A Nonlinear Approach to Modeling of Electrically Stimulated Skeletal Muscle

Henrik Gollee*, David J. Murray-Smith, and Jonathan C. Jarvis

***Abstract*—This paper is concerned with the development and analysis of a nonlinear approach to modeling of the contraction of electrically stimulated skeletal muscle. The model structure is based on a network of locally valid linear models which are blended together by a scheduler. Data are from experiments with rabbit tibialis anterior muscles in which the muscles contracted isometrically while being stimulated by supramaximal impulses with randomly varying inter-pulse intervals. The model accounts for nonlinear effects due to variations of the stimulation frequency, such as the "catch-like" effect. It is shown that this modeling technique is suitable for modeling the contraction of muscles with very different characteristics, such as muscle with a majority of fast motor units and muscle with mainly slow motor units. The approach is also suitable as a basis for the design of muscle stimulation controllers.**

***Index Terms*—Functional electrical stimulation, local model network, muscle modeling, nonlinear system identification.**

I. Introduction

FOR muscle which has lost nervous control, artificial electrical stimulation can be used as a technique aimed at providing muscular contraction and producing a functionally useful movement [28]. This is generally referred to as functional electrical stimulation (FES) and is used in different application areas such as the rehabilitation of paralyzed patients in whom natural neural control of muscular contraction has been lost due to a spinal cord injury [20], [35] and in cardiac assistance where skeletal muscle can be used to support a failing heart [38]. For both these FES applications, a model of the muscle is essential to develop algorithms for its controlled stimulation. Such a model needs to describe the nonlinearities within the behavior of the muscle that are relevant to the intended application. The model parameters need to be easily adjustable, and capable of derivation from the results of standard experiments which do not damage the muscle. The structure of the model needs to be controller-orientated, that is, controller design based on such a model should be possible. The designed controller must not be computationally expensive if it is to be implemented in the form of an implantable device.

An extensive review of various muscle modeling approaches can be found in [43]. Many muscle models are based on an analysis of the physiological principle of muscle contraction, either on a macroscopic or on a microscopic level. Such models have the advantage that their parameters can often be directly related to characteristics of the muscle. On the other hand, such models tend to be complex and thus computationally expensive. Their parameters are often difficult to identify, and their structure is rarely controller orientated.

Another common modeling approach is to use empirical model strategies which aim to describe the input–output (I/O) characteristics of muscle (often limited to conditions common in FES applications) and are purely based on I/O data. The simplest form of an empirical model of muscle contraction is a linear second-order dynamic model which has been found adequate to describe isometric contraction [30], [43] and nonisometric contraction [2], [3] under constant stimulation conditions (e.g., constant pulse energy and frequency).

Nonlinear effects of varying motor-unit recruitment for nonconstant stimulation are often accounted for by adding a static recruitment curve which results in a Hammerstein model [4], [40], [15]. A model with this structure can be transformed easily into a form suitable for linear controller design by shaping the control signal by the inverse static recruitment curve [14], [20]. Generally, however, the process of motor unit recruitment is not static as it includes, for example, hysteresis effects and the assumption that the dynamics of the muscle can be described by a linear time-invariant transfer function does not hold if the level of activation varies over a wide range [21].

Most empirical model structures account only for effects due to varying stimulation levels but are unable to describe changes of the muscle characteristics with varying stimulation frequency, or varying inter-pulse interval (IPI). Controller-orientated models which describe nonlinear properties related to variations of the stimulation frequency should be very useful for FES, as it is important to employ such nonlinear characteristics to stimulate the muscle in a way which is closer to natural stimulation patterns, and this can help to reduce muscle fatigue [27].

It is well known that muscle characteristics vary significantly with the stimulation frequency [9]. The static force (that is, the sustained force of a tetanic contraction) increases in a sigmoidal way for steadily increasing frequency of stimulation (the force-frequency curve). Generally, the force-frequency relationship is not static; the force developed by the muscle depends on the history of the stimulation frequency in a dynamic way [5], [13]. A nonlinear summation of contraction for stimulation pulses with a very short IPI can be observed in many muscles, and the phrase "catch-like" effect is often used to refer to this. The effect is described by, e.g., Burke *et al.* [8], Binder-Macleod, and Barrish [5], and analyzed in [33] and [41]. As the "catch-like" effect is normally initiated by a doublet or triplet of pulses with short IPI, it is sometimes referred to as the "doublet" or "triplet" effect.

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Bobet *et al.* [7] showed that a linear time-varying model can successfully describe muscle contraction under conditions where pulse energy and/or stimulation frequency vary. In their model structure, the muscle force is approximated by a critically damped, linear second-order system which is time-invariant between stimulation pulses. The model parameters are adapted separately for each IPI. This results in an overall model with as many linear models as IPIs. The fact that this approach can approximate isometric muscle contraction under various stimulation conditions shows that a second-order time-variant linear model is an appropriate model structure. The approach was developed further to a linear time-varying Wiener-type model in which a static nonlinearity is placed between two first-order transfer functions [6]. The time-constant of the second transfer function varies with the force. Good matches were found with experimental data for stimulation with varying IPI.

Donaldson *et al.* [11] obtained encouraging results using a radial basis function network to model isometric contraction of muscle which is stimulated with supramaximal pulse trains of varying frequency. This approach was developed further using local descriptions of muscle characteristics by second-order linear models which are valid for certain operating regions [17]. These local models are blended together using a scheduler which selects the model closest to the current operating point, and interpolates between models. As this overall blended structure represents a time-varying linear model, the approach is closely related to to the work presented by Bobet *et al.* in [6] and [7]. The model developed in [17] was, however, found to be suitable only for muscle with a majority of fast motor units. Thus, the approach was extended and generalized, and the model presented in this paper can be used with a wider range of muscles.

