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On the polarimetric variability of bright O-type stars

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Abstract. Polarimetric data associated with multi-parameter observational campaigns of selected bright O-type stars and their variable winds, are analysed in relation to the outcomes of the UV and optical spectroscopic studies. For the stars ξ Per and λ Cep, individual measurement uncertainties are $\Delta p \sim \pm 0.0002$ with nightly mean uncertainties of $\Delta p \sim \pm 0.00007$. Although variability is apparent on a night-to-night basis, with differences in $\delta p \sim 0.0002$, no correlations are found between these and the periodic behaviours associated with the stellar Si IV and Hα lines. Similar polarimetric variability is seen in the data for the standard star ϕ Cas used as a reference in this observing campaign. It is suggested that all of these low level fluctuations are not intrinsic to the stars but are engendered by structured instrumental polarization in the diffraction pattern and depolarization effects in combination with inconsistent target acquisition and with variable seeing conditions in the Earth's atmosphere. Reassessment of older data for λ Cep from Hayes (1978) also supports this thesis.

Key words. polarization – stars: early-type – stars: individual: ξ Per – stars: individual: λ Cep – stars: individual: 68 Cyg

1. Introduction

In order to explore the natures and mechanisms of winds emanating from O-type stars, some recent enterprises have involved coordinated multi-site, multi-parameter campaigns. For these studies, time-resolved contemporaneous observations have been made by the *International Ultraviolet Explorer* (IUE) to monitor the variations of discrete absorption components (DACs) in UV resonance lines, by ground based spectrometry to record the variations of lines such as Hα, and by broad-band photometry and polarimetry to monitor the overall behaviour of the selected stars. Examples of the outcomes from the spectroscopic contributions of such exercises can be found in Kaper et al. (1997), with periods presented for three stars, and in de Jong et al. (1999), with the detection of nonradial pulsations in the stars ξ Per and λ Cep. These papers make mention of the concurrent polarimetric observations at the McDonald Observatory but no comments were made on these latter measurements. The contributions of the polarimetric dimension to the above campaigns and studies will be addressed here.

The relevance of polarimetry to the understanding and modeling of wind inhomogeneities in WR stars in terms of the resultant variability and distributions of measurements in the normalised Stokes parameter (NSP) plane has been discussed by Brown et al. (1995) and by Richardson et al. (1996). The same approach is also applicable to O-type stars. Harries (2000) has also emphasised the importance of polarimetry as a diagnostic for understanding the nature of stellar winds of OB supergiants and has developed a comprehensive computer code to explore several polarigenic mechanisms.

Recently, McDavid (2000) has reported on a general observational survey of nine O-type stars, with variable winds, concluding that their polarimetric behaviours are consistent with interstellar origins. He makes comment, though, on the tantalising evidence remaining for small intrinsic components for some these of stars. One aspect of this is a tentative suggestion that variations in degree of polarization in the V-band, $p(V)$, measured during the 1991 Oct. campaign, correlate with the oscillatory behaviour of the equivalent widths of the Si IV and Hα lines; for 68 Cyg, a possible $p(V)$ oscillation is presented by displaying the measurements according to the determined spectroscopic period by Kaper et al. (1997), while for ξ Per, the $p(V)$ variation is imposed at double the spectroscopic frequency (see McDavid 2000, Figs. 10 and 12

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respectively). Note was also made of the earlier polarimetric study by Hayes (1975), followed by statistical assessments, demonstrating that his data revealed variations at the 3σ level for the two O-type stars, λ Cep (Hayes 1978) and α Cam (Hayes 1984). In later observations, Lupie & Nordsieck (1987) presented evidence for spectropolarimetric variability of the same two stars. Finally, Harries & Howarth (1996) discovered that, across the Hα emission line of ζ Pup, a $p(\lambda)$ variation is evident implying a polarizing wind asymmetry. Analysis by Harries (2000) suggests that the effect is caused by line absorption of continuum photons in a rotating wind rather than by line dilution of any continuum polarization.

In this paper, the issue of whether O-type stars exhibit variable intrinsic polarization in their radiations is addressed by undertaking statistical tests on the data from the campaign of 1989 (October) and by investigating more rigourously the proposal by McDavid (2000) that the polarization data from the 1991 (October) campaign show variations in phase with the observed spectroscopic oscillations. For the star λ Cep, the data of and conclusions made by Hayes (1978) are also re-examined.

2. The data

The reported polarimetric data[1] here were obtained as part of the the multi-site, multi-parameter observational schemes covering the periods 1989, October 17 to 22, and 1991, October 21 to 28. The polarimeter used on the 0.9 m telescope at the McDonald Observatory (Texas) has been described by Breger (1979). For each record, a cyclic routine involved sets of three measurements, interspersed with a sky background record (star 200 s, star 200 s, sky 200 s and star 200 s). The NSP uncertainties of each record, Δq, Δu, include the contributions from photon counting statistics and the noise from the sky background subtraction. A small instrumental polarization ($\sim0.1\%$) based on observations of unpolarized stars has been removed from all the measurements. Summaries of the night-to-night weighted mean NSP values (equatorial frame) and their weighted uncertainties, $\overline{q} \pm \overline{\Delta q}$, $\overline{u} \pm \overline{\Delta u}$, according to the N records made on each night are given in Tables 1 and 2. For the first campaign, B-band polarimetry was performed on the stars λ Cep, and ξ Per; for the second exercise, V-band data for λ Cep, ξ Per and 68 Cyg were obtained. Because of difficulties associated with noise-induced bias affecting the values p and θ, statistical analyses are better performed on NSPs (see Clarke & Stewart 1986) but it may be noted that the associated polarimetric parameters, degree of polarization, p, and position angle, θ, may be obtained from

$$p = (q^2 + u^2)^{1/2} \qquad \text{and} \qquad \theta = \frac{1}{2}\arctan^{-1}\left(\frac{u}{q}\right). \qquad (1)$$

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[1] Listings of the data are available in electronic form at the CDS via anonymous ftp to
`cdsarc.u-strasbg.fr` (130.79.128.5) or via
`http://cdsweb.u-strasbg.fr/cgi-bin/qcat?J/A+A/383/580`

The measurements are of high polarimetric quality and particularly for the first campaign are homogeneous in the sense that $\Delta q \approx \Delta u$, with typical uncertainties in the individual records of $\sim\pm0.00025$ [$\sim\pm0.025\%$] for ξ Per and $\sim\pm0.00019$ [$\sim\pm0.019\%$] for λ Cep, respectively, but with the uncertainties of the nightly means not reducing fully by the factor of $\sqrt{N}$ and displaying disparities between the $\overline{\Delta q}$ and $\overline{\Delta u}$ values. The first campaign data are also fairly extensive (reasonably large values of N) making them more amenable to statistical analyses and period searching.

3. Data analysing procedures

3.1. Statistical tests

In order to detect the presence of low level polarimetric temporal variability of stars, rigorous statistical tests are required. These tests were performed on the NSPs and on the p, θ values, although the latter are less reliable because of possible bias problems as previously mentioned.

