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Ray-optical negative refraction and pseudoscopic imaging with Dove-prism arrays

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Abstract. A sheet consisting of an array of small, aligned Dove prisms can locally (on the scale of the width of the prisms) invert one component of the ray direction. A sandwich of two such Dove-prism sheets that inverts both transverse components of the ray direction is a ray-optical approximation to the interface between two media with refractive indices $+n$ and $-n$. We demonstrate the simulated imaging properties of such a Dove-prism-sheet sandwich, including a demonstration of pseudoscopic imaging.

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1. Introduction

Negative refraction is the unusual bending of light that does not normally occur in nature². The concept was first discussed by Veselago [2], who noticed that materials with negative permittivity and permeability possess a negative refractive index. Such materials have been recently built in the form of metamaterials [3]–[5]—resonant electromagnetic structures periodic on a scale below the wavelength, where they act as a homogeneous optical medium. This has revived interest in negative refraction, leading for example to the ray-tracing visualization of objects with negative refractive index [6].

Ray-optical components such as lenses can also be miniaturized and arranged periodically. We consider here simple combinations of such periodic arrangements. To be clear, these are *not* metamaterials; they affect passing light waves very much like inhomogeneous media. However, they can affect light rays like homogeneous media. In this sense, they can be considered to be *ray-optical* metamaterials.

Negative refraction has already been realized ray-optically in the form of lenslet arrays: pairs of lenslet arrays with a common focal plane bend light rays like the interface between optical materials with refractive indices $+n$ and $-n$. These have been realized in the form of standard [7] and GRIN lenslet arrays [8], and their three-dimensional (3D) imaging properties, including pseudoscopic imaging, have been examined.

We investigate here another way of achieving ray-optical negative refraction, which uses combinations of miniaturized Dove prisms. Our combinations of Dove prisms consist of two periodic Dove-prism arrays, which we call Dove-prism sheets, whereby one sheet is rotated with respect to the other by 90° . Our Dove-prism-sheet sandwiches work differently from the lenslet arrays described above: whereas the lenslet arrays work by forming an intermediate image, our Dove-prism-sheet sandwiches work by successively inverting the ray vector's x - and y -components.

This work is mainly driven by curiosity and the desire to work towards ‘experiencing’ the optics of negative refraction on a macroscopic scale. However, our approach is of additional interest because it can be generalized to rotation angles between the Dove-prism sheets other than 90° , resulting in optical sheets that rotate the *local* ray direction through an arbitrary, but fixed, angle around the sheet normal, which is unprecedented. We will investigate this in future papers.

2. Dove prisms and negative refraction

The basic building block of a Dove-prism sheet is a Dove prism. With the coordinate system chosen as in figure 1, a Dove prism inverts the y -direction of any transmitted light ray. It also offsets the rays, whereby the offset is on the scale of the prism diameter. We are considering here the limit of small Dove prisms, so small in fact that we can ignore this offset. Clearly, wave-optically this limit breaks down as the prism diameter reaches the wavelength of the light. Acceptable compromises for visual purposes could be prism diameters of between $10\ \mu\text{m}$ and $1\ \text{mm}$.

² One example is known of negative refraction occurring in unusual materials under special conditions for very specific wavelengths [1].

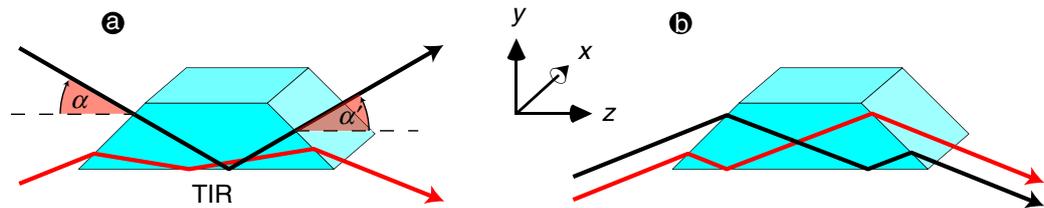


Figure 1. Ray optics of Dove prisms. A Dove prism orientated as shown in (a) inverts the y component of the direction of individual light rays in a light beam passing through it. This happens during total internal reflection (TIR) from the prism's bottom surface. This corresponds to an inversion of the angle the light ray has with respect to the prism's bottom surface: $\alpha = -\alpha'$, where α and α' , respectively, are the angles before and after passage through the prism, as shown in (a). The prism also offsets the y position of each ray; in (b), the prism swaps the positions of the red and black rays.