This paper is structured as follows. In Section II, the experimental setup for the data collection is described. Data for contraction with constant muscle length were collected from two muscles with very different characteristics. The empirical modeling approach which is based on a decomposition of the operating space into smaller sub-regions is introduced in Section III. Results which compare the model output with the experimental data are presented in Section IV. These results are discussed and the model properties are analyzed in Section V. Conclusions are presented in the final section.

II. Methods

The data used in this paper were obtained from experiments with rabbit *tibialis anterior* muscles. The muscles were stimulated indirectly by irregular supramaximal pulse trains using flap electrodes placed around both common peroneal nerves. The term *supramaximal* refers to the fact that the amplitude and length of the stimulation pulses were chosen such that all motoneurons of the muscle were recruited. The activation of the muscles was varied by changes of the IPIs of the stimulation pulses.

The experimental protocol is described in detail in [29]. The following conditions are particularly relevant for our studies and apply to all experiments.

- The muscles were stimulated using electrical impulses of 200-μs duration and an amplitude three times the threshold for muscle stimulation, which ensures supramaximal stimulation.
- The IPIs were varied randomly between 1 and 70 ms.
- The maximum duration of each pulse train was 300 ms. Together with periods of rest of 30 s between the pulse trains, this ensured that the influence of fatigue on the recorded data was minimized.
- A constant-frequency burst of impulses (25-ms IPI, which corresponds to a stimulation frequency of 40 Hz) was delivered every 5 min to check that the preparation did not show progressive deterioration during the experiment.
- The data were recorded with a sampling interval of $T_s =$ 1 ms.

The contractile force of the muscle was measured and recorded while the muscle contracted isometrically, i.e., the muscle length was held constant. In our experimental setup, the muscle pulls a lever which is attached to a servomotor. The muscle length was controlled and the force measured by means of this servomotor which was designed for this purpose (Model 310B, Cambridge Instruments, Watertown Massachusetts). This instrument is capable of setting and holding length with an error of less than 0.01 mm against forces of up to 50 N. The linearity of the force measurement is within 0.2% of the force range, and the resolution of the force signal is 0.01 N. In practice, the resolution of the experimental data was determined by the 12-bit analog-to-digital converter that we used so that the working resolution was approximately 0.02 N for force, and 0.02 mm for length.

The experimental data were preprocessed such that the offset in the measured force was removed, and the input and output data sets were normalized in such a way that they lay in the range $[0, 1]$.

Two types of muscle were used in the experiments, a control muscle and a chronically stimulated muscle.

The *control* muscle is an unchanged rabbit tibialis anterior whose characteristics are determined by a majority of fast motor units. We will, therefore, refer to it as the *fast* muscle. A total of 60 data sets, containing the input pulses and the contractile force were recorded. The duration of each set is 590 ms.

The *chronically stimulated* muscle is a rabbit tibialis anterior which was stimulated at 10 Hz for four weeks. As outlined in [22], such chronic stimulation reduces the contractile speed of the muscle. The muscle characteristics are, therefore, dominated by slow motor units, and we will refer to this muscle as the *slow* muscle. A total of 84 data sets, containing the input pulses and the contractile force was recorded. The duration of each set is 600 ms.

III. Modelling

The basic idea of the modeling approach employed in this work is to divide a complex nonlinear modeling task into smaller and simpler sub-tasks. Each sub-task can then be handled locally by a simpler model. A scheduler is used to decide how relevant the models are for the current operating condition and weights them accordingly. The overall model is the sum of all weighted local models. This approach is referred to as a local model network (LMN) [24]. The relationship between LMNs and other approaches, such as those based on fuzzy logic, is reviewed in [25].

The local models used to form a LMN can generally be of any form, e.g., linear or nonlinear, in I/O or state-space form, empirical or based on physical analysis. It is often straightforward to incorporate *a priori* knowledge when selecting the structure of

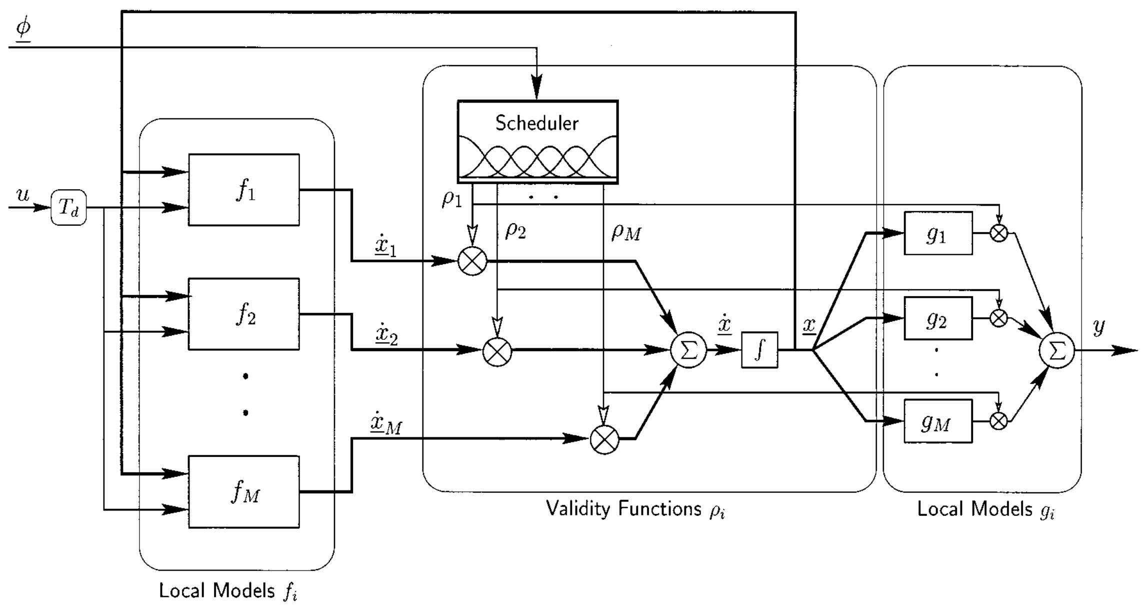


Fig. 1. LMN in state-space representation.

the local models. We will restrict ourselves to local *linear* descriptions, employing the concept of local linearization.

