Supervenience among classes of relations

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1. A case of neglect

For large parts of their history, logic and metaphysics tended to focus on monadic properties at the expense of relations. Until the nineteenth century, logic consisted largely of the theory of the syllogism. That theory has nothing to say about inferences involving relations between individuals. The non-logical terms in the language of the syllogism stand for monadic attributes only. Metaphysics was much preoccupied with the nature of properties, but not with relations. Indeed, typical metaphysical views did not admit relations among the constituents of fundamental reality. One such view is *monadism*, associated with Leibniz, according to which the true logical form of claims apparently of the form Rab is $Fa \wedge Gb$. Another one is *monism*, associated with Spinoza and Bradley, according to which the true logical of Rab is $H(a+b)$, where $a+b$ is a complex subject composed of a and b. The founding fathers of analytic philosophy distinguished themselves from their idealist teachers notably by espousing realism about relations.

The last paragraph presents a simplified version of a story that Russell influentially told in several places.[1] It is debatable whether or not this story contains more than a grain of truth. But at any rate, it does raise an interesting question: are relations reducible to monadic properties? Notoriously, it is a difficult question what exactly constitutes reducibility. If it entails definability of non-monadic predicates in terms of monadic ones, then relations are hardly reducible. In particular, the crude versions of monadism and monism fail, as Russell and several philosophers after him have pointed out. Monadism has the consequence that whenever x is in the domain and y in the converse domain of R, then Rxy. To see this, note that for any binary predicate R expressing a relation, monadism entails the truth of a biconditional of the following form:

(M1) $Rxy \leftrightarrow Fx \wedge Gy$

Suppose now that x is in the domain of R, such that Rxz for some z. Then Fx holds by the left-to-right direction of (M1). If y is in the converse domain of R, then $Rz'y$ for some z', and thus Gy by the left-to-right direction of (M1) again. Hence $Fx \wedge Gy$, and Rxy by the right-to-left direction of (M1). This consequence is unacceptable, since many relations do not relate everything in their domain with everything in their converse domain. That I am taller than someone and someone is taller than you does not entail that I am taller than you, after all.[2]

Monism has the consequence that every relation is symmetric. It entails a biconditional of the following form for every binary R:

(M2) $Rxy \leftrightarrow H(x+y)$

[1] For example, in Russell (1903) (especially paragraphs 212-216), and in chapter 9 of the popular Russell (1912).

[2] For a similarly unacceptable consequence of crude monadism, see Mates (1986: 217).

If Rxy, then $H(x+y)$ by the left-to-right direction of (M2). Hence $H(y+x)$, and Ryx by the right-to-left direction of (M2). This argument assumes that the function $+$ is commutative. But unless told otherwise by the monist, we can assume that $+$ is just ordinary mereological summation, which is commutative.[3] Since taller than is not symmetric, monism can account for it no more than monadism can.

It is tempting to pool the resources of monadism and monism, and hold that the true logical form of Rxy is $Fx \wedge Gy \wedge H(x+y)$. We may call that view *monadicism*.[4] Neither of the two previous objections applies. Nonetheless, the view has the unacceptable consequence that any relation is symmetric among those entities that are in both its domain and its converse domain. For it entails for a biconditional of the following form for any R:

(M3) $Rxy \leftrightarrow Fx \wedge Gy \wedge H(x+y)$

Suppose that for some z and z', Rzx and Ryz'. Then $Fy \wedge Gx$ by the left-to-right direction of (M3). Now suppose Rxy. Then $H(x+y)$ by the left-to-right direction of (M3) again, and $H(y+x)$ by the commutativity of $+$. Hence $Fy \wedge Gx \wedge H(y+x)$, and Ryx by the right-to-left direction of (M3). But again, you may be taller than me, but not vice versa, even though we are both in the domain and the converse domain of *being taller than*.

If the project of showing relations to be reducible requires a translation scheme of the kind suggested by (M1) - (M3), its prospects are dim.[5] But perhaps there is another sense in which relations – or at least some of them – are reducible. Intuitively, it seems that whatever we think about whether all relations are reducible, surely *being taller than* is.

The question what constitutes reducibility has received a great deal of attention in other areas. When looking for a pertinent concept of reducibility for relations, we could do worse than consider work done in the philosophy of mind. As is well known, the concept of *supervenience* has occupied centre stage there. Typically, philosophers accept that supervenience – of one variety or another – is necessary for reducibility. It is widely acknowledged that the supervenience of a class A on a class B is not by itself sufficient for reducibility. Whether it is sufficient when conjoined with certain other claims is a difficult question that I will not address here.

Whatever the exact relationship between supervenience and reducibility, the question whether some relations are reducible to properties naturally leads to the question whether the former supervene on the latter. What would it take for relations to supervene on properties? The extant literature does not contain a sustained and systematic examination of this question, at least as far as I am aware. This is surprising, given that a great deal of work has been done on distinguishing various concepts of supervenience, supplying exact characterizations for them, and finding applications. It is even more surprising in light of the fact that the concept of supervenience is eminently suitable to be applied to relations, as I shall argue.

