A Nonlinear Disturbance Observer for Robotic Manipulators

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Abstract—A new nonlinear disturbance observer (NDO) for robotic manipulators is derived in this paper. The global exponential stability of the proposed disturbance observer (DO) is guaranteed by selecting design parameters, which depend on the maximum velocity and physical parameters of robotic manipulators. This new observer overcomes the disadvantages of existing DO’s, which are designed or analyzed by linear system techniques. It can be applied in robotic manipulators for various purposes such as friction compensation, independent joint control, sensorless torque control, and fault diagnosis. The performance of the proposed observer is demonstrated by the friction estimation and compensation for a two-link robotic manipulator. Both simulation and experimental results show the NDO works well.

Index Terms—Friction, nonlinear estimation, observers, robots.

I. INTRODUCTION

DISTURBANCE observers (DO’s) have been used in robotic manipulator control for a long time. In general, the main objective of the use of DO’s is to deduce external unknown or uncertain disturbance torques without the use of an additional sensor. The use of DO’s in robotic manipulators can be divided into the following categories.

1) Independent joint control is widely used in industrial robots where a multilink manipulator is divided into several independent links with linear dynamics. The performance of these kind of controllers can be improved by the use of a DO. This is accomplished by considering the coupling torques from other links as an unknown external disturbance and using a DO to estimate and compensate for it [1]. This technique has also been extended to deal with parameter variations and unmodeled dynamics whereby it improves the robustness of robots [2];

2) Friction is a common phenomenon in mechanical systems and plays an important role in system performance. Many friction models and compensation methods have been proposed [3]. One of the most promising methods is observer-based control where a DO is used to estimate friction [4];

3) DO’s have been used in robotic manipulators for force feedback and hybrid position/force control where the DO works as a torque sensor [5]–[7]. In this case, it is supposed that the friction and other dynamics are well modeled and compensated for. The use of a DO, rather than several torque sensors (for each link, at least one torque sensor is required), simplifies the structure of the system, reduces the cost, and improves the reliability. With this technique, sensorless torque control can be implemented [5]–[7];

4) Robotic manipulators work in a dynamic highly uncertain environment. In this application, DO’s provide signals for monitoring and trajectory planning rather than for control. For example, in robotic assembly when the component is misinserted, the reaction torque/force is greatly increased and could damage the robotic manipulator. A DO can estimate the reaction torque online and transmit this information to the monitoring or planning level. Chan [8] uses a DO in electronic component assembly, while Ohishi and Ohde [9] give an example of the use of a DO in collision. Although the DO technique has been widely used in robotic manipulator control for various purposes, in almost all cases, the analysis or design is based on linearized models or using linear system techniques. Since a multilink robotic manipulator is a highly nonlinear and coupled system, the validity of using linear analysis and synthesis techniques may be doubtful. Many important properties of existing DO’s have not been established, e.g., unbiased estimation or even global stability. There are, however, some recent results using nonlinear disturbance observers (NDO’s). A variable structure DO has been proposed [10] and a nonlinear observer for a special kind of friction, i.e., Coulomb friction, has been proposed by Friedland and Park [11]. This nonlinear observer is designed based on the model of Coulomb friction, and the global convergence ability has been shown. It has been further modified and implemented on robotic manipulators by Tafazoli et al. [12]. However, a specific model of friction will not be used in this paper, and the whole design is based on the DO concept. That is, similar to other unknown torques, the friction is considered as a disturbance on the control torque.

It should be stressed that the DO rather than a velocity observer of a manipulator is considered in this paper. A velocity observer has been considered by many authors. Bona and Indri have compared and implemented several linear and nonlinear velocity observers on a robotic manipulator [13].
An NDO for multilink robotic manipulators will be presented in this paper. By carefully selecting the observer gain function, it will be shown that global convergence is guaranteed. This result is based on Lyapunov theory.

