Direct measurement of the skew angle of the Poynting vector in a helically phased beam

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Abstract: We measure the local skew angle of the Poynting vector within a helically-phased, $\exp(il\phi)$, beam using a Shack Hartmann wavefront sensor. It is the skew angle of the Poynting vector with respect to the beam axis that gives rise to the orbital angular momentum of a light beam. We confirm that this skew angle is l/kr, corresponding to an orbital angular momentum of $l\hbar$ per photon. Measurement of orbital angular momentum in this way is an alternative to interferometric techniques giving a non-ambiguous result to both the magnitude and sign of l from a single measurement, without any restriction on the optical bandwidth.

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1. Introduction

Beams with helical phase fronts, characterized by an azimuthal phase term $\exp(il\phi)$ possess an orbital angular momentum of $l\hbar$ per photon [1], where ϕ is the azimuthal angle and l is an integer. Unlike spin angular momentum that is linked to circular polarization, and ultimately to the photon spin, the orbital angular momentum is solely a function of the form of the optical phase front. This azimuthal phase structure can be studied with an interferometer, where the azimuthal phase term results in the characteristic spiral interference pattern with $|l|$ radial fringes [2]. The l-fold rotational symmetry can also be utilized within a Mach Zhender interferometer to sort an input beam between two or more outputs depending on the value of $|l|$ [3]. Obviously interferometry requires sub-wavelength experimental precision and eliminating the ambiguity associated with the sign of the gradient of the wavefront strictly requires multiple interferograms and phase stepping techniques. This ambiguity is particularly pertinent to beams with a helical phase since the number of fringes depends only upon the modulus of the orbital angular momentum $|l|$. Determining the sign of the orbital angular momentum, clockwise +ve l or anticlockwise –ve l requires further measurements.

In this paper we report the use of a Shack Hartmann wavefront sensor [4] to measure directly the inclination of the wavefronts confirming the radial dependence of the skew angle and giving both the magnitude and sign of l. Rapid, simple and non-ambiguous determination of the sign of l and azimuthal direction of the momentum is particularly important in optical tweezing experiments [5] especially those involving the transfer of light's angular momentum [6].

2. The Poynting vector in helically phased beams

Within the paraxial approximation, any linearly polarized, helically-phased beam of complex amplitude $u(r,\phi,z) = u(r,z)\exp(il\phi)$, of which Laguerre-Gaussian and high order Bessel beams are both examples, has r-, ϕ- and z-components of linear momentum density, $\mathbf{p} = \varepsilon_0 \mathbf{E} \times \mathbf{B}$, given by [1] (where $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic field strengths respectively, and ε_0 is the dielectric permittivity),

$$p_r = \varepsilon_0 \frac{\omega k r z}{\left(z_R^2 + z^2\right)} |u|^2, \quad p_\phi = \varepsilon_0 \left[\frac{\omega l}{r} |u|^2 \right] \quad \text{and} \quad p_z = \varepsilon_0 \omega k |u|^2 .$$

where ω and k are the angular frequency and the wavenumber of the light and z_r is the Rayleigh range of the Gaussian beam. For a well-collimated beam, $p_r \approx 0$, and p_ϕ / p_z gives the skew angle, γ, of the Poynting vector with respect to the beam axis to be $\gamma = l/kr$ [7]. For $l = 1$, $\lambda = 632\text{nm}$ and $r = 1\text{mm}$, γ is only 0.1 milliradians! Resolving the linear momentum of the photon $\hbar k$, into its corresponding azimuthal and axial components and multiplying by the radius, r, is compatible with the orbital angular momentum around the beam axis being $l\hbar$ per photon. The skew angle of the Poynting vector gives rise to an azimuthal shift of the beam behind an linear obstruction [8] and/or a shift in the inference pattern produced by Young's double slits [9]. However, rather than relying on these subtle effects or inverting potentially ambiguous interferometric data, the Shack Hartmann wavefront sensor can be used to measure the skew angle of the Poynting vector directly.