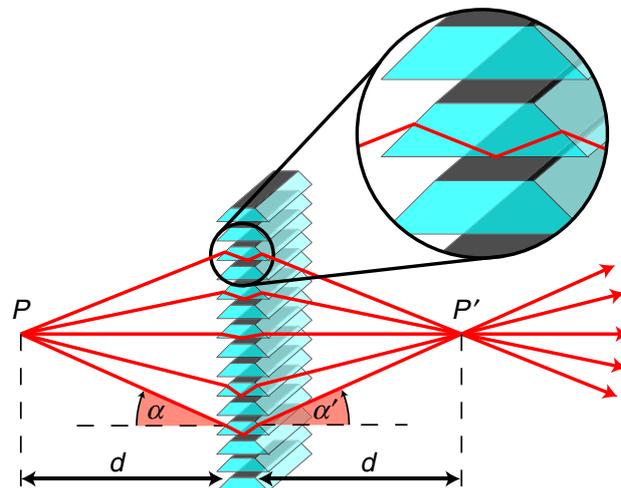


Figure 2. Structure and ray-optics of a Dove-prism sheet. The inversion of the angle with respect to the prism's bottom surface (figure 1) now becomes an inversion of the angle with respect to the Dove-prism sheet normal. In the limit of small prism apertures, the sheet does not significantly offset light rays (red). A point light source, P , placed at a distance d in front of the sheet, creates light rays that intersect again in a point P' the same distance d behind the sheet.

Dove prisms that are stretched in the x -direction (again with the choice of coordinate system shown in figure 1) and stacked on top of each other form a Dove-prism sheet (figure 2). Note that the prisms need to be separated by a few wavelengths to ensure that total internal reflection at the long side (see figure 1(a)) is not frustrated.

The ray optics of such a sheet are simple: in the limit of small Dove prisms the sheet flips the y -direction of individual light rays in a beam passing through it. This implies that for light rays incident in a plane parallel to the (y, z) -plane, the angles of incidence, α_1 , and refraction, α_2 ,

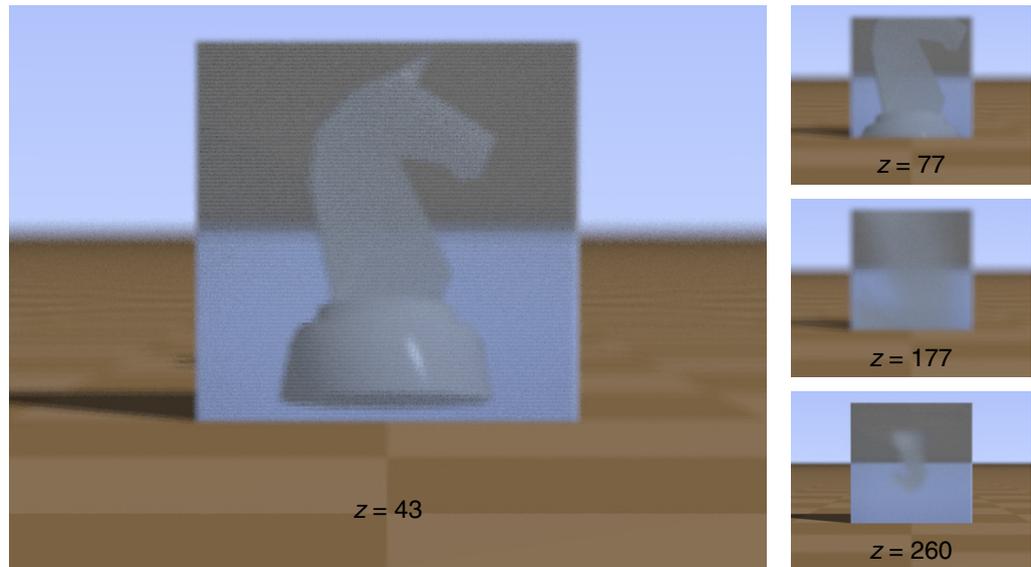


Figure 3. Relationship between object and image distance for crossed Dove-prism sheets. A chess piece—the object—is positioned at a distance z behind the sheets; the crossed Dove-prism sheets image it to a position at a distance z *in front of* the sheets. The different frames show the image of the chess piece for various object distances; the sheets and the camera are stationary. In the first ($z = 43$) and second ($z = 77$) frames the image becomes larger and larger as it moves towards the camera, positioned at a distance of 120 units in front of the sheets. The image then moves through the camera plane and behind it, where it re-appears upside-down and getting smaller. In the first two frames, $z = 43$ and $z = 77$, the camera is focused on to the image of the chess piece; its image can be gleaned by inspection of the position of the focus on the chequered floor, which has a square length of 20 units. In the second two frames simple focusing is not possible as the chess piece is behind the camera, which is roughly focused on to the sheets. The frames are from a movie (MPEG-4, 256 KB, available from stacks.iop.org/NJP/10/023028/mmedia) calculated by performing ray tracing through the detailed prism-sheet structure, using the freely-available software POV-Ray [10].

