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The decay time distributions of $B^0_s$ mesons decaying into the $J/\psi \phi$ final state have been measured to use the parameters $\phi_s$ and $\Delta \Gamma_s = \Gamma_L - \Gamma_H$ of the $B^0_s$ system [1–3]. Here, $\phi_s$ is the $CP$ violating phase equal to the phase difference between the amplitude for the direct decay and the amplitude for the decay after oscillation. $\Gamma_L$ and $\Gamma_H$ are the decay widths of the light and heavy $B^0_s$ mass eigenstates, respectively. The most precise results, presented recently by the LHCb experiment [3],

$$\phi_s = 0.15 \pm 0.18 \text{ (stat) } \pm 0.06 \text{ (syst) rad},$$

$$\Delta \Gamma_s = 0.123 \pm 0.029 \text{ (stat) } \pm 0.011 \text{ (syst) ps}^{-1},$$

show no evidence of $CP$ violation yet, indicating that $CP$ violation is rather small in the $B^0_s$ system. There is clear evidence for the decay width difference $\Delta \Gamma_s$ being non-zero. It must be noted that there exists another solution,

$$\phi_s = 2.99 \pm 0.18 \text{ (stat) } \pm 0.06 \text{ (syst) rad},$$

$$\Delta \Gamma_s = -0.123 \pm 0.029 \text{ (stat) } \pm 0.011 \text{ (syst) ps}^{-1},$$

arising from the fact that the time-dependent differential decay rates are invariant under the transformation $(\phi_s, \Delta \Gamma_s) \leftrightarrow (\pi - \phi_s, -\Delta \Gamma_s)$, together with an appropriate transformation for the strong phases. In the absence of $CP$ violation, $\sin \phi_s = 0$, i.e., $\phi_s = 0$ or $\phi_s = \pi$, the two mass eigenstates also become $CP$ eigenstates with $CP = +1$ and $CP = -1$, according to the relationship between $B^0_s$ mass eigenstates and $CP$ eigenstates given in Ref. [4]. They can be identified by the decays into final states which are $CP$ eigenstates. In $B^0_s \rightarrow J/\psi K^* K^-$ decays, the final state is a superposition of $CP = +1$ and $CP = -1$ for the $K^+ K^-$ pair in the $P$-wave configuration and $CP = -1$ for the $K^0 \bar{K}^0$ pair in the $S$-wave configuration. Higher-order partial waves are neglected. These decays have different angular distributions of the final-state particles and are distinguishable.

Solution I is close to the case $\phi_s = 0$ and leads to the light (heavy) mass eigenstate being almost aligned with the $CP = +1 \ (CP = -1)$ state. Similarly, solution II is close to the case $\phi_s = \pi$ and leads to the heavy (light) mass eigenstate being almost aligned with the $CP = +1 \ (CP = -1)$ state. In Fig. 2 of Ref. [3], a fit to the observed decay time distribution shows that it can be well described by a superposition of two exponential functions corresponding to $CP = +1$ and $CP = -1$, compatible with no $CP$ violation [3]. In this fit, the lifetime of the decay to the $CP = +1$ final state is found to be smaller than that of the decay to $CP = -1$. Thus, the mass eigenstate that is predominantly $CP$ even decays faster than the $CP$ odd state. For solution I, we find $\Delta \Gamma_s > 0$, i.e., $\Gamma_L > \Gamma_H$, and, for solution II, $\Delta \Gamma_s < 0$, i.e., $\Gamma_L < \Gamma_H$. In order to determine if the decay width difference $\Delta \Gamma_s$ is positive or negative, it is necessary to resolve the ambiguity between the two solutions.

Since each solution corresponds to a different set of strong phases, one may attempt to resolve the ambiguity by using the strong phases either as predicted by factorization or as measured in $B^0 \rightarrow J/\psi K^{00}$ decays. Unfortunately, these two possibilities lead to opposite answers [5]. A direct experimental resolution of the ambiguity is therefore desirable.

