Optimal Monetary Policy in a New Keynesian Model with Habits in Consumption*

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Abstract

In this paper we consider the implications of habits for optimal monetary policy, when those habits either exist at the level of the aggregate basket of consumption goods ('superficial' habits) or at the level of individual goods ('deep' habits: see Ravn, Schmitt-Grohe, and Uribe (2006)). External habits generate an additional distortion in the economy and create new trade-offs for optimal policy, as the policy maker does not respond as aggressively to technology shocks in order to avoid exacerbating the habits externality. This can dramatically affect both the parameterization of optimal simple rules, as well as their determinacy properties. These effects are particularly strong when habits are of the deep kind.

- JEL Codes: E30, E57 and E61
- Key Words: consumption habits, nominal inertia, optimal monetary policy

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1 Introduction

Within the benchmark New Keynesian analysis of monetary policy (see, for example, Woodford (2003)), monetary policy typically influences the economy through the impact of interest rates on a representative household's intertemporal consumption decision. It has often been felt that the purely forward-looking consumption dynamics that such basic intertemporal consumption decisions imply, are unable to capture the hump-shaped output response to changes in monetary policy one typically finds in the data. As a means of accounting for such patterns, some authors have augmented the benchmark model with various forms of habits effects in consumption. The habits effects can either be internal (see for example, Fuhrer (2000), Christiano, Eichenbaum, and Evans (2005), Leith and Malley (2005)) or external (see, for example, Smets and Wouters (2007)), the latter reflecting a catching up with the Joneses effect whereby households fail to internalize the externality their own consumption causes on the utility of other households. Both forms of habits behavior can help the New Keynesian monetary policy model capture the persistence found in the data (see, for example Kozicki and Tinsley (2002)), although the policy implications are likely to be different. More recently, Ravn, Schmitt-Grohe, and Uribe (2006) offer an alternative form of habits behavior, which they label 'deep'. Deep habits occur at the level of individual goods rather than at the level of an aggregate consumption basket ('superficial' habits). While this distinction does not affect the dynamic description of aggregate consumption behavior relative to the case of superficial habits, it does render the individual firms' pricing decisions intertemporal and, in the flexible price economy considered by Ravn, Schmitt-Grohe, and Uribe (2006), can produce a counter-cyclical mark-up which significantly affects the responses of key aggregates to shocks. Ravn, Schmitt-Grohe, Uribe, and Uuskula (2010) then extend the analysis of deep habits to a sticky-price environment and find that such a model is able to explain, with moderate degrees of price stickiness and plausible policy rules, the prices puzzle and inflation persistence.[1]

While the focus of the papers listed above is on the dynamic response of economies which feature some form of habits, they do not consider the implications for optimal policy of such an extension. In contrast, Amato and Laubach (2004) consider optimal monetary policy in a sticky-price New Keynesian economy which has been augmented to include internal (but superficial) habits. Since the form of habits is internal (households care about their consumption relative to their own past consumption, rather than the consumption of other households), there is no additional externality associated with consumption habits themselves, and, given an efficient steady-state, the flexible price equilibrium in the neighborhood of that steady-state remains efficient. Accordingly, as in the benchmark New Keynesian model, there is no trade-off between output gap and inflation stabilization in the face of technology shocks and interesting policy trade-offs

[1] It should be noted that Ravn, Schmitt-Grohe, Uribe, and Uuskula (2010) do not consider the optimal policy or determinacy issues that are the focus of this paper.

require the introduction of additional inefficiencies (such as mark-up shocks or a desire for interest rate smoothing, perhaps due to worries over the zero lower bound in nominal interest rates).

In this paper, we extend the benchmark sticky-price New Keynesian economy to include *external* habits in consumption, where these habits can be either superficial or deep.[2] The focus on external habits implies that there is an externality associated with fluctuations in consumption and that the flexible price equilibrium will not usually be efficient, thereby creating an additional trade-off for optimal policy. Essentially, policy makers do not respond to technology shocks as aggressively as they would in the absence of a habits externality, as they wish to avoid exacerbating that externality. This is particularly so in the case of deep habits, where monetary policy affects the firms' discounted profits and thereby their optimal intertemporal markup. In the face of a positive technology shock, the typical monetary policy response of cutting real interest rates induces the firms producing the goods over which consumers form deep habits to cut markups and encourage consumers to consume more than is socially desirable. The loose monetary policy will then be more muted. When we lower the inflation target, we find that the policy maker's concern over the zero lower bound for nominal interests will further reduce the monetary policy response to the same shock. There are also stabilization biases associated with the time-consistent discretionary policy, which not only fails to achieve the price level control observed under commitment, but also fails to mitigate the formation of socially undesirable habits to the same extent as optimal commitment policy.

In addition to examining optimal policy, we also consider how the presence of habits affects the conduct of policy through simple rules. We find that the introduction of deep habits can induce problems of indeterminacy, as the tightening of monetary policy can induce inflation through variations in mark-up behavior, such that an interest rate rule which satisfies the Taylor principle (where nominal interest rates rise more than one for one with increases in inflation above target) may not be sufficient to ensure determinacy of the local equilibrium. We also find that optimal simple rules can come close to mimicking the commitment solution, even if we constrain the rule parameters to lie in a plausible range and avoid the zero lower bound for nominal interest rates. Moreover, as the extent of habits are increased, the optimal rule focuses less on stabilizing inflation and more on eliminating the habits externality and this trade-off is reflected in the optimized rule parameters.

The plan of the paper is as follows: in the next section we outline our model with deep and superficial habits. In section 3, we consider optimal policy under both commitment and discretion, where the policy-maker's objective function is derived from a second order approximation to households' utility. In section 4, we turn to our analysis of simple rules, considering both their determinacy properties and, for rules which can ensure determinacy, their ability to mimic optimal policy. Section 5 summarizes the welfare results, and Section

[2] Throughout the paper, we also contrast external habits with internal habits, although the latter requires additional distortions to make the policy problem interesting.

6 concludes.

2 The Model

The economy is comprised of households, a monopolistically competitive production sector, and the government. There is a continuum of goods that enter the households' consumption basket. Households can either form external consumption habits at the level of each individual good in their basket, Ravn, Schmitt-Grohe, and Uribe (2006) call this type of habits 'deep', or they can form habits at the level of the consumption basket as a whole- 'superficial' habits. Throughout the paper, we use the same terminology. Furthermore, we assume the economy is subject to price inertia. We shall derive a general model, and note when assuming superficial or deep habits alters the behavioral equations. In section 3, we also outline the key features of an economy with internal habits in consumption.

2.1 Households

The economy is populated by a continuum of households, indexed by k and of measure 1. Households derive utility from consumption of a composite good and disutility from hours spent working.

Deep Habits When habits are of the deep kind, each household's consumption basket, X_t^k, is an aggregate of a continuum of habit-adjusted goods, indexed by i and of measure 1,

$$X_t^k = \left(\int_0^1 \left(C_{it}^k - \theta C_{it-1} \right)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}},$$

where C_{it}^k is household k's consumption of good i and $C_{it} \equiv \int_0^1 C_{it}^k dk$ denotes the cross-sectional average consumption of this good. η is the elasticity of substitution between habit-adjusted goods ($\eta > 1$), while the parameter θ measures the degree of external habit formation in the consumption of each individual good i. Setting θ to 0 returns us to the usual case of no habits.

The composition of the consumption basket is chosen in order to minimize expenditures, and the demand for good i is

$$C_{it}^k = \left(\frac{P_{it}}{P_t} \right)^{-\eta} X_t^k + \theta C_{it-1}, \qquad \forall i$$

where P_t represents the overall price index, defined as an average of all goods prices, $P_t \equiv \left(\int_0^1 P_{it}^{1-\eta} di \right)^{1/(1-\eta)}$. Aggregating across households yields the total demand for good i, $i \in [0,1]$,

$$C_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\eta} X_t + \theta C_{it-1}. \tag{1}$$

Due to the presence of deep habits, this demand is dynamic in nature, as it depends not

only on current period elements but also on the lagged value of consumption. This, in turn, will affect the pricing/output decisions of the firms producing these goods, as shown below.

Superficial Habits Habits are superficial when they are formed at the level of the aggregate consumption good. Households derive utility from the habit-adjusted composite good X_t^k,

$$X_t^k = C_t^k - \theta C_{t-1},$$

where household k's consumption, C_t^k, is an aggregate of the continuum of goods $i \in [0,1]$,

$$C_t^k = \left(\int_0^1 \left(C_{it}^k \right)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}},$$

with η the elasticity of substitution between them and $C_{t-1} \equiv \int_0^1 C_{t-1}^k dk$ the cross-sectional average of consumption.

Households decide the composition of the consumption basket to minimize expenditures and the demand for individual good i is

$$C_{it}^k = \left(\frac{P_{it}}{P_t} \right)^{-\eta} C_t^k = \left(\frac{P_{it}}{P_t} \right)^{-\eta} \left(X_t^k + \theta C_{t-1} \right).$$

By aggregating across all households, we obtain the overall demand for good i as

$$C_{it} = \int_0^1 C_{it}^k dk = \left(\frac{P_{it}}{P_t} \right)^{-\eta} C_t. \tag{2}$$

Unlike the case of deep habits, this demand is *not* dynamic.

Remainder of the Household's Problem The remainder of the household's problem is the same irrespective of whether or not habits are deep or superficial. Specifically, households choose the habit-adjusted consumption aggregate, X_t^k, hours worked, N_t^k, and the portfolio allocation, D_{t+1}^k, to maximize expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(X_t^k \right)^{1-\sigma}}{1-\sigma} - \frac{\left(N_t^k \right)^{1+\upsilon}}{1+\upsilon} \right]$$

subject to the budget constraint

$$\int_0^1 P_{it} C_{it}^k di + E_t Q_{t,t+1} D_{t+1}^k = W_t N_t^k (1-\tau) + D_t^k + \Phi_t + T_t \tag{3}$$

and the usual transversality condition. E_t is the mathematical expectation conditional on information available at time t, β is the discount factor $(0 < \beta < 1)$, and σ and υ are the inverses of the intertemporal elasticities of habit-adjusted consumption and work

$(\sigma, \upsilon > 0;\ \sigma \neq 1)$. The household's period-t income includes: wage income from providing labor services to goods producing firms, $W_t N_t^k$, which is subject to a constant tax rate, τ, dividends from the monopolistically competitive firms, Φ_t, and payments on the portfolio of assets, D_t^k. Financial markets are complete and $Q_{t,t+1}$ is the one-period stochastic discount factor for nominal payoffs. T_t are lump-sum transfers received from the government. In the maximization problem, households take as given the processes for C_{t-1}, W_t, Φ_t, and T_t, as well as the initial asset position D_{-1}^k. The tax rate, τ, will be used to finance lump-sum transfers, and is designed to ensure that the long-run equilibrium is efficient in the presence of the habits externality.

The first order conditions for labor and habit-adjusted consumption are

$$\frac{\left(N_t^k\right)^{\upsilon}}{\left(X_t^k\right)^{-\sigma}} = w_t(1-\tau)$$

and

$$Q_{t,t+1} = \beta \left(\frac{X_{t+1}^k}{X_t^k}\right)^{-\sigma} \frac{P_t}{P_{t+1}}, \tag{4}$$

where $w_t \equiv \frac{W_t}{P_t}$ is the real wage (see Appendix A for further details). The Euler equation for consumption can be written as

$$1 = \beta E_t \left[\left(\frac{X_{t+1}^k}{X_t^k}\right)^{-\sigma} \frac{P_t}{P_{t+1}}\right] R_t,$$

where $R_t^{-1} = E_t\left[Q_{t,t+1}\right]$ denotes the inverse of the risk-free gross nominal interest rate between periods t and $t+1$.

2.2 Firms

In this subsection we consider the behavior of firms, which are monopolistically competitive and subject to nominal inertia in the form of quadratic price adjustment costs as in Rotemberg (1982).[3] There is a continuum of such firms, indexed by i and of measure 1. Each firm i produces a unique good using only labor as input in the production process

$$Y_{it} = A_t N_{it}. \tag{5}$$

Total factor productivity, A_t, affects all firms symmetrically and follows an exogenous stationary process, $\ln A_t = \rho \ln A_{t-1} + \varepsilon_t$, with persistence parameter $\rho \in (0,1)$ and random shocks $\varepsilon_t \sim iidN\left(0, \sigma_A^2\right)$. Firms choose the amount of labor that minimizes production costs, $W_t N_{it}$. The minimization problem gives a demand for labor, $N_{it} = \frac{Y_{it}}{A_t}$, and a nominal marginal cost, $MC_t = \frac{W_t}{A_t}$, which is the same across firms. (See Appendix A for more details.)

[3] For a version of the model where nominal inertia is in the form of Calvo (1983) contracts, see Leith, Moldovan, and Rossi (2009).

Firms can reset prices in every period but, in doing so, they must pay a cost of price adjustment, expressed in terms of the produced good as

$$\frac{\varphi}{2}\left(\frac{P_{it}}{\pi P_{it-1}}-1\right)^2 Y_t$$

where $\varphi \geq 0$ measures the magnitude of the price adjustment costs and $\pi \geq 1$ is the gross steady state inflation rate. Nominal profits are then defined as:

$$\Phi_{it} = (P_{it} - MC_t)\, Y_{it} - P_t \left[\frac{\varphi}{2}\left(\frac{P_{it}}{\pi P_{it-1}}-1\right)^2 Y_t\right].$$

Deep Habits When habits are deep, firms face the dynamic demand from households, given by expression (1), where the choice of price affects market share and future profits. Firms then choose processes for Y_{it} and P_{it} to maximize the present discounted value of expected future profits, $E_t \sum_{s=0}^{\infty} Q_{t,t+s}\Phi_{it+s}$, subject to this dynamic demand and the constraint that $C_{it} = Y_{it}$. $Q_{t,t+s}$ is the s-step ahead equivalent of the one-period stochastic discount factor in (4). The first order conditions for Y_{it} and P_{it} are

$$v_{it} = (P_{it} - MC_t) + \theta E_t \left[Q_{t,t+1} v_{it+1}\right] \tag{6}$$

and

$$Y_{it} = v_{it}\left[\eta\left(\frac{P_{it}}{P_t}\right)^{-\eta-1}\frac{X_t}{P_t}\right] + \varphi\left[\left(\frac{P_{it}}{\pi P_{it-1}}-1\right)\frac{P_t Y_t}{\pi P_{it-1}} - E_t Q_{t,t+1}\left(\frac{P_{it+1}}{\pi P_{it}}-1\right)\frac{P_{it+1}}{\pi\left(P_{it}\right)^2}P_{t+1}Y_{t+1}\right] \tag{7}$$

where the Lagrange multiplier v_{it} represents the shadow price of producing an additional unit of good i. This shadow value equals the marginal benefit of additional profits, $(P_{it} - MC_t)$, plus the discounted expected payoffs from higher future sales, $\theta E_t \left[Q_{t,t+1} v_{it+1}\right]$. Due to the presence of habits in consumption, increasing output by one unit in the current period leads to an increase in sales of θ in the next period. In the absence of habits, when $\theta = 0$, the intertemporal effects of higher output disappear and the shadow price simply equals time-t profits. The other first order condition in equation (7) says that an increase in price brings additional revenues, Y_{it}, while simultaneously causing a decline in demand, given by the first term in square brackets and valued at the shadow value v_{it}, to which we must add the net effect of price changes on current and expected future price adjustment costs.[4]

Superficial Habits Under superficial habits, firms face a typical static demand as given in equation (2). However, the nature of the price adjustment costs still renders the profit maximization problem dynamic. Firms are thus choosing prices to maximize the present

[4] An increase in price will raise price adjustment costs in the current period but lower them in the future.

discounted value of future profits, subject to the demand for their good and under the restriction that all demand be satisfied at the chosen price. The optimal price is set so as to balance the marginal benefit of additional revenues with the marginal cost of reduced demand and of increased net price adjustment costs:

$$Y_{it} = \eta \left(P_{it} - MC_t\right) \left(\frac{P_{it}}{P_t}\right)^{-\eta-1} \frac{Y_t}{P_t} + \varphi \left[\left(\frac{P_{it}}{\pi P_{it-1}} - 1\right) \frac{P_t Y_t}{\pi P_{it-1}} - E_t Q_{t,t+1} \left(\frac{P_{it+1}}{\pi P_{it}} - 1\right) \frac{P_{it+1}}{\pi P_{it}^2} P_{t+1} Y_{t+1}\right]$$

2.3 The Government

The government collects labor income taxes which it rebates to households as transfers. There is no government spending per se. The government budget constraint is given by

$$\tau W_t N_t = T_t. \tag{8}$$

In this cashless economy, monetary policy is conducted in optimal fashion, with the nominal interest rate being the central bank's policy instrument. However, we also consider the consequences of the central bank adopting more simple forms of policy, such as Taylor-type interest rate rules, and explore how closely these simple policy rules come to the optimal.

