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Deposited on: 22 June 2010
Multi-objective Optimisation on Motorised Momentum Exchange Tether for Payload Orbital Transfer

Yi Chen, Matthew P. Cartmell

Abstract—The symmetrical motorised momentum exchange tether, is intended to be excited by a continuous torque, so that, it can be applied as an orbital transfer system. The motor drive accelerates the tether, and increases the relative velocity of payloads fitted to each end. In order to access better tether performance, a higher efficiency index needs to be achieved. Meanwhile, the stress in each tether sub-span should stay within the stress limitations. The multi-objective optimisation methods of Genetic Algorithms can be applied for tether performance enhancement. The tether’s efficiency index and stress are used as multi-objectives, and the analysis of the resulting Pareto front suggests a set of solutions for the parameters of the motorised momentum exchange tether when used for payload transfer, in order to achieve relative high transfer performance, and safe tether strength.

I. INTRODUCTION

The symmetrical motorised momentum exchange tether (MMET) was first proposed by Cartmell in 1998 [1]. A conceptual schematic for the motorised momentum exchange tether is shown in Fig. 1. A more developed MMET model was discussed further by Ziegler and Cartmell [2][3], which was motorised by a motor driver and used angular generalised co-ordinates to represent spin and tilt, together with an angular co-ordinate for circular orbital motion, and a further angular co-ordinate defining back-spin of the propulsion motor’s stator components. The payload masses (`payload mass #1’ and ‘payload mass #2’ in Fig.1) are fitted to each end of the tether sub-span, and the system orbits a source of gravity in space, the use of the tether means that all parts of the system have the same angular velocity as the overall centre of mass (COM). As shown in Fig. 1, the symmetrical double-ended motorized spinning tether can be applied as orbital transfer system, in order to exploit momentum exchange to propel and transfer payloads.

Tethered payloads, orbiting a source of gravity in space, possess the same orbital angular velocity as the overall COM. As the tethered system’s acceleration caused by the motor builds up about the COM, eventually the tangential velocity of the payloads reaches the required level and the payloads are released onto a desired tangential path. As the upper payload is released from a spinning tether, it always aligned along the gravity vector, and the upper payload carries more angular velocity than it requires to stay on that circular orbit, but because the upper payload does not have enough energy to escape the Earth’s gravity, the upper payload goes into an elliptical orbit with the release point being the perigee of the orbit, as shown in Fig. 2. In this way, the upper payload can be transferred from low Earth orbit (LEO) to geostationary Earth orbit (GEO). The lower payload is still attached to each remaining free end of the tether. Similarly, the lower payload does not have enough velocity to stay on its circular orbit when it is released, and so it too goes into an elliptical orbit but this time with the release point being the apogee of the orbit. Half an orbit later the upper payload reaches its apogee and is hence further from the Earth than it was at the point of release. Upon reaching the perigee of the orbit, the lower payload is closer to the Earth than it was at release. Thus, the upper and lower masses released from a spinning tether are respectively raised and lowered.

When the tether with a payload at each end is undergoing spin-up, libration or rotating about the COM, the resulting stress will be generated in each tether sub-span. The stress of the tether should be kept within the stress limit of tether.

Most problems in industrial applications may have several (or possibly conflicting) objectives to be satisfied. It is rare that there is a single point that simultaneously optimizes all the objective functions. Therefore, the solutions of ‘trade-off’ need to be found, rather than single solutions when dealing with multi-objective optimisation problems. To maximise the payloads transfer’s performance (apogee altitude gain and perigee altitude loss [2][4]) for the most desirable tether motion, meanwhile, to minimize tether stress, are a case of a multi-objective problem with several objectives which
need to be compromised. Genetic Algorithms are widely applied into appropriating solutions for the multi-objective optimization problems as a set of efficient tools. So, the solutions study for multi-objective problems of tether payload transfer and tether stress will be carried on by multi-objective methods of Genetic Algorithms.

