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Comparing the Delay Effects in Different Channels in Packet-Based Networked Control Systems

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Abstract—The different effects of the sensor-to-controller and the controller-to-actuator delays in networked control systems, are investigated within the packet-based control framework. The study starts with identifying the specific control strategies that make those two delays different for the system. The problem is then carefully formulated and theoretical analysis is conducted, revealing that under certain conditions the sensor-to-controller delay can cause less deterioration of the system performance than the controller-to-actuator delay. This result is verified by a numerical example and has its practical guidance value.

I. INTRODUCTION

NETWORKED control systems (NCSs), i.e., systems that are controlled over the communication network, have gained much attention in recent years. In most cases, the communication network in NCSs refers to the data network like the Internet but not the control-oriented network such as the Control Area Network, or the DeviceNet. Unlike the latter, the Internet has not been designed or optimized for the control applications, meaning that lossless data transmission as assumed in conventional control systems, is not achievable for the Internet. Therefore, despite all the potential applications of NCSs in the remote and distributed control area, the communication constraints in NCSs caused by the inserted communication network, i.e., network-induced delay, data packet dropout, data rate constraint, etc. have to be carefully dealt with before NCSs can be widely applied as a reliable control strategy [1]–[7].

One of the most distinct characteristics among all these communication constraints is the network-induced delay, caused by the imperfect data transmission in NCSs. This is also one of the main topics in the majority of the works done in NCSs [8]–[12]. The reason is obvious: in fact, the delay in NCSs builds a direct bridge between the theory of NCSs and that of time delay systems, thus enabling the latter to be applied to NCSs readily. From the literature to date, it is noticed that most of these works do not distinguish between the delay in either the sensor-to-controller or the controller-to-actuator channel. Indeed, to distinguish between these two delays is not absolutely necessary for the majority of the models used for NCSs in the literature, since they simply assume those two delays affect the system performance in the same way. However, on the basis of a recently reported packet-based control framework for NCSs, it is observed that the delays in different channels can affect the system performance in different ways. This observation thus implies the necessity of investigating the different delay effects in different channels in packet-based NCSs (PBNCSs), a topic neglected by most of the works available to date.

The study starts with identifying the specific control strategies that make those two delays different for the system. It is concluded that, it is not necessary to distinguish between these two delays for most conventional control strategies, whereas the delays in different channels do affect the system performance in different ways in most PBNCSs. Two reasons contribute to this difference, that is, the model-based controller design and the time-dependent feedback gains used in PBNCSs. On the basis of this observation, both theoretical analysis and numerical verification are conducted for the case where the model-based control is designed with a constant feedback gain. The analysis shows that the delay in the sensor-to-controller channel affects the system performance less than that in the controller-to-actuator channel, provided a sufficiently precise model can be obtained for the plant. This conclusion can be regarded as an important design principle in terms of the resource allocation and the control structure design in NCSs.

The remainder of the paper is organized as follows. The conditions when the delay effects in different channels are different is discussed in Section II. Based on the careful problem formulation, the difference is then analysed quantitatively in Section III, which is verified by a numerical example in Section IV. Section V concludes the paper and proposes some works to be done in the future.

II. PACKET-BASED CONTROL FOR NCSs: WHEN ARE THE DELAY EFFECTS DIFFERENT?

The following linear, nominal system is considered in this paper,

\[ x(k+1) = Ax(k) + Bu(k) \]  