To illustrate the modeling concept, we consider the following general time-invariant nonlinear system in state space form

$$\dot{\underline{x}}(t) = f(\underline{x}(t), u(t - T_d)) \tag{1a}$$

$$y(t) = g(\underline{x}(t)). \tag{1b}$$

Here, $f()$ and $g()$ are nonlinear, continuous differentiable functions. For simplicity, we restrict ourselves to single-input–single-output system, i.e., the input u and the output y are scalar. The dimensionality n of the state vector $\underline{x}$ defines the dynamic order of the system as a function of time t, and $\dot{\underline{x}}(t)$ denotes the derivative of the state with respect to time, $d\underline{x}/dt$. The scalar $T_d \in \mathbb{R}$ is a time-delay, and the initial state at $t = 0$ is $\underline{x}_0$.

In the state-space description (1), there are two nonlinear functions, f and g, which can each be approximated by means of a local function decomposition. The system can then be rewritten as a weighted sum of M local models

$$\dot{\underline{x}}(t) = \sum_{i=1}^{M} \rho_i(\underline{\phi}(t)) f_i(\underline{x}(t), u(t - T_d)) \tag{2a}$$

$$y(t) = \sum_{i=1}^{M} \rho_i(\underline{\phi}(t))\, g_i(\underline{x}(t)) \tag{2b}$$

which is a LMN representation of the system (1). Here, $f_i()$ and $g_i()$ are local approximations for $f()$ and $g()$, respectively, for the operating conditions where the corresponding validity function ρ_i is active. The set of validity functions $\{\rho_i\}_{i=1}^{M}$ forms the scheduler, where $\underline{\phi}$ is the scheduling vector. This structure is shown in Fig. 1.

Employing the concept of local linearization for different operating conditions, we choose to work with standard linear local state-space representations [26]. This results in

$$f_i(\underline{x}(t), u(t - T_d)) = \mathbf{A}_i \underline{x}(t) + \underline{b}_i u(t - T_d) + \underline{d}_i^x \tag{3a}$$

$$g_i(\underline{x}(t)) = \underline{c}_i^T \underline{x}(t) + d_i^y. \tag{3b}$$

with $i = 1, \ldots, M$. The overall system can then be approximated as

$$\dot{\underline{x}}(t) = \sum_{i=1}^{M} \rho_i(\underline{\phi}(t)) \left[\mathbf{A}_i \underline{x}(t) + \underline{b}_i u(t - T_d) + \underline{d}_i^x\right] \tag{4a}$$

$$y(t) = \sum_{i=1}^{M} \rho_i(\underline{\phi}(t)) \left[\underline{c}_i^T \underline{x}(t) + d_i^y\right]. \tag{4b}$$

An equivalent discrete-time form which is suitable for implementation in real-time computing hardware, can be obtained by transforming the system (4) into δ operator form [31]

$$\delta \underline{x}(k) = \frac{\underline{x}(k+1) - \underline{x}(k)}{T_s} = \sum_{i=1}^{M} \rho_i(\underline{\phi}(k)) \left[\tilde{\mathbf{A}}_i\, \underline{x}(k) + \tilde{\underline{b}}_i u(k - k_d) + \tilde{\underline{d}}_i^x\right] \tag{5a}$$

$$y(k) = \sum_{i=1}^{M} \rho_i(\underline{\phi}(k)) \left[\tilde{\underline{c}}_i^T \underline{x}(k) + \tilde{d}_i^y\right]. \tag{5b}$$

Here, T_s denotes the sampling period, $k = 1, 2, 3, \ldots$ is the sample index, and k_d is the input delay defined as $k_d = T_d/T_s$.

It should be noted that (4) and (5) represent linear parameter-varying (LPV) systems in which the parameters depend on the scheduling vector $\underline{\phi}$. For (4), we can substitute the interpolated model parameters as functions of $\underline{\phi}$

$$\begin{aligned} \mathbf{A}(\underline{\phi}) &= \sum_{i=1}^{M} \rho_i(\underline{\phi}) \mathbf{A}_i, \quad \underline{b}(\underline{\phi}) = \sum_{i=1}^{M} \rho_i(\underline{\phi}) \underline{b}_i \\ \underline{d}^x(\underline{\phi}) &= \sum_{i=1}^{M} \rho_i(\underline{\phi}) \underline{d}_i^x \\ \underline{c}(\underline{\phi}) &= \sum_{i=1}^{M} \rho_i(\underline{\phi}) \underline{c}_i, \quad d^y(\underline{\phi}) = \sum_{i=1}^{M} \rho_i(\underline{\phi})\, d_i^y. \end{aligned} \tag{6}$$