II. PROBLEM FORMULATION AND A BASIC OBSERVER

A. Problem Formulation

For the sake of simplicity, a two-link robotic manipulator is considered in this paper. The main idea is readily extended to the more general case. The model of a two-link robotic manipulator can be represented by

\[ J(\dot{\theta})\ddot{\theta} + G(\theta, \dot{\theta}) = T + d \]

where \( \theta \in \mathbb{R}^2 \), \( \dot{\theta} \in \mathbb{R}^2 \), and \( T(t) \in \mathbb{R}^2 \) are displacement, velocity, and control vectors, respectively. When only the dynamics of the links are considered in the model, the control input \( T \) is either the torque or force. \( d \in \mathbb{R}^2 \) is a disturbance torque or force vector. It should be noted that \( d \) has different meanings in different observer applications. For example, it can be friction in friction compensation, reaction torque or force in force control, and unmodeled dynamics in independent joint control. Since a general observer will be derived in this paper, all of them are considered as "disturbances." When the first-order dynamics of dc motors are included in the above model, \( d \) is the voltage vector imposed on the motors instead of the torque vector. As a result, the torque disturbance is also equivalent to the disturbance on the voltage on the motors. Hence, \( d \) is the disturbance voltage in this case.

The objective of this paper is to design an observer such that the estimation \( \hat{d} \) yielded by the observer exponentially approaches the disturbance \( d \) under any \( \theta(t) \), \( \dot{\theta}(t) \), and \( t \in [t_0, \infty) \).

B. Initial Observer

A basic idea in the design of observers/estimators is to modify the estimation by the difference between the estimated output and the actual output. Since (1) can be written as

\[ \dot{d} = J(\theta)\ddot{\theta} + G(\theta, \dot{\theta}) - T \]

a DO is proposed as

\[ \dot{\hat{d}} = -L(\theta, \dot{\theta})\hat{d} + L(\theta, \dot{\theta})(J(\theta)\ddot{\theta} + G(\theta, \dot{\theta}) - T) \]

(3)

Since, in general, there is no prior information about the derivative of the disturbance \( d \), it is reasonable to suppose that

\[ \dot{\hat{d}} = 0 \]

(4)

which implies that the disturbance varies slowly relative to the observer dynamics. However, it will be illustrated by simulation and experiment that the observer developed in this paper can also track some fast time-varying disturbances.

Define the observer error

\[ e(t) = d - \hat{d}. \]

Combining (4) with the observer (3) yields

\[ \dot{e} = \dot{d} - \dot{\hat{d}} = L(\theta, \dot{\theta})\ddot{d} - L(\theta, \dot{\theta}) d. \]

(6)

That is, the observer error is governed by

\[ \dot{e} + L(\theta, \dot{\theta})e = 0. \]

(7)

It can be shown that the observer is globally asymptotically stable by choosing

\[ L(\theta, \dot{\theta}) = \text{diag}\{c, c\} \]

(8)

where \( c > 0 \). More specifically, the exponential convergence rate can be specified by choosing \( c \).

III. NDO

The acceleration signal \( \ddot{\theta} \) is not available in many robotic manipulators, and it is also difficult to construct the acceleration signal from the velocity signal by differentiation due to measurement noise. However, although the observer (3) is not practical to implement, it provides a basis for the further nonlinear observer design.

A. Modified Observer

Define an auxiliary variable vector

\[ z = \dot{\hat{d}} - p(\theta, \dot{\theta}) \]

(9)

where \( z \in \mathbb{R}^2 \). The designed function vector \( p(\theta, \dot{\theta}) \) is to be determined.

Let the function \( L(\theta, \dot{\theta}) \) in (3) be given by the following nonlinear equation:

\[ L(\theta, \dot{\theta}) = \left[ \frac{\partial p(\theta, \dot{\theta})}{\partial \theta} \frac{\partial p(\theta, \dot{\theta})}{\partial \dot{\theta}} \right] \begin{bmatrix} \dot{\theta} \\ \dot{\theta} \end{bmatrix}. \]

(10)

Invoking (9) and (10) with (3) yields

\[ \dot{z} = \dot{\hat{d}} - \frac{\partial p(\theta, \dot{\theta})}{\partial \theta} \frac{\partial p(\theta, \dot{\theta})}{\partial \dot{\theta}} \begin{bmatrix} \dot{\theta} \\ \dot{\theta} \end{bmatrix} = -L(\theta, \dot{\theta}) \left( z + p(\theta, \dot{\theta}) \right) + L(\theta, \dot{\theta}) \left( G(\theta, \dot{\theta}) - T \right). \]