are related through the equation

$$\alpha_1 = -\alpha_2. \quad (1)$$

It is particularly interesting to combine a Dove-prism sheet with another, parallel, Dove-prism sheet that is rotated around the z -direction through 90° , and which therefore flips the x -direction of light rays passing through it. Such Dove-prism-sheet sandwiches then flip both transverse ray directions (x and y), and invert the angle of incidence for any plane of incidence. When the two crossed Dove-prism sheets are close together, they lead to no additional ray offset. They therefore act like the interface between two optical media with equal and opposite refractive indices, $+n$ and $-n$: Snell's law, written for this situation, states that

$$n \sin(\alpha_1) = -n \sin(\alpha_2), \quad (2)$$

which (provided that $-90^\circ \leq \alpha_1, \alpha_2 \leq +90^\circ$) is equivalent to equation (1).

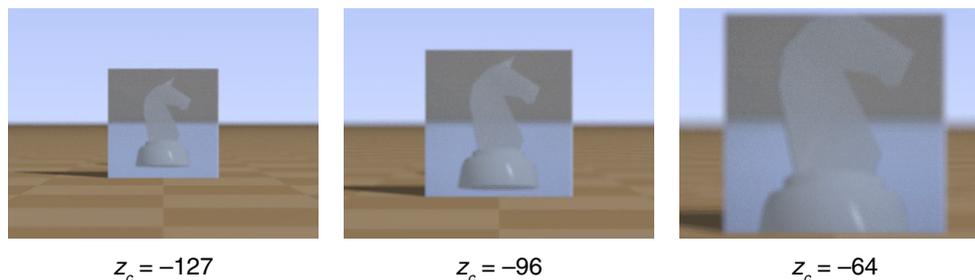


Figure 4. Image of a chess piece created by crossed Dove-prism sheets as seen from different distances. The sheets are in the $z = 0$ plane; the image is at $z = -40$ (that is 40 units in front of the sheet); z_c is the position of the camera. From left to right, the frames show the simulated view as seen with a camera moving closer to the Dove-prism sheets; both the sheets and the chess piece are stationary. Because the distance between camera and image is less than that between camera and sheets, a decrease in both distances by the same absolute amount, that is moving the camera in the direction of image and sheets, decreases the distance to the image by a larger factor than that to the sheets. This means that the angle under which the image of the chess piece is seen grows more than the angle under which the sheets are seen. The frames are from a POV-Ray [10] movie (MPEG-4, 204 KB, available from stacks.iop.org/NJP/10/023028/mmedia).

3. Pseudoscopic imaging

Images produced by single lenses are orthoscopic: if two objects at longitudinal positions z_1 and z_2 are imaged into positions z'_1 and z'_2 , and if the first object is in front of the second, i.e. if $z_1 < z_2$, then the image of the first object will be in front of the image of the second, so $z'_1 < z'_2$. The opposite is true in pseudoscopic imaging [9], where the image of the second object is in front of that of the first, so $z'_1 > z'_2$.

The effect of the inversion of the angle of incidence by crossed Dove-prism sheets is to image any object a distance d behind the sheets to the same distance in front of the sheets (figure 2). In other words, if the longitudinal coordinate z is chosen such that the sheets are at $z = 0$, then an object distance z corresponds to an image distance $-z$. For the two longitudinal object positions with $z_1 < z_2$ discussed above this results in image positions $z'_{1,2} = -z_{1,2}$, and therefore the inverted relationship between the longitudinal image positions $z'_1 > z'_2$. Crossed Dove-prism sheets therefore produce pseudoscopic images.

Figures 3 and 4 demonstrate this pseudoscopic imaging with ray-tracing simulations performed using the software POV-Ray [10]. Both figures visualize imaging of a chess piece through crossed Dove-prism sheets, each comprising 200 Dove prisms. In figure 3 the distance of the chess piece behind this Dove-prism-sheet sandwich is varied; in figure 4 the distance of the (simulated) camera from the sheet sandwich is varied.