Genetic Algorithms (GAs) are global, parallel, stochastic search methods, founded on Darwinian evolutionary principles, by Holland, 1975 [5]. GAs work with a population of potential solutions to a problem, and each individual within the population represents a specific solution to the problem, which is generally expressed as some form of genetic code. Since then, GAs have been frequently applied as optimisers for many engineering applications. Practical problems are often characterised by several non-commensurable and often competing measures of performance, or objectives, with a number of restrictions imposed on the decision variable. Trade-offs exist between some objectives, where advancement in one objective will cause deterioration in another. It is very rare for problems to have a single solution. These problems usually have no unique or perfect solution, but a set of non-dominated, alternative solutions, known as the Pareto-optimal set. The notion of Pareto-optimality is only a first step towards solving a multi-objective optimisation problem. The choice of a suitable compromise solution from all non-inferior alternatives is not only problem-dependent, it generally depends also on the subjective preferences of a decision agent, the Decision Maker (DM). Thus, the final solution to the problem is the result of both an optimisation process and a decision process.

The Multiple Objective Genetic Algorithms (MOGA) was proposed by Fonseca and Fleming in 1993 [6], it is an algorithm that applies Pareto ranking and sharing on fitness objects’ values. In this algorithm, an individual’s rank corresponds to the number of individuals in the current population by which it is dominated. Nondominated individuals are assigned as the same rank, while dominated ones are penalized according to population density in the corresponding region of the trade-off surface. Fitness is assigned by interpolation, e.g., for a linear fitness function, from the best to the worst individuals in the population, assigned values are averaged data between individuals with the same rank.

The Niched-Pareto Genetic Algorithm (NPGA) is proposed by Horn et al. [7]. It uses a tournament selection scheme based on Pareto dominance. Two individuals randomly chosen are compared against a subset from the entire population. When both competitors are either dominated or nondominated, the result of the tournament is decided through fitness sharing in the objective domain (a technique called equivalent class sharing is used in this case).

The Nondominated Sorting Genetic Algorithm (NSGA) is proposed by Srinivas and Deb [8]. It is based on several layers of classifications of the individuals. Nondominated individuals get a certain dummy fitness value and then are removed from the population. The process is repeated until the entire population has been classified. To maintain the diversity of the population, classified individuals are shared with their dummy fitness values.

The Nondominated Sorting Genetic Algorithm II (NSGA-II) is proposed by Deb et al. [9], it is a new version of the Nondominated Sorting Genetic Algorithm (NSGA), which is more efficient (computationally speaking), it uses elitism and a crowded comparison operator that keeps diversity without specifying any additional parameters.

Optimal performance according to one objective if such an optimum exists, often implies unacceptably low performance in one or more of the other objective dimensions, creating the need for a compromise to be reached. The 4 multi-objective GAs methods of MOGA, NPGA, NSGA and NSGAII will be applied into MMET payload transfer optimisations, which is to search for a set of suitable solutions involving 3 conflicting objectives below, and offer acceptable performance in all objective dimensions.

II. CONCEPT OF MMET PAYLOAD TRANSFER

As shown in Fig. 2, when a symmetrical motorised momentum exchange tether is in LEO, the upper payload can be released from a spinning tether, which is always aligned along the local gravity vector. It will carry more angular velocity than it requires to stay on that circular orbit, but it does not contain enough energy to escape away from the Earth’s gravity. The upper payload will go into an elliptical orbit with the release point being the perigee of the orbit in Fig. 2. Half an orbit later the upper payload reaches its apogee and is further from the Earth than it was at the point of release. Similarly, the lower payload does not have enough
energy to stay on its circular orbit when it is released, and it goes into another elliptical orbit with the release point being the apogee of that orbit. On reaching the perigee of its elliptical orbit, the lower payload is closer to the Earth than it was at release. In so doing, the upper and lower payload masses are released from a spinning tether, and raised and lowered, respectively.

