they tend to supply less labour as the income effect dominates the substitution effect.

The overshooting in the relative supply of skilled labour in the short-run, driven by the higher returns to capital that will materialise in the long-run, leads to a fall in the skill premium in the short-run, which, in turn, has positive effects for the unskilled workers. However, over time, the relative supply of skilled labour falls and the quantity of capital increases. Both factors lead to a rising skill premium towards the new steady-state. Overall, the dynamic analysis indicates that, in general equilibrium, the complementarity between capital (or skilled labour) and unskilled labour is higher in the short-run, compared with the long-run.
Therefore, our analysis implies that after the capital tax reform, wage inequality changes initially favour the agents with less wealth, and this works to partially offset the increase in asset income inequality in the short-run.\textsuperscript{30} Therefore, in this model of capital-skill complementarity, the biggest relative gains for the poorest segment of the population after the capital tax cut materialise immediately after the reform, when the increase in the capital stock is lower and the relative skill supply overshoots, such that the wage premium moves favourably for the unskilled workers. Over time, the gains for the unskilled worker are diminishing faster than those for the skilled and wealthier groups, since both wage and asset income inequality now move in the same direction, implying that the welfare gap between the agents rises more in the long-run.

### III.5 Substitutability between capital and unskilled labour

The above results suggest that the elasticity of substitution between capital and unskilled labour is a critical factor in determining the inequality effects of capital tax cuts. Thus, we next explore the quantitative effects of higher elasticities of substitution. As discussed previously, empirical analyses provide a range of estimates for the critical parameter $\alpha$ in the production function. We consider a set of values of $\alpha$ which are consistent with this range and re-parameterise the model to obtain the same factor shares and $B/Y$ ratio as in the pre-reform economy in Table 2.\textsuperscript{31} In Table 6, we present the results for the welfare gains or losses for the three types of agents post-reform for the steady-state and for all periods according to these alternative calibrations. In each case, different capital tax reductions were applied to reach a debt-to-output ratio of 60% in the new steady-state.

[Table 6 about here]

The results in Table 6 suggest that over both time horizons considered, the welfare gains from the reduction in the capital tax to capitalists and skilled workers increase when the sub-
stitutability between capital (or skilled labour) and unskilled labour is increased. In contrast, the welfare gains to the unskilled workers fall. Therefore, the overall welfare inequality effects of capital tax cuts rise in the presence of higher capital skill complementarity, since reductions in the capital tax are skill-biased and thus raise wage inequality. While these qualitative results are expected, the small quantitative changes obtained for the empirically relevant range of parameters considered, lend support to the robustness of the model predictions in Tables 4 and 5.

**IV Conclusions**

Using a heterogeneous agent model allowing for different degrees of complementarity between capital, skilled and unskilled labour, we have evaluated supply-side reforms consistent with a lower public debt-to-GDP ratio. To implement these reforms, we calibrated the model so that the pre-reform steady-state represented the current state of the UK economy and then simulated different permanent changes in tax rates.

Our results suggest that, relative to the other tax reforms, capital tax reductions lead to the highest aggregate welfare but are skill-biased and thus increase inequality in the long-run. Our findings also show that including the transition period in the welfare evaluation lowers the inequality effects of reducing the capital tax since the complementarity between capital and all labour inputs is higher in the short- than in the long-run.

Our results further imply that it may be appropriate to consider redistributive policies alongside capital tax cuts. While these policies have not been studied here, we expect them to be more effective if they aim to raise the productivity of factor inputs and, in particular, enhance social mobility, rather than simply redistribute income towards the income groups that are not favoured by the reform. A careful evaluation of such policies would be an obvious extension to
this work. We leave these issues for future research.
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Notes

1 At the same time, at the aggregate level, there is also an important literature that examines optimal tax policy. A benchmark result from Ramsey optimal taxation is that the tax rate on capital should be zero in the long-run (see e.g. Chamley, 1986, Chari et al., 1994, and Chari and Kehoe, 1999). This result, however, does not necessarily hold in models incorporating market failures (see e.g. Guo and Lansing, 1999) nor in models under time-consistent optimal taxation (see e.g. Klein et al., 2008).

2 Studies that take into account the redistributive effects of capital taxation in designing optimal taxation in heterogeneous agent models are fewer. In Judd (1985), Ramsey-type optimal taxation leads to a zero tax on capital in the long-run. However, this result does not necessarily hold when further types of heterogeneity and market and policy failures are considered (see e.g. Lansing, 1999, Krusell, 2002, Conesa et al., 2009, and Angelopoulos et al., 2011).

3 See for example, the Panel Study of Income Dynamics (PSID) data for wealth inequality in the USA analysed in Garcia-Mila et al. (2010), and the Family Resources Survey data discussed in more detail below for the UK.

4 See e.g. Hornstein et al. (2005), for a review of the literature and empirical evidence on factor- and sector-specific technologies and inequality.

5 The survey is sponsored by the Department for Work and Pensions (see their Table 4.9 for the information reported here).


8 See e.g. Martinez-Mongay (2000), for effective tax rates in European countries. More details on tax and other data used for the calibration are provided later in Section 3.