The first approach was to check if the data were randomly distributed about their mean with a standard Normal behaviour and with variances for the NSP pairs (q, u), simply in keeping with their measurement noise levels. A statistically significant deviation from Normality would point to the presence of polarization variability. Any null test giving a conformity to Normality with the given variance does not categorically exclude variability but makes it unlikely. Further testing of data which are indistinguishable from being Normal may be performed by the LK method (see below) to see how the distribution has been assembled with time.

Quite generally, any polarimetric change is likely to affect one NSP more than the other, this giving an indication of the projected location of the disturbance relative to the stellar centre. Persistent movements in some preferred direction in the q, u plane indicate variability which is confined to particular regions within a stellar system (see Clarke & McGale 1987). Accordingly, possible asymmetry of data distributions in the q, u plane was explored. The scheme involved the application of the standard two-tailed F-Test to the sample variances of the two NSPs. Revised NSPs, q', u', were obtained by rotation of the reference frame and the F-Test re-applied. By doing this in a progressive way, the rotation angle was determined which provided the maximum disparity between the two sample variances associated with q', u' and checks were then made using the appropriate tables to see if the disparity was statistically significant.

Following this procedure, the NSPs in both the original and rotated frames were checked for Normality by a Kolmogorov Test, referred to as the K-Test. This test has been successfully applied previously to stellar polarimetry by Clarke et al. (1993) for investigations of stellar zero-polarization catalogues. Before the determinations of $T = \sup_x |F^*(x) - S(x)|$ can be made see (Conover 1980),

Table 1. A summary of the night-to-night behaviour of the *B*-band polarimetry for ξ Per and λ Cep over the October 1989 campaign. Column 1 provides the HJD (244****.+) of the night, Cols. 2 and 5 register the number of separate values obtained. The weighted mean values of the normalised Stokes parameters, $\overline{q}, \overline{u}$, and their weighted errors are given in Cols. 3/4 and 6/7 respectively for each star. Typical errors associated with the individual measurements from which these means are obtained are $\Delta q : \Delta u = \pm 0.025[\%]$ for ξ Per and $\pm 0.019[\%]$ for λ Cep, with minor fluctuations on these values.

		ξ Per			λ Cep	
HJD	N	$\overline{q}[\%]$	$\overline{u}[\%]$	N	$\overline{q}[\%]$	$\overline{u}[\%]$
7815.+	18	-1.0131 ± 0.0092	-0.7424 ± 0.0054	18	-0.5293 ± 0.0069	$+1.1056 \pm 0.0099$
7816.+	9	-0.9577 ± 0.0149	-0.7181 ± 0.0131	15	-0.5141 ± 0.0106	$+1.1325 \pm 0.0084$
7818.+	14	-0.9584 ± 0.0081	-0.7380 ± 0.0116	12	-0.4508 ± 0.0129	$+1.0939 \pm 0.0070$
7819.+	18	-0.9810 ± 0.0071	-0.7291 ± 0.0054	21	-0.4420 ± 0.0097	$+1.0840 \pm 0.0056$

Table 2. A summary of the night-to-night behaviour of the *V*-band polarimetry for ξ Per, λ Cep and 68 Cyg over the October 1991 campaign. The information is tabulated in the same way as for Table 1.

		ξ Per			λ Cep			68 Cyg	
HJD	N	$\overline{q}[\%]$	$\overline{u}[\%]$	N	$\overline{q}[\%]$	$\overline{u}[\%]$	N	$\overline{q}[\%]$	$\overline{u}[\%]$
8551.+	1	-1.113 ± 0.019	-0.849 ± 0.012	1	-0.396 ± 0.021	$+1.115 \pm 0.034$	1	-0.388 ± 0.034	$+0.523 \pm 0.008$
8552.+	4	-1.104 ± 0.003	-0.845 ± 0.012	3	-0.449 ± 0.015	$+1.097 \pm 0.016$	3	-0.424 ± 0.011	$+0.437 \pm 0.016$
8553.+	3	-1.150 ± 0.011	-0.856 ± 0.007	4	-0.477 ± 0.018	$+1.060 \pm 0.011$	4	-0.403 ± 0.010	$+0.535 \pm 0.016$
8554.+	0			0			1	-0.413 ± 0.027	$+0.511 \pm 0.013$
8555.+	3	-1.107 ± 0.012	-0.885 ± 0.003	4	-0.444 ± 0.010	$+1.046 \pm 0.005$	4	-0.416 ± 0.010	$+0.482 \pm 0.011$
8556.+	3	-1.143 ± 0.011	-0.845 ± 0.008	3	-0.447 ± 0.006	$+1.128 \pm 0.030$	3	-0.396 ± 0.016	$+0.480 \pm 0.010$
8557.+	1	-1.164 ± 0.066	-0.844 ± 0.037	1	-0.507 ± 0.020	$+1.099 \pm 0.054$	1	-0.393 ± 0.036	$+0.498 \pm 0.035$

the appropriate form of the theoretical probability distribution function, $F^*(x)$, must be first established for comparison with the empirical distribution function, $S(x)$. For example, if measurements of q are being assessed as to whether they are representative of an hypothesised Normal distribution, then the underlying true values of the mean $\overline{q}$ and variance σ_q^2 must be known. At best, these defining parameters are only available directly from the measurements as estimates which themselves carry uncertainties. To allow for this, a range of $F^*(q)$ functions need to be considered, so making the applied K-Test more conservative but more meaningful. This was done by providing two extreme $F^*(q)$ curves based on a Normal distribution but with mean values of $\overline{q} + \delta q$ and $\overline{q} - \delta q$ where δq is the standard error of the mean as determined from the measurements. The forms of these functions are essentially identical but are displaced on the q-axis with respect to each other by $2\delta q$. Similarly, a spread of hypothesised cumulative distribution curves was also used for exploring the behaviours of measurements $F^*(q')$, $F^*(u)$ and $F^*(u')$, the variables q', u' corresponding to the NSPs rotated to a frame which produces the maximum difference in their sample variances. No allowance was made for the uncertainty of the variance used in setting up the hypothesised cumulative distribution curves. If this were done, a range of $F^*(q)$ curves might be considered with effective start and finish positions at greater distances from the mean value $\overline{q}$. This point, however, was kept in mind when inspecting the tails of the empirical cumulative distributions.

Although individual groups of data may behave as Normal distributions with parameters defined by measurement noise, questions remain as to whether one group is distinguishable from another. This is relevant, for example, in exploring the presence of night-to-night variations. In the first instance, such an exercise can be done by examining the weighted mean NSP values and their errors as given in Tables 1 and 2, but difficulties arise in providing accurate confidence values to any disparities as only pairs of data sets can be compared at a time and again their associated errors are themselves not absolute, but estimates. The investigation of differences between data sets was undertaken by using the Welch Test (W-Test) as promoted by Clarke & Stewart (1986). This strategy allows data sets of unequal sample size and unequal variance to be investigated to see if the underlying individual means are unequal. According to Brown & Forsythe (1974), in terms of the size and power, this being related to the probability of rejecting the hypothesis of equality of means when it is indeed false, the W-Test is a recommended statistical procedure. As part of the algorithm, each of the component data sets were checked for Normality by determining the skewness and kurtosis coefficients prior to the application of the W-Test. The coefficients were compared with confidence tables related to the range of values that are acceptable as being consistent with Normality (see Brooks et al. 1995). Experience from the investigations with the presented data shows that the K-test is more conservative in detection of non-Normality relative to the application of skewness and kurtosis coefficients, possibly as a result

of the data sets not quite having internal homogeneity. The outcome of the K-test is preferred as, in addition to a numerical confidence level being provided, the form of the data distribution is readily appreciated by eye.