(11)

Hence, the NDO is given by

\[ \dot{z} = -L(\theta, \dot{\theta}) z + L(\theta, \dot{\theta}) \left( G(\theta, \dot{\theta}) - T - p(\theta, \dot{\theta}) \right) \]

(12)

and

\[ \dot{\hat{d}} = z + p(\theta, \dot{\theta}) \]

(13)

where \( L(\theta, \dot{\theta}) \) is given by (10).

B. Stability of the NDO

It follows from (5), and (11)–(13) that the observer error equation is given by

\[ \dot{e} = \dot{d} - \dot{\hat{d}} = -z = -\frac{\partial p(\theta, \dot{\theta})}{\partial \theta} = -L(\theta, \dot{\theta}) e. \]

(14)
The estimation \( \hat{d} \) approaches the disturbance \( d \) if \( L(\theta, \hat{\theta}) \) is chosen such that (14) is asymptotically stable. Hence, the function \( p(\theta, \hat{\theta}) \) must be selected such that the function \( L(\theta, \hat{\theta}) \) given by (10) satisfies this condition. In general, it is not easy to select such a nonlinear function \( p(\theta, \hat{\theta}) \) especially for multi-variable systems. However, for a multilink robotic manipulator, with the help of its characteristics, a systematic method for selecting the nonlinear function \( p(\theta, \hat{\theta}) \), such that the observer with \( L(\theta, \hat{\theta}) \) given by (10) is asymptotically stable, is developed in this paper.

The inertial matrix \( J(\hat{\theta}) \) for a two-link manipulator is given by [14]

\[
J(\hat{\theta}) = \begin{bmatrix}
    j_1 + 2X \cos(\theta_2) & j_2 + X \cos(\theta_2) \\
    j_2 + X \cos(\theta_2) & j_3
\end{bmatrix}
\]  

where \( j_1, j_2, j_3, \) and \( X \) are inertial parameters, which depend on the masses of the links, motors and tip load, and the lengths of the links.

**Theorem:** For the two-link robotic manipulator (1), when the function \( p(\theta, \hat{\theta}) \) in the observer (12) and (13) is chosen as

\[
p(\theta, \hat{\theta}) = c \begin{bmatrix}
    \hat{\theta}_1 \\
    \hat{\theta}_2
\end{bmatrix}
\]  

and \( c \) satisfies

\[
c > X \dot{\theta}_{2m}
\]  

where \( \dot{\theta}_{2m} \) denotes the maximum velocity of the second link, then the observer (12) and (13) is globally asymptotically stable.

**Proof:** Since \( p(\theta, \hat{\theta}) \) is given by (16), it yields

\[
\frac{d}{dt}p(\theta, \hat{\theta}) = \begin{bmatrix}
    \frac{dp(\theta, \hat{\theta})}{\partial \theta} & \frac{dp(\theta, \hat{\theta})}{\partial \hat{\theta}}
\end{bmatrix} \begin{bmatrix}
    \dot{\theta} \\
    \dot{\hat{\theta}}
\end{bmatrix} = c \begin{bmatrix}
    1 & 0 \\
    1 & 1
\end{bmatrix} \dot{\theta}.
\]  

Combining the above equation with (10) gives

\[
L(\theta, \dot{\theta}) = c \begin{bmatrix}
    1 & 0 \\
    1 & 1
\end{bmatrix} J(\theta)^{-1}
\]  

since \( J(\theta) \) is positive definite for all \( \theta \) and \( \dot{\theta} \) and, therefore, invertible.