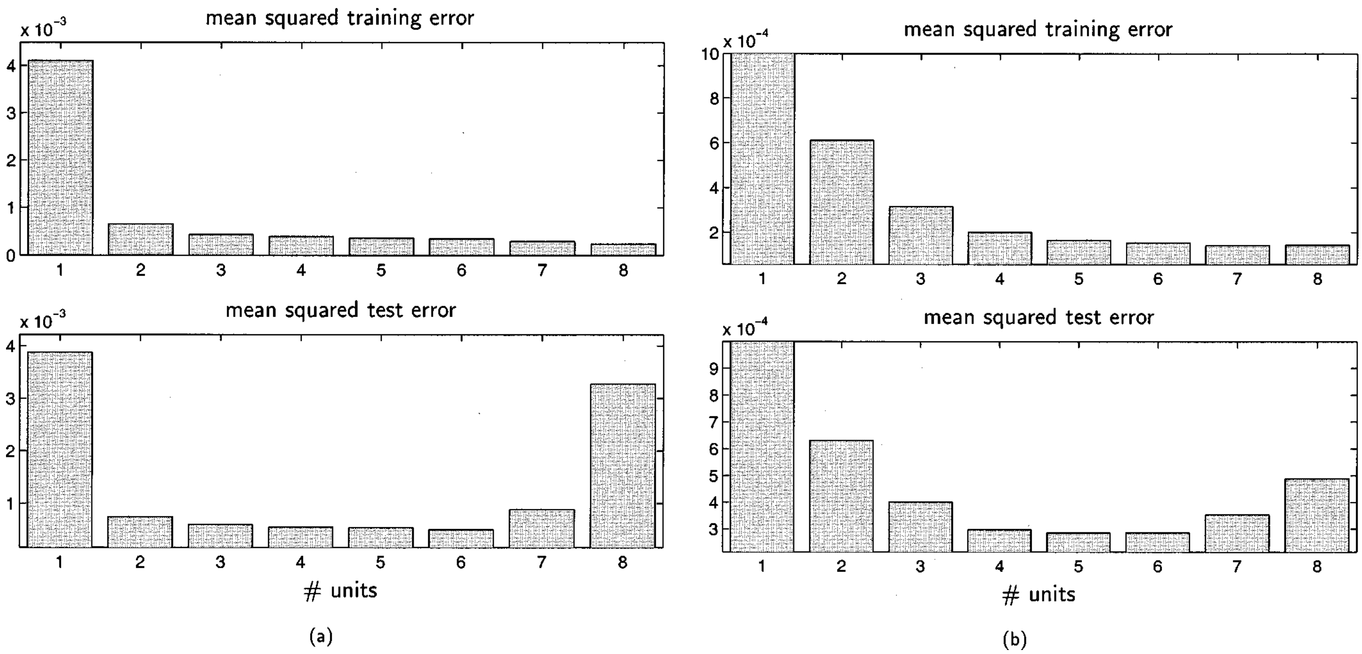


Fig. 2. Training and test mses for LMN structures with one to eight units, simulated with an infinite prediction horizon. (a) Fast muscle. (b) Slow muscle.

The system can then be rewritten as an LPV system

$$\dot{\underline{x}}(t) = \mathbf{A}(\underline{\phi}(t))\underline{x}(t) + \underline{b}(\underline{\phi}(t))\,u(t-T_d) + \underline{d}^x(\underline{\phi}(t)) \quad \text{(7a)}$$

$$y(t) = \underline{c}^T(\underline{\phi}(t))\underline{x}(t) + d^y(\underline{\phi}(t)). \quad \text{(7b)}$$

Equations (5) can be rewritten in a similar way.

The model properties of the system (7) can be analyzed locally using methods which are based on standard linear system analysis [39]. This provides a means of checking the local model properties against known characteristics of the real system.

Local models of second-order were found to be optimal for the given application which is in agreement with other findings, e.g., [43], [7]. A time-delay of $T_d = 5$ ms was detected.

We chose to work with a set of quadratic B-splines [10] for the validity functions $\{\rho_i\}_{i=1}^M$ in (4). This ensured both a localized region of activity for each local model and a smooth interpolation between neighboring models.

The scheduling vector $\underline{\phi}$ should represent the nonlinear changes of the system characteristics. It can generally be the input or the output of the system, or a combination of both. In [17] and [18], the output of the system was used to select which local model is active. In the experiments presented here, this approach did not generally yield satisfying modeling results, as it showed good results only with the fast muscle, but failed for the slow muscle.

Another straightforward choice for the scheduling variable is to use a measurement of the instantaneous stimulation frequency. As the current stimulation frequency cannot be determined for a sequence of pulses with randomly varying IPI, it is approximated by filtering the input pulses using a second-order critically damped low-pass system with the transfer function

$$G_{\mathrm{pre}}(s) = \frac{k}{s^2 + 2\xi\omega_n s + \omega_n^2}. \quad (8)$$

This approach which uses the system input to select which local model is active, was found to give good results for all the investigated muscles. The choice of the filter parameters is straightforward: the damping factor is set to $\xi = 1$ to obtain a critically damped response. The factor k is selected such that the filtered variable lies approximately within the input range of the validity functions, i.e., between zero and one. The natural frequency ω_n was selected empirically. We found that its exact value is not critical as the model structure can compensate for a nonoptimal choice. Thus, for our experiments we chose to work with $\xi = 1$, $\omega_n = 50$, $k = 25 \times 10^3$.

The parameters of the local models in (4) or (5), $\{\mathbf{A}_i, \underline{b}_i, \underline{d}_i^x, \underline{c}_i, d_i^y\}_{i=1}^M$, can be optimized with an infinite prediction horizon using the Levenberg–Marquardt algorithm [36]. The number of local models is determined by successively increasing the network size until an optimum with respect to the mean-squared-error (mse) on test data is reached. Note that this definition of the optimal size does not necessarily find the simplest acceptable structure. This could be achieved by extending the mse criterion with a penalty term for the number of parameters, such as in Akaike's Information Criterion [1].

IV. Results

The LMN approach was used to obtain models for the fast and the slow muscle introduced in Section II. For each muscle, 30 data sets were used for the identification of the model parameters. The models were then validated using the remaining test data sets (30 sets for the fast muscle and 54 sets for the slow muscle). Starting with a single local model (i.e., with a linear model), the number of local models in the structure was steadily increased. Whereas the error on the training data decreases monotonically with the number of units in the LMN, the error on the test data sets starts to increase after the optimal model size has been reached which indicates over-fitting of the data [19]. This is shown in Fig. 2. The structure with the smallest test error is then selected as the optimal one for each muscle.