(1)

where \( x \in \mathbb{R}^n \), \( u \in \mathbb{R}^m \), \( y \in \mathbb{R}^r \), \( A \in \mathbb{R}^{n \times n} \) and \( B \in \mathbb{R}^{n \times m} \). This system is assumed to be controlled over the communication network, with a system setting as illustrated in Fig. 1. From Fig. 1 it is seen that two delays exist in the considered NCS, i.e., the sensor-to-controller delay, \( \tau_{sc,k} \), and the controller-to-actuator delay, \( \tau_{ca,k} \), respectively. The
subscript \( k \) is used here to notify the fact that both delays are time varying, dependent on the current time, \( k \). In this paper the delays are assumed to be upper bounded, by \( \bar{\tau}_{sc} \), \( \bar{\tau}_{ca} \) and \( \bar{\tau} \) for the upper bounds of the sensor-to-controller delay, the controller-to-actuator delay and the round trip delay, respectively. It is immediately clear that \( \bar{\tau} = \bar{\tau}_{sc} + \bar{\tau}_{ca} \) and \( \tau_k = \bar{\tau}_{sc,k} + \bar{\tau}_{ca,k} \) where \( \tau_k \) is the round trip delay.

![Fig. 1. The block diagram of a networked control system.](image)

For the purpose of a clear presentation, in what follows the underlying idea of the packet-based control approach to NCSs is first briefly outlined. The controller obtained is then compared with conventional approaches to NCSs and the conditions when the delay effects in different channels are different are finally concluded.

A. Packet-based control for NCSs: The underlying idea

The essential idea of packet-based control for NCSs is to take advantage of the fact that the data in NCSs is transmitted in the form of data packets via the communication network and, the packet size (denoted by \( B_p \)) is usually relatively large compared with the data size required for encoding one single step of the control signal (denoted by \( B_c \)). More precisely, the following relationship is held for most NCSs,

\[
\bar{\tau} + 1 \leq \left[ \frac{B_p}{B_c} \right]
\]  

(2)

where \( \left[ \frac{B_p}{B_c} \right] \triangleq \max \{ \zeta | \zeta \leq \frac{B_p}{B_c}, \zeta \in \mathbb{N} \} \).

The relationship in (2) implies that a sequence of the forward control signals, or referred to as the “forward control sequence” (FCS), can be packed into one data packet and sent simultaneously to the actuator. The FCS is designed as follows if time-synchronization is unavailable between the sensor and the controller,

\[
U_N(k|k - \tau_{sc,k}) = [u(k - \tau_{sc,k}|k - \tau_{sc,k}) \ldots u(k - \tau_{sc,k} + \bar{\tau}|k - \tau_{sc,k})]
\]  

(3)

In the presence of time-synchronization, the above FCS can be shortened by discarding the clearly outdated control signals from the FCS in (3), as follows,

\[
U_S(k|k - \tau_{sc,k}) = [u(k|k - \tau_{sc,k}) \ldots u(k + \bar{\tau}_{ca}|k - \tau_{sc,k})]
\]  

(4)

The FCS in (4) can be obtained in such a way since the sensor-to-controller delay can be known by the controller in the presence of time-synchronization and with the use of time stamps for the data packets [13]. Notice that in this case the requirement in (2) can be relaxed by replacing \( \bar{\tau} \) by \( \bar{\tau}_{ca} \).

Notice also that in both (3) and (4) \( k \) refers to the time at the controller side.

By sending the FCSs simultaneously to the actuator and designing some auxiliary mechanisms to choose from them the appropriate control signals, the communication constraints in NCSs including network-induced delay, data packet dropout and data packet disorder, can then be actively compensated for. The block diagram of a general PBNCS is illustrated in Fig. 2. For more details of the packet-based control approach, the reader is referred to [13]–[16].

B. Different control laws for NCSs

In this subsection, the design details of PBNCSs and conventional approaches to NCSs are neglected. The focus is on the controllers they actually derive for the whole system. For simplicity, only the case of state feedback with exact state measurement is considered. In addition, unlike in the last subsection \( k \) in this subsection refers to the time at the actuator side.

First notice that for most conventional approaches, the control law can be written as

\[
u(k) = K \hat{x}(k|k - \tau_k)\]  

(5a)

where \( K \) is the constant feedback gain [17]–[19].

