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MACRO AND FINANCIAL MARKETS:
The memory of an elephant?

Karim Abadir and Gabriel Talmain
Background, REStud 2002:

- Model and solution:
  - Micro-founded macro model.
  - Standard RBC + heterogeneity: drop “representative firm” assumption.
  - General Equilibrium yields explicit dynamic equation for GDP etc.
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- When \( \exists \) heterogeneity, aggregation \( \Rightarrow \) long-memory; e.g. Robinson (1978), Granger (1980), and TS lit.

- But in economics, \( \exists \) an inherent nonlinearity. Decompose GDP as
  \[
  Y \equiv Y_1 + Y_2 + \cdots = e^{\log Y_1} + e^{\log Y_2} + \cdots \neq e^{\log Y_1 + \log Y_2 + \cdots}
  \]
• The result is a new auto-correlation function (ACF) $\rho_\tau$:

![Graph of ACF](image)

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![ACF graph]

with different macropolicy implications.

• But if $\exists$ integration, what about the modification of co-integration?
1 UIP and forward premium puzzle

- Fisher (1930): “speculation” equates expected returns after conversion to the same currency (UIP); e.g.
  - £1 invested in domestic (UK) bond yields £(1 + I_t) at maturity; vs.
  - £1 invested in foreign (US) bond converted into $1/S_t, which yield $\left(1 + I_t^*\right) / S_t = £(1 + I_t^*) S_{t+1}/S_t$ at maturity.

- Define $i := \log (1 + I)$, $s := \log S$, and
  $$r_{t+1} := \Delta s_{t+1} + i_t^* - i_t,$$

as the excess return from investing in the foreign asset. Then, $E_t [r_{t+1}] = 0$ and $r_{t+1}$ should not be predictable.
• Typical empirical implementation: regress $r_{t+1}$ on the forward premium $(f_t - s_t)$, where $f$ and $s$ are the forward and spot rates in logs:

$$r_{t+1} = \alpha + \beta (f_t - s_t) + u_{t+1},$$

where $\alpha$ is the average risk premium and $\beta$ is the informational content of the forward premium.

- UIP hypothesis implies $H_0: \alpha = 0$ and $\beta = 0$.
- If one believes $f_t = E_t [s_{t+1}]$, then $(f_t - s_t) < 0$ indicates that the US$ should depreciate.
- Routine finding: $\beta < 0$. As more US$ depreciation is expected, higher returns are actually made on the US$!

 Investors are ready to pay more for an asset which, according to their expectations, should have become less attractive!

Running this regression with our data (3-month rates on US$ and UK£ deposits in London and 3-month forward rate)

\[
\hat{r}_{t+1} = -0.0157 - 3.26 (f_t - s_t)
\]

t-ratio \ (-3.67) \ (-6.14)

HAC t \ [-2.85] \ [-2.90]

where HAC = heteroskedastic and autocorrelation-consistent,

Durbin-Watson statistic : 0.65

ARCH(7) test, \ F(7, 242) : 24.10 \ {0.0\%}

RESET (omitted nonlinearities), \ F(1, 255) : 9.66 \ {0.2\%}
• Scatter plot of data

\[ r_{t+1} = -0.0157 - 3.2615(f_t - s_t) \]
\[ R^2 = 0.1283 \]

• For years, it has defied economic logic to find such a result: how could investors’ expectations be so systematically wrong? Or are they?!
2 Solution to the puzzle: coping with non-linear long-memory

- Co-movements vs. own dynamics.
- Incomplete modelling of dynamics can make the estimation of co-movements biased and inconsistent.
- Example (just an illustration): consider autoregressive process

\[ y_t = \alpha y_{t-1} + \beta x_t + u_t, \]

with \( u_t = \rho u_{t-1} + \varepsilon_t \) and \( \varepsilon_t \sim \text{IID}(0, \sigma^2). \)

Running OLS on the first equation only \( \implies \) biased and inconsistent estimators; e.g. Maddala and Rao (1973, Ecta).

- Usual approaches:
  - GLS on the augmented first equation; or
error correction mechanism (ECM), autoregressive distributed lag (ADL)

\[ y_t = (\alpha + \rho) y_{t-1} - \alpha \rho y_{t-2} + \beta x_t - \beta \rho x_{t-1} + \varepsilon_t, \]
estimating

\[ y_t = a_1 y_{t-1} + a_2 y_{t-2} + a_3 x_t + a_4 x_{t-1} + e_t \]
then testing for the restriction

\[ a_1 = \frac{a_2 a_3}{a_4} - \frac{a_4}{a_3}. \]
— error correction mechanism (ECM), autoregressive distributed lag (ADL)

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\[ a_1 = \frac{a_2 a_3}{a_4} - \frac{a_4}{a_3}. \]

• Long-memory times series models: very persistent time series ("Joseph effect").