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[3] This argument against monism is given in Russell (1903: 225): the whole $a+b$ 'is symmetrical with regard to a and b, and thus the property of the whole will be exactly the same in the case where a is greater than b as in the case where b is greater than a'. Russell claims that Leibniz was already aware of the argument.

[4] Its negation is called 'the doctrine of external relations' by Russell (1924: 373).

[5] For a sophisticated discussion of technical aspects of that question, see Humberstone (1984).

As noted above, supervenience is useful in discussing the relationship between mental and non-mental properties. One reason for this is the putative multiple realizability of the former. If the multiplicitly is infinite, then what is realized is not finitely definable in terms of the realizers, and hence not definable in a language like English in which no infinite disjunctions can be formed. Nonetheless, the realized properties supervene on the class of realizers. Some relations can also be said to have an infinite multiplicity of realizers. Again, *being taller than* serves as an example. That relation cannot be finitely defined in terms of height properties, where those are understood as properties that never differ among things that are of the same height. To put this in the formal mode: no two-place predicate that expresses the relation of *being taller than* can be defined in the language of predicate logic whose only non-logical terms are unary predicates. Nonetheless, *being taller than* supervenes on heights. We can spell out precisely what this means. Let $\langle x, y \rangle$ and $\langle x', y' \rangle$ be two ordered pairs that are indiscernible with respect to height properties – that is, for every height property H, x has H if and only if x' has H, and y has H if and only if y' has H. Then $\langle x, y \rangle$ and $\langle x', y' \rangle$ are indiscernible with respect to *being taller than* – x is greater than y just in case x' is greater than y', and y is greater than x just in case y' is greater than x'. An argument for the truth of that supervenience claim will be given in section 3.

As this example shows, the intuitive idea of supervenience is easily applied to relations. My aim in this paper is not to introduce a new concept; rather, it is to explicate a concept that we already have, and discuss a couple of applications.

2. Global versus local supervenience among relations

The neglect of relations has not been total. Some authors have deployed supervenience when theorizing about relations. For example, supervenience has been invoked in the classification of relations into internal, external and extrinsic, a topic to be discussed in sections 4 and 5. Occasionally, even the general question what it is for classes of relations to be related by supervenience has been raised. However, those who considered the question tended to give up on providing an answer rather quickly. After observing that '[as] Kim (1993) notes, it is far from trivial to apply the individual notions of supervenience directly to relations', Shagrir (2009: 419) claims that '[i]t is more natural to convert relations to properties of individuals that express these relations'. But by and large, theorists of supervenience did not even raise the question in the first place, and have followed the metaphysical tradition in neglecting relations.

What explains such lack of attention? I suspect that it is partly due to the erroneous belief that the work has already been done. The received view, it seems to me, is that supervenience claims involving relations are global supervenience claims, at least implicitly. Since extant accounts of global supervenience are designed to apply to properties and relations alike, they might be thought to provide a satisfactory account of supervenience among classes of relations.[6]

It is hardly an accident that relations tend to be associated with global supervenience. To see whether a certain global supervenience claim holds, we typically need not only compare two

[6] The received view is rarely articulated explicitly. But it seems to be suggested in footnote 6 of Shagrir (2009: 419).

things, but also consider other things besides them. Likewise, to see whether certain relations supervene, we typically need to not only compare two things, but also consider other things besides them. Hence both relations and globality point us beyond simple pairwise comparisons of things. On reflection, though, it is clear that the contrast between monadic and non-monadic is different from the contrast between non-global – or local – and global. There is a clear intuitive difference between global and local supervenience claims. Many relational or extrinsic properties globally, but not locally, supervene on intrinsic ones.[7] For example, the property of *being the oldest person* merely globally supervenes on ages of people. However, not only monadic properties can be relational. Intuitively, a relation is extrinsic or relational if whether it holds or not between its relata depends on other things than the relata. The relation of *being adjacent in age* – of being such that nobody has an intermediate age – globally supervenes on ages, but does not do so locally. It is a relational relation, as it were. For a philosophically more interesting example, consider the relation of reference, holding between mental and linguistic tokens on the one hand and objects on the other. Reference is a broadly intentional relation, and as such widely taken to be globally supervenient on non-intentional properties and relations. However, it arguably does not locally supervene on them, since whether something is a referent of a given token may depend on what other candidates are available. Global and local supervenience thus seem to come apart. In some theoretical contexts, we may be interested in the global concept, and in others in the local one.

In addition to the intuitive difference, there are two further reasons why we may not wish to construe all supervenience claims involving relations as global supervenience claims. First, the concept of global supervenience is not understood as well as the concept of strong supervenience, and the project of explicating it has faced severe difficulties.[8] Second, we may wish to consider supervenience claims involving so-called 'transworld relations' or 'crossworld relations'. Extant accounts of global supervenience only apply to relations whose relata are in the same possible world.[9] It is true that most garden-variety relations are intraworld relations, and only relate worldmates. Presumably, any relation that requires spatiotemporal or causal relatedness will be in that category. But other relations of philosophical interest are crossworld relations. Most famous among them is the relation of identity. Some puzzles about transworld identity raise the question whether it supervenes on qualitative properties, or indeed on any properties that do not in some way involve identity itself. Theorists who deny that there are any true claims of transworld identity have invoked other crossworld relations in the analysis of *de re* modality, such as counterparthood and representation *de re*. We can likewise ask whether these relations supervene on other properties or relations. David Lewis (1986: 221) defined *haecceitism* as the claim that representation *de re* does not supervene on qualitative character, again using our implicit understanding of supervenience as it applies to relations.[10] Depending on how much we are including in the class of qualitative properties – in particular, whether or not we are including extrinsic properties – we get stronger or weaker versions of haecceitism.