3.2. Tests for time dependence and periodicity

In view of the periodicities reported in spectroscopic measurements of O-type stars (see earlier discussion), it is important that the polarimetric data should also be investigated for any cyclic behaviour. It may be noted that even if some polarimetric measurements provide NSP values with a Normal distribution, their assembly as a time sequence may not be random, however. Provided that the data quality is homogeneous, such a notion can be explored by determination of the "string length" according to the statistic (LK) of Lafler & Kinman (1965) without any re-ordering of the data and comparing the result with values that would be obtained if they data were assembled in truly random fashion. Clarke (2001) has shown that this can be done more easily by normalising the original LK statistic so that it is independent of the number of measurements, N. The modified LK statistic may be written as

$$T = \frac{\sum\limits_{j=1}^{N}\left[(x_{j+1}-x_j)^2\right]}{\sum\limits_{j=1}^{N}\left[(x_j-\overline{x})^2\right]} \times \frac{(N-1)}{2N} \tag{2}$$

where $\overline{x}$ is the determined mean value of the measurements and the summation is completed by letting $x_{N+1} = x_1$. For random data, the mean value of $T = 1.0$, this being independent of the underlying variance of the data and the number of measurements. For data providing a value of $T < 1.0$, this offers a possible indication that adjacent values of the time sequence measurements might be correlated. Significance of any departure from 1.0 depends on N.

The above statistic can also be developed to detect any periodicity in the data. Using a grid of periods, for each trial value, P, the data are re-ordered in a sequence, $x_1 \ldots x_i \ldots x_N$, according to their phase value and the $T(x, P)$ value determined using Eq. (2) with the subscript "j" replaced by "i". Such a single parameter "string-length" LK periodogram (SLLK) should have a continuum level value of 1.0 with a noise band according to N. The SLLK algorithm is powerful with small data samples and is appropriate for investigating the behaviour of single parameter measurements such as each of q or u separately. It may be noted that the interval of the period grid should be selected so that the resultant periodogram has no flat sections (i.e., the string-length should change from one selected period to the next).

For multivariate data with parameters, w, x, y, etc., with each measured variable providing independent SLLK periodograms in the form $T(w, P)$, $T(x, P)$, $T(y, P)$, Clarke (2001) has shown that they may be simply combined into a single "rope-length" periodogram (RLLK) by calculating a mean defined by

$$T(Z, P) = \frac{1}{Z}\sum_{k=1}^{Z} T(k, P) \tag{3}$$

where k corresponds to an individual parameter and Z is the number of combined independent variables.

Again, the continuum level through any periodogram defined by Eq. (3) should have a mean level of 1.0 independent of the number of parameters and the number of measurements recorded for each of them. The measurements need not be made for each parameter at the same time. In fact, if the analysis is based on small samples of measurements for each of the parameters, there is positive advantage of not having simultaneity, as windowing and sampling effects may be reduced. Equation (3) provides powerful means of detecting low amplitude periodicity that is weakly present in each of the measured parameters and also allows investigation to see if all the parameters carry the same periodicity. It has obvious application to the analysis of polarimetric data, with its two parameters, q, u, and the data were investigated using the $T(Z = 2, P)$ statistic, written as $T(2, P)$ for brevity. A more comprehensive study of possible periodicity was also undertaken by combining the polarimetric measurements with the contemporaneous data sets from the spectral studies associated with the measurements of line equivalent widths. Calculation of $T(Z = 2)$ using the q, u, simply in their order of collection, also gives an estimate for any joint auto-correlation of the two parameters.

Confidence values on domains where the $T(P)$ values were significantly <1.0 for all of the above LK tests were assigned from repeated exercises involving the replacement of the real measurements with computer simulated data with similar mean values but with random distributions of similar variance. If any well formed, statistically significant depression was noted, the period was determined by the interpolation method of Kwee & van Woerden (1956) as promoted by Fernie (1989) for the LK statistic. It may be noted that this process provides an error estimate for the determined period.

4. Results of the analyses

The polarimetric data from both campaigns are plotted in Figs. 1 and 2 for ξ Per and λ Cep respectively. For ξ Per, the difference between the centers of the measurement distributions along a line joining them to the origin simply reflects the wavelength dependence of the strong interstellar polarization associated with the star, the two runs using different wavelength passbands. There is no immediate indication of any difference resulting from the time interval between the two campaigns. For λ Cep, it can be seen that there is no distinct difference in the polarizations between the two observing runs despite the fact that two different colour passbands were used. According to McDavid (2000), the interstellar polarization of this star
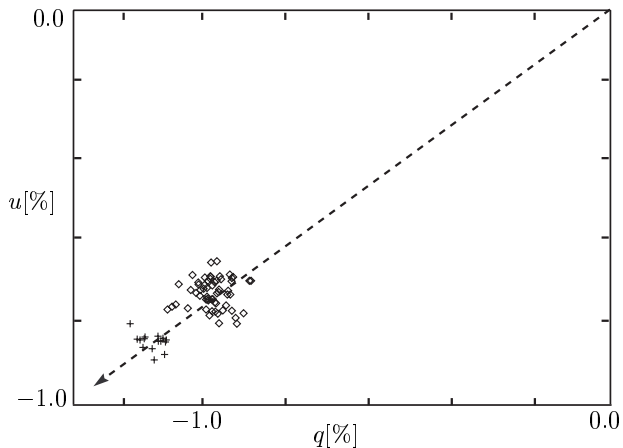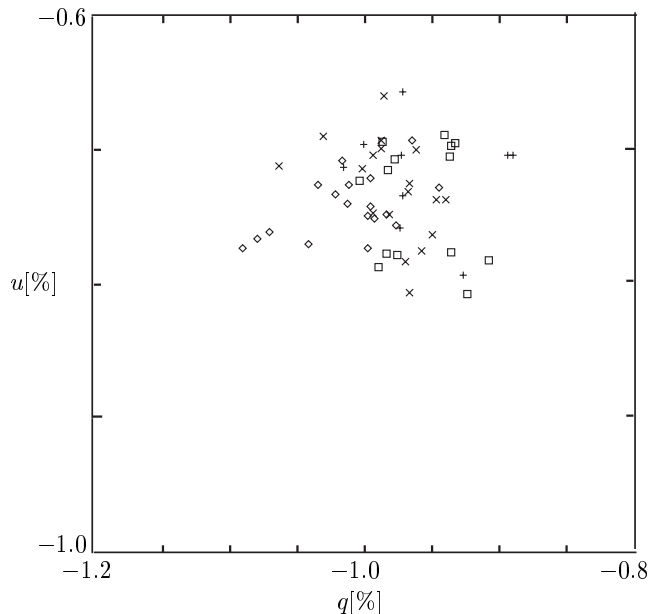
Fig. 1. The polarimetric data of the star ξ Per are plotted as NSPs, q, u, for the two campaigns of 1989 (B-band, marked "$\Diamond$") and 1991 (V-band, marked "$+$"). material]. The centers of gravity of the two distributions are on a line joining them to the frame origin with positions consistent with a purely interstellar cause; the separation of the distributions reflects the $p(\lambda)$ dependence of this interstellar polarization along the line of sight to ξ Per. Any intrinsic polarization associated with this star is very small.
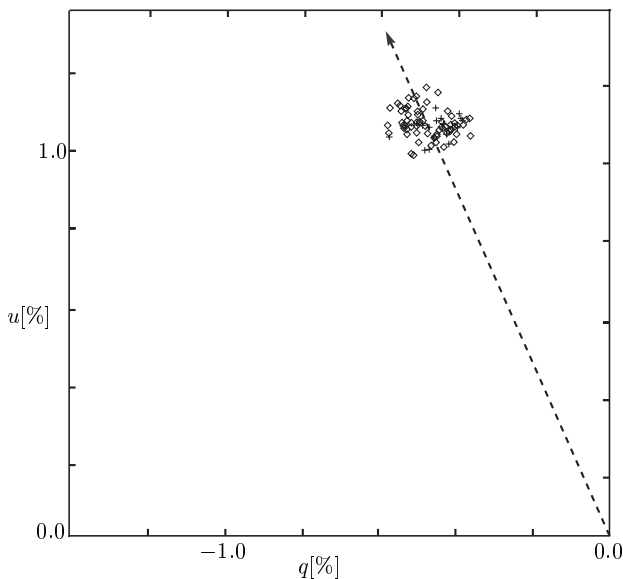
Fig. 2. The polarimetric data of the star λ Cep are plotted as NSPs, q, u, for the two campaigns of 1989 (B-band, marked "$\Diamond$") and 1991 (V-band, marked "$+$"). Again, as for ξ Per, the polarization values are dominated by a strong interstellar component (see McDavid 2000) with essentially the same value of p for the two colours. There is no immediate indication of the presence of any intrinsic polarization associated with λ Cep.