From (15), \( J(\theta) \) can be written as

\[
J(\theta) = \begin{bmatrix}
    1 & 1 \\
    0 & 1
\end{bmatrix} \mathcal{J}(\theta) \begin{bmatrix}
    1 & 0 \\
    1 & 1
\end{bmatrix}
\]  

where

\[
\mathcal{J}(\theta) = \begin{bmatrix}
    j_1 + 2j_2 + j_3 & j_2 + j_3 + X \cos(\theta_2) \\
    j_2 + j_3 + X \cos(\theta_2) & j_3
\end{bmatrix}.
\]  

Hence,

\[
L(\theta) = c \mathcal{J}(\theta)^{-1} \begin{bmatrix}
    1 & -1 \\
    0 & 1
\end{bmatrix}.
\]  

Since \( \mathcal{J}(\theta) \) is also positive definite for all \( \theta \), a Lyapunov function candidate for the observer (12) and (13) can be chosen as

\[
V(c, \theta) = c^2 \mathcal{J}(\theta) c.
\]  

Differentiating the Lyapunov function with respect to time \( t \) along the observer trajectory gives

\[
\frac{dV(c, \theta)}{dt} = \frac{\partial V(c, \theta)}{\partial c} \dot{c} + \frac{\partial V(c, \theta)}{\partial \theta} \dot{\theta}
\]

\[
= -c \mathcal{J}^T \begin{bmatrix}
    1 & -1 \\
    0 & 1
\end{bmatrix} \begin{bmatrix}
    1 & 0 \\
    -1 & 1
\end{bmatrix} c
\]

\[
+ c \mathcal{J} \begin{bmatrix}
    0 & -X \dot{\theta}_2 \sin(\theta_2) \\
    -X \dot{\theta}_2 \sin(\theta_2) & 0
\end{bmatrix} c
\]

\[
= -c \mathcal{J} \begin{bmatrix}
    2c & -c + X \dot{\theta}_2 \sin(\theta_2) \\
    -c + X \dot{\theta}_2 \sin(\theta_2) & 2c
\end{bmatrix} c.
\]  

Hence, \( (dV(c, \theta)/dt) < 0 \) for all \( c \) and \( \theta, \dot{\theta} \) if

\[
\begin{bmatrix}
    2c & -c + X \dot{\theta}_2 \sin(\theta_2) \\
    -c + X \dot{\theta}_2 \sin(\theta_2) & 2c
\end{bmatrix} > 0.
\]  

That is,

\[
(3c - X \dot{\theta}_2 \sin(\theta_2))(c + X \dot{\theta}_2 \sin(\theta_2)) > 0.
\]  

Since \( c \) satisfies (17), the above inequality is met. The equilibrium point \( (c = 0) \) is then globally asymptotically stable.

**C. Convergence Rate of the NDO**

It can be shown the minimum eigenvalue of the matrix in (25) for all \( \theta \) is given by

\[
q_1 = c - X \dot{\theta}_{2m}.
\]  

Thus,

\[
\frac{dV(\theta, c)}{dt} \leq -q_1 ||q||^2.
\]  

Let \( q_2 \) denote the maximum eigenvalue of the matrix \( \mathcal{J}(\theta) \) for all \( \theta \). It then follows from (24) that

\[
V(t) \leq V(t_0) e^{-q_1(t_0/T_0)}.
\]  

The speed of convergence is bounded by \( q_1/q_2 \). For a robotic manipulator, \( q_2 \) and the prescribed maximum velocity are fixed. By choosing the parameter \( c \), the desired exponential convergence rate can be achieved.

**IV. SIMULATION AND EXPERIMENTAL RESULTS**

The proposed NDO is tested in this section. As stated in Section I, the NDO can be used in robotic manipulators for various purposes. In what follows, the NDO is designed as a friction observer. That is, the NDO is used to estimate the friction for a two-link robotic manipulator. The reason for designing a friction observer here is that friction varies rapidly, even discontinuously. It is a challenging task in observer design. The simulation and experiment are based on a two-link manipulator with...
dc motor actuators. The dynamic model of the manipulator including the first-order dc motor dynamics is governed by (1) and is detailed in [15].

A. Friction Simulation

The friction considered is Coulomb and Viscous friction, given by

\[ d(\dot{\theta}) = z \text{ sign}(\dot{\theta}) + k\dot{\theta}. \]  

(30)

The parameters for first and second links in the simulation are given by

\[ z = 0.0541 \text{ N} \cdot \text{m} \]

\[ k = 0.00676 \text{ N} \cdot \text{m/rad/s} \]  

(31)

and

\[ z = 0.0176 \text{ N} \cdot \text{m} \]

\[ k = 0.0088 \text{ N} \cdot \text{m/rad/s} \]  

(32)

respectively.