TABLE I
TETHER PARAMETERS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>tether cross-sectional area</td>
<td>62.83×10⁻⁵ m²</td>
</tr>
<tr>
<td>ρ</td>
<td>tether density</td>
<td>970 kg m⁻³</td>
</tr>
<tr>
<td>T,U</td>
<td>kinetic and potential energy</td>
<td>J</td>
</tr>
<tr>
<td>Mₘ, Mₚ</td>
<td>motor and payload mass</td>
<td>5000,1000 kg</td>
</tr>
<tr>
<td>Δₚₑ</td>
<td>tether performance index</td>
<td>m</td>
</tr>
<tr>
<td>Δₚₑ/L</td>
<td>efficiency index</td>
<td>3.9877848</td>
</tr>
<tr>
<td>Δₚₑ_upper/lower</td>
<td>upper/lower payload efficiency index</td>
<td>50000 m</td>
</tr>
<tr>
<td>L</td>
<td>tether length</td>
<td>m</td>
</tr>
<tr>
<td>l</td>
<td>tether length from COM a point along the tether</td>
<td>0.01 rad/s</td>
</tr>
<tr>
<td>ψ</td>
<td>angular pitch velocity</td>
<td>0.01 rad/s</td>
</tr>
<tr>
<td>θ</td>
<td>angular orbital velocity</td>
<td>√μ/μₜ rad/s</td>
</tr>
<tr>
<td>rₑ</td>
<td>LEO circular orbit radius of COM at payload release</td>
<td>6890 m</td>
</tr>
<tr>
<td>σ₀</td>
<td>axial stress at the end of the tether where it connects to the payload</td>
<td>GPa</td>
</tr>
<tr>
<td>σ</td>
<td>stress</td>
<td>GPa</td>
</tr>
<tr>
<td>σₐₗₜₘ</td>
<td>stress limit</td>
<td>5 GPa</td>
</tr>
<tr>
<td>μ</td>
<td>gravitational constant</td>
<td>3.9877848×10¹⁴</td>
</tr>
<tr>
<td>e</td>
<td>eccentricity</td>
<td>0</td>
</tr>
</tbody>
</table>

\[
\frac{\Delta rₚ_upper}{L} = \frac{\frac{(rₑ + L)^2}{2(rₑ - L)}[(rₑ + L)θ + Lθ]}{L} - rₑ
\]

\[
\frac{\Delta rₚ_lower}{L} = \frac{\frac{(rₑ - L)^2}{2(rₑ - L)}[(rₑ - L)θ - Lθ]}{L} - rₑ
\]

The radial separation of Δₚₑ, which shown in Fig. 2, gives the distance, half an orbit after the tether releases the payload, between the payload and the facility’s orbit at the time of release. It describes how well the tether performance at transferring the payload, and is defined as the tether’s performance index [2]. Δₚₑ/L is defined as the tether’s efficiency index. For the upper payload, it releases at its perigee point, a larger apogee altitude gain of Δₚₑ_upper/L shows better tether transfer performance, given in equation (1). For the lower payload, released at its apogee point, a smaller perigee altitude loss of Δₚₑ_lower/L also shows better tether transfer performance, given in equation (2).

Assuming \( L < rₑ \) and \( L/rₑ < < 1 \), and applying the binomial expansion to equations (1) and (2), they can be re-written as equations (3) and (4), and Table 1 is the parameter list for MMET payload transfer.

\[
\frac{\Delta rₚ_upper}{L} = \frac{7 + 30L}{rₑ} + \frac{4ψ}{θ} + \frac{2Lrₑψ}{μ} \left( 18θ + 5ψ \right)
\]

\[
\frac{\Delta rₚ_lower}{L} = -7 + \frac{30L}{rₑ} + \frac{4ψ}{θ} + \frac{2Lrₑψ}{μ} \left( 18θ + 5ψ \right)
\]