is indistinguishably identical for the B and V-bands and again, no change in the polarization is apparent between the data sets of 3 years apart.

For the campaign of 1989, the data are extensive with sufficient repeated measurements (large N) to allow thorough statistical scrutiny and to search for short term fluctuations ($\sim$minutes to hours) as might be produced by


Fig. 3. The polarimetric data of the star ξ Per are plotted as NSPs, q, u [%], for the campaign of 1989 split into night 1 (marked "$\Diamond$"), night 2 (marked "$+$"), night 3 (marked "$\square$") and night 4 (marked "$\times$") . Inspection of the distributions of the symbols suggests night-to-night variability in the measurements.

inhomogeneities in the stellar wind. For the campaign of 1991, comparatively few measurements were made on each of the nights relative to 1989 and these data lend themselves simply to investigations of night-to-night variations and longer term (days) periodicity. For ease in discussing the analysis of the 1989 campaign, the observations for each of the nights are referred to as "nights 1, 2, 3 and 4", corresponding to HJDs of 7815.+, 7816.+, 7818.+ and 7819.+, respectively.

4.1. Analysis of the data of ξ Per

The data from the first campaign are displayed in Fig. 3, using different symbols for the four nights of the observing run. Cursory inspection of their distribution suggests that there is night-to-night polarimetric variability and inspection of the $\overline{q}, \overline{u}$ values in Table 1 together with the errors also suggests the same. By using Eq. (1), any variation is in keeping with changes in the vector length of the polarization, p, with the variations in the position angle, θ, accountable by the measurement noise. Application of the F-Test for the whole of the sample of 59 measurements, however, provides no indication of the presence of any preferred axis in the distribution.

Results from the W-Test confirm the notion with a confidence $>99\%$ that the underlying means for each of the four nights are not equal. By applying the W-Test to selections of three of the four nights in turn, it was found that the means are statistically indistinguishable for nights 2, 3 and 4; inequality of means was only apparent with the inclusion of the data of night 1. Without knowledge of
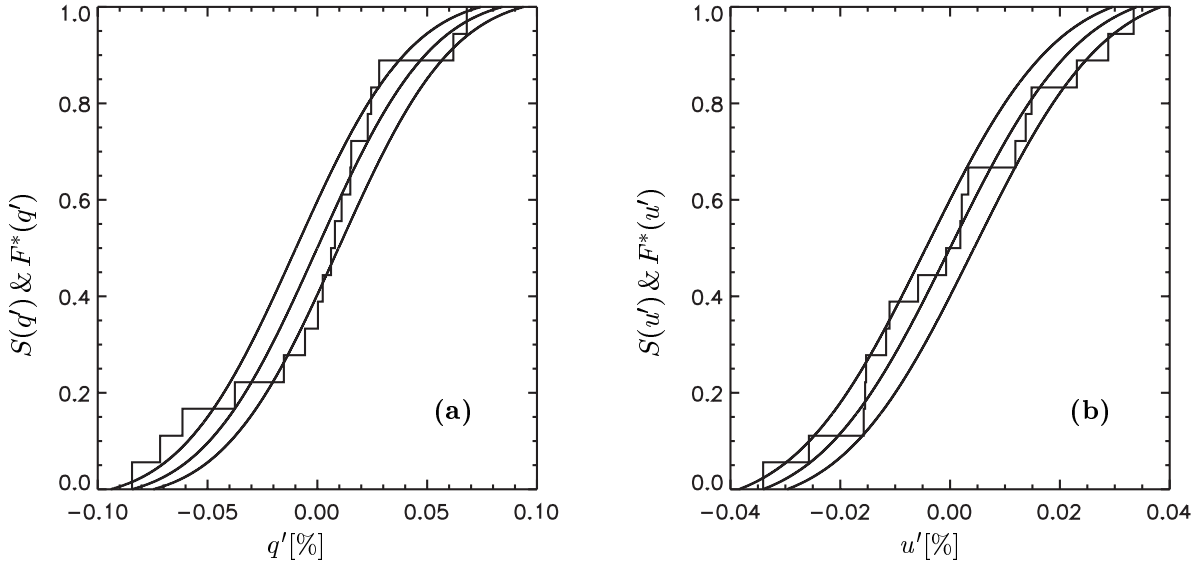
Fig. 4. The empirical distribution functions $S(q')$ and $S(u')$ (stepped curves) for the data of ξ Per on night 1 of the October 1989 campaign are displayed according to a revised reference frame with origin at the night's mean values $(\overline{q}, \overline{u})$ and axes rotated by $20°9$ relative to the equatorial frame. In **a)**, the hypothesised distribution functions based on $F^*(q')$ (central curve), $F^*(q' - \delta q')$ (left hand curve) and $F^*(q' + \delta q')$, (right hand curve), where $\delta q'$ is the determined standard error of the mean, are presented for comparison with $S(q')$; no significant departures from Normality are present. Similar curves are presented for the u' parameter in **b)** for comparison with $S(u')$; again, no significant departures from Normality are present.

the true or stable mean values of the NSPs, it is, however, impossible to say on which nights the data exhibit the greatest departure, although the suspicion lies with night 1. On night 3, the 14 measurements of both q and u give evidence of being kurtose in the sense of providing a narrower distribution than the expected Normal. On night 4, the 18 measurements of q provide a skewed distribution with 95% confidence. No explanation is offered for these findings but they do not significantly affect the outcome of the W-Test.