In this Letter, we resolve this ambiguity using the decay $B^0_s \rightarrow J/\psi K^+ K^-$ with $J/\psi \rightarrow \mu^+ \mu^-$. The total decay amplitude is a coherent sum of $S$-wave and $P$-wave contributions. The phase of the $P$-wave amplitude, which can be described by a spin-1 Breit-Wigner function of the invariant mass of the $K^+ K^-$ pair, denoted by $m_{KK}$, rises rapidly through the $\phi(1020)$ mass region. On the other hand, the phase of the $S$-wave amplitude should vary...
relatively slowly for either an $f_0(980)$ contribution or a nonresonant contribution. As a result, the phase difference between the $S$-wave and $P$-wave amplitudes falls rapidly with increasing $m_{KK}$. By measuring this phase difference as a function of $m_{KK}$ and taking the solution with a decreasing trend around the $\phi(1020)$ mass as the physical solution, the sign of $\Delta \Gamma_{\phi}$ is determined and the ambiguity in $\phi_{s}$ is resolved [6]. This is similar to the way the BABAR Collaboration measured the sign of $\cos 2\beta$ using the decay $B^0 \rightarrow J/\psi K^0_S \pi^0$ [7], where $2\beta$ is the weak phase characterizing mixing-induced $CP$ asymmetry in this decay.

The analysis is based on the same data sample as used in Ref. [3], which corresponds to an integrated luminosity of 0.37 fb$^{-1}$ of $pp$ collisions collected by the LHCb experiment at the Large Hadron Collider at the center-of-mass energy of $\sqrt{s} = 7$ TeV. The LHCb detector is a forward spectrometer and is described in detail in Ref. [8]. The trigger, event selection criteria, and analysis method are very similar to those in Ref. [3], and here we discuss only the differences. The fraction of $K^+ K^- S$-wave contribution measured within $\pm 12$ MeV of the nominal $\phi(1020)$ mass is $0.042 \pm 0.015 \pm 0.018$ [3]. (We adopt units such that $c = 1$ and $h = 1$.) The $S$-wave fraction depends on the mass range taken around the $\phi(1020)$. The result of Ref. [3] is consistent with the CDF limit on the $S$-wave fraction of less than 6% at 95% C.L. (in the range 1009–1028 MeV) [2], smaller than the D0 result of (12 $\pm$ 3)% (in 1010–1030 MeV) [9] and consistent with phenomenological expectations [10]. In order to apply the ambiguity resolution method described above, the range of $m_{KK}$ is extended to 988–1050 MeV. Figure 1 shows the $\mu^+ \mu^- K^+ K^-$ mass distribution where the mass of the $\mu^+ \mu^-$ pair is constrained to the nominal $J/\psi$ mass. We perform an unbinned maximum likelihood fit to the invariant mass distribution of the selected $B^0_s$ candidates. The probability density function (PDF) for the signal $B^0_s$ invariant mass $m_{J/\psi KK}$ is modeled by two Gaussian functions with a common mean. The fraction of the wide Gaussian and its width relative to that of the narrow Gaussian is fixed to values obtained from simulated events. A linear function describes the $m_{J/\psi KK}$ distribution of the background, which is dominated by combinatorial background.

This analysis uses the sWeight technique [11] for background subtraction. The signal weight, denoted by $W_i(m_{J/\psi KK})$, is obtained using $m_{J/\psi KK}$ as the discriminating variable. The correlations between $m_{J/\psi KK}$ and other variables used in the analysis, including $m_{KK}$, decay time $t$, and the angular variables $\Omega$ defined in Ref. [3], are found to be negligible for both the signal and background components in the data. Figure 2 shows the $m_{KK}$ distribution where the background is subtracted statistically using the sWeight technique. The range of $m_{KK}$ is divided into four intervals: 988–1008, 1008–1020, 1020–1032, and 1032–1050 MeV. Table I gives the number of $B^0_s$ signal and background candidates in each interval.

In this analysis, we perform an unbinned maximum likelihood fit to the data using the sFit method [12], an extension of the sWeight technique, that simplifies fitting in the presence of background. In this method, it is only necessary to model the signal PDF, as background is canceled statistically using the signal weights.