2.4 Equilibrium

In a symmetric equilibrium, the dynamics of the economy are characterized by the following set of equations:

Consumers:

$$X_t = C_t - \theta C_{t-1} \tag{9}$$

$$\frac{N_t^{\upsilon}}{X_t^{-\sigma}} = w_t(1-\tau) \tag{10}$$

$$X_t^{-\sigma} = \beta E_t \left[X_{t+1}^{-\sigma} \frac{R_t}{\pi_{t+1}}\right] \tag{11}$$

Government:

$$\tau W_t N_t = T_t \tag{12}$$

Firms:

$$Y_t = A_t N_t \tag{13}$$

$$mc_t = \frac{w_t}{A_t} \tag{14}$$

$$\ln A_t = \rho \ln A_{t-1} + \varepsilon_t \tag{15}$$

Subject to quadratic price adjustment costs, the firms' choice of price under superficial

habits is

$$\left[1-\eta\left(1-\frac{1}{\mu_t}\right)\right]Y_t=\varphi\left(\frac{\pi_t}{\pi}-1\right)\frac{\pi_t}{\pi}Y_t-\varphi\beta E_t\left[\left(\frac{X_{t+1}}{X_t}\right)^{-\sigma}\left(\frac{\pi_{t+1}}{\pi}-1\right)\frac{\pi_{t+1}}{\pi}Y_{t+1}\right]. \tag{16}$$

In contrast, when habits are deep and firms face a dynamic demand curve for their product, the price setting behavior is described by the following two equations (where we have written the shadow price of production in real terms, i.e. $\omega_t \equiv \frac{\upsilon_t}{P_t}$),

$$\omega_t=\left(1-\frac{1}{\mu_t}\right)+\theta\beta E_t\left[\left(\frac{X_{t+1}}{X_t}\right)^{-\sigma}\omega_{t+1}\right] \tag{17}$$

$$Y_t=\eta\omega_t X_t+\varphi\left(\frac{\pi_t}{\pi}-1\right)\frac{\pi_t}{\pi}Y_t-\varphi\beta E_t\left[\left(\frac{X_{t+1}}{X_t}\right)^{-\sigma}\left(\frac{\pi_{t+1}}{\pi}-1\right)\frac{\pi_{t+1}}{\pi}Y_{t+1}\right] \tag{18}$$

In both cases, the markup is the inverse of the real marginal cost,

$$\mu_t=\frac{1}{mc_t}. \tag{19}$$

To these equations, we add the aggregate resource constraint (obtained by combining the aggregate version of the household's budget constraint (3) with the government budget constraint (8) and the definition of aggregate profits, $\Phi_t = P_tY_t - W_tN_t - \frac{\varphi}{2}\left(\frac{\pi_t}{\pi}-1\right)^2 P_tY_t$)

$$Y_t=C_t+\frac{\varphi}{2}\left(\frac{\pi_t}{\pi}-1\right)^2 Y_t \tag{20}$$

and the monetary policy specification (detailed in Sections 3 and 4 below).

2.5 Solution Method and Model Calibration

In the absence of a closed-form solution, the model's equilibrium conditions are log-linearized around the efficient deterministic steady state. The efficiency of the steady state, obtained through the tax on labor income, allows us to obtain an accurate expression for welfare involving only second-order terms and implies that we can compare welfare across different types of habits without results being affected by variations in the steady-state mark-up.

In order to solve the model, we must select numerical values for some key structural parameters. Table 1 reports our choices, which are similar to those of other studies using a New Keynesian economy with habits in consumption. The model is calibrated to a quarterly frequency. We assume an annual real rate of interest of 4%, which implies a discount factor β of 0.9902. The risk aversion parameter σ is set at 2.0, while υ equals 0.25.[5] Consistent with the empirical evidence, the degree of market power is

[5] υ is the inverse of the Frisch labor supply elasticity. While micro estimates of this elasticity are rather small, they tend not to fit well in macro models. Here, we follow the macroeconomic literature and choose a larger value of 4.0.

Parameter	Value	Description
$1/\beta$	$(1.04)^{1/4}$	Real interest rate
σ	2	Inverse of intertemporal elasticity of substitution
ϵ_{Nw}	4.0	Frisch labor supply elasticity
η	11.0	Elasticity of substitution between goods
φ	100	Price adjustment cost parameter
θ	0.6	Degree of habit formation
π	$(1.036)^{1/4}$	Gross inflation rate
ρ	0.78	Persistence of technology
σ_A	0.021	Standard deviation of technology process

Table 1: Parameter values used in simulations

1.1. The steady state value of the markup in the case of internal or superficial habits is $\mu = \left(1 - \frac{1}{\eta}\right)^{-1}$, while in the case of deep habits is given as, $\mu = \left[1 - \frac{(1-\theta\beta)}{(1-\theta)\eta}\right]^{-1}$, and depends on both the elasticity of substitution between final goods η and the degree of habit formation θ. However, the impact of θ on the markup, μ, is small and we therefore set $\eta = 11$ across all model variants, implying a steady-state mark-up of 10% in the case of internal or external superficial habits, and 10.16% in the case of deep habits for our benchmark calibration. For the habits formation parameter θ, we use a benchmark value of 0.6, which falls within the range of estimates identified in the literature.[6] However, we allow θ to vary in the $[0, 1)$ interval as we conduct a sensitivity analysis of our results. We also set the income tax rate τ so as to ensure an efficient steady state. In the case of deep or superficial habits a labor income tax of, $\tau = 1 - \mu(1 - \theta\beta)$, ensures that the steady-state replicates the social planner's allocation, by compensating for the net effect of the habits externality and the distortion due to imperfect competition. In the case of internal habits it is only necessary to compensate for the steady-state distortion due to imperfect competition since there is no longer any habits externality and the subsidy is given by, $\tau = 1 - \mu$. Accordingly, for our benchmark calibration, the income tax is 55% in the case of deep and superficial external habits and -10% in the case of internal habits. In other words our benchmark calibration actually requires the use of a tax to replicate the social planner's allocation in steady state as the habits externality dominates the imperfect competition distortion.[7] The use of this time invariant tax/subsidy implies the steady-state is the same across habits variants facilitating welfare comparisons.

Following Schmitt-Grohe and Uribe (2005), we calibrate the parameters of the technology process (ρ, σ_A) and the parameters of a simple Taylor-type monetary policy rule to match the persistence and standard deviations of output and inflation in the U.S.

[6] Macro-based estimates of habits formation of the superficial type range from 0.59 as in Smets and Wouters (2003) to very high values of 0.98 as reported by Bouakez, Cardia, and Ruge-Murcia (2005). For the deep type of habits, Ravn, Schmitt-Grohe, and Uribe (2006) and Ravn, Schmitt-Grohe, Uribe, and Uuskula (2010) give a value of 0.86 and 0.85, respectively. Micro-based estimates (see, for example, Ravina (2007)) are substantially lower, with a range of 0.29-0.5.

[7] The balancing of the two distortions is used as a device by Levine, Pearlman, and Pierse (2008) to achieve an efficient steady-state while avoiding the need to introduce a steady-state subsidy often found in New Keynesian analyses of optimal monetary policy.

data, over the period 1960:Q1-2010:Q4. Inflation is measured as the percent change in the implicit GDP deflator. We obtain a persistence parameter $\rho = 0.78$ and a standard deviation $\sigma_A = 0.021$. Finally, as a benchmark we set the steady state level of inflation at the data average of 3.57% (annual rate).[8]

2.6 Log-linear Representation

Upon log-linearizing and combining the relevant equilibrium conditions, we obtain a system of equations which characterize the dynamics of the economy in the neighborhood of the efficient steady state. Firstly, we have the IS curve in terms of habit-adjusted consumption,

$$\widehat{X}_t = E_t\widehat{X}_{t+1} - \frac{1}{\sigma}\widehat{R}_t + \frac{1}{\sigma}E_t\widehat{\pi}_{t+1}. \tag{21}$$

Then, when habits are of the deep kind, the New Keynesian Phillips Curve (NKPC) is given by

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \frac{1}{\varphi}\left(\widehat{Y}_t - \widehat{\omega}_t - \widehat{X}_t\right), \tag{22}$$

where the shadow value of production $\widehat{\omega}_t$ behaves according to the following relationship

$$\widehat{\omega}_t = \frac{1}{\mu\omega}\widehat{\mu}_t + \theta\beta E_t\widehat{\omega}_{t+1} + \theta\beta\sigma\left(\widehat{X}_t - E_t\widehat{X}_{t+1}\right), \tag{23}$$

and the markup is inversely related to the real marginal cost,

$$\widehat{\mu}_t = -\widehat{mc}_t, \tag{24}$$

which is given by,

$$\widehat{mc}_t = \sigma\widehat{X}_t + \upsilon\widehat{Y}_t - (1+\upsilon)\,\widehat{A}_t. \tag{25}$$

Under superficial habits, the NKPC takes the more familiar form,

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \frac{\eta - 1}{\varphi}\widehat{mc}_t$$

with the same definition of marginal costs (25). Finally, we have the expression defining habit-adjusted consumption $\widehat{X}_t$,

$$\widehat{X}_t = \frac{1}{1-\theta}\left(\widehat{Y}_t - \theta\widehat{Y}_{t-1}\right). \tag{26}$$

3 Optimal Policy

In this section, we consider the nature of optimal monetary policy in response to technology shocks, under both cases of commitment and discretion by the monetary authority.

[8] In Section 3.4 below, we consider the implications for optimal policy of an alternative long-run inflation level (or target) of 2% (at an annual rate).

The central bank's objective function is given by a second order approximation to the representative households' utility, written in "gap" form as (see Appendix F for details)

$$\Gamma_0 = -\frac{1}{2}\overline{N}^{1+\upsilon} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\sigma(1-\theta)}{1-\theta\beta} \left(\widehat{X}_t - \widehat{X}_t^*\right)^2 + \upsilon \left(\widehat{Y}_t - \widehat{Y}_t^*\right)^2 + \varphi \widehat{\pi}_t^2 \right\} + tip + O[2]. \tag{27}$$

The gap variables are defined as deviations of the decentralized equilibrium allocation from that which would be implemented by a benevolent social planner (see appendix E for the social planner's problem). We note that this welfare measure has the same basic elements (output and inflation) as the benchmark New Keynesian model, but the "output gap" component is more complex, reflecting the presence of habits formation effects. This objective function applies whether or not habits are deep, superficial or internal.

In considering alternative policies, we measure the welfare cost of a particular policy A as the fraction of the consumption path under the Ramsey allocation that must be given up in order to equalize welfare under the two types of policy, $E \sum_{t=0}^{\infty} \beta^t u\left(X_t^A, N_t^A\right) = E \sum_{t=0}^{\infty} \beta^t u\left((1-\xi) X_t^R, N_t^R\right)$, where the R superscript denotes the Ramsey allocation. Given the utility function adopted, the expression for ξ in percentage terms is

$$\xi = \left[1 - \left(\frac{W^A - W_N^R}{W_X^R}\right)^{\frac{1}{1-\sigma}}\right] \times 100 \tag{28}$$

where W^A represents the unconditional expectation of lifetime utility in the economy under monetary policy A, $W^A \equiv E \sum_{t=0}^{\infty} \beta^t u\left(X_t^A, N_t^A\right)$, while W_X^R and W_N^R are welfare components associated with the economy under the full commitment policy, $W_X^R = E \sum_{t=0}^{\infty} \beta^t \frac{\left(X_t^R\right)^{1-\sigma}}{1-\sigma}$ and $W_N^R = -E \sum_{t=0}^{\infty} \beta^t \frac{\left(N_t^R\right)^{1+\upsilon}}{1+\upsilon}$.

3.1 Internal Habits

To better understand the effects of external habits formation on the optimal conduct of monetary policy, we contrast this environment with the case when habits are internal to the households' decision making process.[9] When habits are internal, each household's habits-adjusted consumption is given by,

$$X_t^k = C_t^k - \theta C_{t-1}^k,$$

where the reference level of consumption, C_{t-1}^k, is now that of the individual household rather than the average of all households.

Anticipating that current consumption decisions affect the stock of habits entering

[9] This is the case considered in Amato and Laubach (2004). A more detailed comparison of our results with those in Amato and Laubach (2004) is included in Section 3.4 below.

into the future, the consumption Euler equation is given by,

$$X_t^{-\sigma} - \theta\beta E_t X_{t+1}^{-\sigma} = \beta E_t \left[\left(X_{t+1}^{-\sigma} - \theta\beta E_{t+1} X_{t+2}^{-\sigma} \right) \frac{R_t}{\pi_{t+1}} \right]$$

and the labor supply condition by

$$\frac{(N_t)^{\upsilon}}{\left(X_t^{-\sigma} - \theta\beta E_t X_{t+1}^{-\sigma} \right)} = w_t(1-\tau).$$

These log-linearize as,

$$\widehat{X}_t - \theta\beta E_t \widehat{X}_{t+1} = E_t \left(\widehat{X}_{t+1} - \theta\beta E_{t+1} \widehat{X}_{t+2} \right) - \frac{1-\theta\beta}{\sigma} \left(\widehat{R}_t - E_t \widehat{\pi}_{t+1} \right)$$

and

$$\upsilon \hat{N}_t + \frac{\sigma}{1-\theta\beta} \left(\widehat{X}_t - \theta\beta E_t \widehat{X}_{t+1} \right) = \hat{w}_t$$

The stochastic discount factor used by firms to discount profits, $Q_{t,t+s} = \beta^s \left(\frac{X_{t+s}^{-\sigma} - \theta\beta E_{t+s} X_{t+s+1}^{-\sigma}}{X_t^{-\sigma} - \theta\beta E_t X_{t+1}^{-\sigma}} \right) \frac{P_t}{P_{t+s}}$, differs from the other definitions of habits, but the effects of this are eliminated in the log-linearization. As a result, the NKPC and the rest of the model are the same as in the case of (superficial) external habits,

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \frac{\eta - 1}{\varphi} \widehat{mc}_t,$$

although the definition of marginal costs is affected by the change in the labor supply condition under internal habits. In gap form, this can be written as:

$$\begin{aligned} \widehat{mc}_t &= \widehat{w}_t - \widehat{A}_t \\ &= \frac{\sigma}{1-\theta\beta} \left[\left(\widehat{X}_t - \widehat{X}_t^* \right) - \theta\beta E_t \left(\widehat{X}_{t+1} - \widehat{X}_{t+1}^* \right) \right] + \upsilon \left(\widehat{Y}_t - \widehat{Y}_t^* \right). \end{aligned} \tag{29}$$

3.1.1 The trade-off between output and inflation stabilization

The absence of a trade-off between output stabilization and inflation in the case of internal habits can be seen by considering the objective function, (27). If we imagine a policy maker who was unconcerned with inflation such that the NKPC ceased to be a constraint on their actions, then they would seek to minimize,

$$\frac{1}{2} \overline{N}^{1+\upsilon} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\sigma(1-\theta)}{1-\theta\beta} \left(\widehat{X}_t - \widehat{X}_t^* \right)^2 + \upsilon \left(\widehat{Y}_t - \widehat{Y}_t^* \right)^2 \right\} \tag{30}$$

by varying interest rates to influence consumption decisions, subject to the definition of habits adjusted consumption,

$$\widehat{X}_t - \widehat{X}_t^* = \frac{1}{1-\theta} \left[\left(\widehat{Y}_t - \widehat{Y}_t^* \right) - \theta \left(\widehat{Y}_{t-1} - \widehat{Y}_{t-1}^* \right) \right]$$

implying the following first order condition,

$$\frac{\sigma}{1-\theta\beta}\left[\left(\widehat{X}_t-\widehat{X}_t^*\right)-\theta\beta E_t\left(\widehat{X}_{t+1}-\widehat{X}_{t+1}^*\right)\right]+\upsilon\left(\widehat{Y}_t-\widehat{Y}_t^*\right)=0 \tag{31}$$

This first order condition, in conjunction with the rest of description of the economy, will determine the optimal path of output and consumption for a policy maker unconcerned with inflation. However, from the log-linearized definition of marginal costs under internal habits, (29), we see that this implies that $\widehat{mc}_t = 0$ and this optimal path for output/consumption has no inflationary repercussions. In other words, under internal habits, the policy maker can implement their preferred output/consumption path without suffering any inflationary repercussions. (This is shown formally for the full policy problem in Appendix D).[10]

This is not true of the external forms of habits, since the household's consumption/labor supply decisions do not internalize the consumption externality, such that the policy maker faces a trade-off in stabilizing inflation and the habits externality. This can be seen by examining the forcing variable in the NKPC under either superficial (25) or deep (22)-(25) external habits, where the path for output/consumption implied by (31) will not generally be consistent with zero inflation, such that the policy maker who cares about inflation faces a trade-off in minimizing inflation variability around its target and offsetting the consumption externality.

3.2 Optimal Policy under Commitment

If the monetary authority can credibly commit to following its policy plans, it then chooses the policy that maximizes households' welfare subject to the private sector's optimal behavior, as summarized in equations (21) - (26), and given the exogenous process for technology (see Appendix G for details of the policy problem under commitment). We analyze the implications of this policy in terms of impulse responses to exogenous technology shocks.

Optimal policy faces a trade-off between output and inflation stabilization in the face of technology shocks, which would not be present with internal habits. As noted above, with internal habits, policy would be loosened to ensure the flexible price equilibrium was recreated without generating any inflation. This is illustrated by the dotted impulse responses in Figure 1.[11] However, when habits are external, such that one household does not take account of the impact their increased consumption has on the utility of others, then with only one policy instrument available, the monetary authority cannot

[10] If, however, as in Amato and Laubach (2004), the policy maker is constrained in their ability to change interest rates, due to, for example, a desire to smooth interest rates, then the policy maker will be unable to replicate the flexible price allocation, even if habits are internal. We consider this case in Section 3.4 below.