When the data were considered as nightly sets, the F-Tests indicated that for night 1, there was a preferred axis at $20°9$ to that of the equatorial frame with a 99% confidence level that the individual q' and u' distributions cannot be considered as coming from parent distributions with the same variance. The F-Test on data for night 4 also provided a preferred axis at $167°6$ at the lower confidence level of 95%. For the data from night 1, a revised reference frame with origin at the night's mean, $(\overline{q}, \overline{u})$, and axes rotated by $20°9$. The K-Test was then applied to the revised q' and u' values with the outcome displayed in Fig. 4 in which the empirical distribution functions cannot be distinguished from the theoretical cumulative distribution functions based on the measured means and sample variances. Thus the q', u' data are indistinguishable from having Normal distributions, even though the variances are statistically significantly different. It may be noted that by maximising the ratio of the q' and u' variances by rotation of $20°9$, the direction of the q'-axis is close in direction to the strong interstellar polarization component.

From analysis of data from the October 1989 campaign of the He I $\lambda4713$ Å line, de Jong et al. (1999) detected periods of $3°45$ and $2°63$ for ξ Per. The 59 polarimetric
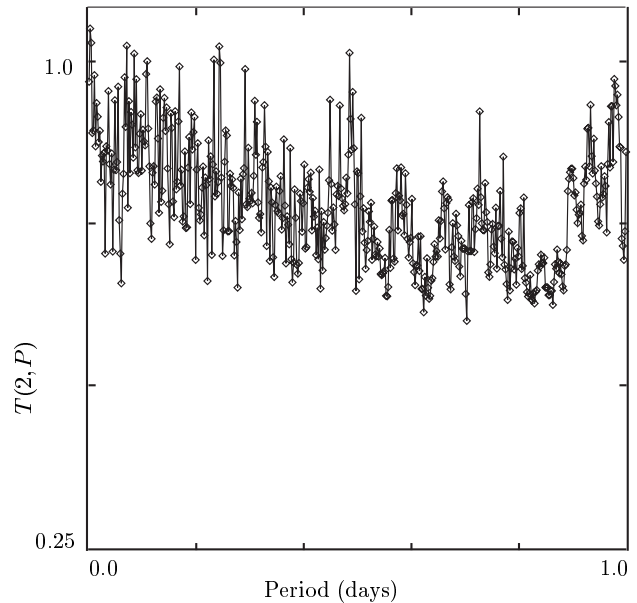
Fig. 5. The string-length periodogram, obtained from the joint analysis of the q and u parameters for ξ Per through the October 1989 campaign, displays a noisy behaviour. The lowest value is at $0°7029$ but this has no real significance.

measurements covering the same campaign were investigated for periodicity using the algorithm of Eq. (3). In the first instance, the test on the data without re-ordering gave no indication of there being correlations between the variability of q and u. The periodogram involving the simultaneously measured parameters, q, u, covering the range 0.0 to 1.0 days, is shown in Fig. 5, showing no ordered minima. The deepest value appears at $0°702(9)$ or
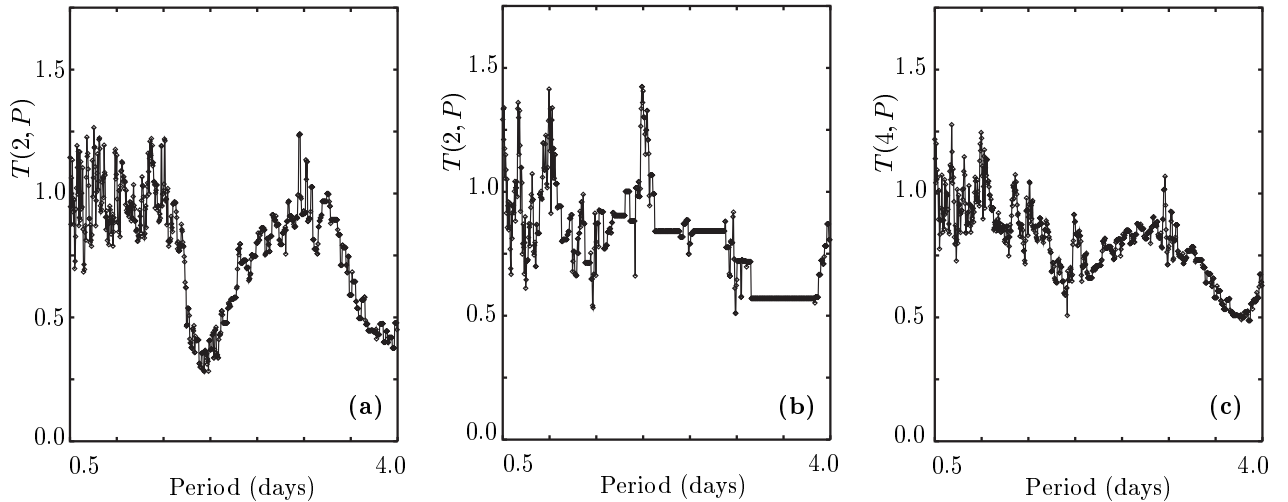
Fig. 6. The RLLK periodogram, $T(2, P)$, obtained from the joint analysis of the spectral line equivalent widths of Si IV and Hα for ξ Per from the October 1991 campaign, is displayed in **a)** showing a strong periodicity at $1^{\rm d}974$. The RLLK periodogram, $T(2, P)$, obtained from the joint analysis of the contemporaneous q and u measurements, displayed in **b)**, shows a noisy behaviour with no obvious periodicity showing. In **c)**, the RLLK periodogram, $T(4, P)$, obtained from the joint analysis of the four parameters corresponding to the Si IV and Hα line measurements and the q and u values, is displayed; in relation to **a)**, the presence of periodicity has been "washed out" indicating a lack of correlation between line variability and any polarimetric variation.

$16^{\rm h}87$ but this feature is not statistically significant. Even with the high accuracy of polarimetry reported here, periodicity is absent and no links can be made to the periodic behaviour associated with non-radial pulsations as perhaps might be expected from such a polarigenic mechanism. Extending the periodogram beyond 1.0 days to the limit of the total length of the observational run, to cover periods associated with stellar rotation or DAC phenomena, also offered no hint of periodicity (see Fig. 6b). It may be noted that there are flat sections to the periodogram with variations only becoming apparent around 2, 2.5, and 3 days, these being a consequence of the limited observing window and the sampling pattern within it. (Similar behaviour was apparent for the periodograms of the other monitored stars.) There is no hint of any periodicity around 2 days and no connections can be made to the $2^{\rm d}086$ period found by the campaign of de Jong et al. (2001). Estimates on the detection probability show that if the amplitude of any variation had been $\delta p \sim 0.00015$, the chance of not detecting it is less than 5%.