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[7] I am using 'relational' loosely here. For a proposal about what distinction to mark by the pair 'extrinsic' and 'relational', see Humberstone (1996).

[8] Some of these are discussed in Leuenberger (2009) and in the works referred to there.

[9] I do not wish to claim that accounts of global supervenience could not be modified to apply to crossworld-relations.

[10] Lewis formulated it as a global supervenience claim. He was not fully explicit about how that claim ought to be understood.

But discussions of crossworld relations are not restricted to theories of *de re* modality. Similarity is a crossworld relations, and we can ask whether it supervenes on intrinsic properties. Mental representation gives rise to other putative cases of crossworld relations. I stand in the relation of *thinking about the species of* to any other-wordly human. Further, one might argue that any other-worldly human stands in the relation of *making true* to my mental tokening of the proposition that humans are possible. Last but not least, the relation of indiscernibility used to define strong supervenience is a crossworld relation.

The earlier literature on supervenience occasionally distinguished between possible-worlds versions and modal-operator versions. The versions are not equivalent, although they are co-extensive if we restrict ourselves to classes of properties that have certain closure properties. By and large, philosophers have preferred to work with the possible-worlds versions. But in ignoring crossworld relations, they have failed to exploit one important advantage of those versions. As is well known, the standard language of modal operators does not have the power to express claims about crossworld relations, while the possibilist language does.

In this section, I hope to have provided some motivation for extending the standard definition of supervenience to relations. In the next section, I will formally define such a notion.

3. Defining strong supervenience for classes of relations

The familiar definition of strong supervenience, which applies to classes of monadic properties, can be broken down into three steps. First, an equivalence relation of indiscernibility is defined relative to every property F. The field of the relation of indiscernibility consists of ordered pairs $\langle x, w\rangle$, with x an individual and w a world. The pair $\langle x, w\rangle$ is F-indiscernible from $\langle x', w'\rangle$ just in case x has F in w iff x' has F in w'. Given the assumption that individuals are world-bound and only have properties in worlds where they exist, we could take the field to consist of individuals. But despite the simplification that it would offer, I will not make this assumption in this section. Second, indiscernibility relative to a class A of properties is defined in terms of indiscernibility relative to each of its members: $\langle x, w\rangle$ is A-indiscernible from $\langle x', w'\rangle$ just in case it is F-indiscernible for every $F \in A$. Since we may identify indiscernibility relations with classes of ordered pairs, as is customary in formal logic, we can define A-indiscernibility to be the intersection of the relations of F-indiscernibility for every member F of A. Third, supervenience of A on B is defined by comparing the relations of indiscernibility generated by A and B, respectively. That is, A strongly supervenes on a class B just in case B-indiscernibility is a sub-class of A-indiscernibility. In other words, A strongly supervenes on B just in case the partition generated by B is a refinement of the partition generated by A. Or in yet other words: A strongly supervenes on class B just in case all B-indiscernible elements are A-indiscernible.

When we generalize this definition to apply to classes that may include monadic properties as well as relations of any finite adicity – henceforth just ‘classes of relations’ – the second and the third step will be exactly the same. A-indiscernibility will be the intersection of all the relations of R-indiscernibility, for any $R \in A$, and A will be said supervene on B if the partition generated by B is a refinement of the partition generated by A. Only the first step requires modification. We need to define a relation of R-indiscernibility for a given relation R, of any adicity.

The field of indiscernibility relations used in the definition of strong supervenience for properties consists of ordered pairs. However, the question whether $\langle x, w\rangle$ and $\langle x', w'\rangle$ are indiscernible relative to a dyadic relation R does not immediately seem to make sense.[11] Rather, we need to consider pairs of such ordered pairs, and ask whether $\langle\langle x, w\rangle, \langle y, v\rangle\rangle$ is R-indiscernible from $\langle\langle x', w'\rangle, \langle y', v'\rangle\rangle$. Clearly, it is a necessary condition for their R-indiscernibility that x in w bears R to y in v if and only if x' in w' bears R to y' in v'. But for the notion of R-indiscernibility pertinent here, that condition is not sufficient: it is also required that y in v bears R to x in w if and only if y' in v' bears R to x' in w'.

That is, if R is admiration and @ the actual world, then $\langle\langle \text{John}, @\rangle, \langle \text{Mary}, @\rangle\rangle$ is R-indiscernible from $\langle\langle \text{Peter}, @\rangle, \langle \text{Martha}, @\rangle\rangle$ only if John admires Mary just in case Peter admires Martha, and Mary admires John just in case Martha admires Peter. Of course, though, it is not required for the R-indiscernibility of those pairs that John admires Mary just in case Martha admires Peter.