[11] Of course, this is also the case in the New Keynesian model without habits - in the face of technology shocks, the monetary policy maker can eliminate the output gap without generating inflation.

simultaneously ensure output is at its efficient level and inflation is eliminated. Instead, while nominal inertia points to a relaxation of policy in the face of a positive technology shock to boost output, the consumption externality suggests that the higher consumption this entails need not be desirable.[12] The solid lines in Figure 1 show that, when habits are superficial, the optimal response of the economy to a positive persistent technology shock is a positive output gap and an initial decline followed by an increase in inflation. To achieve this outcome, the monetary authority reduces the nominal interest rate to boost demand to the socially optimal level. Because the policy is expansionary, we can implicitly say that the inefficiency due to price stickiness is dominating in this case.

We turn to the case of deep habits, illustrated by the dashed impulse responses in Figure 1. Holding the monetary policy response constant, we would expect the slackening of monetary policy to increase the discounted profits of firms, encouraging them to cut current markups, generating consumption habits in their goods and thereby widening the output gap further. In other words the trade-off implied by the habits externality is more pronounced in the case of deep habits, as a result of firms' desire to intertemporally manipulate mark-ups. Consequently, the optimal monetary response does not slacken real interest rates by as much in order to discourage the reduction in mark-ups. As the degree of importance of habits increases, the relaxation of policy is significantly reduced as policy makers seek to dampen the initial rise in consumption which imposes an undesirable externality on households. This can be seen in Figure 2, where the dashed lines depict impulse responses under the benchmark value of deep habits ($\theta = 0.6$) and circles the responses under an alternative higher value of $\theta = 0.8$.

3.3 Optimal Policy with Discretion

The previous sub-section examined policy under commitment. It is well known that not being able to commit to a time-inconsistent policy can give rise to a stabilization bias in the New Keynesian economy, whereby policy makers cannot obtain the most favorable trade-offs between output gap and inflation stabilization.[13] In our economy with a consumption externality, there may be additional sources of stabilization bias which make it interesting to assess the importance of having access to a commitment technology. Appendix G defines the inputs to the iterative algorithm used to compute time-consistent policy in Soderlind (1999).

Figure 3 contrasts policy under discretion and commitment when habits are superficial. Aside from failing to exploit the expectational benefits of price level control, the discretionary policy also fails to achieve the initial relative tightening of policy which mitigates the generation of undesirable habits effects. The desirability of undertaking the commitment policy emerges in the significantly different paths for inflation across com-

[12] Ljungqvist and Uhlig (2000) show how contractionary tax policy can be used for the purpose of aligning output with the efficient level in response to technology shocks.

[13] It should be noted that the stabilization bias can exist in economies which do not contain steady-state distortions, and are therefore not subject to the familiar inflation bias.

mitment and discretion. The welfare cost of not being able to commit to future policies amounts to 0.00069% of consumption levels under commitment, in the benchmark case of $\theta = 0.6$.[14]

When we undertake the same comparison in the case of deep habits (see Figure 4), the time inconsistency problem is even more significant than in the case of superficial habits. This is because under deep habits there is a stronger desire to tighten policy initially in order to prevent an undesirable increase in consumption habits, exacerbated by the profit-maximizing cuts in mark-ups by goods producing firms. Since such a policy is designed to improve policy trade-offs in the future, it is not possible to engineer such a monetary tightening under time consistent policy. The costs of not having access to a commitment technology are correspondingly higher under deep habits, where the welfare costs of discretion are 0.0047% of the consumption levels under the Ramsey policy, for the benchmark degree of habits.

3.4 Alternative Inflation Target

In our benchmark calibration, steady-state inflation was set to be consistent with observed (annualized) inflation in the U.S. over the period 1960:Q1-2010:Q4 of 3.57%. At this rate of inflation, optimal policy does not generally breach the zero lower bound (ZLB) for nominal interest rates, in the sense that twice the standard deviation of the nominal interest rate is less than the steady-state nominal interest rate, so that we would not expect to reach the ZLB at least 97.5% of the time.[15] However, if we lower the inflation target to 2% along the lines of many central banks' objectives, then the ZLB can become an issue. In this case, the policy maker faces additional trade-offs between the desire to stabilize inflation, the need to respect the ZLB and the wish to offset any externality in habits formation.

In order to account for the ZLB, we follow Amato and Laubach (2004) and add a quadratic term in the deviation of the nominal interest rate from steady-state to the policy maker's objective function. The weight attached to this term is sufficient to ensure that, under the optimal policy, the ZLB is respected with 97.5% probability. The impulse responses under the three types of habits formation are given in Figure 5. The first line shows the impulse response under internal habits with and without a concern for the ZLB. The first thing to note is that, as in Amato and Laubach (2004), there is now a meaningful policy trade-off even when habits are internal and policy makers wish to avoid the ZLB, as policy makers moderate their response to the technology shock. Given the expectational benefits of returning the price level to base following shocks, the inflationary effects of

[14] The welfare results across all experiments are collected together in Table 2, which is discussed in Section 5.

[15] We follow Schmitt-Grohe and Uribe (2007) in assessing the likelihood of a breach of the ZLB in this way. The one case in which the zero lower bound may be a problem more often than our self-imposed limit of 2.5% is the case of discretionary policy when habits are deep, where we may meet the zero lower bound with a probability of 0.0281. At 3.57% steady-state inflation, all other policies are comfortably clear of the ZLB.

the moderation of monetary policy are subsequently undone. The second row of Figure 5 reveals a similar pattern for superficial habits. While in the case of deep habits, the third row, the monetary policy response to technology shocks is already muted by the desire to moderate the accumulation of socially undesirable habits, such that there is no ZLB problem even when the inflation target is reduced to 2%.

Interestingly, when we turn to time-consistent discretionary policy, adding a desire to smooth interest rates as in Amato and Laubach (2004) does not resolve the ZLB problem as policy makers are forced to use large initial interest rate movements, as they cannot commit to an ongoing sustained interest rate response to the shock.

In terms of welfare, we find that, for internal habits, accounting for the ZLB problem within the commitment policy raises the costs of shocks to 0.00037% of the level of consumption under the Ramsey policy, while for superficial habits the corresponding increase is smaller, at 8.26E-05%. While in the case of deep habits, as noted above, there is no ZLB problem. Accordingly, the muting of the policy response to technology shocks as a result of the habits externality is qualitatively similar in nature to the moderation policy makers undertake when faced with a ZLB problem.

4 Simple Rules

Having derived the optimal policy under commitment and discretion for our new Keynesian economy with either superficial or deep habits, we now turn to consider the following simple monetary policy rule, akin to that of Schmitt-Grohe and Uribe (2007),

$$\widehat{R}_t = \phi_\pi \widehat{\pi}_t + \phi_y \widehat{Y}_t + \phi_R \widehat{R}_{t-1},$$

where $\widehat{R}_t$ is the nominal interest rate, $\widehat{\pi}_t$ is the rate of inflation and $\widehat{Y}_t$ is the current (observable) level of output. We begin by considering the determinacy properties of our simple rule, under the three forms of habits formation. We then turn to consider the welfare maximizing parameterization of the rule, and assessing to what extent this can mimic the optimal policy under commitment described above.

Figure 6 details the determinacy properties of this rule when habits are of the superficial form. Each sub-plot details the combinations of ϕ_π and ϕ_y which ensure determinacy (light grey dots), indeterminacy (blanks) and instability (dark grey stars).[16] Moving from left to right across subplots increases the degree of interest rate inertia in the rule, ϕ_R, while moving down the page increases the extent of habits formation, θ. Consider the

[16] Following Blanchard and Kahn (1980), we write the dynamic model in matrix form as $AE_t x_{t+1} = Bx_t$ where x_t is an $n \times 1$ vector of the model's endogenous and exogenous variables. A and B are square matrices of size $n \times n$. Let us define $J = A^{-1}B$, m as the number of non-predetermined variables in x, $n - m$ the number of predetermined variables in x, and q the number of eigenvalues of J that are greater than one in absolute value, i.e. explosive eigenvalues. If $q = m$, the system is determinate (determinacy). In other words the solution to $AE_t x_{t+1} = Bx_t$ is unique and converges to the steady state. If $q < m$ there are an infinite number of solutions to $AE_t x_{t+1} = Bx_t$, the system is therefore indeterminate (indeterminacy). Ultimately, if $q > m$ there is no solution to $AE_t x_{t+1} = Bx_t$ and the system is unstable (instability).

first sub-plot in the top left hand corner with $\phi_R = 0$ and $\theta = 0$, which re-states the stability properties of the original Taylor rule. Here the importance of the Taylor principle is revealed as $\phi_\pi > 1$ in combination with a non-negative response to output is a sufficient condition for determinacy. Within this region, there is limited scope for compensating for failing to fulfil the Taylor principle through increasing the positive response of the interest rate to output, and there is slightly greater scope for using a more aggressive monetary policy response to compensate for a mildly negative interest rate response to output. It is also interesting to note that a second region of determinacy exists where the interest rate rule fails to satisfy the Taylor principle, such that $\phi_\pi < 1$, and the response to output is strongly negative. This region is not often discussed in the literature, but is mentioned in Rotemberg and Woodford (1999). Typically, when monetary policy fails to satisfy the Taylor principle, inflation can be driven by self-fulfilling expectations which are validated by monetary policy. However, when the output response is sufficiently negative there is an additional destabilizing element in policy, which overturns the excessive stability generated by a passive monetary policy, implying a unique saddlepath where any deviation from that saddlepath will imply an explosive path for inflation.

As we move down the sub-plots in the first column of Figure 6 we increase the degree of superficial habits. This means that the output response to both policy and shocks is more muted as current consumption is increasingly tied to past levels of consumption. This has two effects on the determinacy properties of the basic Taylor rule. Firstly, a rule which satisfies the Taylor principle will do so even if the response to output is increasingly negative. Secondly, the additional instability caused by adopting a negative interest rate response to output becomes insufficient to move a passive interest rate rule to a position of determinacy. Accordingly, the importance of the Taylor principle is enhanced when consumption is subject to superficial habits effects.

As we move across the page from left to right we increase the extent of interest rate inertia in the rule. In this case, as Woodford (2001) shows, the Taylor principle needs to be rewritten in terms of the long-run interest rate response to excess inflation, $\frac{\phi_\pi}{1-\phi_R} > 1$. As a result, the determinacy region in the positive quadrant spreads further into the adjacent quadrants since a given level of instantaneous policy response to inflation ϕ_π has a far greater long-run effect.

Finally, when we combine superficial habits effects with interest rate inertia, it becomes possible to induce instability in our economy when the rule is passive, $\frac{\phi_\pi}{1-\phi_R} < 1$, and the interest rate response to output is negative, $\phi_y < 0$. Essentially, the slow evolution of consumption under habits combined with interest rate inertia and a perverse policy response to output and inflation serves to induce a cumulative instability in the model.

Figure 7 constructs a similar set of sub-plots when habits are of the deep, rather than superficial, kind. If the extent of habits formation is relatively low, the determinacy properties of the model are similar to those observed under superficial habits. However, as the degree of habits formation rises, then significant differences emerge. Firstly, the usual determinacy region in the positive quadrant disappears and becomes indeterminate. This

indeterminacy is linked to the additional dynamics that arise under deep habits formation, when firms further adjust markups to account for market share. Suppose economic agents expect an increase in inflation. Given an active interest rate rule, $\phi_\pi > 1$, this will give rise to a tightening of monetary policy. Typically, such a policy would lead to a contraction in aggregate demand, invalidating the inflation expectations. However, in the presence of deep habits, the higher real interest rates will encourage firms to raise current mark-ups as they discount the lost future sales such price increases would imply more heavily. If the degree of habits effects is sufficiently large, then this increase in mark-ups can validate the initial increase in inflationary expectations, leading to self-fulfilling inflationary episodes and indeterminacy.

Independently of this paper, Zubairy (2010) also considers the determinacy properties of interest rate rules for a sticky price model featuring deep habits, but which, differently from the current paper, contains capital. Nevertheless, her findings generally accord with ours in finding that deep habits can generate determinacy issues which can be reduced by responding to output and/or introducing interest rate inertia. She also finds that writing the rules in terms of lagged rather than contemporaneous variables mitigates the indeterminacy problem. One interesting difference in our results to those of Zubairy (2010) is that she finds that in a model with costless capital adjustment a stronger response to inflation in the interest rate rule can also reduce the indeterminacy problems associated with deep habits. In our model without capital this effect is not present and it would be interesting in future research to reconcile these results in a model with capital adjustment costs.

In the case of internal habits (see Appendix H), the results are very similar to those under superficial external habits, with the key difference that, because households are more measured in their consumption response to shocks since they internalize the impact of their behavior on habit formation, the regions of instability found under superficial habits are reduced.

Optimal Simple Rules Having explored the determinacy properties of the simple rule described above when embedded in our economy featuring either superficial or deep habits, we now turn to consider the optimal parameterization of the rule in each case.[17] In recognizing the desirability that our simple rule be also 'implementable', we restrict the magnitude of the coefficients ϕ_π and ϕ_y to be less than 5 in absolute value, while at the same time ensuring that the rule satisfies a zero lower bound restriction. In order to do so, when searching over the policy rule parameter space, we exclude any parameter combination which results in a rule which implies that twice the standard deviation of nominal interest rates is greater than the steady-state level of nominal interest rates. As in Schmitt-Grohe and Uribe (2007), this implies that our rule will not fall foul of the ZLB

[17] We search across the rule parameter space using the Simplex method employed by the Fminsearch algorithm in Matlab (see, Lagarias, Reeds, Wright, and Wright (1998)) in order to minimize the unconditional welfare losses associated with the rule.

at least 97.5% of the time, and in practice our rules are even more robust to the ZLB than this limit implies.

In the case of superficial habits and under the benchmark value of $\theta = 0.6$, the optimal simple rule implies a moderate level of interest rate smoothing, with a strong positive response to inflation but a negative response to output ($\phi_\pi = 5$, $\phi_y = -0.23$, and $\phi_R = 0.49$). With this type of optimal monetary policy rule, the economy's response to a technology shock essentially comes close to replicating the response obtained under a full commitment policy, as shown in Figure 8 and discussed in Section 5 below. To explore the intuition underpinning this result, Figure 9 shows how the optimal policy rule parameters vary with the degree of habits formation. If we were to allow our rules to have the same form, but to be unconstrained, then there is only a negligible difference between the rule's performance and full commitment policy.[18]

In the absence of habits effects, in a New Keynesian economy, a positive technology shock leads to a decrease in inflation and, due to the nominal inertia, an insufficiently large increase in output. Optimally, a decrease in the nominal interest rate stimulates demand by reducing the real interest rate. This can be achieved by having a very large coefficient on inflation relative to all other parameters, which essentially allows the simple policy rule to achieve the flex-price equilibrium with a zero output gap and no additional inflation. As we introduce superficial habits effects, in the face of the same shock households over-consume and the output gap becomes positive suggesting that policy be tightened rather than relaxed. This trade-off, which is not present in the model without external habits, affects the optimal parameterization of the simple policy rule. Specifically, as we increase the degree of habits formation, the optimal parameter on inflation in the simple rule falls and the extent of interest rate inertia increases. Furthermore, the negative coefficient on output also falls, eventually turning positive.

A key feature of optimal policy under commitment is price level control, where the optimal policy achieves expectational benefits in seeking to ensure that price level returns to base following any shock. As the degree of superficial habits formation is increased, this price level control can be achieved most effectively through a combination of interest rate inertia and output response. Consider the impact of the positive technology shock depicted in Figure 8. Essentially, the rule is able to maintain a cut in real interest rates, even when inflation is slightly positive (to undo the price level effects of the initial fall in inflation), by responding negatively to the persistent increase in output and maintaining that stance for longer by increasing the amount of interest rate inertia. When the degree of habits formation becomes sufficiently large, the coefficient on output becomes positive in order to reduce the initial relaxation of policy, and the degree of interest rate inertia is increased to ensure price level control.

[18] The unconstrained optimal rule parameters across alternative levels of superficial and deep habits are presented in Appendix H. Allowing the coefficients on the rule to be unconstrained tends to imply that the coefficient on inflation is significantly higher, but decreasing in the level of habits, for the reasons discussed in the main text.

	$\sigma[Y]$	$\sigma[\pi]$	$\sigma[R]$	Welfare cost (ξ)
	Internal Habits			
Commitment	1.2037	0	3.7058	–
ZLB Commitment	1.2008	0.1025	2.9526	0.00037
	Superficial Habits			
Commitment	1.5368	0.0862	3.2029	–
Discretion	1.5352	0.1596	3.7159	0.00069
Strict Inflation Targeting	1.5724	0	3.7066	0.00033
ZLB Commitment	1.5272	0.1181	2.9377	8.26E-05
Optimal rule	1.4778	0.2318	2.7250	0.00086
	Deep Habits			
Commitment	1.5169	0.1268	2.6743	–
Discretion	1.4991	0.4052	3.9092	0.00466
Strict Inflation Targeting	1.6450	0.0131	3.7500	0.00133
Optimal rule	1.4650	0.2029	2.5302	0.00035

Table 2: Standard deviations of selected variables and the welfare costs of technology shocks.

Figure 10 plots the optimal parameters of the simple rule, when the economy features an increasing level of deep habits formation.[19] When habits are deep, there is less of a desire to cut interest rates initially, to prevent firms cutting their mark-ups and generating even greater consumption externalities. This implies that at high levels of deep habits, there is a decline in the desire to stabilize inflation and a greater concern with the habits externality, which manifests itself as a reduction in the response to both inflation and the degree of interest rate inertia, as well as an increase in the response to output. Despite, the simplicity of the rule and the constraints on the rule's parameters, the simple rule in all cases comes relatively close to achieving the welfare levels observed under commitment - see Appendix H for an illustration of impulse responses to a technology shock under the benchmark calibration of $\theta = 0.6$, where the ability of the rule to mimic commitment policy is clear.