Period analysis was also applied to the data of the October 1991 campaign. For this exercise, the spectroscopic data were also included. Commencing with the data (36 measurements) of the equivalent widths of the Si IV line, the SLLK periodogram, gives a deep minimum corresponding to a period $\sim$2 days, confirming the outcome of the power spectrum analysis of Kaper et al. (1997). There is also a minimum $\sim$4 days but care must be taken in interpreting its significance as its value is close to the window length of the data set. The SLLK periodogram resulting from the 15 equivalent width measurements of Hα is much noisier as a result of the more sparsely sampled data but again it displays a minimum at approximately the same period as for the Si IV line. The RLLK

combination periodogram, $T(2, P)$, with equal weights for the two lines is displayed in Fig. 6a with the best period fit at $1^{\rm d}974 \pm 0^{\rm d}002$. The strength of the minimum justifies the conclusions of Kaper et al. (1997) of the "phenomenological correspondence between the systematic variability observed in deep-seated optical lines and the known behaviour of DACs in UV resonance lines".

The SLLK analysis individually for q and u and their RLLK periodogram, $T(2, P)$, (see Fig. 6b) of the 15 measurement pairs (summarised in Table 2) offers no hint at all of any periodicity. When the RLLK method is applied to the full combination of the two spectral lines (Si IV and Hα) and the two NSPs (q and u) with equal weight, the periodogram, $T(4, P)$, is "washed out" and the period $\sim$2 days is no longer significant (see Fig. 6c). At the accuracy of the reported measurements here, it is concluded that there is no correlation or connection between broadband polarimetry and the periodic behaviour clearly seen in the spectral lines. Because of the nature of the phase functions, some stellar geometries involving scattering density enhancements in orbit about the illuminating star can to give rise to polarimetric periodicities which vary at twice the orbital frequency. To check that this is not an issue here, RLLK analysis was also undertaken by combining periodograms for the spectral variations with those for q, u but with the latter doubled in frequency. This resulted in an even flatter periodogram again confirming that no polarimetric periodicity was detectable. Thus, direct searching for periodicity in the polarimetric data provides a null result suggesting that the placing of the $p(V)$ data by McDavid (2000) on a sinusoidal curve with a period twice that of the spectral variation is somewhat arbitrary. It may be noted that parts of the small sized data set in relation to the suggested period based on
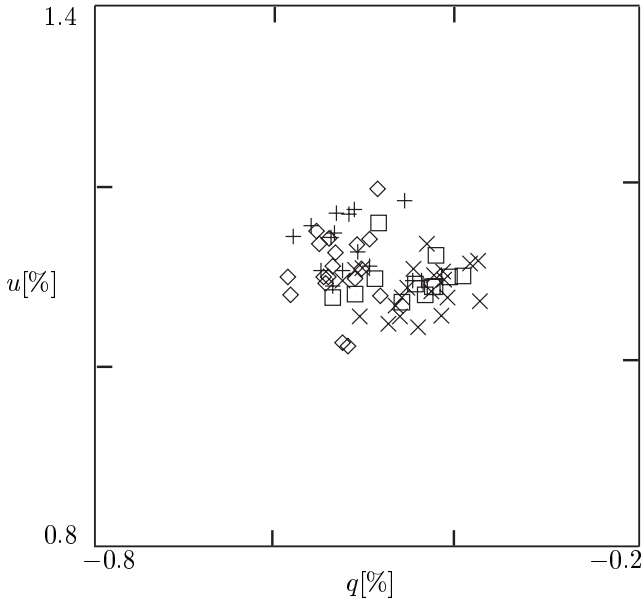
Fig. 7. The polarimetric data of λ Cep are plotted as NSPs, q, u [%], with the campaign of 1989 split into night 1 (marked "$\diamond$"), night 2 (marked "+"), night 3 (marked "$\square$") and night 4 (marked "$\times$"). The distributions of the symbols "+" and "$\times$" suggest night-to-night variability in the measurements.

spectrometry are essentially in anti-phase with the Si IV variation (see McDavid 2000, Fig. 12 at JD 2448555.+). It may be noted too that the Si IV and Hα line variations are apparently out of phase at this time, suggesting that the relationship between the behaviours of these two lines is itself not completely coherent.

4.2. Analysis of the data of λ Cep

The 66 measurements of λ Cep, summarised in Table 1, are displayed in Fig. 7. Although overall they appear to provide a fairly uniform distribution, an F-Test on the complete data set reveals a preferred axis at $-14^\circ\!.5$ with a confidence level >99%, the significance of this, however, is obscure. Figure 7 indicates evidence that the distributions for nights 2 and 4 are displaced with respect to each other, suggesting a night-to-night variability. A comparison of the $\overline{q}, \overline{u}$ values in Table 1 also suggests this; the nightly means show that the values for nights 1 and 2 are indistinguishable, as also are the means for nights 3 and 4. The shift between nights 1/2 and 3/4 shows a reduction in polarization essentially along the p vector shown by McDavid (2000) to have an interstellar origin. Confirmation that the complete set of data cannot be considered as originating from a single parent distribution is given by the W-Test.

The skewness coefficients for nights 1 and 3 provide evidence of non-Normality in the u (95%) and q (99%) parameters respectively. F-Tests applied to the data for individual nights show marginal asymmetry between the q' and u' distributions on nights 3 and 4 with the axis at $4^\circ\!.0$ and $8^\circ\!.6$ respectively. K-Tests on the data, however,

for the separate nights all provide null results in terms of departures from Normality for both q and u.

SLLK and RLLK exercises were also performed to investigate the possible presence of periodicity in the polarimetric data. From the October 1989 campaign, a series of minima in the RLLK periodogram for q, u, the deepest at $0^\mathrm{d}\!.635(0)$ ($15^\mathrm{h}\!.24$) but with no significant confidence and allowing no direct polarimetric link to be made with the non-radial pulsation periods ($12^\mathrm{h}\!.3(5)$ and $6^\mathrm{h}\!.6(3)$) as observed in the He I line by de Jong et al. (1999).

For the October 1991 run, the LK exercise involving spectral data could only include the data for Hα as the Si IV line suffers saturation. The most pronounced minimum in the SLLK periodogram, strongly influenced by the limited size of the data set (11 measurements), was at $0^\mathrm{d}\!.538 \pm 0.002$ with another indeterminate feature with a flat bottom covering $1^\mathrm{d}\!.27$ to $1^\mathrm{d}\!.35$; nothing significant corresponded to the peak ($1.25 \pm 0.06\,\mathrm{day}^{-1}$) in the power spectrum noted by Kaper et al. (1997). No periodicity was detectable in the RLLK analysis of q and u as a data pair or in combination with the Hα equivalent width measurements. Estimates on the detection probability show that if a variation with amplitude $\delta p \sim 0.0001$ had been present, the chance of not detecting it is less than 5%.