There are some problems in using the friction model (30) in simulation directly. One is due to the discontinuity of the friction characteristics at zero velocity—a very small step size is required for testing zero velocity. The other is that when the velocity is zero, or the system is stationary, the friction is indefinite and depends on the controlled torque. In the simulation, to improve the numerical efficiency, a revised friction model, which is modified from [16], is adopted. The structure for the revised friction model is given by Fig. 1. It can be described by the following mathematical model:

\[ d_a(\dot{\theta}) = d + (T_a - d) e^{-c(\dot{\theta})^2} \]  

(33)

where \( d \) is given by (30), \( d_a \) is the revised friction and \( I \) is a small positive scalar, and \( T_a \) is given by

\[ T_a = \begin{cases} 
Z, & T > Z \\
T, & -Z \leq T \leq Z \\
Z, & T < -Z \end{cases} \]  

(34)

When the velocity is within a very small area near zero, defined by \( I \), the friction \( d_a \) is equal to the applied torque \( T \). When the velocity is greater than this, the second term in the above expression vanishes and the friction \( d_a \) given by this revised model is equal to the friction given by (30). To compare the accuracy of the revised friction model (33), the friction defined by the classic Coulomb and viscous friction model (30) and the revised model (33) for first link is shown in Fig. 2. In this figure, \( I \) is chosen as 0.001. The frictions given by these two models are almost indistinguishable. However, experience has shown that using the revised model greatly improves the computational efficiency for simulation.

B. Simulation Results

A controller is designed by computed torque control where the disturbance is not taken into account. When square-wave command references are given for first and second links, respectively, the velocity and friction histories are shown in Figs. 3 and 4. It can be seen that the friction varies very rapidly with the velocity.

A friction observer is designed by the NDO technique developed in this paper. The design parameter \( c \) is chosen as 50. The estimation given by the observer (12) and (13) is shown by Figs. 5 and 6. The observer exhibits excellent tracking performance. It successfully estimates the friction on-line and no friction model is required.
C. Experimental Results

The proposed NDO is implemented on a two-link robotic manipulator in the laboratory. The experimental results for a computed torque controller with and without the NDO are compared. The control structure, which combines a computed torque controller with the NDO, is shown in Fig. 7, where the effect of the friction is compensated for by the outputs of the NDO via feedforward. With the NDO, the parameter $c$ is chosen as one.

Figs. 8–11 show the results of computed torque control with and without the NDO, in each case showing both the first and second links. Fig. 8 shows the reference signals (dotted–dashed) and the position tracking performance for both the computed torque control without the NDO (dashed) and with the NDO (solid line).

The friction estimated by the NDO is plotted in Fig. 9. When the friction of the robot is compensated by outputs of the NDO, under the same controller and reference signal as used for the case without the NDO, the tracking performance is shown in Fig. 8 by a solid line. Compared with the tracking performance of the computed torque controller without the NDO, the performance is greatly improved using the friction estimation and compensation and the steady tracking error disappears. The outputs of the tachometers are also shown in Fig. 10 and the input voltages to the motors are shown in Fig. 11. It should be mentioned that, due to the measurement noise in tachometers, as in the computed torque controller, the velocity signals are filtered before being used in the NDO.
V. Conclusion

This paper has presented a procedure for the design of an NDO for robotic manipulators. Following the procedure presented in this paper, a condition for convergence was established. The speed of the convergence can be specified by the design parameters. The proposed observer was tested by simulation and experiment. Even though the theory is developed for constant disturbances, it was shown that, for a rapid time-varying signal like friction, the observer exhibits satisfactory performance. By feedforward compensation of the estimated friction yielded by the DO, the performance, in particular, the steady-state performance, was significantly improved. The NDO proposed in this paper can also be applied in independent joint control, sensorless torque control, and fault diagnosis in robotics.

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References

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