The parameters of the $B^0_s \rightarrow J/\psi K^+ K^-$ decay time distribution are estimated from a simultaneous fit to the four intervals of $m_{KK}$ by maximizing the log-likelihood function

$$\ln L(\Theta_p, \Theta_\delta) = \sum_{k=1}^{4} W_{p,k} \sum_{i=1}^{N_k} W_{i}(m_{J/\psi KK,i})$$

$$\times \ln P_{\text{sig}}(t_i, \Omega_i, q_i, \omega_i; \Theta_p, \Theta_\delta),$$

where $N_k = N_{\text{sig},k} + N_{\text{bkg},k}$ is the number of candidates in the $m_{J/\psi KK}$ range of 5200–5550 MeV for the $k$th interval. $\Theta_p$ represents the physics parameters independent of $m_{KK}$.

![FIG. 1 (color online). Invariant mass distribution for $B^0_s \rightarrow \mu^+ \mu^- K^+ K^-$ candidates, with the mass of the $\mu^+ \mu^-$ pair constrained to the nominal $J/\psi$ mass. The result of the fit is shown with signal (dashed curve) and combinatorial background (dotted curve) components and their sum (solid curve).](241801-2)
including $\phi_s$, $\Delta \Gamma_s$, and the magnitudes and phases of the $P$-wave amplitudes. Note that the $P$-wave amplitudes for different polarizations share the same dependence on $m_{KK}$. $\Theta_s$ denotes the values of the $m_{KK}$-dependent parameters averaged over each interval, namely, the average fraction of $S$-wave contribution for the $k$th interval, $F_{S;k}$, and the average phase difference between the $S$-wave amplitude and the perpendicular $P$-wave amplitude for the $k$th interval, $\delta_{S,L;k}$. $P_{\text{sig}}$ is the signal PDF of the decay time $t$, angular variables $\Omega$, initial flavor tag $q_i$, and the mistag probability $\omega$. It is based on the theoretical differential decay rates [6] and includes experimental effects such as decay time resolution and acceptance, angular acceptance, and imperfect identification of the initial flavor of the $B^0_s$ particle, as described in Ref. [3]. The factors $W_{p;k}$ account for loss of statistical precision in parameter estimation due to background dilution and are necessary to obtain the correct error coverage. Their values are given in Table I.

The fit results for $\phi_s$, $\Delta \Gamma_s$, $F_{S;k}$, and $\delta_{S,L;k}$ are given in Table II. Figure 3 shows the estimated $K^+K^-$ $S$-wave and $P$-wave contributions in the four $m_{KK}$ intervals. The shape of the measured $P$-wave $m_{KK}$ distribution is in good agreement with that of $B^0_s \to J/\psi \phi$ events simulated using a spin-1 relativistic Breit-Wigner function for the $\phi(1020)$ amplitude. In Fig. 4, the phase difference between the $S$-wave and the perpendicular $P$-wave amplitude is plotted in four $m_{KK}$ intervals for solution I and solution II.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Solution I</th>
<th>Solution II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_s$ (rad)</td>
<td>$0.167 \pm 0.175$</td>
<td>$2.975 \pm 0.175$</td>
</tr>
<tr>
<td>$\Delta \Gamma_s$ (ps$^{-1}$)</td>
<td>$0.120 \pm 0.028$</td>
<td>$-0.120 \pm 0.028$</td>
</tr>
<tr>
<td>$F_{S,1}$</td>
<td>$0.283 \pm 0.113$</td>
<td>$0.283 \pm 0.113$</td>
</tr>
<tr>
<td>$F_{S,2}$</td>
<td>$0.061 \pm 0.022$</td>
<td>$0.061 \pm 0.022$</td>
</tr>
<tr>
<td>$F_{S,3}$</td>
<td>$0.044 \pm 0.022$</td>
<td>$0.044 \pm 0.022$</td>
</tr>
<tr>
<td>$F_{S,4}$</td>
<td>$0.269 \pm 0.067$</td>
<td>$0.269 \pm 0.067$</td>
</tr>
<tr>
<td>$\delta_{S,L;1}$ (rad)</td>
<td>$2.68^{+0.35}_{-0.42}$</td>
<td>$0.46^{+0.42}_{-0.35}$</td>
</tr>
<tr>
<td>$\delta_{S,L;2}$ (rad)</td>
<td>$0.22^{+0.15}_{-0.13}$</td>
<td>$2.92^{+0.13}_{-0.15}$</td>
</tr>
<tr>
<td>$\delta_{S,L;3}$ (rad)</td>
<td>$-0.11^{+0.16}_{-0.18}$</td>
<td>$3.25^{+0.18}_{-0.16}$</td>
</tr>
<tr>
<td>$\delta_{S,L;4}$ (rad)</td>
<td>$-0.97^{+0.28}_{-0.43}$</td>
<td>$4.11^{+0.43}_{-0.28}$</td>
</tr>
</tbody>
</table>