Further analysis using the joint auto-correlation as determined from $T(Z = 2)$, with a progression of q', u' values, again suggests that night-to-night polarimetric variability has been detected in keeping with the notion that the value of p changes but with the position angle being maintained within the limits set by the measurement noise.

This is also confirmed by re-analysis of the B-band data for λ Cep collected by Hayes (1978) over some 32 months. He conducted statistical tests on all of the 74 observations of this star from March 1974 to October 1976 by comparing the sample variance with the theoretical variances derived from photon shot noise, concluding that there was an intrinsic variability. On some particular nights when repeated measurements were obtained, the observed and theoretical variances were found to differ at confidence levels greater than 95%.

The exploration of there being any preferred axis in the data by the F-Test provided a 99% confidence level for a difference in variances between q' and u' when the reference frame is rotated by $\sim 61^\circ$, this being close to the mean position angle of $56^\circ\!.4$, as determined by Hayes (1978). It may be noted that Hayes (loc. cit.) estimated the interstellar polarization position angle as $57^\circ \pm 2^\circ$ from 64 stars located within an area of sky 8° square centered on the galactic co-ordinates of λ Cep, commenting that the value was consistent with a fairly uniform single large interstellar cloud in the environment of the line of sight to the target star. The near coincidence of the measured values of θ to that estimated for the interstellar component prompted Hayes (loc. cit.) to conclude that the time average of the intrinsic polarization is probably close to zero. By applying a revised version of Serkowski's Law (see Wilking et al. 1980) to his $p(\lambda)$ measurements, McDavid (2000) has also
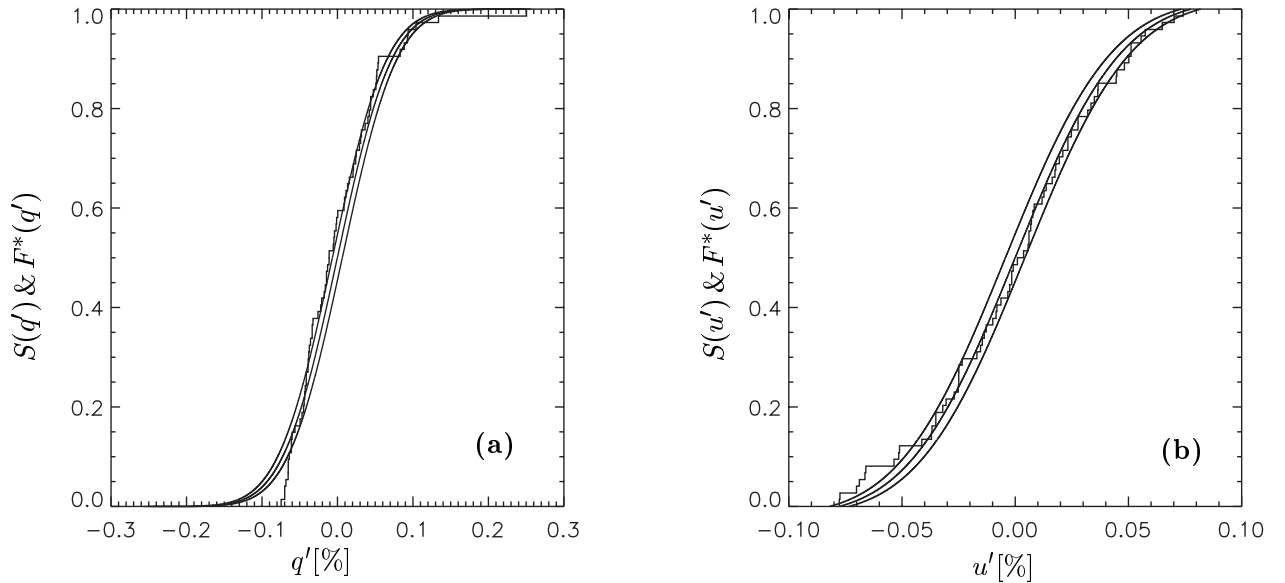
Fig. 8. The empirical distribution functions $S(q')$ and $S(u')$ (stepped curves) for the Hayes' data of λ Cep are displayed according to a revised reference frame with origin at the mean values $(\overline{q}, \overline{u})$ and axes rotated by $61\overset{\circ}{.}0$ relative to the equatorial frame. In **a)**, the hypothesised distribution functions based on $F^*(q')$ (central curve), $F^*(q' - \delta q')$ (left hand curve) and $F^*(q' + \delta q')$, (right hand curve), where $\delta q'$ is the determined standard error of the mean, are presented for comparison with $S(q')$; in **b)**, similar curves are presented for u' for comparison with $S(u')$. No significant departures from Normality are present for the q', u' parameters but with q' displaying a significantly larger variance.

demonstrated that the position angle of the basic measurements is essentially co-incident with the interstellar polarization.

When Hayes' data were investigated using the K-Test, there was no evidence that the empirical distribution functions for q' and u' may not be modeled well by the Normal distributions $F^*(q')$ and $F^*(u')$ respectively. It is apparent, however, from the F-test as mentioned above and from the cumulative distribution functions (see Fig. 8) that their variances are not the same. When described in a frame with origin corresponding to the mean values of the data, $\overline{q}, \overline{u}$, and rotated by $61°$, the measurements of q' are contained within limits of $\pm 0.15\%$ whereas the measurements of u' lie within the limits of $\pm 0.08\%$. It will be noted that for q', there are no data with values less than -0.1%, implying a departure from the Normal distribution in the wing corresponding to low values (see Fig. 8a). The K-Test, however, showed that this feature is not significant.

The overall conclusion here with respect to the newly presented data and the re-examination of the measurements of Hayes (1978), suggests that the observed polarization of λ Cep has an essentially interstellar origin and that any variability is very small with a distribution essentially along the mean position angle direction.

4.3. Analysis of the data of 68 Cyg

The analysis of data for the star 68 Cyg is limited to the October 1991 campaign. The SLLK periodogram for the data of the Si IV provides periods $\sim 1\overset{d}{.}4$ and $\sim 2\overset{d}{.}8$, the latter being slightly more significant. For the Hα data,

the SLLK periodogram is not well defined but minima are apparent again at $\sim 1\overset{d}{.}4$ and $\sim 2\overset{d}{.}8$, the former being marginally more significant. By combining the two periodograms, the resulting values are $1\overset{d}{.}382 \pm 0\overset{d}{.}006$ and $2\overset{d}{.}857 \pm 0\overset{d}{.}006$, with equal statistical significance. The former value is consistent with the periods highlighted by Kaper et al. (1997) for Si IV and the Hα line core. For the polarimetric measurements, the SLLK analysis of the u parameter data provides undistinguished minima in the range $1\overset{d}{.}39$ to $1\overset{d}{.}44$, but with no statistical significance. From the RLLK exercise involving combinations of spectral measurements with polarimetry, no connections could be made between the period and harmonic clearly seen in the spectral data.

4.4. Instrumental stability

As part of the observational study, unpolarized and polarized standard stars were measured, allowing removal of the instrumental offsets and calibration of the instrumental reference axis relative to the standard equatorial frame. Repetitive measurements of polarized standard stars also provides means of assessing the instrumental stability and allows direct data comparisons with the target stars. The star selected for such a study was ϕ Cas, this having similar brightness to λ Cep and with $p \sim 3.2\%$.