5 Welfare

Table 2 draws together the welfare costs of technology shocks across our various permutations of policy and types of habit, where these costs are defined as the fraction of consumption that must be given up in order to equate welfare in the stochastic economy under alternative policies to that under Ramsey policies. We also report the standard deviations of output, inflation and interest rates in each case. We consider the case of internal habits first, where in the absence of a ZLB constraint, policy is perfectly able to replicate the flexible price equilibrium and eliminate inflation volatility. Reducing the inflation target such that the ZLB becomes a concern, implies that policy cannot respond

[19] A similar plot for the case where the parameters of the rule are not constrained to be less than 5 is presented in Appendix H.

as aggressively to technology shocks so that the variability of inflation rises, while that of interest rates falls, and the welfare costs of the shock increase to 0.00037%. However, it remains the case that optimal policy reduces inflation volatility to a fraction of that observed in the data (where the standard deviation of inflation is 2.44% and that of output 1.54%).[20]

When we turn to external habits of the superficial kind, commitment policy materially reduces the variability of inflation relative to that found in the data, with little impact on output variability. In the absence of an ability to commit, the policy maker over-stabilizes output, and allows inflation and interest rate volatility to rise relative to the commitment case. This raises the welfare costs to 0.00069% of the level of consumption found under the Ramsey policy. This tendency is aggravated when the policy maker faces a ZLB problem, and the reduced responsiveness of policy to technology shocks lowers output volatility, but raises inflation variability with a corresponding deterioration in welfare. Similarly, simple rules which are constrained in terms of the acceptable size of their coefficients are not able to stabilize inflation as aggressively as full commitment policy, and tend to over-stabilize output instead. However, a policy of strict inflation targeting which effectively ignores the habits externality, is also damaging in welfare terms, as inflation is stabilized at the expense of output and interest rate volatility.

Finally, in the case of deep habits, under optimal policies there is a reduction in both output and inflation volatility relative to the data. An inability to commit leads to an excessive stabilization of output, at the expense of the smallest reduction in inflation volatility relative to the data across any of our habits variants, for the reasons discussed above. This implies a correspondingly high welfare cost of 0.0047% of Ramsey consumption levels. Simple rules improve on the time-consistent solution, but are still slightly limited in their ability to stabilize inflation relative to the full commitment case. While a policy of strict inflation targeting (moderated by the need to respect the ZLB)[21] implies excessive output and interest rate volatility relative to the case of the Ramsey policy. Across all alternative policies, the welfare costs of failing to implement the Ramsey policy are greatest in the case of deep habits.

6 Conclusion

In this paper we considered the optimal policy response to technology shocks in a New Keynesian economy subject to habits effects in consumption. When these effects were assumed to be external, one household fails to take account of the impact their consumption behavior has on other households, as each household seeks to 'catch up with the Joneses'.

[20] Similar results are reported for the comparable calibration in Table 1 of Amato and Laubach (2004) who also consider optimal commitment policy in a sticky price economy with internal habits and a ZLB problem.

[21] Under deep habits, a policy of strict inflation targeting breaches the ZLB due to the volatility of interest rates used to achieve the target. We account for this in the numbers presented which explains why the strict inflation target is not actually achieved.

This consumption externality needs to be traded-off against the monetary policy maker's usual desire to stabilize inflation (a trade-off which we demonstrated would not exist if habits were internal) and generates a new form of stabilization bias, as time consistent policy is unable to mimic the initial policy response under commitment. This framework is further enriched by allowing the habits effects to be either superficial (at the level of the household's total consumption) or deep (at the level of individual consumption goods). Under deep habits, firms face dynamic demand curves which imply an intertemporal dimension to price setting and endogenous mark-up behavior, beyond that associated with price stickiness. In both cases, but particularly under deep habits due to the enhanced dynamic price setting behavior, the policy maker does not cut interest rates in response to a positive technology shock to the extent required to mimic the flex-price allocation, as they wish to mitigate the undesirable habits formation that would otherwise arise.

In addition to considering optimal policy, we also analyzed the stabilizing properties of simple rules. We investigate the determinacy properties of such rules and find that superficial habits effects tend to increase the range of parameters consistent with determinacy, provided the Taylor principle is satisfied. However, for sufficiently large measures of deep habits (which fall within the range of econometric estimates) the Taylor principle ceases to be either a necessary or sufficient condition for determinacy. We demonstrate that optimally parameterized determinate simple implementable rules can typically come close to achieving the welfare levels observed under optimal commitment policy. Moreover, the optimal parameters within the simple rules are highly dependent on the degree of habits and vary in a manner consistent with the reduced emphasis on inflation stabilization and the increasing concern with the habits externality, observed as the degree of habits increases. Overall, our work suggests that the choice of internal or external habits effects will have non-trivial implications for optimal policy, even if the implied dynamics of the model when policy is described by an ad-hoc rule could be similar (Kozicki and Tinsley (2002)).

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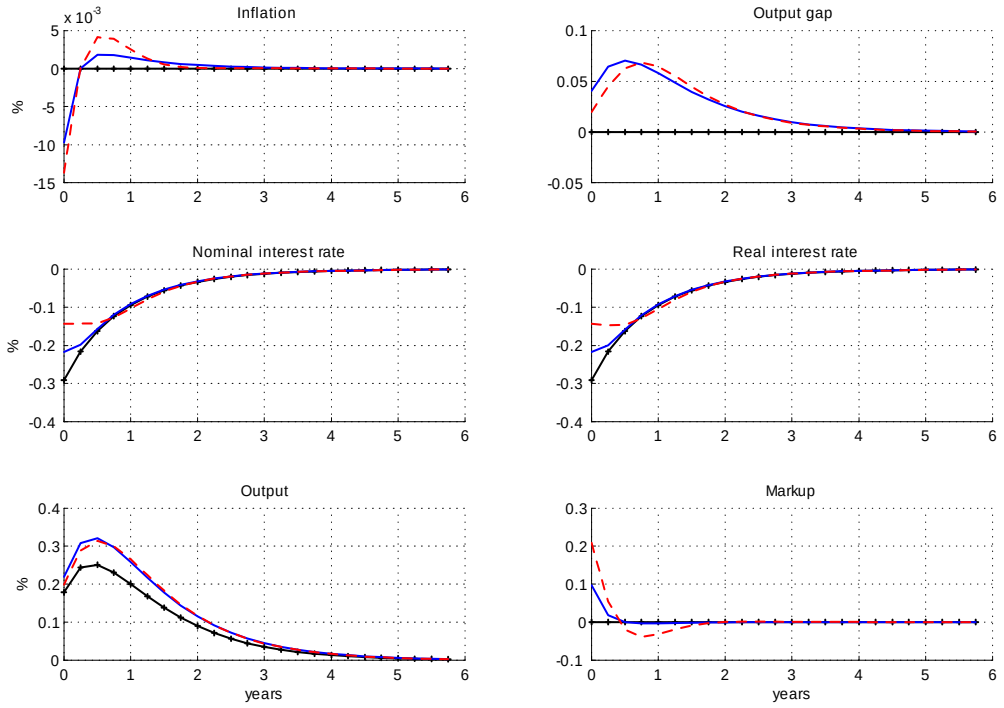

Figure 1: Impulse responses to a 1% positive technology shock under *optimal commitment policy*: internal habits (pluses) and external habits: superficial (solid lines) and deep (dash lines).

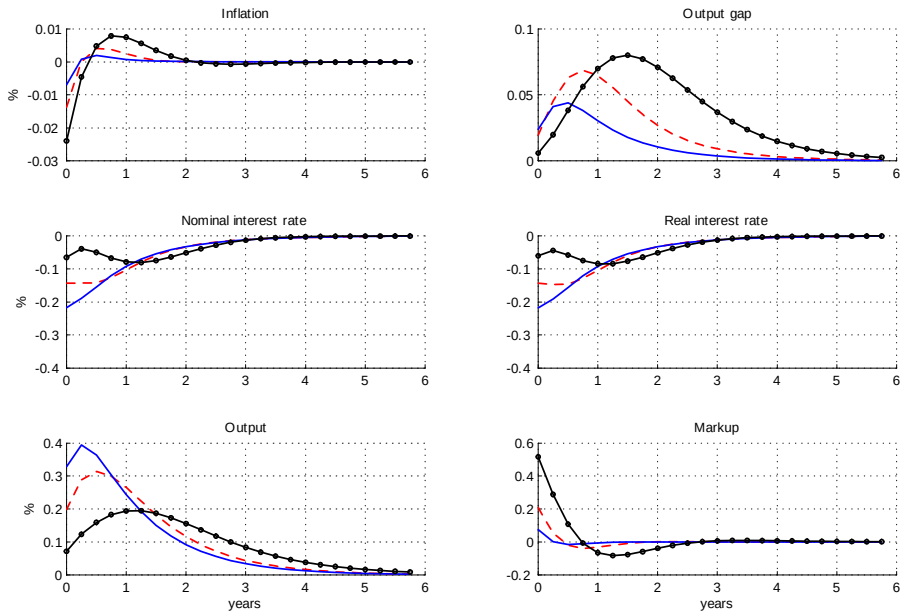

Figure 2: Impulse responses to a 1% positive technology shock under *optimal commitment policy*, with deep habits: $\theta = 0.4$ (solid lines), $\theta = 0.6$ (benchmark value, dash lines), $\theta = 0.8$ (circles).

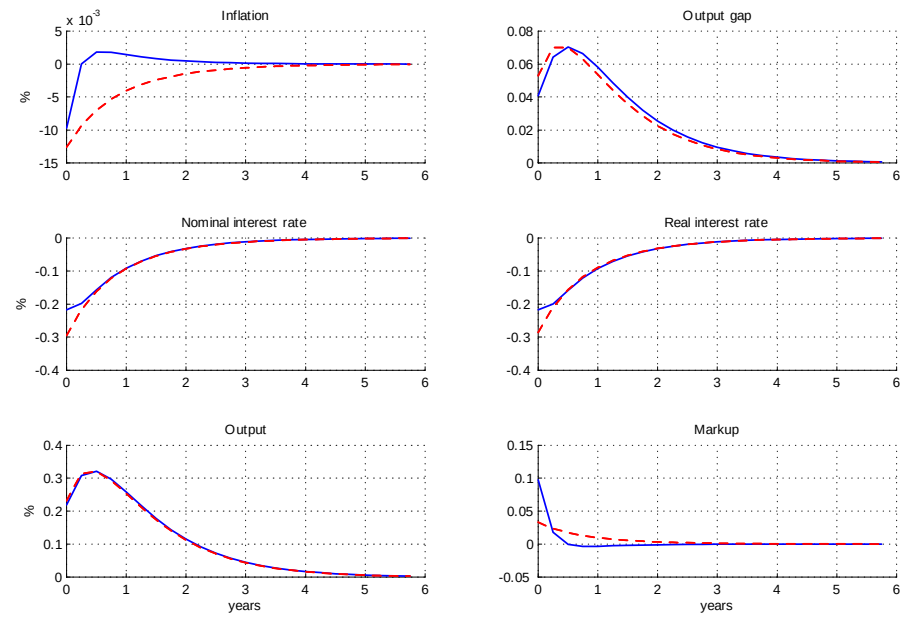

Figure 3: Impulse responses to a 1% positive technology shock in the case of *superficial habits* under optimal policy with *commitment* (solid lines) and with *discretion* (dash lines).

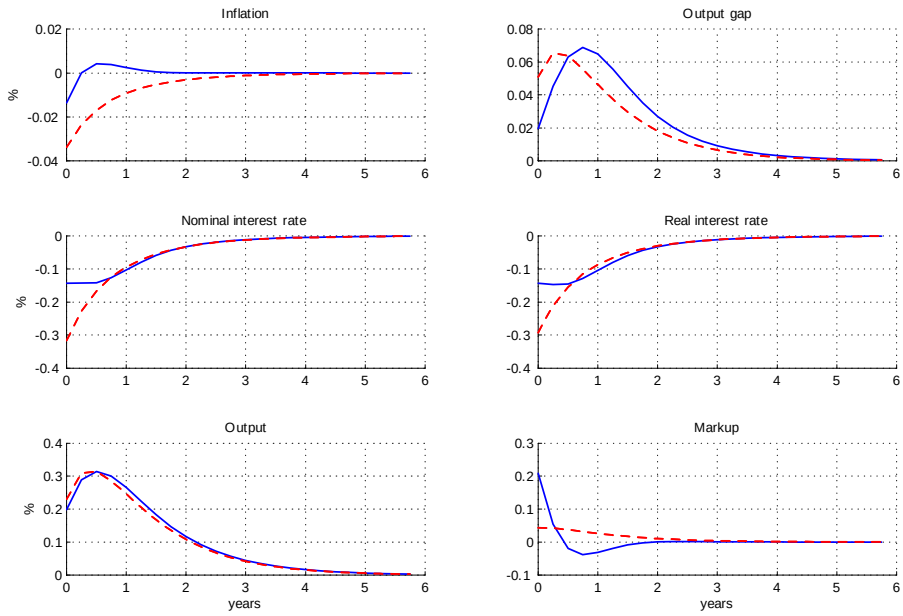

Figure 4: Impulse responses to a 1% positive technology shock in the case of *deep habits* under optimal policy with *commitment* (solid lines) and with *discretion* (dash lines).

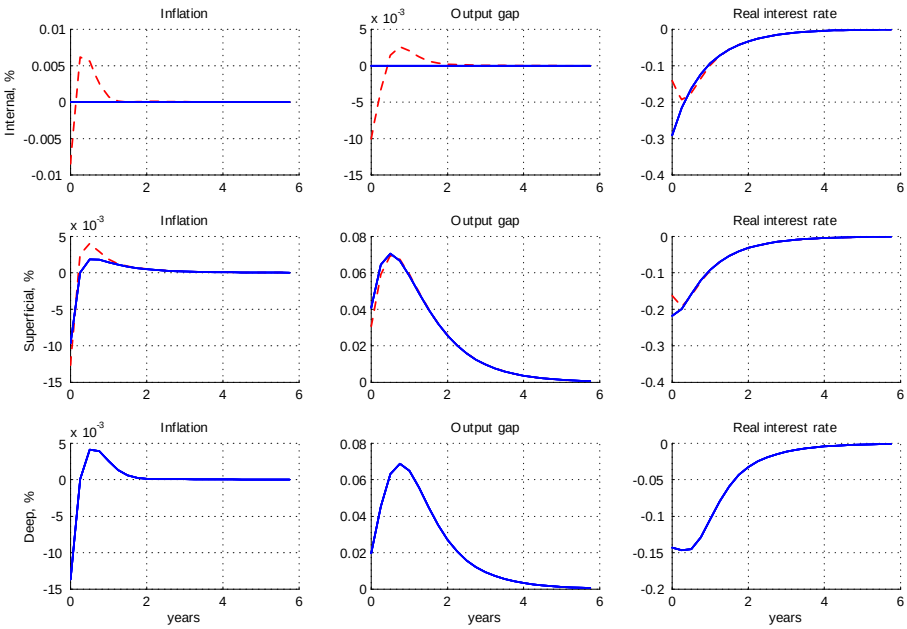

Figure 5: Impulse responses to a 1% positive technology shock under commitment policy with a concern for the ZLB (dash lines) and without (solid lines).

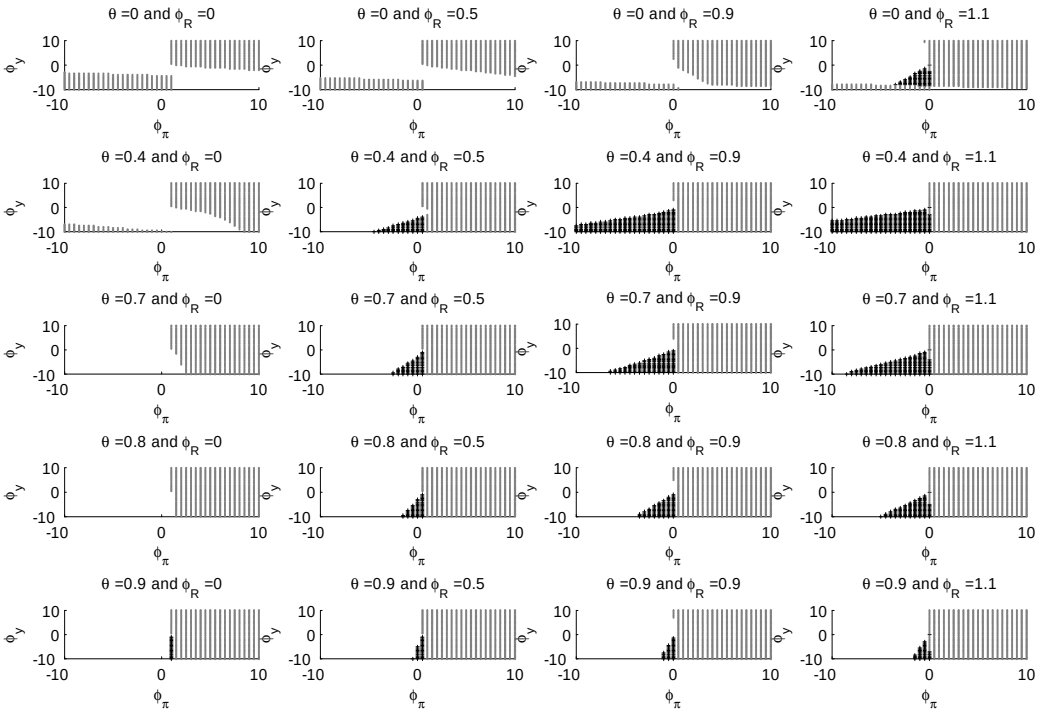

Figure 6: Determinacy properties of the model with *superficial habits*, when monetary policy follows the rule $\widehat{R}_t = \phi_\pi \widehat{\pi}_t + \phi_y \widehat{Y}_t + \phi_R \widehat{R}_{t-1}$: determinacy (light grey dots), indeterminacy (blanks), and instability (dark grey stars).