What is it for two such ordered pairs of ordered pairs to be F-indiscernible for a monadic property F? This case has in effect already been considered by Kim (1993: 161), and I can adopt his suggestion: $\langle\langle x, w\rangle, \langle y, v\rangle\rangle$ is F-indiscernible from $\langle\langle x', w'\rangle, \langle y', v'\rangle\rangle$ iff x in w has F just in case x' in w' has F, and y in v has F just in case y' in v'has F. That is, elements at corresponding places in the sequence need to be F-indiscernible in the sense familiar from the definition of strong supervenience.

We can now generalize the foregoing to sequences that have more than two members. Let an n-selector ρ be a function that takes in a denumerable sequence σ and yields a sequence of length n whose elements are all in σ, and that operates on all such sequences in the same way: for any σ and σ', any natural number j and any $i \leq n$, if $(\rho\sigma)_i = \sigma_j$, then $(\rho\sigma')_i = \sigma'_j$. ($(\rho\sigma)_i$ is the i-th element of the sequence $\rho\sigma$, and σ_j is the j-th element of σ.) We can represent ρ by the n-tuple $\langle \rho_{1,} \dots, \rho_n\rangle$ of natural numbers determined by the condition that $(\rho\sigma)_i = \sigma_{\rho_i}$ for all σ and all $1 \leq i \leq n$. For example, if σ is the denumerable sequence $\langle\langle \text{Socrates}, @\rangle, \langle \text{Plato}, @\rangle, \langle \text{Aristotle}, @\rangle, \langle 4, @\rangle, \langle 5, @\rangle, \dots\rangle$, and $\rho\sigma$ the triple $\langle\langle \text{Plato}, @\rangle, \langle \text{Socrates}, @\rangle, \langle \text{Aristotle}, @\rangle\rangle$, then ρ can be represented by $\langle 2, 1, 3\rangle$.

The class of elements on which R-indiscernibility is to be defined includes all denumerable sequences of ordered pairs of individuals and worlds.[12] Such sequences σ and σ' are R-indiscernible, for n-ary R, iff for all n-selectors $\rho = \langle \rho_1, \dots, \rho_n\rangle$, $\rho\sigma = \langle \sigma_{\rho_1}, \dots, \sigma_{\rho_n}\rangle$ exemplifies R iff $\rho\sigma' = \langle \sigma'_{\rho_1}, \dots, \sigma'_{\rho_n}\rangle$ does. This relation is easily verified to be an equivalence relation. (In my terminology, R-indiscernibility is not the same as agreement on R: n-tuples agree on n-ary R iff either both or neither exemplify R.)

This completes the first step in the definition of strong supervenience for classes A and B that may include relations. The second step is predictable: sequences σ and σ' are A-indiscernible iff they are R-indiscernible for every $R \in A$. The third step, defining strong supervenience, is again familiar.

[11] Kim (1993) discusses different ways in which we might give it a sense.

[12] A generalization to deal with relations with infinitely many relata will not be given here, and neither will a generalization to relations whose relata do not belong to any one particular world (transworld fusions, for example).

Strong supervenience

A strongly supervenes on B iff any two B-indiscernible sequences σ and σ' are also A-indiscernible.

This explication of a concept of strong supervenience that applies to relations has a number of welcome features. First, it is a proper generalization of the familiar explication of strong supervenience, which it entails as a special case. Second, it equips the defined concept with the formal features that we expect a concept of supervenience to have. Third, it yields the welcome verdict that relations strongly supervene on their converses, and vice versa. Fourth, it classifies paradigm cases correctly. Fifth, it is discriminatory, avoiding the consequence that the non-contingent supervenes on everything.

I shall elaborate on these five features in turn.

Generalization. The biconditional standardly used to define strong supervenience for monadic A and B is entailed as a special case. Suppose that there are pairs $\langle x, w\rangle$ and $\langle x', w'\rangle$ that are B-indiscernible but A-discernible according to the standard definition, for some A and B that have only monadic members. Then the two sequences which respectively have those pairs in all places are likewise B-indiscernible but A-discernible, and hence A fails to strongly supervene on B according to the above definition. For the other direction, suppose now that A does not strongly supervene on B according to the above definition. Then there are B-indiscernible sequences σ and σ' that are A-discernible. Since A has only monadic members, there is some $F \in A$ and a natural number i such that $\langle i\rangle\sigma$ and $\langle i\rangle\sigma'$ do no agree on F. It follows that the pairs σ_i and σ'_i are B-indiscernible but A-discernible according to the standard definition.

Formal features. It turns out that strong supervenience has many formal features that we normally want supervenience to have: it is transitive, monotonic (if A strongly supervenes on B and B is a subclass of B' then A strongly supervenes on B') and accumulative (if A and A' both strongly supervene on B, then so does the union of A and A'). All these results are immediate from the above definition and the fact that A-indiscernibility is an equivalence relation for every A. We also obtain the result that any relation strongly supervenes on its negation.[13]

Converses. The fact about negations just mentioned is familiar from the case of properties. But there are also specific consequences for relations. Notably, it turns out that every binary relation strongly supervenes on its converse.[14] This is a welcome result, since it has long been recognized that relations and their converses are intimately related. Indeed, it has been argued that this intimate relation is identity (Williamson 1985). In this case as in many others, we could consider a supervenience claim as a fall-back position if the corresponding identity

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[13] Strictly speaking, this is a result about the corresponding unit classes. I omit the phrase 'the unit class of' when there is no danger of confusion.