Unlike the control law in (5a), for PBNCSs the general control law is obtained as follows, whichever FCSs in (3) or (4) is used,

\[
u(k) = u(k|k - \tau_k)\]  

(5b)

where the reader is advised to refer to [13]–[16] for how (5b) is obtained. The control law in (5b) implies that although this control signal is based on delayed state information at \( k - \tau_k \), it is particularly designed for current time \( k \). This makes it an active compensation scheme for the communication constraints in NCSs and different from the control law in (5a) [13].

In the early development of PBNCSs [20], the controller is designed based on the model based control method, yielding the following control law

\[
u(k) = K \hat{x}(k|k - \tau_k)\]  

(5c)

where \( \hat{x}(k|k - \tau_k) \) is the estimated state at time \( k \) based on the state at time \( k - \tau_k \).

A recent development of PBNCSs gives rise to a more flexible structure of PBNCSs, where the controller design methods can be any that could result in a good system performance. The control law is obtained as follows with the use of the FCS in (3),

\[
u(k) = K_{\tau_k} x(k|k - \tau_k)\]  

(5d)

where the feedback gain \( K_{\tau_k} \) is dependent on the round trip delay \( \tau_k \). With the use of the FCS in (4), the control law is defined by

\[
u(k) = K_{\tau_{sc,k},\tau_{ca,k}} x(k|k - \tau_k)\]  

(5e)
where the feedback gain $K_{\tau_{sc,k},\tau_{ca,k}}$ is dependent on both the sensor-to-controller delay, $\tau_{sc,k}$, and the controller-to-actuator delay, $\tau_{ca,k}$.

The most important feature of the control laws in (5d) and (5e) is that the feedback gains are now time-varying and dependent on the current network condition. It thus brings more design freedom for the control engineers to compensate for the communication constraints in NCSs. A better system performance can also be expected [13].

C. When are the delay effects different?

The packet-based control approach derives different control laws for NCSs from conventional approaches. What is interested in these control laws is to check whether different delays in different channels can bring different effects to the system. For simplicity of analysis the effects here are specified by the control signal, $u(k)$, and the round trip delay, $\tau_k$ is assumed to be the same for all the control laws in (5). The question can then be stated as: given the same round trip delay, $\tau_k$, for all the control laws in (5), check if they give rise to different control signals in the presence of different sensor-to-controller and controller-to-actuator delays.

In fact, it can be readily found out that: 1) the control laws in (5a) and (5d) are totally based on the round trip delay, $\tau_k$; and 2) the control law in (5e) is explicitly dependent on both the sensor-to-controller and controller-to-actuator delays. Therefore, the answers to the above question would be: NO for control laws in (5a) and (5d) and YES for the control law in (5e).

The control law in (5c) is a bit complicated. By the analysis presented in the next section, it is realized that the control law in (5c) is designed based on a model of the plant. This model-based control law presents two factors that would affect the system performance, that is, the model inaccuracy and the error in the model prediction. The sensor-to-controller and the controller-to-actuator delays are related to these two factors in different ways, meaning that these two delays potentially present different delay effects for the system.

To sum up, under the same round trip delay, different delays in different channels do not affect the system performance in the presence of the control laws in (5a) and (5d); however the system performance can be different in the presence of the control laws in (5e) and (5c), due to the following reasons, respectively:

- Model inaccuracy and prediction error for the control law in (5c);
- Time-dependent feedback gains that are varying with different sensor-to-controller and controller-to-actuator delays for the control law in (5e).

Remark 1: As for the control law in (5c), it makes a difference in terms of the delay effects whenever a model-based controller is designed for the system. This means that besides the packet-based control approach, other model-based methods could also suffer from different delay effects for different sensor-to-control and controller-to-actuator delays, such as the approaches proposed in [21], [22]. On the other hand, the idea of using time-dependent feedback gains has been seen in other models used for NCSs [23], despite the missing of the practical design support. It is clear that the results obtained in what follows are also applicable to these models. Therefore, to a certain extent the problem considered in this paper is relatively universal in NCSs.