3. Generation of helically phased beams and the Shack Hartmann wavefront sensor

The helically phased beams investigated in this study were generated using a spatial light modulator (SLM) [10] configured as a diffractive optical element. When programmed with a l−forked diffraction grating [11], the first-order diffracted beam has helical phase fronts described by $\exp(il\phi)$. The Laguerre-Gaussian (LG) modes are a convenient basis set from which to describe beams with helical phase fronts, and are given by [12]

$$u_{pl}^{LG} = \frac{C_{pl}^{LG}}{w(z)}\left(\frac{r\sqrt{2}}{w(z)}\right)^{|l|} \exp\left[-\frac{r^2}{w^2(z)}\right] L_p^{|l|}$$

$$\exp\left(-\frac{ikr^2 z}{2\left(z^2+z_R^2\right)}\right)\exp(-il\phi)\exp\left(i\left(2p+|l|+1\right)\tan^{-1}\frac{z}{z_R}\right),$$

where C_{pl}^{LG} is the normalisation constant; $L_p^{|l|}\left(\frac{2r^2}{w^2(z)}\right)$ is a generalised Laguerre polynomial;

$w(z)$ is the radius of the beam at position z, where $w(z)^2 = \frac{2}{k}\frac{z_R^2+z^2}{z_R}$; $\left(2p+|l|+1\right)\tan^{-1}\frac{z}{z_R}$

is the Gouy phase. p and l are mode indices, where l corresponds to the azimuthal phase terms and p is the number of radial nodes.

A Shack Hartmann wavefront sensor comprises an array of lenses such that an incident plane wave results in a corresponding array of spots focused on to the plane of a detector array. Transverse displacement of any of these spots corresponds to a local inclination of the wavefront [4]. Within this work we use lenses with a 70mm focal length arranged on a square array with a pitch of 300µm. A local wavefront inclination of 0.1 milliradian gives a displacement of the focused spot by 7 microns, comparable to the spot diameter. The software used for measuring the spot displacement can determine the relative spot position to 700nm giving a precision for measuring the local direction of the Poynting vector of 0.01 milliradians.

Figure 1(a) shows the experimental arrangement of the HeNe laser, beam expander to ensure illumination of the SLM aperture, and the Shack Hartmann wavefront sensor. The SLM is a computer controlled diffractive optical element that can be used to adjust the phase of the reflected light. A *4f* imaging system (where *f* is the focal length of one of the lenses) was used to image the plane of the SLM and control the size of the beam incident on the lenslet array. In order to generate pure *LG* beams, it was necessary to control both the intensity and the phase of the incident light. This was achieved by adjusting the local contrast on the SLM to shape the intensity of the light beam accordingly [13, 14]. Figure 1(b) shows an example of such a computer-generated hologram.
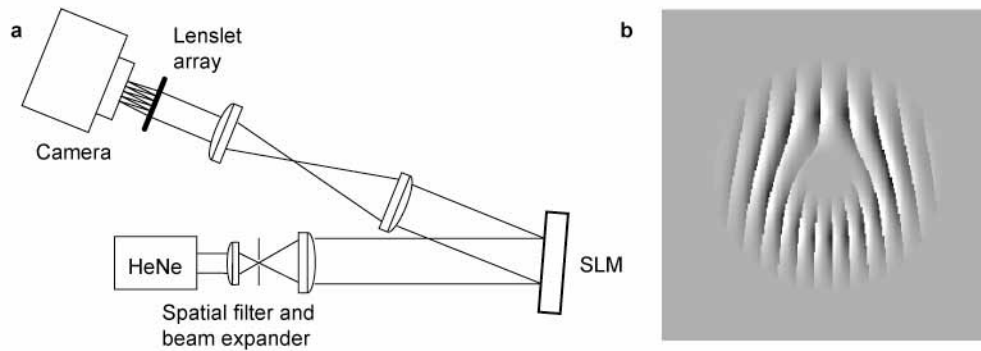
Fig. 1. (a) Experimental configuration measuring the local inclination of the Poynting vector of a beam produced with a spatial light modulator using a Shack Hartman wave front sensor. (b) Sample of a forked grating used to create a LG beam. The example shown was used to create an $l = 5$ beam.