In fact, something slightly stronger holds: replacing some or all members of A and B with their negations never changes the truth-value of the supervenience claim.

[14] Proof : Suppose σ and σ' are $\breve{R}$-discernible, where $\breve{R}$ is the converse of R. Then for some pair $\rho = \langle \rho_1, \rho_2\rangle$, $\rho\sigma$ and $\rho\sigma'$ do not agree on $\breve{R}$. It follows that if $\rho' = \langle \rho_2, \rho_1\rangle$, $\rho'\sigma$ and $\rho'\sigma'$ do not agree on R, and hence σ and σ' are R-indiscernible as well.

claim turned out to be untenable. The result about converses extends straightforwardly to any permutation of a relation.[15]

Paradigms. In section 1, we encountered the relation of *being taller than* as a paradigm of a relation that is intuitively reducible to monadic properties. We can now verify that according to the above definition, that relation does indeed supervene on the class of height properties B. For suppose that σ and σ' are B-indiscernible. Then for any i and any $H \in B$, σ_i has H (i.e. the first member of σ_i has H in the world which is the second member of σ_i) just in case ${\sigma'}_i$ has H. Let $\rho = \langle j, k \rangle$ be any 2-selector. We distinguish three jointly exhaustive cases, and show that in each of them, σ_j is taller than σ_k iff ${\sigma'}_j$ is taller than ${\sigma'}_k$. Case (i): σ_j is taller than σ_k. Then these elements have height properties, say H_1 and H_2 respectively, such that everything with H_1 is taller than anything with H_2. Due to the B- indiscernibility of σ and σ', ${\sigma'}_j$ also has H_1 and ${\sigma'}_k$ also has H_2. It then follows that ${\sigma'}_j$ is taller than ${\sigma'}_k$. Case (ii): σ_j and σ_k both have heights, but the former is not taller than the latter. By reasoning parallel to that in case (i), we can show that ${\sigma'}_j$ is not taller than ${\sigma'}_k$. Case (iii): at least one of σ_j and σ_k does not have a height, such that it is not the case that σ_j is taller than σ_k. Due to the B-indiscernibility of σ and σ', one of ${\sigma'}_j$ and ${\sigma'}_k$ also lacks a height, and hence it is not the case that ${\sigma'}_j$ is taller than ${\sigma'}_k$.

Discrimination. It is often claimed that relations of supervenience fail to make discriminations in the realm of the non-contingent, since everything non-contingent trivially supervenes. Such claims are in need of qualification. It is true that necessarily universal relations and necessarily empty relations supervene on all classes. But not all intuitively non-contingent relations do. There is a good sense in which the relation of identity is non-contingent, according to orthodoxy: if $x = y$ is true, it is necessarily true, and if it is false, it is necessarily false. However, identity does not trivially supervene. For let σ be the sequence with $\langle \text{Socrates}, @ \rangle$ in all places, and σ' the sequence that is like σ in all places except the first, where it has $\langle \text{Plato}, @ \rangle$. Then σ and σ' are discernible with respect to identity, but indiscernible with respect to the universal relation. Hence identity does not strongly supervene on the universal relation.[16]

4. Internal relations

In the first section, I suggested that a notion of supervenience that applies to relations may be of help in formulating broadly reductionist theses about relations – theses that have at least a fighting chance of being true for important classes of relations, unlike monadism, monism,

[15] Kit Fine (2000) argued that there are neutral relations, which lack a direction or sense, and explored so-called ‘positionalist’ and ‘anti-positionalist’ accounts of them. Relations that have converses distinct from them are ‘biased’, in his terminology. The account of supervenience given here applies to biased relations. I shall not discuss whether and how it could be adapted to apply to neutral ones.

[16] The proposal for defining strong supervenience among classes of relations in Leuenberger (2008b) has the first four of these features, but is less discriminating: it entails that identity strongly supervenes on every class of relations. For this reason, the present proposal now seems to me superior.

and monadicism. After having defined such a notion of supervenience, I will now explore such an application.

The words 'internal' and 'external' have notoriously been used to mark various different distinctions among relations. Some of them were prominent in debates between British idealists and their opponents.[17] More recently, one such distinction has been revived by David Lewis. Relations that are internal in his sense may be considered as those that are reducible to monadic properties. Here, I will focus on his account. For ease of exposition, I will follow Lewis in assuming that individuals are world-bound.

In his analysis of internality, Lewis helps himself to the notion of an intrinsic property – roughly speaking, a property that a thing has in virtue of how it is in itself. Intuitively, internal relations are those that are particularly intimately bound up with intrinsic properties. Again, our paradigmatic example is *being taller than*: whether one thing bears that relation to another is entirely a matter of how these things are intrinsically. In contrast, *being two meters apart* and *having a common friend* are not internal, since whether one thing bears them to another is not just a matter of the intrinsic properties of the two things.