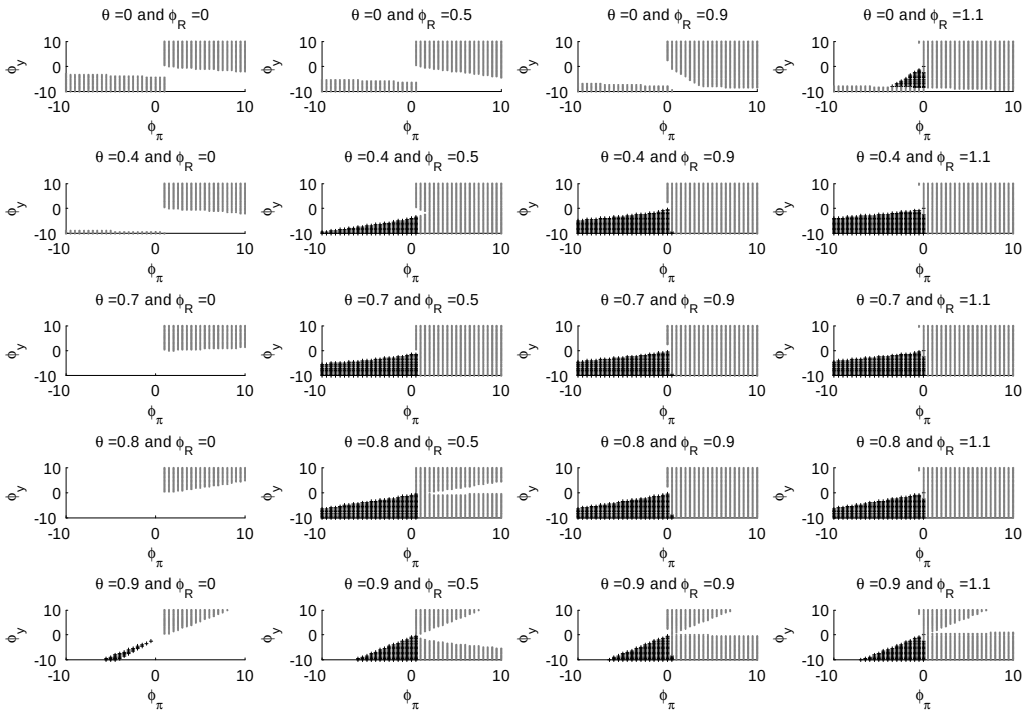

Figure 7: Determinacy properties of the model with *deep habits*, when monetary policy follows the rule $\widehat{R}_t = \phi_\pi \widehat{\pi}_t + \phi_y \widehat{Y}_t + \phi_R \widehat{R}_{t-1}$: determinacy (light grey dots), indeterminacy (blanks), and instability (dark grey stars).

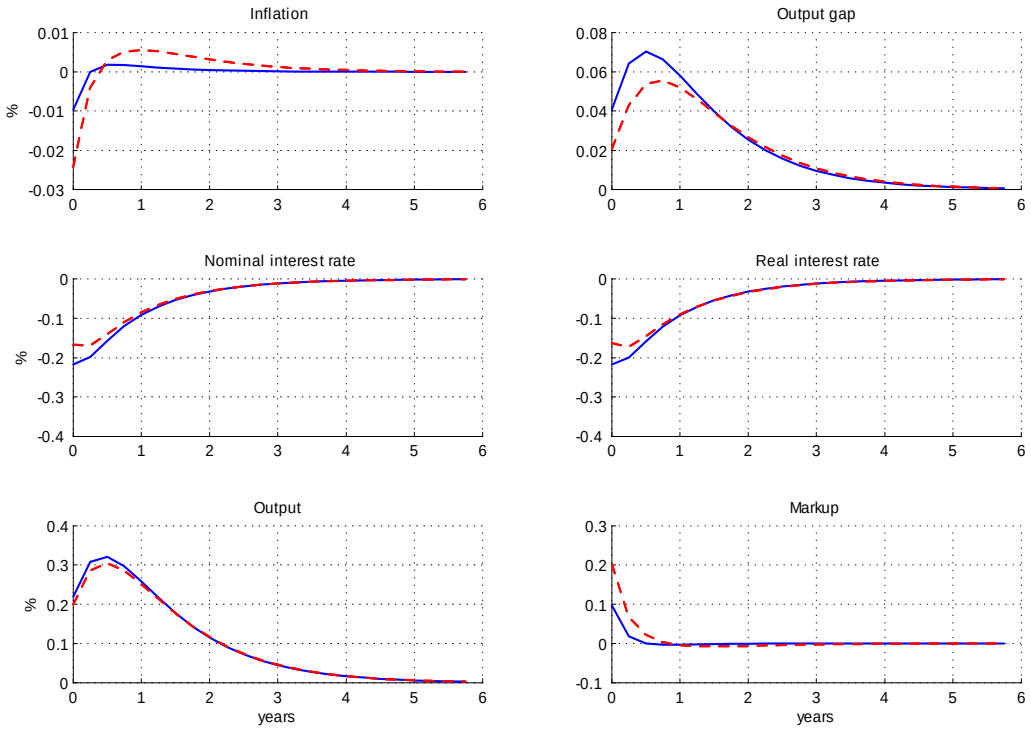

Figure 8: Impulse responses to a 1% positive technology shock in the model with *superficial habits* with $\theta = 0.6$, under the optimal Taylor rule (dash lines) and optimal commitment policy (solid lines).

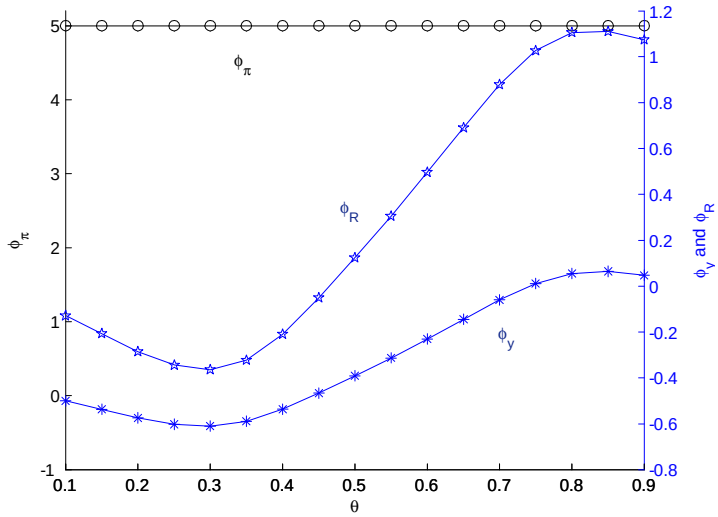

Figure 9: Optimal policy rule parameters for varying degrees of *superficial habits*.

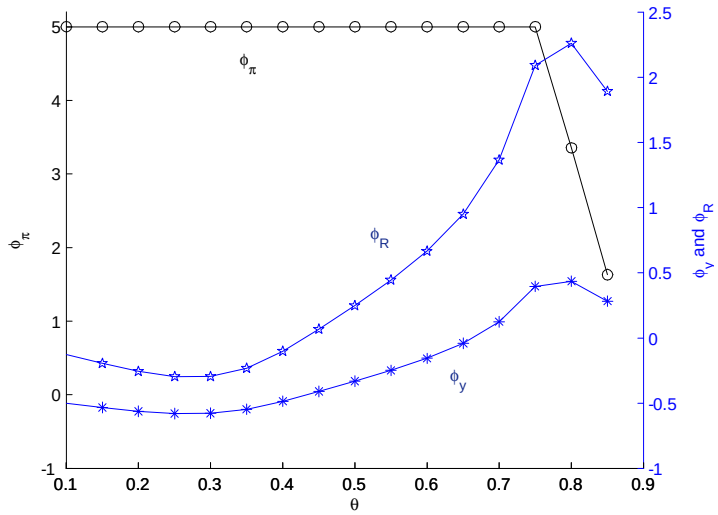

Figure 10: Optimal policy rule parameters for varying degrees of *deep habits*.

Technical Appendix - Not for Inclusion in Journal.

A Analytical Details

A.1 Households

Cost Minimization Households decide the composition of the consumption basket to minimize expenditures

$$\min_{\{C_{it}^k\}_i} \int_0^1 P_{it} C_{it}^k di$$

$$s.t. \left(\int_0^1 \left(C_{it}^k - \theta C_{it-1} \right)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}} \geq \overline{X}_t^k$$

The demand for individual goods i is

$$C_{it}^k = \left(\frac{P_{it}}{P_t} \right)^{-\eta} X_t^k + \theta C_{it-1},$$

where P_t is the overall price level, expressed as an aggregate of the good i prices, $P_t = \left(\int_0^1 P_{it}^{1-\eta} di \right)^{\frac{1}{1-\eta}}$.

Utility Maximization The solution to the utility maximization problem is obtained by solving the Lagrangian function,

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left[u\left(X_t^k, N_t^k \right) - \lambda_t^k \left(P_t X_t^k + P_t \vartheta_t + Q_{t,t+1} D_{t+1}^k - W_t N_t^k (1-\tau) - D_t^k - \Phi_t - T_t \right) \right].$$

In the budget constraint, we have re-expressed the total spending on the consumption basket, $\int_0^1 P_{it} C_{it}^k di$, in terms of quantities that affect the household's utility,

$$\int_0^1 P_{it} C_{it}^k di = P_t X_t^k + P_t \vartheta_t,$$

where under deep habits ϑ_t is given as $\vartheta_t \equiv \theta \int_0^1 \left(\frac{P_{it}}{P_t} \right) C_{it-1} di$, while under superficial habits it takes the simpler form, $\vartheta_t \equiv \theta C_{t-1}$. Households take ϑ_t as given when maximizing utility.

The first order conditions are then,

$\left(X_t^k \right)$: $\quad u_X(t) = \lambda_t^k P_t$

$\left(N_t^k \right)$: $\quad -u_N(t) = u_X(t) \frac{W_t}{P_t} (1-\tau)$

$\left(D_t^k \right)$: $\quad 1 = \beta E_t \left[\frac{u_X(t+1)}{u_X(t)} \frac{P_t}{P_{t+1}} \right] R_t$

where $R_t = \frac{1}{E_t[Q_{t,t+1}]}$ is the one-period gross return on nominal riskless bonds.

With utility given by $u(X,N) = \frac{X^{1-\sigma}}{1-\sigma} - \frac{N^{1+\upsilon}}{1+\upsilon}$, the first derivatives are

$$u_X(\cdot) = X^{-\sigma} \qquad and \qquad u_N(\cdot) = -N^{\upsilon}.$$

A.2 Firms

The cost minimization involves the choice of labor input N_{it} subject to the available production technology

$$\min_{N_{it}} W_t N_{it}$$
$$s.t.\ A_t N_{it} = Y_{it}$$

The minimization problem implies a labor demand, $N_{it} = \frac{Y_{it}}{A_t}$, and a nominal marginal cost which is the same across all brand producing firms, $MC_t = (1-\varkappa)\frac{W_t}{A_t}$. Profits are defined as:

$$\begin{aligned}\Phi_{it} &\equiv P_{it}Y_{it} - W_t N_{it} - \frac{\varphi}{2}\left(\frac{P_{it}}{\pi P_{it-1}} - 1\right)^2 P_t Y_t \\ &= (P_{it} - MC_t)\, Y_{it} - \frac{\varphi}{2}\left(\frac{P_{it}}{\pi P_{it-1}} - 1\right)^2 P_t Y_t\end{aligned}$$

where the last term represents the nominal costs of adjusting prices, as in Rotemberg (1982), and π is the steady state inflation.

Each firm then chooses processes for P_{it} and Y_{it} to maximize the present discounted value of profits, under the restriction that all demand be satisfied at the chosen price ($C_{it} = Y_{it}$):

$$\begin{aligned}\max_{\{P_{it},\ Y_{it}\}} E_t \sum_{s=0}^{\infty} Q_{t,t+s}\Phi_{it+s} &= E_t \sum_{s=0}^{\infty} Q_{t,t+s}\left[(P_{it+s} - MC_{t+s})\, Y_{it+s} - \frac{\varphi}{2}\left(\frac{P_{it+s}}{\pi P_{it+s-1}} - 1\right)^2 P_{t+s}Y_{t+s}\right] \\ s.t. Y_{it+s} &= \left(\frac{P_{it+s}}{P_{t+s}}\right)^{-\eta} X_{t+s} + \theta Y_{it+s-1} \\ Q_{t,t+s} &= \beta^s \left(\frac{X_{t+s}}{X_t}\right)^{-\sigma} \frac{P_t}{P_{t+s}}\end{aligned}$$

The first order conditions are:

$$v_{it} = (P_{it} - MC_t) + \theta E_t\left[Q_{t,t+1} v_{it+1}\right]$$

and

$$Y_{it} = v_{it}\left[\eta\left(\frac{P_{it}}{P_t}\right)^{-\eta-1}\frac{X_t}{P_t}\right] + \left[\varphi\left(\frac{P_{it}}{\pi P_{it-1}} - 1\right)\frac{P_t Y_t}{\pi P_{it-1}} - \varphi E_t Q_{t,t+1}\left(\frac{P_{it+1}}{\pi P_{it}} - 1\right)\frac{P_{it+1}}{\pi (P_{it})^2} P_{t+1}Y_{t+1}\right]$$

where v_{it} is the Lagrange multiplier on the dynamic demand constraint and represents the shadow price of producing good i.

B Equilibrium Conditions

B.1 Aggregation and Symmetry

Aggregate output: In this setup, all firms and all households are symmetric. This implies that aggregate output is given by

$$Y_t = A_t N_t$$

Aggregate resource constraint: Aggregate profits are

$$\Phi_t = P_t Y_t - W_t N_t - \frac{\varphi}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2 P_t Y_t$$

and the household budget constraint becomes in equilibrium (note: $P_t X_t + P_t \vartheta_t$ reduces to $P_t C_t$)

$$P_t C_t = W_t N_t (1-\tau) + \Phi_t + T_t$$

Combine the household budget constraint with the government budget constraint ($\tau W_t N_t = T_t$) and the definition of profits to obtain the aggregate resource constraint

$$C_t + \frac{\varphi}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2 Y_t = Y_t$$

B.2 System of Non-linear Equations

$$X_t = C_t - \theta C_{t-1} \tag{32}$$

$$N_t^{\upsilon} X_t^{\sigma} = \frac{W_t}{P_t} \equiv w_t(1-\tau) \tag{33}$$

$$X_t^{-\sigma} = \beta E_t \left[X_{t+1}^{-\sigma}\ R_t \pi_{t+1}^{-1}\right] \tag{34}$$

$$Y_t = \eta \omega_t X_t + \varphi\left(\frac{\pi_t}{\pi} - 1\right)\frac{\pi_t}{\pi} Y_t - \varphi\beta E_t\left[\left(\frac{X_{t+1}}{X_t}\right)^{-\sigma}\left(\frac{\pi_{t+1}}{\pi} - 1\right)\frac{\pi_{t+1}}{\pi} Y_{t+1}\right] \tag{35}$$

$$\omega_t = \left(1 - \frac{1}{\mu_t}\right) + \theta\beta E_t\left[\left(\frac{X_{t+1}}{X_t}\right)^{-\sigma}\omega_{t+1}\right] \tag{36}$$

$$Y_t = A_t N_t \tag{37}$$

$$Y_t = C_t + \frac{\varphi}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2 Y_t \tag{38}$$

$$mc_t = \frac{w_t}{A_t} \tag{39}$$

$$\mu_t = \frac{1}{mc_t} \tag{40}$$

$$\ln A_t = \rho \ln A_{t-1} + \epsilon_t \tag{41}$$

B.3 The Deterministic Steady State

The non-stochastic long-run equilibrium is characterized by constant real variables and nominal variables growing at a constant rate. The equilibrium conditions (32) - (41) reduce to:

$$X = (1-\theta)\, C \tag{42}$$

$$N^{\upsilon} X^{\sigma} = w(1-\tau) \tag{43}$$

$$1 = \beta\left(R\pi^{-1}\right) = \beta r \tag{44}$$

$$Y = \eta\omega X \tag{45}$$

$$\mu = \left[1-(1-\theta\beta)\,\omega\right]^{-1} \tag{46}$$

$$Y = AN \tag{47}$$

$$Y = C \tag{48}$$

$$mc = \frac{w}{A} \tag{49}$$

$$\mu = \frac{1}{mc}$$

$$A = 1$$

Table 1 contains the imposed calibration restrictions. We assume values for the real interest rate, the Frisch labor supply elasticity, steady state inflation, and the parameters σ, η, φ, and θ. The discount factor β matches the assumed real rate of interest, $\beta = r^{-1}$, while the nominal interest rate is $R = r\pi$. Given the specification of the utility function, υ is the inverse of the Frisch labor supply elasticity, $\epsilon_{Nw} = \frac{1}{\upsilon}$.