Test results for some typical data sets which were not used for the identification of the model parameters are presented for both muscles in Fig. 3.

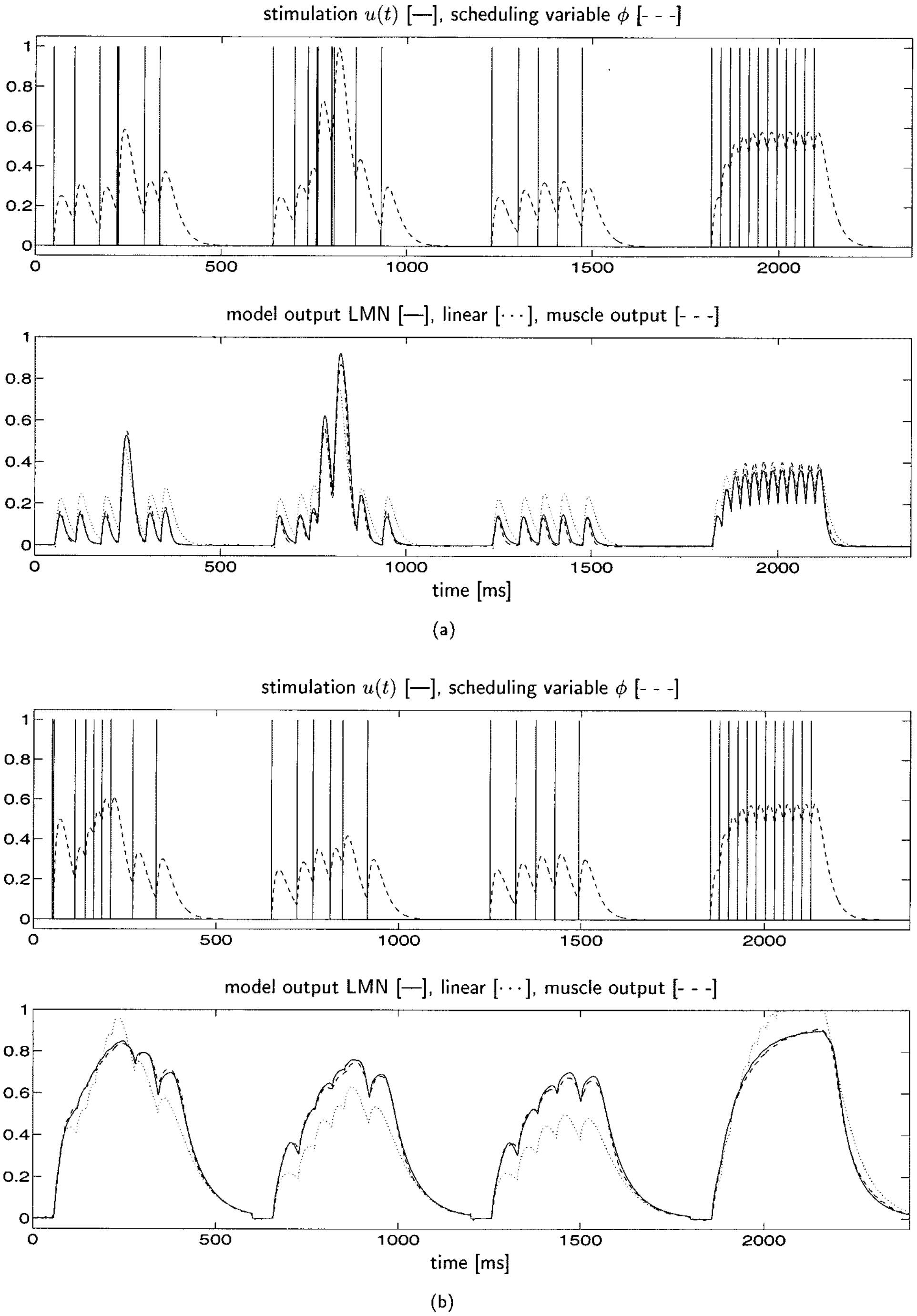
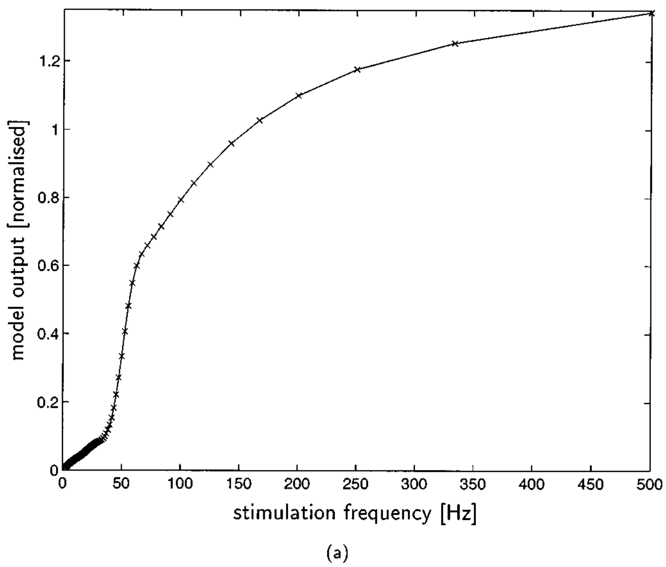


Fig. 3. Modeling results. Comparison of the simulated LMN model output (lower panels, solid line) and the output of the muscle (lower panels, dashed line) for typical test input sequences (upper panels, solid line). The corresponding scheduling variable is also shown (upper panels, dashed line). Additionally, simulation results with linear models are shown (lower panels, dotted line). The data sets are shown concatenated; the simulation is restarted after each set. (a) Fast muscle. (b) Slow muscle.