According to Lewis (1986: 62), an internal relation 'is one that supervenes on the intrinsic properties of its *relata*'. Prima facie, there is more than one way to cash this out precisely. Two different analysanda can be distinguished: first, the notion of an internal relation; and second, the notion of a relation being internal to given relata. This distinction is analogous to the one that Humberstone (1996) drew between global intrinsicality – a property of properties – and local intrinsicality – a relation between a particular and a property. (These labels should not be taken to suggest an interesting connection to the contrast between local and global supervenience.) If R is any dyadic relation, and I the class of (globally) intrinsic properties, we obtain the following biconditionals:

> R is globally internal iff for all x, y, x' and y', if x and x' are I-indiscernible and y and y' are I-indiscernible, then Rxy iff $Rx'y'$.
> R is locally internal to x and y iff for all x' and y', if x and x' are I-indiscernible and y and y' are I-indiscernible, then $Rx'y'$.

It is a consequence of these definitions that a relation is globally internal iff it is locally internal to all x and y that are related by it.[18] However, a relation may be locally internal to some things but not to others, and hence not globally internal. Let R be any contingently exemplified globally internal relation. Presumably, the relation of being twice as massive is as good a candidate as any. Consider the relation R' that holds between worldmates x and y if R is exemplified in the world that they inhabit. Then R' is locally internal to any worldmates that stand in R, but is not globally intrinsic, presumably: two things may stand in R in one world, while their intrinsic duplicates in another world do not. To be sure, this example is contrived. But there may be more natural ones as well. One could argue that the relation of representation holds in some cases in virtue of resemblance in intrinsic matters,

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[17] For a survey of ten different distinctions, see Ewing (1934: 117-142).

[18] The left-to-right direction is straightforward to verify. For the right-to-left direction, suppose that R is not globally internal. Then there are x, y, x', and y' such that x and x' are I-indiscrnible and y and y' are I-indiscernible and yet Rxy but not $Rx'y'$. Then x and y are related by R, but R is not locally internal to them.

and in other cases in virtue of a causal relationship. Since resemblance is internal, representation may be locally internal to cases of the first sort, but not globally internal. Further, consider the relation that holds between x and y iff x is disposed to open y if x is turned. That relation is locally internal to some key and some door, perhaps, and may hold between some switch and some door without being locally internal to them.

In the rest of this section, I will only discuss global internality, and will drop the qualifier 'global'. Internality, as characterized above, can be captured in a simple way using the notion of strong supervenience formally defined in section 3. The first of the above biconditionals is equivalent to the claim that a dyadic relation R is globally internal iff R supervenes on I.[19] Moreover, this account of global internality can be straightforwardly generalized from dyadic ones to relations of any finite adicity: a relation is globally internal iff it supervenes on the class of intrinsic properties.[20]

Is this account of internal relations plausible? One might suspect that it will be vulnerable to a number of standard objections to explications in terms of supervenience.

First, all relations that relate universally come out as internal. Supposing that arithmetical facts are necessary, the relation R that holds between x and y just in case x is such that there are infinitely many prime numbers supervenes on every class of properties or relations, and is thus internal according to the proposal. Prima facie, it might not seem internal. Second, internality comes out as intensional rather than hyperintensional. As a consequence, a dyadic relation R' is internal just in case $R \vee R'$ is, where R is universal. Again, intuitions may disagree, wishing to classify R and $R \vee R'$ differently. Third, the account does not guarantee a certain asymmetry – in this case, between internal relations and intrinsic properties. For the supervenience of internal relations on intrinsic properties is compatible with the supervenience of intrinsic properties on internal relations.

I will not discuss the first two objections, since the relevant considerations seem to be the same as in the discussion of broadly modal accounts of intrinsic properties. The third objection merits separate discussion here, though. We should note that not only is there no guarantee of asymmetry, but indeed a guarantee of symmetry, under the assumption that

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[19] This is established by showing that the supervenience of R on I is equivalent to the claim that for all x, y, x', y', if x and x' are I-indiscernible and x' and y' are I-indiscernible, then Rxy iff $Rx'y'$.

Left-to-right: To show the contrapositive, suppose that there are x, y, x', y' such that x and x', and y' and y', are respectively I-indiscernible, but Rxy and not $Rx'y'$. Let σ be the sequence starting with x followed by y in all other places, and σ' the sequence with x' in the first and y' in all other places. Then σ and σ' are I-indiscernible but R-discernible, and hence R does not supervene on I.

Right-to-left: Suppose that R does not supervene on I. Then there are denumerable sequences σ and σ' that are I-indiscernible but not R-indiscernible, and there is a 2-selector $\rho = \langle \rho_1, \rho_2 \rangle$ such that it is not the case that $\rho\sigma = \langle \sigma_{\rho_1,} \sigma_{\rho_2,} \rangle$ exemplifies R iff $\rho\sigma' = \langle {\sigma'}_{\rho_1,} {\sigma'}_{\rho_2,} \rangle$ exemplifies R. Since σ_{ρ_1} is I-indiscernible from ${\sigma'}_{\rho_1}$ and σ_{ρ_2} is I-indiscernible from ${\sigma'}_{\rho_2}$, the claim on the right-hand side is false as well.