III. TETHER STRENGTH CRITERION

The symmetrical motorised momentum exchange tether with a payload at one end will experience centripetal acceleration when rotating about the facility. A resulting tensile force and stress is generated by the rotation. If it is assumed that the gravity gradient effects are negligible, the stress in the tether can be given in equation (5)[2], where, \( l \) is the variable of tether length, \( l \in [0, L] \). The equations (5) and (6) are showing the symmetrical stress distribution within symmetrical tether sub-spans, and Fig. 3 [4] gives the stress distribution within the tether.

\[
\sigma = \frac{1}{A} \psi>L \left( \frac{M_P + 1/2 ρAL}{\psi} \right)
\]

\[
\sigma_0 = \sigma|_{l=L} = \frac{1}{A} \psi>2 L \left( \frac{M_P + 1/2 ρAL}{\psi} \right)
\]

As can be seen from Fig. 3, equations (5) and (6), the max stress point is where the payloads connect to the tether, and the max stress of the full tether sub-span is given by equation (6). The max tether stress of \( \sigma_0 \) should stay within the safe stress limit of the tether, i.e., \( \sigma_0 < \sigma_{LIM} \).

IV. PAYLOAD TRANSFER MULTI-OBJECTIVE DEFINITIONS

The MMET payload transfer and design strength problems are defined as 3 conflicting objectives which need to be compromised in the optimisation study, and the 4 multi-objective methods of MOGA, NPGA, NSGA and NSGAI are used as the design optimizers. Then, The practical multi-objective problems of MMET payload transfer can be defined as follows:
A. Tether strength

\[ \text{Min } y_1 = \sigma_0 \] (7)

B. Upper payload transfer

\[ \text{Max } y_2 = \frac{\Delta r_{\text{upper}}}{L} \] (8)

C. Lower payload transfer

\[ \text{Min } y_3 = \frac{\Delta r_{\text{lower}}}{L} \] (9)

Subject to:

\[ \sigma_0 \leq \sigma_{\text{LIM}} \] (10)

If the GAs are used as the optimisers for the practical problems, the practical objectives need to be converted into proper fitness functions for GAs. For the MMET payload transfer application, the MAX fitness functions can be defined as following equations (11), (12) and (13), which are transformed from equations (7), (8) and (9), respectively.

\[ f_1 (\dot{\psi}, L, A) = \sigma_{\text{LIM}} - \sigma_0 \] (11)

\[ f_2 (\dot{\psi}, L) = \frac{\Delta r_{\text{upper}}}{L} \] (12)

\[ f_3 (\dot{\psi}, L) = \frac{1}{L} + \frac{\Delta r_{\text{lower}}}{L} + \varepsilon \] (13)

where, \( \varepsilon \) is a small value parameter, which helps to avoid \( f_3 \to \infty \), and \( \varepsilon \to 0 \).

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Value</th>
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<tbody>
<tr>
<td></td>
<td>max generations</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>crossover probability</td>
<td>0.8</td>
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<tr>
<td></td>
<td>mutation probability</td>
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<td></td>
<td>population</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>selection operator</td>
<td>tournament</td>
</tr>
<tr>
<td></td>
<td>crossover operator</td>
<td>single point</td>
</tr>
<tr>
<td></td>
<td>mutation operator</td>
<td>single point</td>
</tr>
<tr>
<td></td>
<td>encoding method</td>
<td>binary</td>
</tr>
<tr>
<td>( x_1 )</td>
<td>( \psi )</td>
<td>([1 \times 10^{-5}, 1])</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>( L )</td>
<td>([1 \times 10^{3}, 2.5 \times 10^{5}])</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>( A )</td>
<td>([5 \times 10^{-4}, 5 \times 10^{-1}])</td>
</tr>
<tr>
<td>( y_1 )</td>
<td>tether stress object function</td>
<td>Eq(7)</td>
</tr>
<tr>
<td>( y_2 )</td>
<td>upper efficiency index object function</td>
<td>Eq(8)</td>
</tr>
<tr>
<td>( y_3 )</td>
<td>lower efficiency index object function</td>
<td>Eq(9)</td>
</tr>
<tr>
<td>( f_1 )</td>
<td>fitness function 1</td>
<td>Eq(11)</td>
</tr>
<tr>
<td>( f_2 )</td>
<td>fitness function 2</td>
<td>Eq(12)</td>
</tr>
<tr>
<td>( f_3 )</td>
<td>fitness function 3</td>
<td>Eq(13)</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>a small value parameter, which helps to avoid ( f_3 \to \infty ).</td>
<td>eps:the floating point relative accuracy</td>
</tr>
</tbody>
</table>