For the fast muscle, an LMN with six local models was found to be optimal. Test results are shown in Fig. 3(a). The LMN model output matches the muscle output with great accuracy for almost all operating conditions. A small model error is present in the first part of the fourth data set. The "catch-like" effect which is present in this muscle is modeled accurately (middle of first data set at 250 ms, fourth and fifth pulse of the second data set at 750 ms).

For the slow muscle, a structure comprising five local models performed best. Results for typical test data are shown in Fig. 3(b). The LMN model output matches the muscle output for all operating conditions. The "catch-like" effect is not present in this muscle as can be seen at the beginning of the first data set.

For comparison, the results obtained with linear models are included for both muscles in Fig. 3. Although these models provide a response which follows the muscle output on average, large errors can be observed in particular for low and for high stimulation levels.



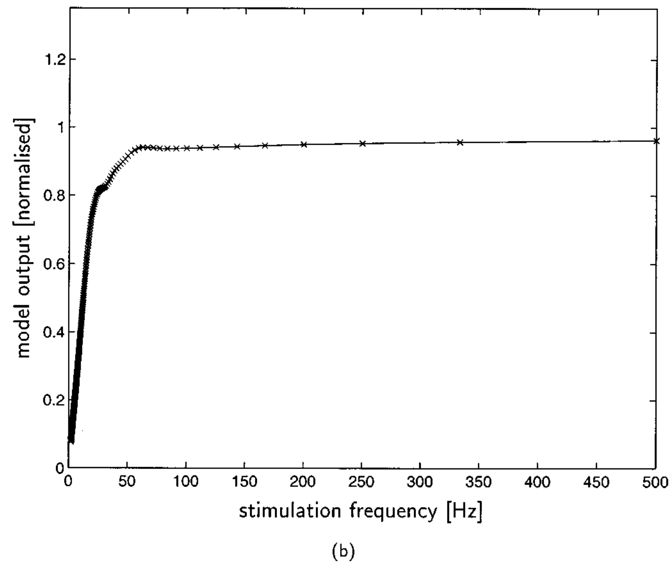


Fig. 4. Force-frequency curve for monotonically decreasing IPI. (a) Fast muscle. (b) Slow muscle.

V. DISCUSSION

The result plots in Fig. 3 show an excellent match between the outputs of the nonlinear models and the muscle forces for all operating conditions present in the data. The different characteristics of the muscles for varying stimulation conditions are modeled accurately. In particular, the "catch-like" effect (i.e., the response to two or three closely spaced stimulation pulses) is represented correctly in the model of the fast muscle. The LMN approach is, therefore, suitable to model the I/O characteristics of muscle stimulated with varying IPI.

The top plot in Fig. 3(a) shows that the obtained scheduling variable ϕ resembles a scaled linear approximation of the output of the fast muscle. Although this was not intended when the parameters of the prefilter, (8), were selected, it is a direct results of choosing to work with a second-order filter. As scheduling on the system output worked well for the fast muscle in [17] and [18], using a scheduling variable which has similar characteristics as the output of the fast muscle is expected to be suitable for both the fast and the slow muscle.

The validation of a model by comparing its output with experimental data is useful to characterize the model performance for the operating conditions present in the data. Generalized statements about the model performance can, however, only be made if data are available for all operating conditions. This is normally not possible for nonlinear models as an unlimited amount of data would be required in this case. For a proper analysis of the model performance, it is, therefore, useful to employ additional means of model validation. We will, therefore, first analyze the models with respect to the force-frequency characteristics of the muscle, and then analyze model properties such as steady-state gain and time-constants, and relate them to known properties of the muscles.

A. Force-Frequency Characteristic

In Fig. 4, the outputs of the models are shown for a monotonically increasing stimulation frequency. These curves correspond to the force-frequency characteristics of the muscle, the qualitative shape of which is known. For a steadily increasing stimulation frequency, the muscle output force increases monotonically until a saturation level is reached. A further increase of the stimulation frequency does not lead to a higher force generation. This saturation effect can be observed clearly for the slow muscle model [Fig. 4(b)]; it is less strong for the model of the fast muscle [Fig. 4(a)].

Owing to the domination of slow motor units in the slow muscle, we expect that this muscle will reach its saturation level at frequencies which are lower than the saturation frequency of the fast muscle. Comparing Fig. 4(a) and (b), we can conclude that this is also the case with the corresponding muscle models.