[20] Leuenberger (2008b) touches on this point briefly.

relations are relatively abundant: intrinsic properties supervene on internal relations, as well as the other way round. In the case of dyadic relations, the required assumption of relative abundance is that for any intrinsic property F, there exists a relation R that x bears to y iff both x and y are F.[21]

Why is the lack of asymmetry of supervenience thought to be objectionable? Some authors have tried to press supervenience into service for explicating relations such as metaphysical explanation, or grounding, or metaphysical priority. Such attempts have met with criticism. For present purposes, we need not settle the question whether its lack of asymmetry disqualifies supervenience from deployment in explicating those relations. What matters here is that some uses of supervenience do not require it to be asymmetric. To me, the present one seems to be among them.

The thesis that a given relation is internal may be accompanied by the claim that its holding is metaphysically explained, or grounded, by intrinsic features of the relata. For example, Bernard Bosanquet (1911: 277) takes it to be characteristic of internal relations that they are grounded in the relata (albeit by their nature rather than their intrinsic properties): he has a discussion of 'what have been called "internal relations", i.e. relations grounded in the nature of the related terms'. Moreover, it would arguably be in the spirit of David Lewis' metaphysical system to say that internal relations hold in virtue of the exemplification of some more natural intrinsic properties. Nonetheless, internality need not involve grounding by the intrinsic properties or the nature of the relata. The notion characterized here is available even to philosophers who are sceptical of the intelligibility of talk about grounding, or those who wish to remain neutral about the direction of explanation in that particular case. More interestingly, it is available to philosophers who take the intrinsic features of the relata to be metaphysically explained, or grounded, by the holding of the internal relation. Such a thesis may be in the spirit of certain structuralist or structural realist views – although I am not aware that the term 'internal relations' is used in the pertinent literature. Possibly, such a thesis can even be found in the idealist tradition, which made extensive use of the term. At any rate, some formulations do suggest it. As Ewing (1934: 130) puts it in his survey of that tradition, 'an internal relation is often described as a relation which "makes a difference to" its terms'. On one reading of that phrase, it entails that the relata have their intrinsic nature at least partly in virtue of the fact that the relation holds between them. Such a claim would be compatible with the supervenience of the relation on intrinsic properties. Perhaps the holding of the relation modifies the relata in such a way that they are intrinsically different from any entities not standing in the relation. If so, the relation supervenes on the intrinsic properties.

In light of this, it seems to me that the third of the above objections can be answered: there is at least one useful notion of internality which does not require an asymmetry between intrinsic properties and internal relations, and it is a virtue of the explication that it fails to entail asymmetry. I conclude that an analysis of internality in terms of supervenience has promise.

5. External relations

[21] To show that every intrinsic property supervenes on dyadic internal relations, suppose that H is any intrinsic property. Let R hold between x and y just in case both x and y have H. It is easy to verify that R supervenes on H, and is thus internal. Likewise, it is easy to verify that H supervenes on R.

Even if we suppose that internal relations are those that supervene on intrinsic properties, there remains the question what external relations are. Intuitively, external relations are those whose exemplification is not a matter of how the relata are intrinsically. A first stab would be to say that a relation is external iff it is not internal. However, this may be too liberal, since it allows relations with internal conjuncts or disjuncts to be external. A second stab is due to Lewis: external relations are those that are not internal but still intrinsic rather than extrinsic. What, then, are intrinsic relations? The concept of intrinsicality, familiar in the domain of properties, can be applied to relations too. Intuitively, a dyadic relation is intrinsic if it such that whenever x bears it to y, it does so in virtue of how things stand with respect to x and y, independently of how the rest of the world is; and mutatis mutandis whenever x does not bear it to y. There is no need to discuss the concept of an intrinsic relation in detail here. For our purposes, it is enough to assume that the class of intrinsic relations includes all internal ones, and is closed under supervenience.

Paradigmatic examples of external relations are spatial ones, such as being two meters apart, as well as temporal and spatiotemporal ones – at least as these relations are typically conceived of by metaphysicians.[22] What is distinctive of such relations is that they are independent of the intrinsic nature of the relata: the masses and charges of two point particles neither metaphysically necessitate nor preclude any particular distance between them. It is natural to take this as a general account of external relations: they are those that are independent of the intrinsic properties. This would be consistent with how earlier authors conceived of external relations. For example, Taylor (1903: 141) seems to have an independence account in mind when he asks 'whether there are or are not merely external relations (i.e. relations which are independent of the special qualities of their terms)'.

When Lewis defined external relations as those that are intrinsic but not internal, he did so in a context in which it was dialectically useful for him to have internality, externality and extrinsicality to come out trichotomous. However, if we follow him in taking every intrinsic relation to be either internal or external, we fail to do justice to the idea that external relations are independent of intrinsic properties. If R is internal and R' any other relation, then neither their conjunction nor their disjunction enjoy such independence. However, if R is internal and R' external in Lewis' sense – *weakly external*, from now on – then either their conjunction or their disjunction is also weakly external.[23]

There is a more demanding sense of externality such that certain conjunctions or disjunctions of an internal and an external relation come out as neither internal nor external, since they are not suitably independent from intrinsic properties.[24] However, it turns out that such a

[22] Bricker (1993) contends that this conception is wrong. He draws on the General Theory of Relativity to argue that actual spatiotemporal relations are extrinsic, not external.