The inversion of the z -coordinate during imaging implies that crossed Dove-prism sheets produce pseudoscopic images. Figure 5 demonstrates various properties of these pseudoscopic images. Specifically, it shows that pseudoscopic images appear to be ‘inside out’; the pseudoscopic image of a convex chess piece, for example, is concave. When looking at this image from different directions, the image appears to have rotated, just like the hollow face

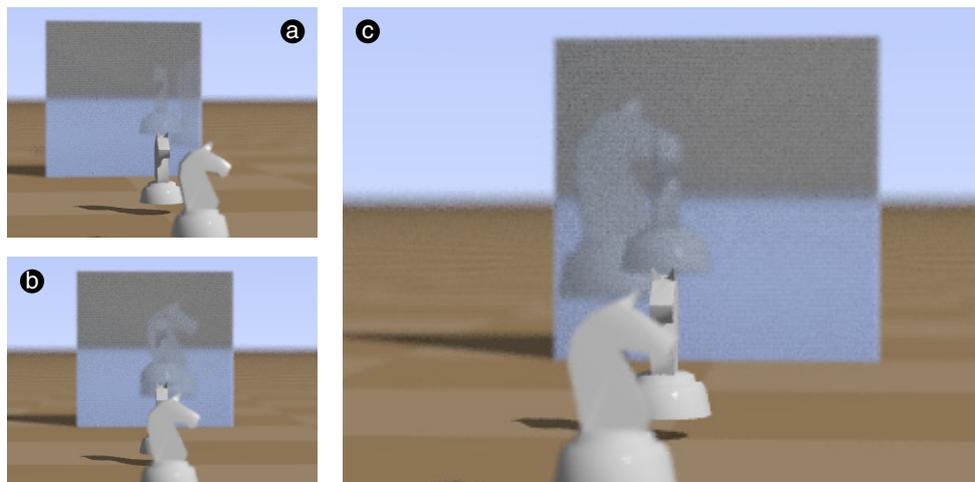


Figure 5. Orthoscopic objects and pseudoscopic images. The frames show two chess pieces in front of crossed Dove-prism sheets, and the images of two chess pieces behind the sheets, as seen from three different viewing positions. The pieces are arranged such that one image is at the same distance as one of the chess pieces in front of the sheet, the other image is at the same distance as the other piece in front of the sheet. This can be seen by one chess piece always being below one image, independent of viewing angle, which means they are always undergoing the same parallax, which in turn implies that they are at the same distance from the camera. However, while the left side of the front piece is visible from the left-most viewing point (a) and the right side from the right-most viewing point (c), the opposite is true for the pseudoscopic images. Also, while the piece in front (which, of course, appears bigger) obscures the piece behind it, the image in front (again the bigger image) is *obscured* by the image behind it. The frames are from a movie (MPEG-4, 848 KB, available from stacks.iop.org/NJP/10/023028/mmedia) calculated by performing ray tracing through the detailed prism-sheet structure, using the freely-available software POV-Ray [10].

mask in the famous hollow-face (or ‘Bust of the Tyrant’) illusion [11]. In the case of the chess piece shown in figure 5, looking at the pseudoscopic image of one of the chess pieces placed behind the Dove-prism sheets from the left lets us see the right side of the chess piece, not the left side, as is the case with the chess piece placed in the same longitudinal position for comparison. Figure 4 in [11] shows the same effects in the hollow-face illusion.

Figure 5 also demonstrates another striking property of pseudoscopic images. If two objects are placed behind one another, the object in front obscures the object behind. In the pseudoscopic images of two objects placed behind one another, the image *behind* obscures the image in front.

4. Conclusions and future work

Transferring a basic idea from metamaterials-research—miniaturization and repetition of interesting electro-magnetic components—to ray optics, we have investigated the effect of

miniaturizing and repeating an optical component with interesting ray-optical properties, the Dove prism. Using ray-tracing simulations, we have demonstrated that the resulting Dove-prism sheets can ray-optically act like the interface between optical media with refractive indices of the same magnitude but opposite sign. We have also demonstrated some of the unusual properties of their pseudoscopic imaging.

We are currently generalizing the ideas from this paper, for example varying the rotation angle between the sheets. We are also investigating the possibility of building Dove-prism sheets about the size of an A4 piece of paper. Such sheets would be suitable for demonstration experiments and allow the optics of negative refractive indices to be ‘experienced’.

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