B. Analysis of Model Properties

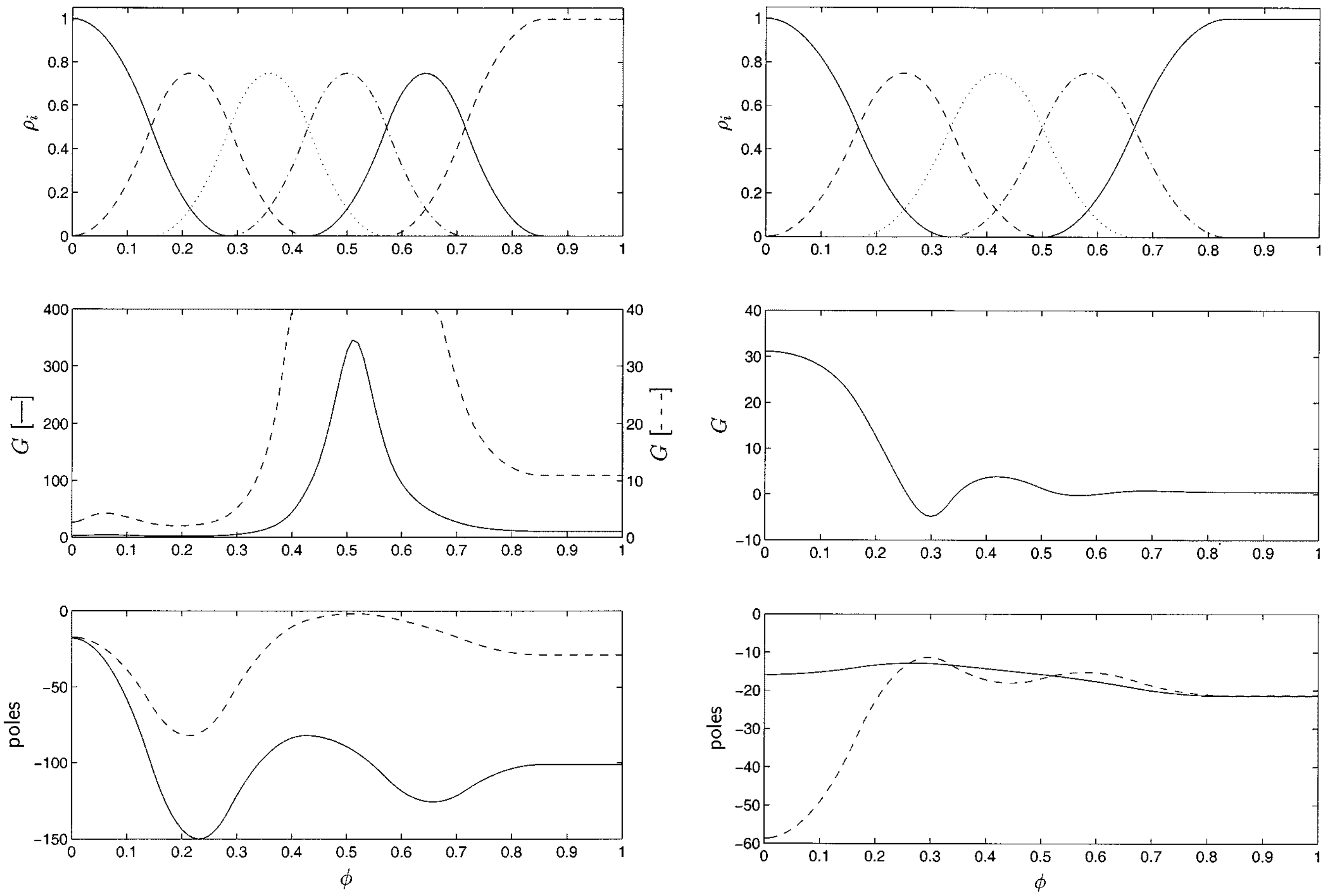
As mentioned in Section III, the form of LMNs used here represents a linear parameter-varying system, cf. (7). Thus, linear system characteristics can be extracted for each level of the scheduling variable ϕ. These characteristics can then be interpreted and related to properties of the physical system. A detailed system-theoretical analysis, with this muscle modeling application as an example, can be found in [39]. For the present paper, we restrict ourselves to a straightforward interpretation of the changes of the steady-state gains, $G(\phi)$, and of the location of the poles, $p(\phi)$, with varying scheduling variable, i.e., with varying muscle activation. These are defined as

$$G(\phi) = \underline{c}^T(\phi)\,\mathbf{A}^{-1}(\phi)\underline{b}(\phi) \tag{9a}$$

$$p(\phi) = \mathrm{eig}\mathbf{A}(\phi) \tag{9b}$$

and are shown for both models in Fig. 5. These characteristics can be interpreted as follows.

- The steady-state gain [Fig. 5 (middle plots)] is the integral of the system response to an impulse. For the present application it, therefore, represents the amount of force generated over time, per pulse, i.e., the force-time integral (FTI)/pulse.



(a)

(b)

Fig. 5. Static analysis for the optimal LMN structures. (upper plots) Activations of the validity functions, ρ_i. In the middle plots, the steady-state gains, G, of the interpolated models are depicted. In the middle plots, values of the two poles of the interpolated models are shown. (Note that the poles are real for all operating conditions.) These properties are shown as functions of the scheduling variable ϕ which is related to the muscle stimulation. (a) Fast muscle. In the middle plot, the solid line corresponds to the scaling of the left y axis and the dashed line relates to the scaling of the right y axis. (b) Slow muscle.

- The location of the poles [Fig. 5 (lower plots)] characterizes the dynamic response of the system. The absolute magnitude of the poles is related to the time-constants of the system: the larger the magnitude, the smaller is the corresponding time-constant, and the faster is the system response. The system is stable if all its poles have a negative real part. If all poles are real (i.e., they do not have an imaginary component) then the system response is over-damped.

It should be noted that this interpretation is generally only valid for a slowly changing scheduling variable as it neglegts transient effects which can occur due to rapid changes of operating regimes [39]. The simplified interpretation provides nevertheless useful insight into the model characteristics.

For both models, the characteristics vary smoothly with the scheduling variable. Together with the analysis of the test and training errors in Fig. 2, this indicates that the models do not over-fit the data.

For the model of the fast muscle, the gain varies over a wide range, with a distinct maximum at $\phi \approx 0.5$. Note that this value of the scheduling variable is achieved by stimulation with a doublet with an IPI of 2 ms. The gain for small activation is relatively small. For high activation, the gain remains constant and nonzero. Thus, saturation is not present for large activations.

The large variation of the gain for the models of the fast muscle indicates that, for this muscle, the FTI/pulse depends strongly on the activation. This is expected as the fast muscle shows the "catch-like" effect. The large value of the gain for $\phi \approx 0.5$ can be related to the initial response of the model to two closely spaced input pulses (the response to a "doublet").

For the model of the slow muscle, the gain varies over a much smaller range than the gain of the model for the fast muscle. It has a distinct maximum for small activation, and quickly decreases as the activation becomes larger. The gain becomes slightly negative for $\phi \approx 0.3$. This could indicate that for this activation, stimulation will decrease the model output, but it is also possible that transient effects are present here (which we neglegt in our simplified interpretation). For large activation, the gain is close to zero which corresponds to saturation.