4. Results

Figure 2 (and associated movie file) shows a series of vector fields illustrating the measured skew angle of the Poynting vector for *LG* modes with *l* values ranging between +5 and -5. The length and direction of each vector arrow corresponds to the movement of the spot produced by a single lenslet within the Shack Hartman array. Where the intensity was too low to make an accurate measurement of the spot position, the arrow has been omitted.
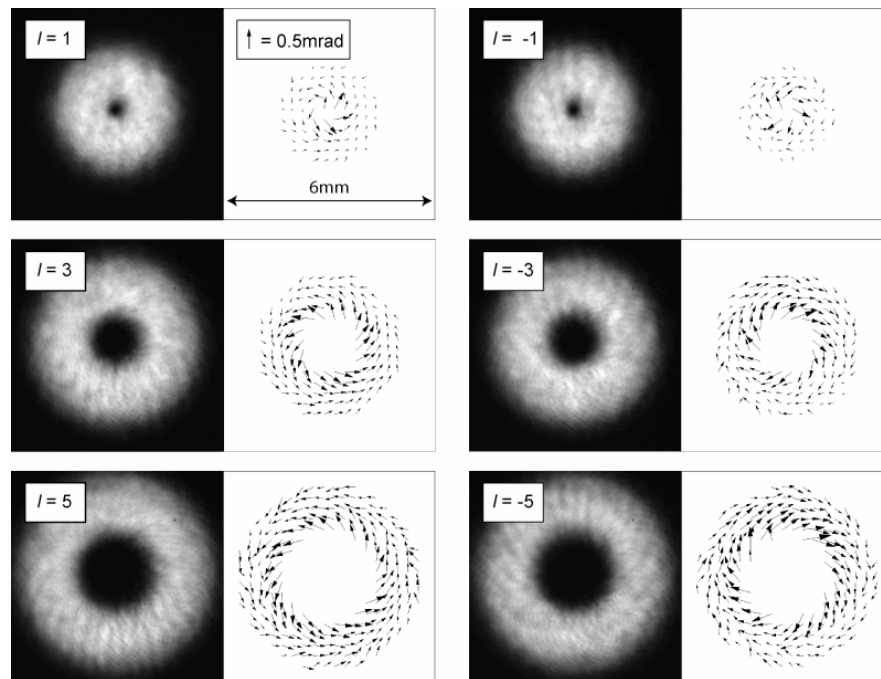

Fig. 2. Intensity and vector plots showing the measured inclination of the Poynting vector for different values of *l*. Both positive and negative values of *l* are shown.