III. COMPARING THE DIFFERENT DELAY EFFECTS IN PBNCSs–A PRELIMINARY RESULT

Although the control structure of PBNCSs is now clear, the controller design methods within this framework can still be various, see (5c)-(5e). Particularly, the control law in (5c) itself can be various due to different predictive methods used to obtain $\hat{x}(k|k-\tau_k)$. Examples of this variance can be seen in [20] for a simple dynamics-based approach and in [13] for a receding horizon based approach. As a preliminary result, simple quantitative analysis is done for the case in [20] with the use of the control law in (5c). However, more work is still necessary to be done for other cases in the near future.

A. The dynamic-based approach in PBNCSs

To implement the control law in (5c), it is essential to obtain the predicted state $\hat{x}(k|k-\tau_k)$. This is done in [20] using a dynamic-based approach. The basic idea is to use a model for the plant at the controller side, which gives
the predicted states based on delayed state information. The model used can be written as

\[ \hat{x}(k + 1) = \hat{A}\hat{x}(k) + \hat{B}\hat{u}(k) \tag{6} \]

where \( \hat{A} \) and \( \hat{B} \) are not equivalent to \( A \) and \( B \) in general due to the modelling error. Furthermore, the control signals \( \hat{u}(k) \) may not be the same as the real ones, \( u(k) \), since the latter is usually not fully accessible to the controller.

For simplicity of notation in what follows let \( \alpha \triangleq \tau_{sc,k} \), \( \beta \triangleq \tau_{ca,k} \), and \( \tau \triangleq \tau_k \). Notice that \( k \) in (5c) is the time at the actuator side. The time for generating the FCS at the controller side is thus \( k - \beta \) and the FCS is calculated based on the delayed state information at time \( k - \tau \).

The dynamic-based approach estimates the state \( \hat{x}(k|\tau - \tau_k) \) based on the available delayed state \( \hat{x}(k - \tau) \), using the following two steps.

1) Estimate from \( \hat{x}(k - \tau + 1|k - \tau) \) to \( \hat{x}(k - \beta|k - \tau) \). In this step it is assumed that the real control signals applied to the plant from \( u(k - \tau) \) to \( u(k - \beta - 1) \) are available to the controller, that is, \( \hat{u}(k - \tau + i) = u(k - \tau + i), i = 0, 1, \ldots, \beta + 1 \). This assumption is realized to be difficult to be implemented in practice in [24] and a better approach is proposed to deal with this difficulty. However in this paper we keep this assumption unchanged for simplicity of analysis. Based on this assumption and the predictive model in (6), the dynamics of the predictive model can be written as

\[ \hat{x}(k - \tau + i|k - \tau) = \hat{A}\hat{x}(k - \tau + i - 1|k - \tau) + \hat{B}\hat{u}(k - \tau + i - 1), \quad i = 1, \ldots, \alpha \tag{7} \]

where \( \hat{x}(k - \tau|k - \tau) = x(k - \tau) \). This yields

\[ \hat{x}(k - \beta|k - \tau) = \hat{A}^\alpha\hat{x}(k - \tau) + \sum_{j=1}^{\alpha} \hat{A}^{\alpha-j}\hat{B}\hat{u}(k - \tau + j - 1) \tag{8} \]

2) Estimate from \( \hat{x}(k - \beta + 1|k - \tau) \) to \( \hat{x}(k|k - \tau) \). In this step the control signal is assumed to be given by \( \hat{u}(k - \beta + i) = u(k - \beta + i|k - \tau) = K\hat{x}(k - \beta + i|k - \tau) \), as the real ones are clearly not available. Based on this assumption, the predictive model in (6) turns to be

\[ \hat{x}(k - \beta + i|k - \tau) = (\hat{A} + BK)\hat{x}(k - \beta + i - 1|k - \tau), \quad i = 1, \ldots, \beta \tag{9} \]

which gives

\[ \hat{x}(k|k - \tau) = (\hat{A} + BK)^\beta\hat{x}(k - \beta|k - \tau) \tag{10} \]

B. Performance comparison

What we are interested in this study is to compare the different delay effects in different channels. This can be done by comparing the control inputs, or equivalently, in the case of state feedback with constant feedback gains, comparing the estimated state, \( \hat{x}(k|k - \tau) \) and the real one, \( x(k|k - \tau) = x(k) \).