The limited and smooth variation of the gain for the models of the slow muscle indicates that the FTI/pulse does not change significantly depending on the activation. We expect such behavior for this muscle as it does not show the "catch-like" effect. The fact that the gain is almost zero for activations above

$\phi \approx 0.7$ corresponds to a decrease of the FTI/pulse for intensive stimulation.

The poles of the model of the fast muscle vary over a wide range. A fast and a slow pole can be distinguished. The magnitude of the poles is maximal for $\phi \approx 0.2$ (i.e., the system response is fastest) and the value of the slow pole approaches zero for $\phi \approx 0.5$. This can be related to the initially very fast response of the model to two closely spaced input pulses (the response to a "doublet") where the response becomes slower once the large force has been reached. For the model of the slow muscle, the poles are significantly slower than the poles of the model for the fast muscle. The slow pole changes only slightly, whereas the second pole becomes close to the slow pole for increased activation.

This analysis shows that known properties of the muscles can be related to characteristics of the models. This verifies that the models have not just performed a simple mapping of I/O data, but have captured the muscle characteristics correctly. The analysis reveals also that there is no strong saturation present for the model of the fast muscle. This corresponds with the shape of the force-frequency curve shown in Fig. 4(a). Further analyses show that the model of the fast muscle is indeed limited in that it is unable to predict correctly the saturated muscle response to long trains of constant stimulation with small IPI. Such input patterns were not present in the experimental data used here.

The modeling approach uses only I/O data of the muscle from simple experiments which are designed to excite all dynamic modes of the system. No explicit knowledge of the physiological processes of muscle contraction was used. However, the modeling technique differs from a black-box approach in that *a priori* knowledge about the expected muscle behavior was used for the selection of the scheduling variable and for the analytical validation of the models obtained. Thus, the approach described here could be termed a "grey-box" technique.

The fact that knowledge of the physiological properties of muscle is not used directly in the modeling approach is, however, also a potential disadvantage. All information necessary to describe the characteristics of the muscle needs to be presented in the experimental I/O data. It is, therefore, necessary to ensure that all operating conditions of interest for the intended application are represented in the data. The limitations of the modeling technique become obvious for the models of the fast muscle which cannot predict the saturation present in real muscle for stimulation with long trains of constant high frequency. One way to over-come this limitation would be to include long trains of input stimulation with different constant frequencies in the experimental data.

VI. Conclusion

In this paper, a novel approach to modeling of electrically stimulated muscle under conditions of isometric contraction has been presented. The model is nonlinear and its structure is based on a network of locally valid linear models which are blended together by a scheduler to form a LMN. The model accounts for nonlinear effects due to variations of the stimulation frequency, such as the "doublet" effect. It was shown that this modeling technique is suitable for modeling the contraction of muscles with very different characteristics, such as muscle with a majority of fast motor units and muscle with mainly slow motor units.

The LMNs used here represent linear time-varying systems. Such systems have been found to be suitable for describing muscle contraction under varying conditions [6], [7], [17]. The technique to change the model parameters depending on the muscle activation using a scheduling variable obtained from the model inputs can be used for muscles with very different characteristics.

The approach can be interpreted as a generalized form of a Hammerstein model [21]. In the Hammerstein model structure, only the gain of the model varies. In our approach, additionally the dynamics of the model vary for different operating regions. The analysis of the model properties shows that the variation of both the gain and the dynamics (i.e., location of poles) of the model are required to adequately describe the characteristics of both muscle types.

The model structure is controller orientated. An application of local control techniques based on the muscle model introduced here is described in [17] and developed further in [16]. The simplicity of the model from the system theoretical point of view is a great advantage compared to more complex muscle models. Controllers which are designed based on the model presented here are able to generate stimulation patterns with varying IPI and can, therefore, employ related nonlinear effects (such as the "catch" property) for the same mechanical response to be elicited by fewer impulses compared to stimulation with constant IPI. As such stimulation reduces the number of repetitive stimulations of the same motor units it is thought to reduce fatigue [27]. Jarvis *et al.* [23] have also shown that for chronically stimulated muscle, a lower stimulation frequency can have positive effects on the contractile speed and power of the muscle.

The model structure presented here can be implemented in a relatively simple way if the validity functions are stored in the form of a lookup table. The models can be evaluated in a discrete-time simulation which can be implemented easily in digital computing hardware. In contrast to various models which are based on physiological properties of muscle [12], [37], no numerical solution of continuous differential equations is necessary. The system can, therefore, easily be simulated in real-time. In the implementation used in this work, which is based on the C programming language, the LMNs of the muscles can be simulated approximately 50 times faster than real-time on a Pentium II, 266 MHz.

Future work will focus on the extension of this approach for more general muscle contraction, such as stimulation with varying IPI and varying pulse intensity, and for nonisometric contraction. The problem to obtain sufficient information for the identification of the model parameters from experimental I/O data will become more important for these general stimulation conditions. Generalized recommendations on the amount of data required to identify a good model are difficult to make since the important aspect is that the data obtained cover all operating regions which are of interest. Thus, the experiments to obtain the data need to be planned carefully to ensure that all relevant operating conditions are covered. The performance of the model on the data used to identify the parameters should be similar to that on unseen test data which indicates that the model did not simply memorise the data but is able to generalize.

The structure of the model can be extended by extra scheduling variables to account for additional nonlinear characteristics. Nonlinearities due to varying pulse intensity can be taken into account by scheduling on this variable. Changes of the muscle characteristics with changing length during nonisometric contraction can be accounted for by using the muscle length as a scheduling variable. Thus, the present model structure can be adapted to represent those muscle characteristics relevant for the intended application of the model.

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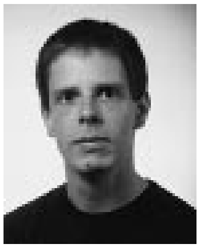

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