The data distributions in the q, u plane are displayed in Fig. 9 and have similar characteristics to ξ Per (Fig. 3) and λ Cep (Fig. 7). Although the W-Test does not provide a high confidence for the detection of night-to-night variability in either q or u, there are suggestions that the data tend to group about different mean values on the
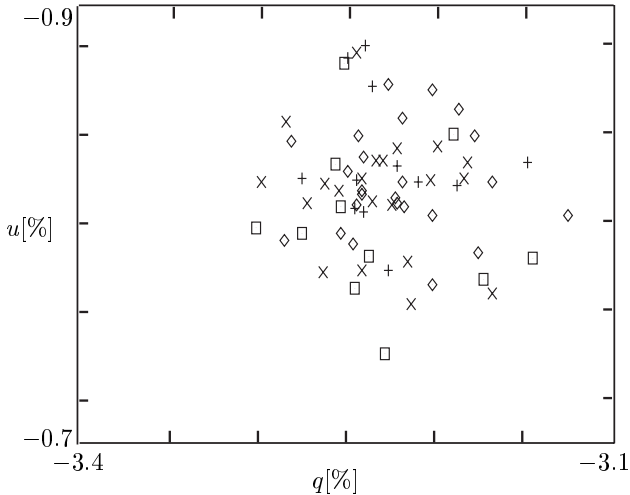
Fig. 9. The polarimetric data of the standard star ϕ Cas are plotted as NSPs, q, u [%], for the campaign of 1989 split into night 1 (marked "$\Diamond$"), night 2 (marked "+"), night 3 (marked "$\Box$") and night 4 (marked "$\times$") . Inspection of the distributions suggests night-to-night variability in the measurements, e.g., compare the data of nights 1 and 3, the latter being non-Normal, particularly in the q parameter.

four nights of the 1989 campaign. On night 3, the K-Test reveals non-Normality for both NSPs – q (99% confidence) and u (90% confidence).

In summary, the statistical behaviours of repeated measurements of this standard star and the target stars show small variable instabilities just above the level set by the associated photon counting noise, this providing uncertainties in $p \sim \pm 0.0002$ over each observational interval.

Possible problems related to the polarimetric stability of ϕ Cas have been raised previously by Dolan & Tapia (1986) and by Bastien et al. (1988). It may be noted that the specific conclusions of the latter group were based on five different observing runs on different telescopes using two different polarimeters, this not being ideal for such a study (see the criticisms by Clarke & Naghizadah-Khouei 1994 on the treatment of these data). There is also anecdotal evidence for the apparent stochastical polarimetric variability of a variety of the supposed standard stars such as 55 Cyg and o Sco when the measurement accuracy is pushed to values of $\Delta P \sim \pm 0.0001$ and better. The question of the origin of the apparent instabilities as to whether they are intrinsic to the stars or to experimental sources needs to be addressed more fully in the future (see below).

5. Discussion

Both the presented polarimetric measurements reported here for ξ Per and λ Cep and those by Hayes (1978) of λ Cep reveal small but significant night-to-night changes. The general behaviour suggests that the variations are chiefly associated with the p parameter, with small movements apparent along its vector when displayed in the NSP plane. These variations are undoubtedly present in

the data, but it is not clear that they have any physical origin related to the variability of stellar winds.

Following the analysis by Serkowski (1958), it is well known that p is a biased parameter and that, for measurements made with a given signal-to-noise ratio, the data follow a non-Normal distribution. Additional scatter in the determined values of p may therefore ensue if the measurements are made with different signal-to-noise ratios (p/σ_p). Individual measurements are more likely to be nearer the most probable value of the underlying distribution rather than its mean, the difference between the two being dependent on p/σ_p. Such problems may well be apparent when, p/σ_p is less than 10 (see Simmons & Stewart 1985) but would be insensibly small for the measurements ($p/\sigma_p \sim 100$) reported here. In any case, the statistical investigations were conducted on the NSPs and they do not suffer from such noise induced bias (see Clarke & Stewart 1986). Thus, variable Serkowski bias is unlikely to be the cause of the apparent polarimetric variability.

As demonstrated by McDavid (2000), by far the greatest component of the observed polarization of the discussed stars is generated in the interstellar medium. If the line-of-sight paths through the galaxy suffer drifts in density of the integrated dust column or temporal changes of the grain alignment, variations in p may be experienced. Such notions were raised by Bastien et al. (1988) in regard to measurements of so-called polarization standard stars, but it was shown by Clarke & Naghizadah-Khouei (1994) that the presented data sample suffered from effects of inhomogeneities and that the statistical analyses contained flaws. Fluctuations in the line-of-sight dust columns would also affect photometric measurements and colour excess but no such effects have been recorded.

The proposal made here suggests that the observed fluctuations originate nearer to home. For stellar polarimetry, observational procedures involve the determination and subtraction of the instrumental polarization, usually dominated by the telescope, and the polarimeter's frame is calibrated by measuring the position angle of the polarization of a highly polarized standard star. Traditionally, the telescope induced instrumental polarization has been characterised by a single Mueller matrix relating the Stokes vector of the collected radiation to that describing the radiation at the telescope's focus. Sánchez Almeida & Martinez Pillet (1992) have shown, however, that this approach becomes inadequate when accuracies of 10^{-4} or better are pursued. Their theoretical analysis demonstrates that the focal plane image of a point source carries polarimetric structure. Even for perfectly aluminised mirrors without degraded areas, the theoretical diffraction pattern reveals cross-talk between the q and u parameters over the image structure and also a depolarization. In a later paper, Sánchez Almeida (1994) quantifies the very significant influence that atmospheric seeing has on the telescope's polarization behaviour.

For the two main stars under scrutiny in this paper, the polarization is relatively high ($p \sim 0.01$ or just >1%) as a result of the interstellar medium. The behaviour of

the discussed data could be described in terms of this polarization being made apparently variable by changes or fluctuations in the depolarization effect, perhaps accompanied with small changes in q and u as might occur as a result of a variable crosstalk between the parameters. The magnitude of the effects are in keeping with the predicted instrumental and seeing induced effects described by Sánchez Almeida & Martinez Pillet (1992) and Sánchez Almeida (1994). The data behaviour could result from inaccuracies in keeping the observed star constantly and exactly at the centre of the field of view and from variation in the average seeing quality from night to night. It may be noted that these theoretical analyses on variable instrumental polarization are based on diffraction theory associated simply with circular telescope apertures. In reality, variable depolarization effects and cross-talk problems may well be greater than the results obtained from the above analyses if the diffraction effects of the spider supporting the secondary mirror are taken into account. This inherent problem associated with high accuracy polarimetry depends on the individual design of the telescope. Certainly, if future observational schemes are envisaged to investigate the potential of polarimetry at high accuracy to stellar wind diagnosis, the role that the effects of inconsistent centering of the target star in the field stop, imperfect telescope tracking and variable atmospheric seeing play needs careful investigation.

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