With no price adjustment costs in the deterministic steady state, $C = Y$. Then, using equations (42) and (45), the steady state value of the shadow price ω is

$$\omega = \left[\eta\,(1-\theta)\right]^{-1},$$

while the markup μ is given by equation (46) and the marginal cost is its inverse, $mc = \mu^{-1}$.

To determine the steady state value of labor, we substitute for X in terms of Y in (43) and then, using the aggregate production function, we obtain the following expression,

$$N^{\sigma+\upsilon}\left[(1-\theta)\,A\right]^{\sigma} = w(1-\tau), \tag{50}$$

which can be solved for N. Note that this expression depends on the real wage w, which can be obtained from equation (49). However, in order to assess the level of taxation needed to make the long-run equilibrium efficient, we substitute for real wages using the

steady-state condition for marginal costs, $mc = w/A = 1/\mu$

$$N^{\sigma+\upsilon}\left[(1-\theta)A\right]^{\sigma} = \frac{A}{\mu}(1-\tau), \tag{51}$$

In order for this condition to match the social planner's allocation (65) it must be the case that $\tau = 1 - \mu(1-\theta\beta)$. See Appendix E for the social planner's problem. Finally, equations (47) and (42) can be solved for aggregate output Y (or consumption C) and habit-adjusted consumption X.

B.4 System of Log-linear Equations

Log-linearizing the equilibrium conditions (32) - (41) around the efficient deterministic steady state gives the following set of equations:

$$\widehat{X}_t = (1-\theta)^{-1}\left(\widehat{C}_t - \theta\widehat{C}_{t-1}\right)$$

$$\sigma\widehat{X}_t + \upsilon\widehat{N}_t = \widehat{w}_t$$

$$\widehat{X}_t = E_t\widehat{X}_{t+1} - \frac{1}{\sigma}\left(\widehat{R}_t - E_t\widehat{\pi}_{t+1}\right)$$

$$\widehat{Y}_t = \widehat{\omega}_t + \widehat{X}_t + \varphi\left(\widehat{\pi}_t - \beta E_t\widehat{\pi}_{t+1}\right) \tag{52}$$

$$\widehat{\omega}_t = \frac{1}{\mu\omega}\widehat{\mu}_t + \theta\beta E_t\widehat{\omega}_{t+1} + \theta\beta\sigma\left(\widehat{X}_t - E_t\widehat{X}_{t+1}\right) \tag{53}$$

$$\widehat{Y}_t = \widehat{A}_t + \widehat{N}_t$$

$$\widehat{Y}_t = \widehat{C}_t$$

$$\widehat{mc}_t = \widehat{w}_t - \widehat{A}_t$$

$$\widehat{\mu}_t = -\widehat{mc}_t$$

$$\widehat{A}_t = \rho\widehat{A}_{t-1} + \varepsilon_t$$

The New Keynesian Phillips Curve is given by the pricing equation (52)

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \frac{1}{\varphi}\left(\widehat{Y}_t - \widehat{\omega}_t - \widehat{X}_t\right)$$

where the evolution of the shadow value $\widehat{\omega}_t$ is given by equation (53).

C The Case of Superficial External Habits

C.1 Households

Habits are "superficial" when they are formed at the level of the aggregate consumption good. Households derive utility from the habit-adjusted composite good X_t^k,

$$X_t^k = C_t^k - \theta C_{t-1},$$

where household k's consumption, C_t^k, is an aggregate of a continuum of final goods, indexed by $i \in [0,1]$,

$$C_t^k = \left(\int_0^1 \left(C_{it}^k \right)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}},$$

with $\eta > 1$ the elasticity of substitution between them and $C_{t-1} \equiv \int_0^1 C_{t-1}^k dk$ the cross-sectional average of consumption.

Cost Minimization Households decide the composition of the consumption basket to minimize expenditures

$$\min_{\{C_{it}^k\}_i} \int_0^1 P_{it} C_{it}^k di$$
$$s.t. \left(\int_0^1 \left(C_{it}^k \right)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}} \geq \overline{C}_t^k.$$

The demand for individual goods i is

$$C_{it}^k = \left(\frac{P_{it}}{P_t} \right)^{-\eta} C_t^k,$$

where $P_t \equiv \left(\int_0^1 P_{it}^{1-\eta} di \right)^{\frac{1}{1-\eta}}$ is the consumer price index. The overall demand for good i is obtained by aggregating across all households

$$C_{it} = \int_0^1 C_{it}^k dk = \left(\frac{P_{it}}{P_t} \right)^{-\eta} C_t. \tag{54}$$

Unlike in the case of deep habits, this demand is *not* dynamic.

C.2 Firms

The firms' cost minimization problem is unchanged, while the profit maximization is still dynamic (due to the nature of price inertia) but subject to the static demand (54). The

price is set optimally to satisfy the following relationship:

$$\left[1-\eta\left(1-\frac{1}{\mu_t}\right)\right]Y_t=\varphi\left(\frac{\pi_t}{\pi}-1\right)\frac{\pi_t}{\pi}Y_t-\varphi\beta E_t\left[\left(\frac{X_{t+1}}{X_t}\right)^{-\sigma}\left(\frac{\pi_{t+1}}{\pi}-1\right)\frac{\pi_{t+1}}{\pi}Y_{t+1}\right]. \tag{55}$$

C.3 Equilibrium

In this setup, we obtain the familiar looking New Keynesian Phillips Curve,

$$\widehat{\pi}_t=\beta E_t\widehat{\pi}_{t+1}+\frac{\eta-1}{\varphi}\widehat{mc}_t \tag{56}$$

to which we add the IS curve,

$$\widehat{X}_t=E_t\widehat{X}_{t+1}-\frac{1}{\sigma}\widehat{R}_t+\frac{1}{\sigma}E_t\widehat{\pi}_{t+1}, \tag{57}$$

and two equations defining the habit-adjusted consumption and the real marginal cost,

$$\widehat{X}_t=\frac{1}{1-\theta}\left(\widehat{Y}_t-\theta\widehat{Y}_{t-1}\right) \tag{58}$$

$$\widehat{mc}_t=\sigma\widehat{X}_t+\upsilon\widehat{Y}_t-(1+\upsilon)\,\widehat{A}_t. \tag{59}$$

D The Case of Internal Habits

D.1 Households

Habits are internal and superficial when they are formed at the level of the aggregate consumption bundle but households endogenize the effects of their current consumption choices on future utility. Each household k derives utility from a habit-adjusted composite good X_t^k,

$$X_t^k = C_t^k - \theta C_{t-1}^k$$

where C_t^k is the time-t aggregate of a continuum of goods, indexed by $i \in [0,1]$,

$$C_t^k = \left(\int_0^1 \left(C_{it}^k \right)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}}$$

with $\eta > 0$ the elasticity of substitution between them, and C_{t-1}^k is the previous period consumption of household k. Since households are symmetric and, in this case, we also do not need to distinguish between individual and aggregate variables, in what follows we drop the k superscript.

Cost Minimization The households' cost minimization problem yields a typical static demand for each good i,

$$C_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\eta} C_t$$

where $P_t \equiv \left(\int_0^1 P_{it}^{1-\eta} di \right)^{\frac{1}{1-\eta}}$.

Utility maximization The Lagrangian function is

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left[\left(\frac{(C_t - \theta C_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\upsilon}}{1+\upsilon} \right) - \lambda_t \left(P_t C_t + E_t Q_{t,t+1} D_{t+1} - W_t N_t (1-\tau) - D_t - \Phi_t - T_t \right) \right]$$

where $R_t = \frac{1}{E_t[Q_{t,t+1}]}$ is the one-period gross return on nominal riskless bonds. Anticipating that current consumption decisions affect the stock of habits entering into the future, the consumption Euler equation is given by,

$$X_t^{-\sigma} - \theta \beta E_t X_{t+1}^{-\sigma} = \beta E_t \left[\left(X_{t+1}^{-\sigma} - \theta \beta E_{t+1} X_{t+2}^{-\sigma} \right) \frac{R_t}{\pi_{t+1}} \right]$$

and the labor supply condition by,

$$\frac{(N_t)^{\upsilon}}{\left(X_t^{-\sigma} - \theta \beta E_t X_{t+1}^{-\sigma} \right)} = w_t (1-\tau)$$

In log-linear form, the first order condition for labor and the Euler equation become

$$\upsilon \widehat{N}_t = \widehat{w}_t - \frac{\sigma}{1-\theta\beta}\left(\widehat{X}_t - \theta\beta E_t \widehat{X}_{t+1}\right) \tag{60}$$

and

$$\widehat{X}_t - \theta\beta E_t \widehat{X}_{t+1} = E_t\left(\widehat{X}_{t+1} - \theta\beta E_{t+1}\widehat{X}_{t+2}\right) - \frac{1-\theta\beta}{\sigma}\left(\widehat{R}_t - E_t\widehat{\pi}_{t+1}\right) \tag{61}$$

D.2 Firms

The firms' behavior is unchanged, except that the stochastic discount factor in their profit maximization problem is given by,

$$Q_{t,t+s} = \beta^s\left(\frac{\lambda_{t+s}}{\lambda_t}\right) = \beta^s\left(\frac{X_{t+s}^{-\sigma} - \theta\beta E_{t+s}X_{t+s+1}^{-\sigma}}{X_t^{-\sigma} - \theta\beta E_t X_{t+1}^{-\sigma}}\right)\frac{P_t}{P_{t+s}}$$

and the firms' FOC for price is then

$$[1-\eta(1-mc_t)]\,Y_t = \varphi\left(\frac{\pi_t}{\pi}-1\right)\frac{\pi_t}{\pi}Y_t - \varphi\beta E_t\left[\left(\frac{X_{t+1}^{-\sigma} - \theta\beta E_{t+1}X_{t+2}^{-\sigma}}{X_t^{-\sigma} - \theta\beta E_t X_{t+1}^{-\sigma}}\right)\left(\frac{\pi_{t+1}}{\pi}-1\right)\frac{\pi_{t+1}}{\pi}Y_{t+1}\right]$$

In a deterministic steady state, this relationship still reduces to $1 = \eta(1-mc)$, and the log-linearized equation, which is essentially the NKPC, is the same as under external superficial habits.

The marginal cost relationship is slightly modified due to the different marginal rate of substitution between consumption and labor:

$$\begin{aligned}\widehat{mc}_t &= \widehat{w}_t - \widehat{A}_t \\ &= \left[\frac{\sigma}{1-\theta\beta}\left(\widehat{X}_t - \theta\beta E_t\widehat{X}_{t+1}\right) + \upsilon\widehat{N}_t\right] - \widehat{A}_t \\ &= \frac{\sigma}{1-\theta\beta}\left(\widehat{X}_t - \theta\beta E_t\widehat{X}_{t+1}\right) + \upsilon\widehat{Y}_t - (1+\upsilon)\widehat{A}_t \end{aligned} \tag{62}$$

Hence, the system of relevant equations includes the IS curve (61), the NKPC (56), the marginal cost relationship (62), and the usual definition of the habit-adjusted consumption (58).

In gap form

In the case of internal habits, it is easy to write these relationships in 'gap' form. Let $\widehat{Y}_t^g \equiv \widehat{Y}_t - \widehat{Y}_t^*$ and $\widehat{X}_t^g \equiv \widehat{X}_t - \widehat{X}_t^*$ denote the relevant 'gap' variables. Using the equations describing the social planner's allocation below, the marginal cost equation can be written

as

$$\begin{aligned}
\widehat{mc}_t &= \frac{\sigma}{1-\theta\beta}\left(\widehat{X}_t - \theta\beta E_t\widehat{X}_{t+1}\right) + \upsilon\widehat{Y}_t - (1+\upsilon)\,\widehat{A}_t \\
&= \frac{\sigma}{1-\theta\beta}\left(\widehat{X}_t - \theta\beta E_t\widehat{X}_{t+1}\right) + \upsilon\widehat{Y}_t - \frac{\sigma}{1-\theta\beta}\left(\widehat{X}_t^* - \theta\beta E_t\widehat{X}_{t+1}^*\right) - \upsilon\widehat{Y}_t^* \\
&= \frac{\sigma}{1-\theta\beta}\left[\left(\widehat{X}_t - \widehat{X}_t^*\right) - \theta\beta E_t\left(\widehat{X}_{t+1} - \widehat{X}_{t+1}^*\right)\right] + \upsilon\left(\widehat{Y}_t - \widehat{Y}_t^*\right) \\
&= \frac{\sigma}{1-\theta\beta}\left(\widehat{X}_t^g - \theta\beta E_t\widehat{X}_{t+1}^g\right) + \upsilon\widehat{Y}_t^g
\end{aligned}$$

and the NKPC is then,

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \frac{\eta-1}{\varphi}\left[\frac{\sigma}{1-\theta\beta}\left(\widehat{X}_t^g - \theta\beta E_t\widehat{X}_{t+1}^g\right) + \upsilon\widehat{Y}_t^g\right] \tag{63}$$

The IS curve can also be written using gap variables and additional terms involving only the social planner's allocation. From the social planner's problem, $\widehat{X}_t^* - \theta\beta E_t\widehat{X}_{t+1}^* = \frac{1-\theta\beta}{\sigma}\left[(1+\upsilon)\,\widehat{A}_t - \upsilon\widehat{Y}_t^*\right]$, which then allows us to write the IS curve as (add and subtract the time t and $(t+1)$ expressions from the LHS and RHS of the IS equation):

$$\widehat{X}_t^g - \theta\beta E_t\widehat{X}_{t+1}^g = \left\{\begin{array}{c} E_t\left(\widehat{X}_{t+1}^g - \theta\beta E_{t+1}\widehat{X}_{t+2}^g\right) - \frac{1-\theta\beta}{\sigma}\left(\widehat{R}_t - E_t\widehat{\pi}_{t+1}\right) \\ +\frac{1-\theta\beta}{\sigma}\left[(1+\upsilon)(\rho-1)\,\widehat{A}_t - \upsilon\left(E_t\widehat{Y}_{t+1}^* - \widehat{Y}_t^*\right)\right] \end{array}\right\}$$

D.3 Optimal Policy: internal habits

In the case of internal (superficial) habits, we can show analytically that in response to technology shocks the Ramsey planner can set the nominal interest rate so as to match the social planner's allocation, without creating inflation. Using the link between gap variables, $\widehat{X}_t^g = \frac{1}{1-\theta}\left(\widehat{Y}_t^g - \theta\widehat{Y}_{t-1}^g\right)$, we first express all relevant equations in terms of inflation and output gap. The welfare function Γ_0 is

$$\Gamma_0 = -\frac{1}{2}\Omega E_0\sum_{t=0}^{\infty}\beta^t\left\{\delta\left(\widehat{Y}_t^g - \theta\widehat{Y}_{t-1}^g\right)^2 + \upsilon\left(\widehat{Y}_t^g\right)^2 + \varphi\widehat{\pi}_t^2\right\} + tip + O[2]$$

and the New Keynesian Phillips curve (63) is

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} - \frac{\eta-1}{\varphi}\delta\left[\theta\beta E_t\widehat{Y}_{t+1}^g - \zeta\widehat{Y}_t^g + \theta\widehat{Y}_{t-1}^g\right] \tag{64}$$

where Ω, δ, and ζ are defined as: $\Omega \equiv \overline{N}^{1+\upsilon}$, $\delta \equiv \frac{\sigma}{(1-\theta\beta)(1-\theta)}$, and $\zeta \equiv \left(1+\theta^2\beta+\frac{\upsilon}{\delta}\right)$.

With the IS curve not binding, the Lagrangian is

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} -\frac{1}{2}\Omega \left[\delta \left(\widehat{Y}_t^g - \theta \widehat{Y}_{t-1}^g \right)^2 + \upsilon \widehat{Y}_t^g + \varphi \widehat{\pi}_t^2 \right] \\ -\gamma_t \left[\widehat{\pi}_t - \beta \widehat{\pi}_{t+1} + \frac{\eta-1}{\varphi} \delta \left(\theta \beta E_t \widehat{Y}_{t+1}^g - \zeta \widehat{Y}_t^g + \theta \widehat{Y}_{t-1}^g \right) \right] \end{array} \right\}$$

and the first order conditions for the output gap and inflation are:

$$\left(\widehat{Y}_t^g \right) \quad : \quad \zeta \left(\widehat{Y}_t^g - \frac{\eta - 1}{\varphi \Omega} \gamma_t \right) = \theta \beta E_t \left(\widehat{Y}_{t+1}^g - \frac{\eta - 1}{\varphi \Omega} \gamma_{t+1} \right) + \theta \left(\widehat{Y}_{t-1}^g - \frac{\eta - 1}{\varphi \Omega} \gamma_{t-1} \right)$$

$$(\widehat{\pi}_t) \quad : \quad \widehat{\pi}_t = -\frac{1}{\varphi \Omega} \left(\gamma_t - \gamma_{t-1} \right)$$

with the additional restriction that, under full commitment, the central bank ignores past commitments in the first period and sets all pre-existing conditions to zero, $\widehat{Y}_{-1}^g = \gamma_{-1} = 0$. By varying interest rates to eliminate the output gap, the value of the Lagrange multiplier associated with the NKPC is zero, $\gamma_t = 0$, and the policy maker can achieve the flexible-price allocation which is desirable since there are no frictions other than nominal inertia in this economy featuring internal habits.