[23] Both $R \wedge R'$ and $R \vee R'$ are intrinsic: they supervene on the intrinsic R and R', and the class of intrinsic relations is closed under supervenience. Moreover, at least one of $R \wedge R'$ and $R \vee R'$ fails to be internal. For reductio, suppose they are. Then R, $R \wedge R'$ and $R \vee R'$ are all internal. Since R' is cointensional with a truth-functional combination of those three relations, it supervenes on the class consisting of them. Using the closure of internal relations under supervenience, we can conclude that R' is also internal, contrary to the assumption that it is weakly external.

[24] There is a precedent for this in the analogous case of intrinsic properties: Lewis (1999) was happy to allow for properties which are neither intrinsic nor extrinsic, but intermediate.

demanding notion of externality cannot be defined in terms of supervenience. Full independence implies lack of supervenience, but lack of supervenience does not imply full independence. However, it can be defined from the same conceptual tools that have been used to define supervenience. Lewis (1998: 117) introduced a notion of orthogonality of subject matters: 'Two subject matters M_1 and M_2 are *orthogonal* iff, roughly, any way for M_1 to be is compatible with any way for M_2 to be.' Orthogonal subject matters are independent of each other. We can define a notion of orthogonality that stands to strong supervenience like Lewis' orthogonality stands to global supervenience. Say that n-ary R and m-ary R' are *orthogonal* iff for any sequences σ and σ', there is a sequence σ'' that is R-indiscernible from σ and R'-indiscernible from σ'. If R and R are orthogonal, any way to be with respect to R is compatible with any way to be with respect to R'. The following definition of strong externality then suggests itself: R is *strongly external* iff it is orthogonal to the class of intrinsic properties.[25]

Since any intrinsic relation is either internal or weakly external, it is trivially the case that any intrinsic relation supervenes on the internal and the weakly external relations together. However, it is a substantive metaphysical question whether any intrinsic relation supervenes – strongly or even only globally – on the internal and the strongly external relations together. It is conceivable, after all, that there are weakly external but no strongly external relations. Imagine that distance relations are intrinsic, but constrained with metaphysical necessity by some intrinsic property of the relata. Perhaps very massive point-particles cannot be very close to each other. If so, distance relations are not strongly external. Since they are generally taken to be a prime candidate, we may doubt whether there are any strongly external relations. Furthermore, some philosophers might think that being abstract is an intrinsic property, and that abstract objects cannot stand in spatial relations.[26]

If we take some relations to be uncontroversially external, we may think that the above analysis undergenerates. However, there is another respect in which it might be thought to overgenerate: it entails that if R' supervenes on a strongly external R, then R' is likewise strongly external.[27] For if σ and σ' are any sequences, then the strong externality of R guarantees that there exists a sequence σ'' that is R-indiscernible from σ and I-indiscernible from σ'. Since R' supervenes on R, σ'' is also R'-indiscernible from σ, and thus also

[25] Given this formal explication of independence in hand, we can now prove that independence is not definable from supervenience, by describing two models M_1 and M_2 that agree on all supervenience claims but not on all orthogonality claims. In M_1, a has F and G, b has F but not G, and c has G but not F. M_2 is just like M_1, except that there is an additional element d that has neither F nor G. It is easy to verify that in both models, $\{F\}$ fails to supervene on $\{G\}$, and $\{G\}$ fails to supervene on $\{F\}$. It follows that the models agree on all supervenience claims. However, $\{F\}$ and $\{G\}$ are orthogonal only in M_2, but not in M_1.

[26] To be sure, metaphysicians with Humean inclinations will be disposed to dismiss such scenarios, on the grounds that they involve a brute necessary connection. But Humeanism iss itself a substantive metaphysical claim.

[27] This is not to say that the class of strongly external relations is closed under supervenience. Supervenience on a more than one-membered class of strongly external relations does not guarantee strong externality. Perhaps a certain profile of intrinsic properties is compatible with two things being at any spatial distance from each other, and compatible with them being at any temporal distance from each other, but incompatible with them having a certain spatial distance and a certain temporal distance together.

orthogonal to I. As a consequence, relations that supervene on any relation turn out to be strongly external as well as internal. Surprisingly, then, the classes of internal and of strongly external relations are not disjoint. Moreover, the proposal may yield intuitively questionable results for some relations that hold between more than two things. Consider equidistance, i.e. the relation R that holds between x, y, and z iff the distance between x and z is equal to the distance between y and z. Since equidistance supervenes on distance, it is strongly external if distance is. However, there is a sense in which that three-place relation is not fully external – it is determined by intrinsic relations holding between some but not all of its relata. For that reason, we may define a relation of adicity $n + 1$ as *purely external* iff it is orthogonal to all intrinsic properties and relations of adicity at most n.

I will not try to settle the question whether external relations are adequately analysed in terms of orthogonality and intrinsicality. However, I hope to have made it plausible that both supervenience and the related concept of orthogonality can fruitfully be applied to relations.[28]

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