Figure 4 shows a clear decreasing trend of the phase difference between the $S$-wave and $P$-wave amplitudes in the $\phi(1020)$ mass region for solution I, as expected for the physical solution. To estimate the significance of the result, we perform an unbinned maximum likelihood fit to the data by parametrizing the phase difference $\delta_{S,L;k}$ as a linear function of the average $m_{KK}$ value in the $k$th interval. This leads to a slope of $-0.050^{+0.013}_{-0.020}$ rad/Mev for solution I and the opposite sign for solution II, where the uncertainties are statistical only. The difference of the lnL value between this fit and a fit in which the slope is fixed to be zero is 11.0. Hence, the negative trend of solution I has a significance of 4.7 standard deviations. Therefore, we conclude that solution I, which has $\Delta \Gamma_s > 0$, is the physical solution. The trend of solution I is also qualitatively consistent with that of the phase difference between the $K^+K^-$ $S$-wave and $P$-wave amplitudes versus $m_{KK}$ measured in the decay $D^+_s \to K^+K^-\pi^+$ by the BABAR Collaboration [13].

![Figure 3](color online). Distribution of (a) $K^+K^-$ $S$-wave signal events and (b) $K^+K^-$ $P$-wave signal events, both in four invariant mass intervals. In (b), the distribution of simulated $B^0_s \to J/\psi \phi$ events in the four intervals assuming the same total number of $P$-wave events is also shown (dashed lines). Note that the interference between the $K^+K^-$ $S$-wave and $P$-wave amplitudes integrated over the angular variables has a vanishing contribution in these distributions.
Several possible sources of systematic uncertainty on the phase variation versus $m_{KK}$ have been considered. A possible background from decays with similar final states such as $B^0 \to J/\psi K^{*0}$ could have a small effect. From simulation, the contamination to the signal from such decays is estimated to be 1.1% in the $m_{KK}$ range of 988–1050 MeV. We add a 2.2% contribution of simulated $B^0 \to J/\psi K^{*0}$ events to the data and repeat the analysis. The largest observed change is a shift of $\delta_{S\perp}$ by 0.06 rad, which is only 20% of its statistical uncertainty and has a negligible effect on the slope of $\delta_{S\perp}$ versus $m_{KK}$. The effect of neglecting the variation of the values of $F_3$ and $\delta_{S\perp}$ in each $m_{KK}$ interval is determined to change the significance of the negative trend of solution I by less than 0.1 standard deviations. We also repeat the analysis for different $m_{KK}$ ranges, different ways of dividing the $m_{KK}$ range, or different shapes of the signal and background $m_{J/\psi KK}$ distributions. The significance of the negative trend of solution I is not affected. To measure precisely the $S$-wave line shape and determine its resonance structure, more data are needed. However, the results presented here do not depend on such detailed knowledge.

In conclusion, the analysis of the strong interaction phase shift resolves the ambiguity between solution I and solution II. Values of $\phi_s$ close to zero and positive $\Delta \Gamma_s$ are preferred. It follows that, in the $B^0$ system, the mass eigenstate that is almost $CP$ even is lighter and decays faster than the state that is almost $CP$ odd. This is in agreement with the standard model expectations (e.g., [14]). It is also interesting to note that this situation is similar to that in the neutral kaon system.

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