Figure 3 is a graph derived from the same data as displayed in Fig. 2 showing the relationship between the measured skew angle of the Poynting vector, the beam radius and the azimuthal mode index. As anticipated there is a close agreement between these measurements and the predicted value of $\gamma = l/kr$.
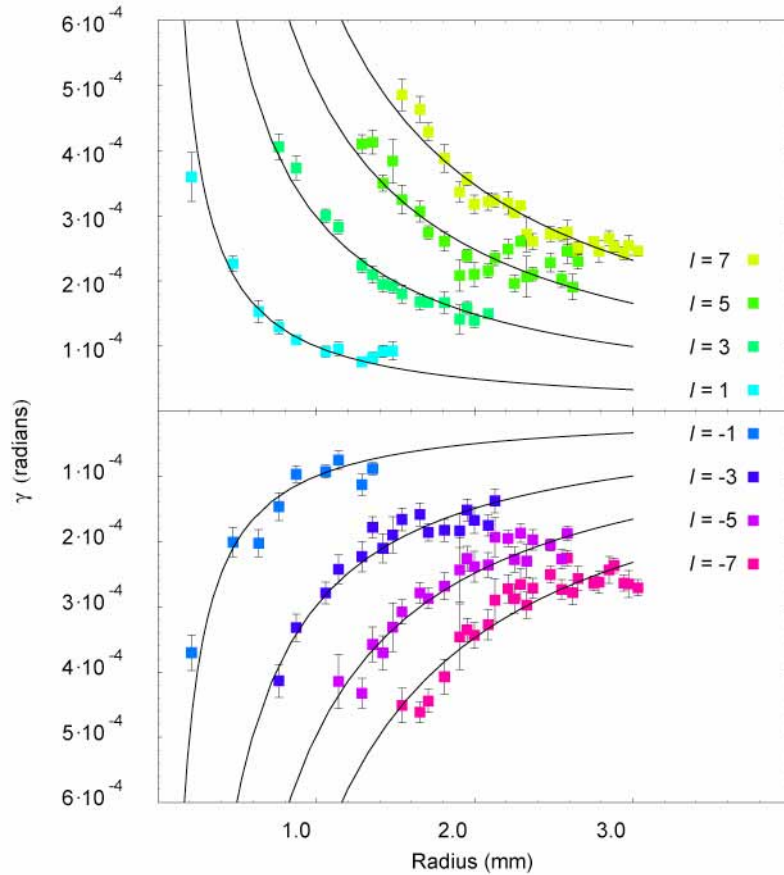

Fig. 3. The relationship between the measured skew angle of the Poynting vector, the beam radius and azimuthal mode index, l, is in close agreement with the predicted value, $\gamma = l/kr$.

The technique can also be used to measure the l value of a Laguerre-Gaussian mode. For each lenslet within the array, the expression for the skew angle of the Poynting vector can be rearranged to give $l = \gamma kr$. The value of γkr can then be averaged over all the lenslets to give a measure of l and hence the orbital angular momentum per photon, see Fig. 4.
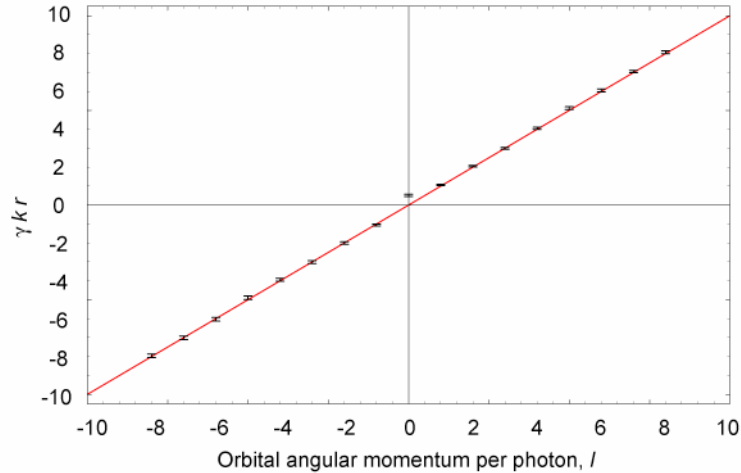
Fig. 4. The mean value of γkr calculated from the data in Figure 3. The error bars show the error in the mean value.

Note that the error in the mean value of is typically <0.1, demonstrating that a Shack Hartmann lenslet array can be used as a unambiguous method for measuring the orbital angular momentum per photon of LG beams.

5. Discussion and conclusions

We have shown that the local skew angle of the Poynting vector within a helically-phased, $\exp(il\phi)$, beam can be measured using a Shack Hartmann wavefront sensor. We have confirmed that this skew angle is in close agreement with that expected, corresponding to an orbital angular momentum of $l\hbar$ per photon. Beyond a simple confirmation of the structure of helically phased modes, the technique is particularly suited for making unambiguous measurements of the sense of the orbital angular momentum in experiments based on momentum transfer. This non-interferometric technique requires only a single, exposure and is hence ideally suited to use with pulsed or white light sources. It is also applicable to analysis of more complex modal superpositions with differing values of l, such as elliptical [15] or spiral [16] beams that, although possessing orbital angular momentum, do not have a single value of l.

Acknowledgments