• Problem of ECM/ADL for data with long memory: too many lags.

• How about the GLS route?
• We would like to estimate the relation

\[ z_t = \tilde{z}_t + u_t, \quad t = 1, 2, \ldots T, \]

where the \( T \times 1 \) vector \( \tilde{z} = X\beta \) is the “fundamental” value of \( z \), but \( z \) and \( \tilde{z} \) (hence possibly \( u \)) have long memory which needs to be accounted for. A possible 2-step procedure (à la GLS):

− decompose the autocorrelation matrix of \( z \) as \( R = LL' \), where \( L \) is lower-triangular and invertible;
− the transformed data \( L^{-1}z \) and \( L^{-1}\tilde{z} \) do not contain long memory and can be regressed by traditional methods.
• Unfortunately, estimating \( \mathbf{R} \) requires estimating \( T - 1 \) parameters: same as infeasible GLS!

Solution: estimate the ACF of \( z \), using a variant of the functional form in Abadir and Talmain (2002, R E Stud)

\[
\rho_\tau \approx \frac{1 - a [1 - \cos (\omega \tau)]}{1 + b \tau^c},
\]

with only 4 parameters to fit. (Note: denominator controls decay of memory.)
**UIP example:** fit is excellent for $s$

![Actual vs fitted ACF of the spot rate $\$-\pounds$](image_url)
- The estimated $T \times T$ correlation matrix is then
\[
\begin{pmatrix}
1 & \hat{\rho}_1 & \hat{\rho}_2 & \cdots & \hat{\rho}_{T-2} & \hat{\rho}_{T-1} \\
\hat{\rho}_1 & 1 & \hat{\rho}_1 & \cdots & \cdots & \hat{\rho}_{T-2} \\
\hat{\rho}_2 & \hat{\rho}_1 & \cdots & \cdots & \cdots & \cdots \\
\vdots & \vdots & \vdots & \ddots & \hat{\rho}_1 & \hat{\rho}_2 \\
\hat{\rho}_{T-2} & \cdots & \cdots & \hat{\rho}_1 & 1 & \hat{\rho}_1 \\
\hat{\rho}_{T-1} & \hat{\rho}_{T-2} & \cdots & \hat{\rho}_2 & \hat{\rho}_1 & 1
\end{pmatrix}
\]
- Find the Cholesky decomposition (matlab) $\hat{\mathbf{R}} = \hat{\mathbf{L}}\hat{\mathbf{L}}'$; and
- calculate $\mathbf{s}^{\text{acf}} = \hat{\mathbf{L}}^{-1}\mathbf{s}$ and $\mathbf{f}^{\text{acf}} = \hat{\mathbf{L}}^{-1}\mathbf{f}$. 
The scatter plot of the transformed data is a nice spherical cloud.
and the regression with transformed data becomes

\[ \hat{r}_{t+1}^{\text{acf}} = 0.00582 + 0.0604 \left( f_t^\text{acf} - s_t^\text{acf} \right) \]

\[
\begin{align*}
\text{t-ratio} & \quad (0.14) \quad (0.05) \\
\text{HAC t} & \quad [0.19] \quad [0.04]
\end{align*}
\]

Durbin-Watson statistic : 2.20

ARCH(7) test, \( F(7, 242) : 6.54 \ \{0.0\%\} \)

RESET (omitted nonlinearities), \( F(1, 255) : 0.82 \ \{36.6\%\} \).
3 Stock market application

- Unit roots?! (NB: unit root = permanent memory and no mean-reversion to any regular pattern or trend!)
- How about Stock indices?
- Jegadeesh and Titman (1993 and 2001, J Fin) find momentum;
- De Bondt and Thaler (1985 and 1987, J Fin) find long cycles;
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• GE theory: long-run proportionality relationship between the aggregate real value of firms and GDP. On a balanced growth path:
  – real interest rate is a function of capital/output, which is constant
    ⇒ rate at which future aggregate profits are discounted is fixed;
  – but share of aggregate profits in GDP is constant;
  – hence the discounted stream of future profits (i.e. capitalized value of the stock market) is proportional to GDP.
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• Empirically-testable version: log of stock market index (S&P500) corrected for inflation, $s_t$, should be in a long-term proportionality with the log of real GDP, $y_t$. 
• Error-correction model of $s_t$ on $y_t$,

$$
\Delta s_t = \alpha + (\beta_1 \Delta s_{t-1} + \cdots + \beta_m \Delta s_{t-m}) + (\gamma_0 \Delta y_t + \cdots + \gamma_n \Delta y_{t-n}) - \delta (s_{t-1} - y_{t-1}) + \delta_1 y_{t-1} + \varepsilon_t.
$$