V. RESULTS

A. Simulation

Table II is the GAs parameter setting for the MMET payload transfer multi-objective optimisation simulation. The Simple Genetic Algorithms Laboratory (SGALAB) toolbox for MATLAB is the GAs simulation tool for MMET payload transfer optimisation.

Figs. 4, 5, 6 and 7 are the diagrams of the mean value of fitness function 1, 2 and 3 within max generation for the methods of MOGA, NPGA, NSGA and NSGAII, respectively. Performance information is shown in these figures, all the fitness lines reach a steady state in high convergent rates. The convergent generations for the 4 multi-objective methods are about 10 for the MOGA, 4 for the NPGA, 4 for the NSGA and 4 for the NSGAII, and this means, the MOGA need more time or generations to reach convergence and it shows its work efficiency is lower than the other multi-objective methods, but still get similar results. The GAs optimizer starts from a random initial values for each variables of \( \dot{\psi} \), \( L \) and \( A \) within corresponding specific variable ranges in Table II. This leads to a random initial fitness values. Unlike single objective problem, and the multi-objective GAs reach the convergent state by compromising each fitness functions, and fitness lines perform ups and downs in Figs. 4, 5, 6 and 7.

Fig. 4 is the mean value of fitness diagram for MOGA, the convergent point of generation is about 10, and then fitness lines stay in a steady state. Within the 1 to 10 generation, there are some local fluctuations for fitness function 1, 2 and 3, these local fluctuations are the MOGA’s stochastic searching outputs.

Fig. 4. Fitness function 1,2,3 vs. generation by MOGA

Figs. 5, 6, 7 are the mean value of fitness diagrams for the NPGA, NSGA and NSGAII, respectively, and the convergent point of the generation is about 4. Then fitness lines stay in a steady state without any local fluctuations, this means, for the current 3 objective problems of MMET
payload transfer, NPGA, NSGA and NSGAII can converge to a higher performance than the MOGA.

For 'trade-off', solutions need to be provided for the 3 multi-objective MMET payload transfer problems, rather than single point solutions that can optimize all the objectives simultaneously. The notion of 'optimality' is defined as Pareto-optimal[], if there exists no feasible decision variables which would increase some criterion without causing a simultaneous decrease in at least one other criterion. And, this concept almost gives not one single solution, but a set of solutions of 'Pareto optimal'. The decision variables corresponding to the solutions included in the Pareto-optimal set are so-called non-dominated solutions. The diagram of the Pareto-optimal set under the multi-objective functions is so-called Pareto-front. The Pareto-front can also show the performance information of GAs multi-objective methods, it provides a way of evolving a specific region of the search space, and this allows the decision maker to focus on a region of the Pareto-front by GAs. Fig. 8, 9 and 10 are the Pareto-fronts of 3 MMET payload transfer fitness functions for the MOGA, NPGA, NSGA and NSGAII, they are fitness 1 vs. fitness 2, fitness 1 vs. fitness 3, and fitness 2 vs. fitness 3, respectively.