E The Social Planner's Problem

The subsidy level that ensures an efficient long-run equilibrium is obtained by comparing the steady state solution of the social planner's problem with the steady state obtained in the decentralized equilibrium. The social planner ignores the nominal inertia and all other inefficiencies and chooses real allocations that maximize the representative consumer's utility subject to the aggregate resource constraint, the aggregate production function, and the law of motion for habit-adjusted consumption:

$$\max_{\{X_t^*, C_t^*, N_t^*\}} E_0 \sum_{t=0}^{\infty} \beta^t u\left(X_t^*, N_t^*\right)$$

$$\begin{aligned} s.t.\ Y_t^* &= C_t^* \\ Y_t^* &= A_t N_t^* \\ X_t^* &= C_t^* - \theta C_{t-1}^* \end{aligned}$$

The optimal choice implies the following relationship between the marginal rate of substitution between labor and habit-adjusted consumption and the intertemporal marginal rate of substitution in habit-adjusted consumption

$$\frac{\chi\left(N_t^*\right)^{\upsilon}}{\left(X_t^*\right)^{-\sigma}} = A_t\left[1 - \theta\beta E_t\left(\frac{X_{t+1}^*}{X_t^*}\right)^{-\sigma}\right].$$

The steady state equivalent of this expression can be written as,

$$\chi\left(N^*\right)^{\upsilon+\sigma}\left[\left(1-\theta\right)A\right]^{\sigma} = A\left(1-\theta\beta\right). \tag{65}$$

The dynamics of this model are driven by technology shocks to the system of equilibrium conditions composed of the Euler equation, the resource constraint, and the evolution of habit-adjusted consumption. In log-linear form, these are:

$$\widehat{X}_t^* = \theta\beta E_t \widehat{X}_{t+1}^* + \frac{1-\theta\beta}{\sigma}\left(-\upsilon\widehat{N}_t^* + \widehat{A}_t\right)$$

$$\widehat{Y}_t^* = \widehat{A}_t + \widehat{N}_t^*$$

$$\widehat{X}_t^* = \frac{1}{1-\theta}\left(\widehat{Y}_t^* - \theta\widehat{Y}_{t-1}^*\right),$$

which combined yield the following dynamic equation

$$\zeta\widehat{Y}_t^* = \theta\beta E_t\widehat{Y}_{t+1}^* + \theta\widehat{Y}_{t-1}^* + \left(\frac{1+\upsilon}{\delta}\right)\widehat{A}_t$$

where $\zeta \equiv \left(1+\theta^2\beta+\frac{\upsilon}{\delta}\right)$ and $\delta \equiv \frac{\sigma}{(1-\theta\beta)(1-\theta)}$. In the absence of deep habits, $\theta = 0$, the model reduces to the basic New Keynesian model where $\widehat{Y}_t^* = \left(\frac{1+\upsilon}{\sigma+\upsilon}\right)\widehat{A}_t$.

F Derivation of Welfare

Individual utility in period t is

$$\frac{X_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\upsilon}}{1+\upsilon}$$

where $X_t = C_t - \theta C_{t-1}$ is the habit-adjusted aggregate consumption. Before considering the elements of the utility function, we need to note the following general result relating to second order approximations

$$\frac{Y_t - Y}{Y_t} = \widehat{Y}_t + \frac{1}{2}\widehat{Y}_t^2 + O[2]$$

where $\widehat{Y}_t = \ln\left(\frac{Y_t}{Y}\right)$ and $O[2]$ represents terms that are of order higher than 2 in the bound on the amplitude of the relevant shocks. This will be used in various places in the derivation of welfare. Now consider the second order approximation to the first term,

$$\frac{X_t^{1-\sigma}}{1-\sigma} = \overline{X}^{1-\sigma}\left(\frac{X_t - \overline{X}}{\overline{X}}\right) - \frac{\sigma}{2}\overline{X}^{1-\sigma}\left(\frac{X_t - \overline{X}}{\overline{X}}\right)^2 + tip + O[2]$$

where *tip* represents 'terms independent of policy'. Using the results above this can be rewritten in terms of hatted variables

$$\frac{X_t^{1-\sigma}}{1-\sigma} = \overline{X}^{1-\sigma}\left\{\widehat{X}_t + \frac{1}{2}(1-\sigma)\widehat{X}_t^2\right\} + tip + O[2].$$

In pure consumption terms, the value of X_t can be approximated to second order by:

$$\widehat{X}_t = \frac{1}{1-\theta}\left(\widehat{C}_t + \frac{1}{2}\widehat{C}_t^2\right) - \frac{\theta}{1-\theta}\left(\widehat{C}_{t-1} + \frac{1}{2}\widehat{C}_{t-1}^2\right) - \frac{1}{2}\widehat{X}_t^2 + O[2]$$

To a first order,

$$\widehat{X}_t = \frac{1}{1-\theta}\widehat{C}_t - \frac{\theta}{1-\theta}\widehat{C}_{t-1} + O[1]$$

which implies

$$\widehat{X}_t^2 = \frac{1}{(1-\theta)^2}\left(\widehat{C}_t - \theta\widehat{C}_{t-1}\right)^2 + O[2]$$

Therefore,

$$\frac{X_t^{1-\sigma}}{1-\sigma} = \overline{X}^{1-\sigma}\left\{\frac{1}{1-\theta}\left(\widehat{C}_t + \frac{1}{2}\widehat{C}_t^2\right) - \frac{\theta}{1-\theta}\left(\widehat{C}_{t-1} + \frac{1}{2}\widehat{C}_{t-1}^2\right) + \frac{1}{2}(-\sigma)\widehat{X}_t^2\right\} + tip + O[2]$$

Summing over the future,

$$\sum_{t=0}^{\infty}\beta^t\frac{X_t^{1-\sigma}}{1-\sigma} = \overline{X}^{1-\sigma}\sum_{t=0}^{\infty}\beta^t\left\{\frac{1-\theta\beta}{1-\theta}\left(\widehat{C}_t + \frac{1}{2}\widehat{C}_t^2\right) - \frac{1}{2}\sigma\widehat{X}_t^2\right\} + tip + O[2].$$

The term in labour supply can be written as

$$\frac{N_t^{1+\upsilon}}{1+\upsilon} = \overline{N}^{1+\upsilon}\left\{\widehat{N}_t + \frac{1}{2}(1+\upsilon)\widehat{N}_t^2\right\} + tip + O[2]$$

Now we need to relate the labor input to output which, in this case without price dispersion, is simply,

$$N_t = \frac{Y_t}{A_t}$$

and can be approximated to first order,

$$\widehat{N}_t = \widehat{Y}_t - \widehat{A}_t$$

which implies

$$\widehat{N}_t^2 = \left(\widehat{Y}_t - \widehat{A}_t\right)^2$$

so we can then write

$$\frac{N_t^{1+\upsilon}}{1+\upsilon} = \overline{N}^{1+\upsilon}\left\{\widehat{Y}_t + \frac{1}{2}(1+\upsilon)\widehat{Y}_t^2 - (1+\upsilon)\widehat{Y}_t\widehat{A}_t\right\} + tip + O[2]$$

Welfare is then given by

$$\begin{aligned}\Gamma_0 &= \overline{X}^{1-\sigma}E_0\sum_{t=0}^{\infty}\beta^t\left\{\frac{1-\theta\beta}{1-\theta}\left(\widehat{C}_t + \frac{1}{2}\widehat{C}_t^2\right) - \frac{1}{2}\sigma\hat{X}_t^2\right\} \\ &\quad -\overline{N}^{1+\upsilon}E_0\sum_{t=0}^{\infty}\beta^t\left\{\widehat{Y}_t + \frac{1}{2}(1+\upsilon)\widehat{Y}_t^2 - (1+\upsilon)\widehat{Y}_t\widehat{A}_t\right\} \\ &\quad + tip + O[2]\end{aligned}$$

From the social planner's problem we know, $\overline{X}^{-\sigma}(1-\theta\beta) = \overline{N}^{\upsilon}$ such that $\overline{X}^{1-\sigma}(1-\theta\beta) = (1-\theta)\overline{N}^{1+\upsilon}$. If we use the appropriate subsidy to render the steady-state efficient and also use the second order approximation to the national accounting identity,

$$\widehat{C}_t + \frac{1}{2}\widehat{C}_t^2 = \widehat{Y}_t + \frac{1}{2}\widehat{Y}_t^2 - \frac{\varphi}{2}\widehat{\pi}_t^2 + O[2],$$

we can eliminate the level terms and write the sum of discounted utilities as:

$$\Gamma_0 = -\frac{1}{2}\overline{N}^{1+\upsilon}E_0\sum_{t=0}^{\infty}\beta^t\left\{\frac{\sigma(1-\theta)}{1-\theta\beta}\widehat{X}_t^2 + \upsilon\left(\widehat{Y}_t - \frac{1+\upsilon}{\upsilon}\widehat{A}_t\right)^2 + \varphi\widehat{\pi}_t^2\right\} + tip + O[2] \quad (66)$$

The welfare function can be also expressed in the usual "gap" form. To do so, we employ the social planner's solution in log-linear form to re-write the output term in the

welfare function as,

$$\upsilon\left(\widehat{Y}_t-\frac{1+\upsilon}{\upsilon}\widehat{A}_t\right)^2=\upsilon\left(\widehat{Y}_t-\widehat{Y}_t^*\right)^2+2\frac{\sigma}{1-\theta\beta}\widehat{Y}_t\left(\widehat{X}_t^*-\theta\beta E_t\widehat{X}_{t+1}^*\right)+tip$$

Summing across time periods, we have

$$E_0\sum_{t=0}^{\infty}\beta^t\upsilon\left(\widehat{Y}_t-\frac{1+\upsilon}{\upsilon}\widehat{A}_t\right)^2=E_0\sum_{t=0}^{\infty}\beta^t\left[\upsilon\left(\widehat{Y}_t-\widehat{Y}_t^*\right)^2-2\frac{\sigma}{1-\theta\beta}\widehat{Y}_t\left(\widehat{X}_t^*-\theta\beta E_t\widehat{X}_{t+1}^*\right)\right]+tip+O\left[2\right]$$

Then note that we can write

$$\widehat{X}_t^2=\left(\widehat{X}_t-\widehat{X}_t^*\right)^2+2\widehat{X}_t\widehat{X}_t^*-\left(\widehat{X}_t^*\right)^2$$

and, keeping only the terms relevant for policy, the welfare function becomes,

$$\Gamma_0=-\frac{1}{2}\overline{N}^{1+\upsilon}E_0\sum_{t=0}^{\infty}\beta^t\left\{\begin{array}{c}\frac{\sigma(1-\theta)}{1-\theta\beta}\left(\widehat{X}_t-\widehat{X}_t^*\right)^2+2\frac{\sigma(1-\theta)}{1-\theta\beta}\widehat{X}_t\widehat{X}_t^*+\upsilon\left(\widehat{Y}_t-\widehat{Y}_t^*\right)^2\\ -2\frac{\sigma}{1-\theta\beta}\widehat{Y}_t\left(\widehat{X}_t^*-\theta\beta E_t\widehat{X}_{t+1}^*\right)+\varphi\widehat{\pi}_t^2\end{array}\right\}+tip+O[2]$$

Finally, we can show that

$$E_0\sum_{t=0}^{\infty}\beta^t\frac{\sigma}{1-\theta\beta}\widehat{Y}_t\left(\widehat{X}_t^*-\theta\beta E_t\widehat{X}_{t+1}^*\right)=\left[E_0\sum_{t=0}^{\infty}\beta^t\frac{\sigma\left(1-\theta\right)}{1-\theta\beta}\widehat{X}_t\widehat{X}_t^*\right]+tip$$

which allows to write the welfare function in gap form as follows

$$\Gamma_0=-\frac{1}{2}\overline{N}^{1+\upsilon}E_0\sum_{t=0}^{\infty}\beta^t\left\{\frac{\sigma\left(1-\theta\right)}{1-\theta\beta}\left(\widehat{X}_t-\widehat{X}_t^*\right)^2+\upsilon\left(\widehat{Y}_t-\widehat{Y}_t^*\right)^2+\varphi\widehat{\pi}_t^2\right\}+tip+O[2]$$

F.1 Welfare Measure

We measure welfare as the unconditional expectation of lifetime utility, approximated as

$$\begin{aligned}W&=E\sum_{t=0}^{\infty}\beta^t u\left(X_t,N_t\right)\\&=\frac{1}{1-\beta}\left\{\overline{u}+\frac{1}{2}\left[\left(\Lambda_C+\Lambda_{C_{-1}}\right)var\left(\widehat{Y}_t\right)+2\Lambda_{CC_{-1}}\ cov\left(\widehat{Y}_t,\widehat{Y}_{t-1}\right)+\Lambda_N var\left(\widehat{N}_t\right)+\Lambda_\pi var\left(\widehat{\pi}_t\right)\right]\right\}\end{aligned}$$

with $\overline{u}$ the steady-state level of the momentary utility and the Λ coefficients defined as

$$\begin{cases} \Lambda_C \equiv \frac{1}{1-\theta}\left(1-\frac{\sigma}{1-\theta}\right)\overline{X}^{1-\sigma} \\ \\ \Lambda_{C_{-1}} \equiv -\frac{\theta}{1-\theta}\left(1+\frac{\sigma\theta}{1-\theta}\right)\overline{X}^{1-\sigma} \\ \\ \Lambda_{CC_{-1}} \equiv \frac{\sigma\theta}{(1-\theta)^2}\overline{X}^{1-\sigma} \end{cases} \quad and \quad \begin{cases} \Lambda_N \equiv -\left(1+\upsilon\right)\overline{N}^{1+\upsilon} \\ \\ \Lambda_\pi \equiv -\varphi\overline{X}^{1-\sigma} \end{cases}$$

The welfare terms associated with the Ramsey policy that are involved in computing the welfare costs of alternative policies, as in expression (28) in the text, are given by,

$$\begin{aligned} W_X^R &= E\sum_{t=0}^{\infty}\beta^t\frac{\left(X_t^R\right)^{1-\sigma}}{1-\sigma} \\ &= \frac{1}{1-\beta}\left\{\frac{\left(\overline{X}\right)^{1-\sigma}}{1-\sigma}+\frac{1}{2}\left[\left(\Lambda_C+\Lambda_{C_{-1}}\right)var\left(\widehat{Y}_t^R\right)+2\Lambda_{CC_{-1}}\ cov\left(\widehat{Y}_t^R,\widehat{Y}_{t-1}^R\right)+\Lambda_\pi var\left(\widehat{\pi}_t^R\right)\right]\right\} \end{aligned}$$

and

$$W_N^R = -E\sum_{t=0}^{\infty}\beta^t\frac{\left(N_t^R\right)^{1+\upsilon}}{1+\upsilon} = \frac{1}{1-\beta}\left\{-\frac{\left(\overline{N}\right)^{1+\upsilon}}{1+\upsilon}+\frac{1}{2}\Lambda_N var\left(\widehat{N}_t^R\right)\right\}$$

G Optimal Policy: Commitment

Upon substitution of the habit-adjusted consumption term, the central bank's objective function becomes

$$\frac{1}{2}\Omega E_0 \sum_{t=0}^{\infty} \beta^t \left[(\delta + \upsilon) \widehat{Y}_t^2 - 2\theta\delta \widehat{Y}_t \widehat{Y}_{t-1} + \theta^2 \delta \widehat{Y}_{t-1}^2 - 2(1+\upsilon) \widehat{Y}_t \widehat{A}_t + \varphi \widehat{\pi}_t^2 \right]$$

where $\Omega \equiv \overline{N}^{1+\upsilon}$ and $\delta \equiv \frac{\sigma}{(1-\theta\beta)(1-\theta)}$ and we re-write the constraints as,

$$\left(\frac{1+\theta}{1-\theta}\right) \widehat{Y}_t = \frac{1}{1-\theta} E_t \widehat{Y}_{t+1} + \frac{1}{\sigma} E_t \widehat{\pi}_{t+1} + \frac{\theta}{1-\theta} \widehat{Y}_{t-1} - \frac{1}{\sigma} \widehat{R}_t$$

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} - \kappa_2 \widehat{Y}_t + \kappa_2 \widehat{Y}_{t-1} - \kappa_1 \widehat{\omega}_t$$

$$\mu\omega \widehat{\omega}_t = \mu\omega\theta\beta E_t \widehat{\omega}_{t+1} - \gamma_1 \beta E_t \widehat{Y}_{t+1} + \gamma_2 \widehat{Y}_t + \gamma_3 \widehat{Y}_{t-1} + (1+\upsilon) \widehat{A}_t$$

where

$$\begin{cases} \kappa_1 \equiv \frac{1}{\varphi} \\ \\ \kappa_2 \equiv \kappa_1 \frac{\theta}{1-\theta} \end{cases} \qquad \textit{and} \qquad \begin{cases} \gamma_1 \equiv \mu\omega \frac{\sigma\theta}{1-\theta} \\ \\ \gamma_2 \equiv \gamma_1 \beta (1+\theta) - \left(\frac{\sigma}{1-\theta} + \upsilon \right) \\ \\ \gamma_3 \equiv \frac{\sigma\theta}{1-\theta} (1 - \mu\omega\theta\beta) \end{cases}$$