- The term $-\delta (s_{t-1} - y_{t-1}) + \delta_1 y_{t-1}$ is the ECM.
- It represents the long-run ‘equilibrium’ relationship between $s$ and $y$:

$$
s_e = \left(1 + \frac{\delta_1}{\delta}\right) y_e, \quad \delta \neq 0.
$$

- $H_0$: $\delta_1 = 0$ for long-run proportionality between $S_e$ and $Y_e$.
- Define $d_{t-1} := s_{t-1} - s_e$ as the deviation of $s_{t-1}$ from its long-term value $s_e$.
- ECM: this deviation will pull $s_t$ back towards its long-term equilibrium value by $\delta d_{t-1}$, where $\delta > 0$. 
• For S&P500 over 1958-2000, we obtained the regression

\[
\Delta s_t = -0.728 + 0.539 \Delta s_{t-4} + 0.380 \Delta s_{t-6} \\
(-2.33) (4.35) (2.83)
\]

\[
+ 3.11 \Delta y_t - 1.72 \Delta y_{t-1} + 1.84 \Delta y_{t-2} - 1.15 \Delta y_{t-6} \\
(4.30) (-2.41) (2.58) (-1.77)
\]

\[-0.112 (s_{t-1} - y_{t-1}) + 0.0396 y_{t-1} \\
(-2.11) (0.92)
\]

where the t-ratios are in parentheses, and we have \( R^2 = 57.5\% \),

\[
\text{AR(2) test, } F(2, 32) : 0.43 \{65.4\%\}
\]

\[
\text{ARCH(1) test, } F(1, 32) : 0.17 \{68.1\%\}
\]

\[
\text{RESET, } F(1, 33) : 1.36 \{25.3\%\}
\]

• \( H_0: \delta_1 = 0 \) is supported, but:
by the end of the period, 1995-2000, the fit is poor;
the coefficient of $y_{t-1}$ is unstable and $H_0$ would be rejected on a sample ending in 1994;
including more lags of $\Delta s$ in the regression worsens rather than improves stability, while not improving the fit.

The recursive parameter estimates are...
• Remember memory?!

– Fit the ACF of $s$;
run the regression with transformed variables over 1960-2000

\[ \Delta s_{t}^{\text{acf}} = -0.363 + 2.52 \Delta y_{t}^{\text{acf}} - 1.41 \left( s_{t-1}^{\text{acf}} - y_{t-1}^{\text{acf}} \right) - 0.301 y_{t-1}^{\text{acf}} \]

\[ (-0.27) \quad (3.67) \quad (-10.0) \quad (-0.49) \]

where \( R^2 = 76.3\% \),

AR(2) test, \( F(2, 35) : 2.47 \{ 9.9\% \} \)

ARCH(1) test, \( F(1, 35) : 1.72 \{ 19.9\% \} \)

RESET, \( F(1, 33) : 0.72 \{ 40.2\% \} \).
$-H_0: \delta_1 = 0$ is supported throughout the sample
– deviations from cycles around the long-run fundamental values are restored well within a year;
– good fit.
4 Extensions

- The two-step procedure is not the most efficient estimation method:
  - we provide formulae for full GLS, QMLE, etc.;
  - but qualitative results remain unchanged.

Example: QMLE case. For any given $R$, define

$$
\hat{\beta}_R \equiv \left( X' R^{-1} X \right)^{-1} X' R^{-1} z
$$

as a function of $R$. The QMLE of $R$ is obtained by maximizing

$$
- \log \left| \left( z - X \hat{\beta}_R \right)' R^{-1} \left( z - X \hat{\beta}_R \right) \right|
$$

with respect to the parameters of the ACF: the optimization of the joint likelihood (for $R$ and $\beta$) now depends on only 4 parameters that determine the whole autocorrelation matrix $R$. Once the optimal value $\hat{R}$ of $R$ is obtained, the MLE of $\beta$ is $\hat{\beta} \equiv \hat{\beta}_{\hat{R}}$. 