By (1) \( x(k|k - \tau) \) is given by

\[ x(k|k - \tau) = A^\tau x(k) + \sum_{j=1}^{\tau} A^{\tau-j}Bu(k - \tau + j - 1) \tag{11} \]

which is based on the state at time \( k - \tau \).

Define \( e(k|k - \tau) \triangleq x(k|k - \tau) - \hat{x}(k|k - \tau) \). From (8), (10) and (11) \( e(k|k - \tau) \) can be explicitly expressed, which in general is a function of the sensor-to-actuator delay, \( \alpha \),

\[ e(k|k - \tau) = \Gamma_{\tau,K}(\alpha) \tag{12} \]

Although it is possible to investigate the explicit expression of \( e(k|k - \tau) \) in (12) directly, it is too complicated to derive any valuable results. As the main purpose of the paper is to study the effects in the presence of different delays in different channels, it is thus possible to study the effects indirectly from two different dynamics of \( e(k|k - \tau) \), based on (7) and (9). On the basis of this observation, the following result is obtained.

**Proposition 1:** With the use of the dynamic based control law in [20], the sensor-to-controller delay, \( \tau_{sc,k} \), affects the system performance less than the controller-to-actuator delay, \( \tau_{ca,k} \), provided the predictive model in (6) is sufficiently precise.

**Proof:** In order to demonstrate the above result, the error dynamics \( e(k|k - \tau) \) is analysed based on the aforementioned two steps in the dynamic based approach. From \( k - \tau \) to \( k - \beta \), the error dynamics is obtained as follows, based on (7) and (11),

\[ e_{\alpha}(i) \triangleq e(k - \tau + i|k - \tau) = (A - \hat{A})x(k - \tau + i - 1|k - \tau) + \hat{A}e(k - \tau + i - 1|k - \tau) + (B - \hat{B})u(k - \tau + i - 1), \quad i = 1, \ldots, \alpha \tag{13} \]

with \( e(k - \tau|k - \tau) = 0 \).

On the other hand, from \( k - \beta + 1 \) to \( k \), the error dynamics is obtained based on (9) and (11), as follows,

\[ e_{\beta}(i) \triangleq e(k - \beta + i|k - \tau) = (A - \hat{A} - BK)x(k - \beta + i - 1|k - \tau) + (\hat{A} + BK)e(k - \beta + i - 1|k - \tau) + Bu(k - \beta + i - 1), \quad i = 1, \ldots, \beta \tag{14} \]

It is noticed that the error \( e_{\alpha}(\cdot) \) is purely dependent on the sensor-to-controller delay, \( \alpha \), and is accumulated with the increase of \( \alpha \). On the other hand, although \( e_{\beta}(\cdot) \) is mainly affected by the controller-to-actuator delay, it is also affected
by the sensor-to-controller delay, since its initial state, \( e(k - \beta \mid k - \tau) \), is obtained in (13).

Now suppose we have an exact model of the plant, i.e., \( A = \hat{A}, B = \hat{B} \). It immediately follows that \( e_\alpha(i) \equiv 0, i = 1, \ldots, \alpha \), and in particular the initial state for (14), \( e(k - \beta \mid k - \tau) = e_\alpha(0) = 0 \). Therefore, in this case the sensor-to-controller delay does not affect the system performance at all. On the other hand, it is readily seen that \( e_\beta(i) \neq 0 \) in general and will accumulate with the increase of \( \beta \). Based on this observation, it is therefore fair to claim the statement made in this proposition.