Fig. 8 shows the 4 multi-objective methods of MOGA, NPGA, NSGA and NSGAII have searched all the regions within 'Fitness 1 → Max' and 'Fitness 2 → Max' axes, with searching policy of 'trade-off' between max of fitness 1 and max of fitness 2. The marker of the MOGA data is '.' point, it is scattered over a larger area than other multi-objective methods, and will cost the MOGA more searching time, and the fluctuations in Fig.4 also shows this situation. The 4 methods can reach similar regions of solutions, the position with higher scatter data density means the Pareto optimal of 'trade-off' solutions are near to the 'Fitness 1 → Max' axis direction.

Fig. 9 and Fig. 10 are the similar Pareto-fronts for the 4 multi-objective methods, they have searched all the regions within 'Fitness 1 → Max' and 'Fitness 3 → Max' axes with searching policy of 'trade-off' between max of fitness 1 and max of fitness 2, and, 'Fitness 2 → Max' and 'Fitness 3 → Max' axes with searching policy of 'trade-off' between max of fitness 2 and max of fitness 3, respectively. The 4 methods can reach similar region of solutions, the position with higher scatter data density means the Pareto optimal of 'trade-off' solutions are near to the 'Fitness 1 → Max' axis direction in Fig. 9, and there are 3 higher density areas in Fig. 10, and there are more suitable 'trade-off' which can be reached between fitness 2 and fitness 3.

The Pareto-fronts are showing the corresponding relations for the 3 fitness functions, and their evolution tendencies of 3 fitness functions. The 4 multi-objective methods can show high performance in the evolution process, and a stable dataset can be reached quickly, then the Pareto-fronts suggest comparable solutions for MMET payload transfer applications within a safe stress design criterion.
B. Comparison with the existing design

Table III shows a comparison of fitness values for simulation results and existing design, the simulation fitness values are from Table II, and current existing design fitness values are from Table I. The simulation fitness value of its decision vectors which are not dominated by any current exits fitness values of its decision vector, they can be a set of Pareto-optimal. And, there may be a special interest in finding or approximating the Pareto-optimal set mainly to gain deeper insight into the problem and knowledge about alternate solutions, respectively. According to practical MMET applications, if there are more specific criterions are given, the practical results from Pareto dataset of solutions can be provided.

There are some small differences of fitness values between GAs results and existing design in Table III, this helps to validate the current existing design, it works in a practical solution of optimisation. Otherwise, the practical existing solution also help to prove the practical and applicable performance of multi-objective GAs methods, and they can be applied into further more complicated MMET payload transfer designs.

VI. CONCLUSION AND FUTURE WORK

The multi-objective optimisation on MMET payload transfer problems by GAs has been performed. The conflicting problems were to maximize tether’s upper and lower efficiency indexes, and to minimize tether stress or within the tether strength limit. By converting the 3 practical design criterions into fitness functions for GAs, 4 multi-objective methods were applied into the optimisation study.

The 4 multi-objective methods of MOGA, NPGA, NSGA and NSGAII showed flexible optimisation ability on the MMET practical objectives, and the Pareto dataset suggests a series of solutions for MMET designing for the strength criterion and payload transfer performance. The simulation example helps to validate the current MMET payload transfer design, and more specific results can be obtained by adding more specific or practical conditions for each MMET application.

The extreme Pareto solutions are found to be physically reasonable and the centres of the Pareto fronts give a good compromise. The results confirm the feasibility of the multi-objective approaches for MMET tether payload transfer.
optimization.
Given the advantage of multi-objective GAs, they can also be applied as follows:
- more multi-objective GAs methods will be applied into the current MMET payload transfer study.
- some optimisation studies for multiple orbit MMET payload transfers.
- intelligent real-time controller design for MMET payload transfer or navigation.
- dynamical system modelling for MMET.

ACKNOWLEDGMENT
The authors would like to acknowledge the support provided to the first author by the Overseas Research Students Awards Scheme and the Scholarship awarded by the University of Glasgow’s Faculty of Engineering.

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