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \frac{1}{2}\Omega \left[-(\delta+\upsilon) \widehat{Y}_t^2 + 2\theta\delta \widehat{Y}_t \widehat{Y}_{t-1} - \theta^2 \delta \widehat{Y}_{t-1}^2 + 2(1+\upsilon) \widehat{Y}_t \widehat{A}_t - \varphi \widehat{\pi}_t^2 \right] \\ -\chi_t \left[\left(\frac{1+\theta}{1-\theta} \right) \widehat{Y}_t - \frac{1}{1-\theta} \widehat{Y}_{t+1} - \frac{1}{\sigma} \widehat{\pi}_{t+1} - \frac{\theta}{1-\theta} \widehat{Y}_{t-1} + \frac{1}{\sigma} \widehat{R}_t \right] \\ -\psi_t \left[\widehat{\pi}_t - \beta \widehat{\pi}_{t+1} + \kappa_2 \widehat{Y}_t - \kappa_2 \widehat{Y}_{t-1} + \kappa_1 \widehat{\omega}_t \right] \\ -\varsigma_t \left[\mu\omega \widehat{\omega}_t - \mu\omega\theta\beta E_t \widehat{\omega}_{t+1} + \gamma_1 \beta E_t \widehat{Y}_{t+1} - \gamma_2 \widehat{Y}_t - \gamma_3 \widehat{Y}_{t-1} - (1+\upsilon) \widehat{A}_t \right] \end{array} \right\}$$

The government chooses paths for $\widehat{R}_t$, $\widehat{Y}_t$, $\widehat{\pi}_t$, and $\widehat{\omega}_t$. The first order condition with respect to the nominal interest rate gives:

$$-\sigma^{-1} E_0 \beta^t \chi_t = 0, \qquad \forall t \geq 0 \tag{67}$$

which implies that the IS curve is not binding and it can therefore be excluded from the optimization problem. Once the optimal rules for the other variables have been obtained, we use the IS curve to determine the path of the nominal interest rate. So, the central bank now chooses $\left\{ \widehat{Y}_t, \widehat{\pi}_t, \widehat{\omega}_t \right\}$. The Lagrangian takes the form:

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \frac{1}{2}\Omega \left[-(\delta+\upsilon) \widehat{Y}_t^2 + 2\theta\delta \widehat{Y}_t \widehat{Y}_{t-1} - \theta^2 \delta \widehat{Y}_{t-1}^2 + 2(1+\upsilon) \widehat{Y}_t \widehat{A}_t - \varphi \widehat{\pi}_t^2 \right] \\ -\psi_t \left[\widehat{\pi}_t - \beta \widehat{\pi}_{t+1} + \kappa_2 \widehat{Y}_t - \kappa_2 \widehat{Y}_{t-1} + \kappa_1 \widehat{\omega}_t \right] \\ -\varsigma_t \left[\mu\omega \widehat{\omega}_t - \mu\omega\theta\beta E_t \widehat{\omega}_{t+1} + \gamma_1 \beta E_t \widehat{Y}_{t+1} - \gamma_2 \widehat{Y}_t - \gamma_3 \widehat{Y}_{t-1} - (1+\upsilon) \widehat{A}_t \right] \end{array} \right\}.$$

The first order condition for the shadow value $\widehat{\omega}_t$ gives the relationship between the two Lagrange multipliers,

$$\kappa_1 \psi_t = -\mu\omega \left(\varsigma_t - \theta \varsigma_{t-1}\right)$$

while for inflation we have the rather usual expression

$$\widehat{\pi}_t = -\frac{1}{\varphi\Omega}\left(\psi_t - \psi_{t-1}\right).$$

The first order condition for output gives

$$-\Omega\delta\zeta\widehat{Y}_t + \Omega\delta\theta\widehat{Y}_{t-1} + \Omega\left(1+\upsilon\right)\widehat{A}_t - \kappa_2\psi_t + \gamma_2\varsigma_t - \gamma_1\varsigma_{t-1} + \beta E_t\left[\Omega\delta\theta\widehat{Y}_{t+1} + \kappa_2\psi_{t+1} + \gamma_3\varsigma_{t+1}\right] = 0$$

where, as defined before, $\zeta \equiv \left(1 + \theta^2\beta + \frac{\upsilon}{\delta}\right)$.

Under full commitment, the central bank ignores past commitments in the first period by setting all pre-existing conditions to zero, $\widehat{Y}_{-1} = 0$ and $\psi_{-1} = \varsigma_{-1} = 0$. To find the solution, solve the system of equations composed of the first order conditions, the three constraints, and the technology process.

G.1 Optimal Policy: Discretion

In order to solve the time-consistent policy problem we employ the iterative algorithm of Soderlind (1999), which follows Currie and Levine (1993) in solving the Bellman equation. The per-period objective function can be written in matrix form as $Z_t' Q Z_t$, where $Z_{t+1} = \begin{bmatrix} \widehat{A}_{t+1} & \widehat{Y}_t & E_t\widehat{Y}_{t+1} & E_t\widehat{\omega}_{t+1} & E_t\widehat{\pi}_{t+1} \end{bmatrix}'$ and

$$Q = \frac{1}{2}\Omega \begin{bmatrix} 0 & 0 & -\left(1+\upsilon\right) & 0 & 0 \\ 0 & \theta^2\delta & -\theta\delta & 0 & 0 \\ -\left(1+\upsilon\right) & -\theta\delta & \left(\delta+\upsilon\right) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \varphi \end{bmatrix}$$

and the structural description of the economy is given by,

$$Z_{t+1} = AZ_t + Bu_t + \xi_{t+1},$$

where $u_t = \left[\widehat{R}_t\right]$, $\xi_{t+1} = \begin{bmatrix} \varepsilon_{t+1}^A & 0 & 0 & 0 & 0 \end{bmatrix}'$, $A \equiv A_0^{-1}A_1$, $B \equiv A_0^{-1}B_0$,

$$A_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \gamma_1\beta & 0 & 0 \\ 0 & 0 & \frac{1}{1-\theta} & 0 & \sigma^{-1} \\ 0 & 0 & 0 & 0 & \beta \end{bmatrix}, \qquad A_1 = \begin{bmatrix} \rho & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1+\upsilon & \gamma_3 & \gamma_2 & -\mu\omega & 0 \\ 0 & -\frac{\theta}{1-\theta} & \frac{1+\theta}{1-\theta} & 0 & 0 \\ 0 & -\kappa_2 & \kappa_2 & \kappa_1 & 1 \end{bmatrix}$$

and

$$B_0 = \begin{bmatrix} 0 & 0 & 0 & \sigma^{-1} & 0 \end{bmatrix}'.$$

This completes the description of the required inputs for Soderlind (1999)'s Matlab code which computes optimal discretionary policy.

H Additional Figures

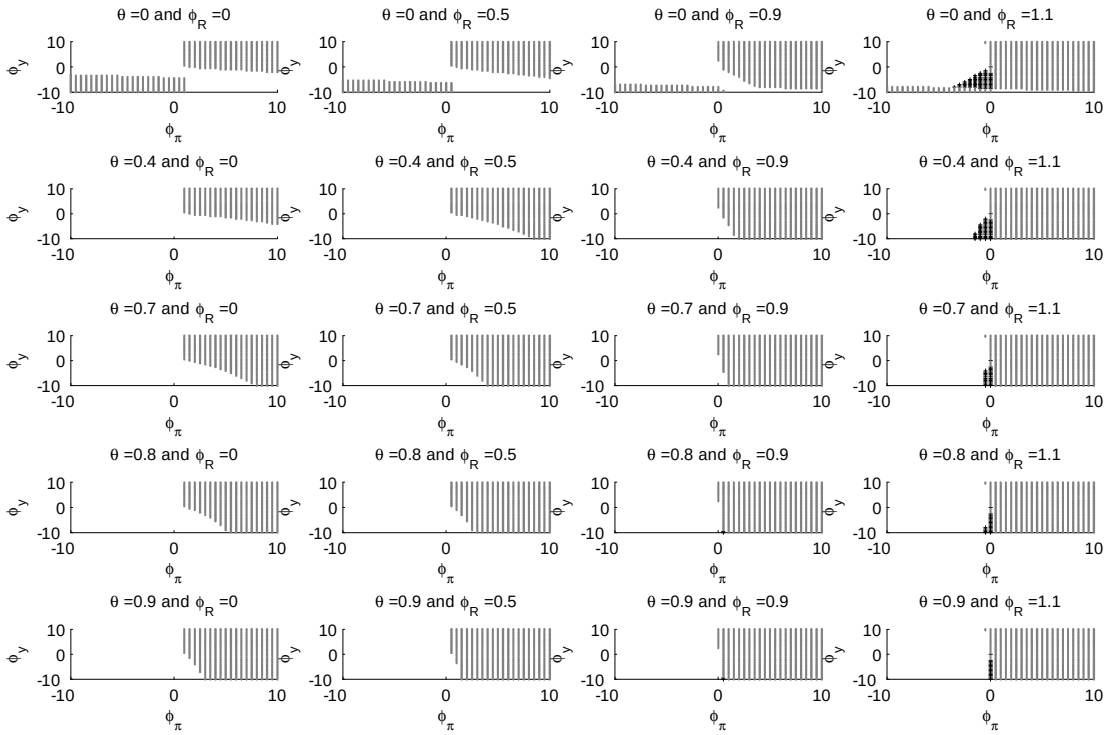

Figure 11: Determinacy properties of the model with *internal habits*, when monetary policy follows the rule $\widehat{R}_t = \phi_\pi \widehat{\pi}_t + \phi_y \widehat{Y}_t + \phi_R \widehat{R}_{t-1}$: determinacy (light grey dots), indeterminacy (blanks), and instability (dark grey stars).

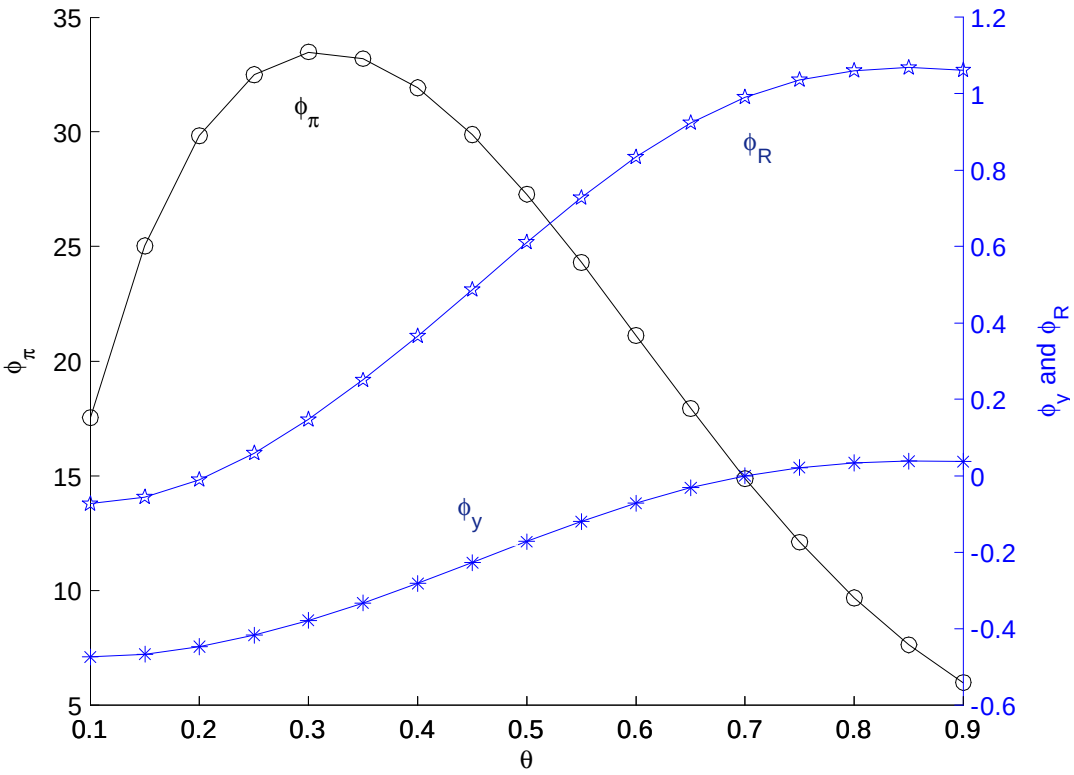

Figure 12: Optimal policy rule parameters for varying degrees of *superficial habits, the unconstrained case.*

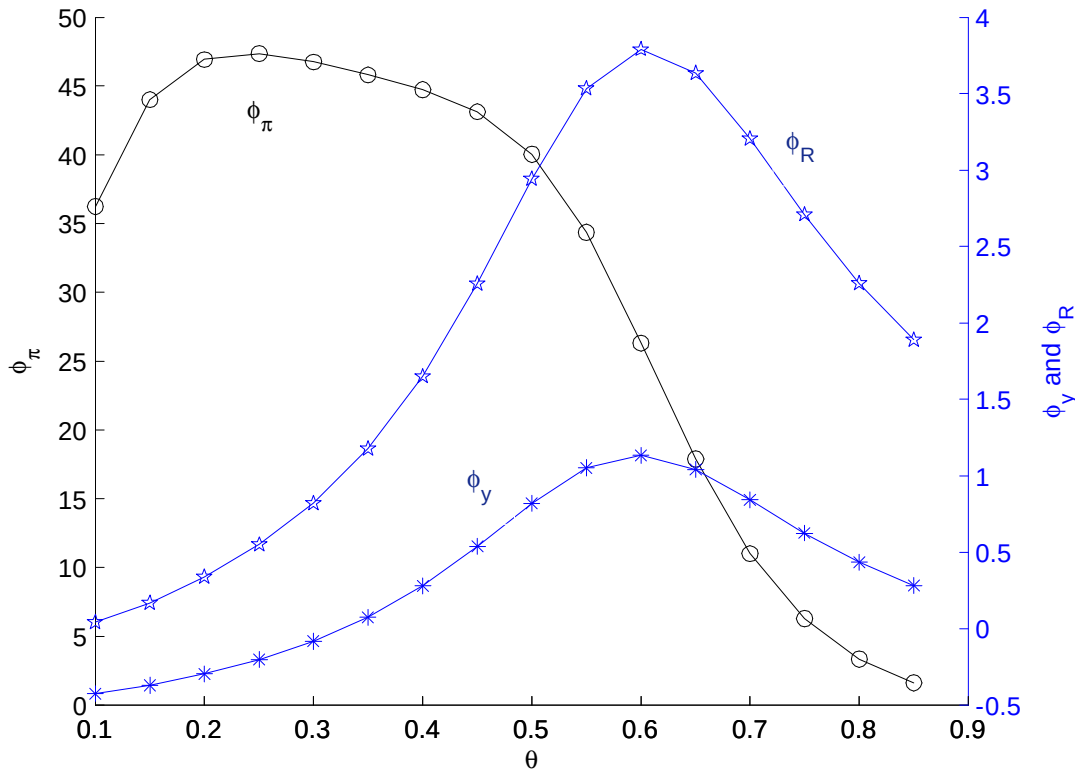

Figure 13: Optimal policy rule parameters for varying degrees of *deep habits, the unconstrained case.*

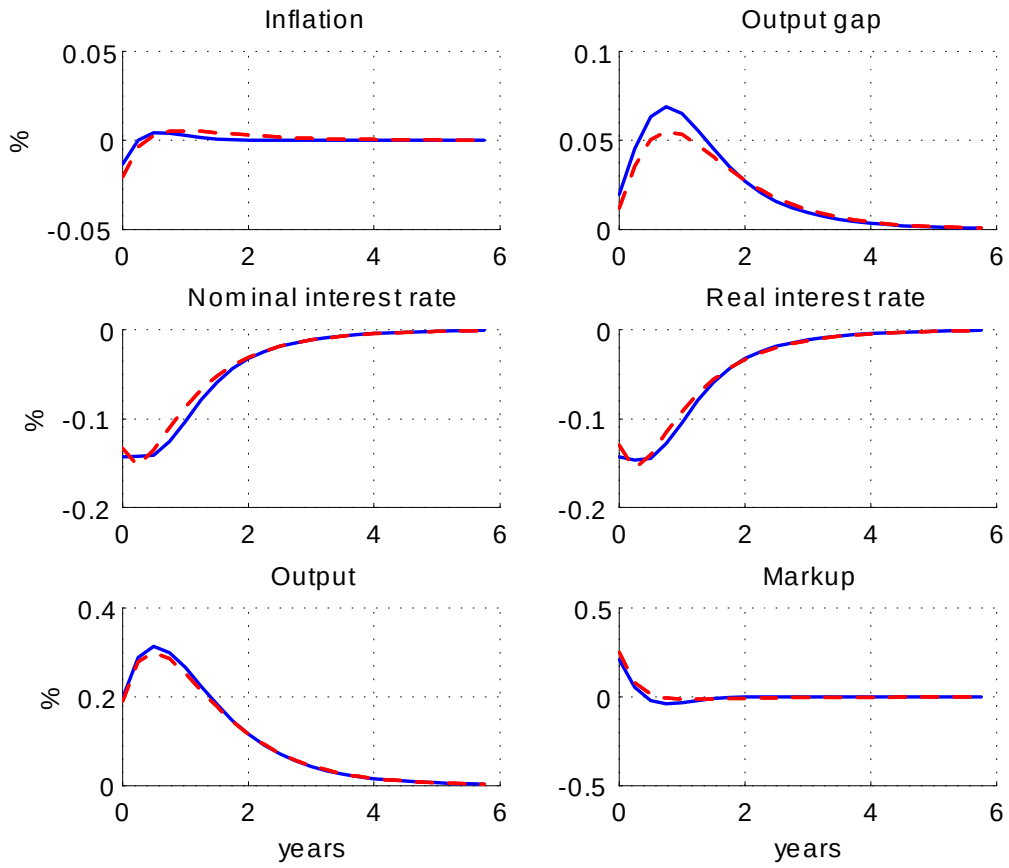

Figure 14: Impulse responses to a 1% positive technology shock in the model with *deep habits* with $\theta = 0.6$, under the optimal Taylor rule (dash lines) and optimal commitment policy (solid lines).