Remark 2: Proposition 1 implies that, under certain conditions, it can result in a better system performance to place the controller as close to the actuator as possible, if the system allows us to do so. In this sense Proposition 1 has its practical guidance value. However, Proposition 1 is based on the nominal system and it could be wrong in the presence of fairly large model inaccuracy, measurement error, or any other type of uncertainties in the system. Indeed, as stated in the proof, the sensor-to-controller delay affects both \( e_\alpha(.) \) and \( e_\beta(.) \) while the controller-to-actuator delay affects only \( e_\beta(.) \). Therefore, if the system setting allows the sensor-to-controller delay to take effect, it is very likely that this delay could affect the system performance more severely than that of the controller-to-actuator delay. This implies that Proposition 1 has its rigid conditions of applicability and is far from a general rule for all NCSs.

IV. NUMERICAL EXAMPLE

Consider the system in (1) with the following system matrices borrowed from [20],

\[
A = \begin{pmatrix} 1.010 & 0.271 & -0.488 \\ 0.482 & 0.100 & 0.240 \\ 0.002 & 0.3681 & 0.7070 \end{pmatrix}, \quad B = \begin{pmatrix} 5 & 5 \\ 3 & -2 \\ 5 & 4 \end{pmatrix}
\]

As in [20], the initial state is set as \( x_0 = [0.1 \ 0.1 \ 0.1]^T \) and the constant feedback gain is given by

\[
K = \begin{pmatrix} 0.5858 & -0.1347 & -0.4543 \\ -0.5550 & 0.0461 & 0.4721 \end{pmatrix}
\]

Different from the system setting in [20], it is assumed that the system states of the above system can be obtained exactly and therefore the measurement system and the observer are not necessary. The control signal is assumed to be zero before the arrival of the first FCS. In addition, in order to focus on the delay effects in different channels, the delays are all set to be time-invariant.

The simulations of the above system prove the observation in this paper. Under the same round trip delay, \( \bar{\tau} = 3 \), Fig. 3 shows that the system is stable with \( \tau_{ca} = 1 \) while unstable with \( \tau_{ca} = 2 \). This proves the result in Proposition 1, that is, the smaller the controller-to-actuator delay is, the better the system performance will be. Further examples can be seen in Figs. 4 and 5. With \( \tau_{ca} = 1 \) and \( \tau_{sc} = 1 \) respectively, the system is stable even with \( \tau_{sc} = 12 \) while only stable for \( \tau_{ca} < 2 \). This clearly shows that the sensor-to-controller delay has a less negative effect on the system performance.

In order to simulate the delay effects in the presence of the modeling error, a particular case is shown in Fig. 6, where the inaccurate system matrices are defined as \( \hat{A} = (1 + \epsilon)A \) and \( \hat{B} = (1 - \epsilon)B \) with \( \epsilon = 0.16 \). For this particular case it shows that the sensor-to-actuator delay could affect the system performance more severely. This proves the statement made in Remark 2. However, it is worth pointing out that with inaccurate models, the sensor-to-actuator delay could still be possible to affect the system performance more lightly. This implies that with the modeling error in presence, the delay effects in different channels are complicated and no general results exist.

![Fig. 3. State responses with different delays in different channels.](image)

![Fig. 4. State responses with the same controller-to-actuator delay.](image)

V. CONCLUSION AND FUTURE WORK

A packet-based control framework is proposed for NCSs in recent years. Within this framework, the controllers are designed with explicate compensation for the communication constraints in NCSs. Consequently, it is observed that different delays in different channels can affect the system
performance in different ways. Preliminary analysis of this phenomenon is conducted based on an early development of the framework. The obtained results show that under certain conditions, the sensor-to-actuator delay can affect the system performance less than the controller-to-actuator delay. This result has its practical guidance value.

The obtained results are preliminary. More rigid mathematical analysis of general packet-based control approaches, as shown in both (5c) and (5e), is still to be done in the near future.

REFERENCES


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Fig. 5. State responses with the same sensor-to-controller delay.

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Fig. 6. State responses in the presence of modeling error.