9 Note that we discuss how our results may be affected by this assumption in Section 4.

10 Note that, in equilibrium, profits, \( \Pi_f \), are driven to zero due to perfect competition.

11 See e.g. Aghion et al. (1999), for a microeconomic rationalisation of credit constraints that do not allow agents to participate in asset markets.

12 Table 1 indicates that the numeric parameters are obtained by: (i) assumption (i.e. they are either adopted from estimation studies or generally accepted values from the literature); (ii) calibration (i.e. they are set in order to match some equilibrium targets in the data); and (iii) referring directly to the data.

13 As discussed in Krusell et al. (2000) and Hornstein et al. (2005), these estimates cohere well with the microeconometric evidence reported in the literature.

14 This is found by calculating the average income tax rate that applies approximately to the lower 30% and
the upper 70% of the tax payers. We then add the national insurance contribution rate of 11% and calculate the ratio of these two effective average tax rates.

To obtain this we divide total hours worked by total hours available for work or leisure, following Ho and Jorgenson (2001). They assume that there are 14 hours available for work or leisure per day with the remaining 10 hours accounted for by physiological needs.

See http://budgetresponsibility.independent.gov.uk.

This is calculated as $\frac{(N^c + N^s)}{H} = (N^c * C_c)/C$. The same formula is used below for similar quantities.

The ratio is calculated as $\frac{H_{nc}}{(N^c + N^s)} + \frac{H_{ns}}{(N^c + N^s)}$. The data refer to average actual weekly hours of work by industry sector from 1997-2012. Unskilled and skilled hours are obtained respectively by averaging over industries A-I and J-Q reported in the UK LFS.

The long-run utility of agent $i$ is given by $U_i = \frac{1}{1-\pi} \bar{u}_i$, for $i = c, s, u$, where $\bar{u}_i$ is the welfare of agent $i$ calculated at the steady-state. Also note that $U_a = n^c U_c + n^s U_s + n^u U_u$.

Given that we seek to evaluate the distributional effects of tax reforms and not the optimal size of the government or government debt, we take this debt target as given. Hence, we do not evaluate the potential welfare benefits from reducing the debt-to-GDP ratio, in the form of, for instance, lowering the cost of borrowing for the government and reassuring financial markets that there is no risk of default.

Notice that, conditional on the model being at steady-state after $T$ periods, infinite discounted sums involved in the welfare calculations can be computed exactly (see also e.g. Garcia-Milà et al., 2010). In our case the model fully converges to the post-reform steady-state for all variables after 140 periods.

This is despite the use of different models. Trabandt and Uhlig (2012) use a representative agent model, with a Cobb-Douglas production function and allow for monopolistic competition in the product market.

These are not presented to save space but will be made available on request.

A critical condition for this is that a Laffer curve exists with respect to total tax revenue. Further note that Schmitt-Grohé and Uribe (1997) also discuss the parameter range under which some of these equilibria can be indeterminate. For our model and the calibrated parameters for the UK, all solutions obtained below are saddle-path stable.

Note that by reducing interest payments in the steady-state, the tax cuts considered here imply an additional channel through which they affect the agents differently. Namely, debt in the steady-state represents assets to skilled workers and capitalists. Hence, its reduction implies, ceteris paribus, a reduction in an income source for these two agents, but not for unskilled workers. This hurts capitalists and skilled workers, especially when the tax rate on unskilled labour falls.
26 Note also that the policy experiments are different, since He and Liu (2008) consider a capital tax cut that is met by a labour tax rise, whereas we isolate the effects of the capital tax cut, by allowing the level of debt and interest payments on debt to adjust.

27 As a robustness check, we also calculated the transition paths and welfare after a tax reform under adaptive learning as in e.g. Giannitsarou (2006). Given that the tax reforms under analysis imply relatively small changes in the tax rates, the results in all these are similar with those obtained under the rational expectations solution reported here. This implies that these tax reforms are effectively learnable.

28 Note that the literature on tax reforms (see e.g. Domeij and Heathcote, 2004, and Garcia-Milà et al., 2010), has emphasised that capital tax cuts will lead to welfare losses for those households whose resources depend predominantly on labour income, when the elimination of the capital tax cut is met by a rise in the labour tax to balance the budget. We confirm that this is obtained in this model as well, for a similar tax reform. Results are available upon request but are not shown here, since, to save on space, we focus on the productivity gains after a capital tax cut.

29 To save space we do not present the Figures associated with the remaining tax reforms reported in Table 5 but these are available on request.

30 In the tax reforms considered in models that do not allow for capital-skill complementarity (e.g. Domeij and Heathcote, 2004, Greulich and Marcet, 2008; and Garcia-Milà et al., 2010), the productivity gains and thus the benefits to the workers from a capital tax cut are stronger in the long-run, as the capital stock is built up. However, these models do not allow for an evaluation of the wage inequality following a capital tax cut.

31 See e.g. Cantore and Levine (2012) on "re-parameterisation" with CES production functions. The re-calibration considered here ensures that the values at the pre-reform steady-state when $\alpha = \{0.42, 0.45, 0.50\}$ are the same as those reported in Table 2 (i.e. when $\alpha = 0.401$) up to the third decimal place. For this purpose, $\mu$ took the values $\{0.281, 0.290, 0.303\}$ while $\rho$ took the values $\{0.645, 0.645, 0.635\}$, respectively.
References


