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Loads Transfer Principles of a Four-Bladed Scissors Rotor

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I. Introduction

A scissors rotor, or non-orthogonal rotor, usually consists of two pairs of rotor blades with unequal inter-blade angles and a vertical separation between the blade pairs. In 1970s, the potential of scissors rotors in reducing rotor noise had been recognized [1]. The tests of a four-bladed experimental scissors rotor showed that the maximum rotor thrust, and the Figure of Merit could be improved compared with a conventional rotor [2]. The noise of a helicopter with a four-bladed scissors rotor during flyovers was clearly lower than the helicopter with a conventional three bladed rotor [2]. Several researchers also investigated the noise performance of scissors rotors [3, 4], and currently, scissors tail rotors are used in the AH-64, Mi-28 and Z-10 helicopters. Past research usually focused on the noise characteristics of scissors rotors.

Different from conventional rotors, the blades of scissors rotors are not distributed evenly, and the phase angles between adjacent blades are different. The load transfer mechanism of this kind of rotor is bound to be different from that of conventional rotors. Walter and Drees pointed out that the scissors rotor resembles more a two-bladed rotor than a four-bladed rotor [1]. The fuselage vibrations with a scissors rotor contained two-per-rev components, which usually can be filtered by a conventional four-bladed rotor. The loads transfer principles of four-bladed scissors rotors are different from those of conventional four-bladed rotors, and this has not been investigated.

This work presents a theoretical analysis of the loads transfer principles of four-bladed scissor rotors. The blade root forces and moments are expressed as Fourier series, and the rotor hub forces and moments are obtained by summing these forces and moments. The relations between the blade loads and rotor hub loads are derived, and the corresponding transfer principles are also analyzed. The study aims to provide a simple analytical tool to quickly account for the effect of the inter-blade angle to the loads transferred to the rotor hub. This could be a starting point for more detailed analyses and optimizations of scissors rotors with blade element or CFD methods.

II. Out-of-plane Force

For a helicopter rotor blade operating in steady state, the blade root forces in the rotating coordinate frame (blade attached coordinate frame), shown in Figure 1, can be written as Fourier series at every blade azimuth ψ_k

$$\begin{cases} X_{bk} = X_0 + \sum_{n=1}^{\infty} X_{nc} \cos n\psi_k + \sum_{n=1}^{\infty} X_{ns} \sin n\psi_k \\ Y_{bk} = Y_0 + \sum_{n=1}^{\infty} Y_{nc} \cos n\psi_k + \sum_{n=1}^{\infty} Y_{ns} \sin n\psi_k \\ Z_{bk} = Z_0 + \sum_{n=1}^{\infty} Z_{nc} \cos n\psi_k + \sum_{n=1}^{\infty} Z_{ns} \sin n\psi_k \end{cases} \quad (1)$$

In the above, k denotes the index of a blade, and n denotes the harmonic order. The subscript nc denotes the n th cosine harmonic component, and ns denotes the n th sine harmonic component.

Figure 1 Coordinate frames

The rotor thrust is obtained by summing the root vertical shear forces over all N_b blades

$$Z_s = \sum_{k=1}^{N_b} Z_{bk} = N_b Z_0 + \sum_{n=1}^{\infty} Z_{nc} \sum_{k=1}^{N_b} \cos(n\psi_k) + \sum_{n=1}^{\infty} Z_{ns} \sum_{k=1}^{N_b} \sin(n\psi_k) \quad (2)$$

For a four-bladed scissors rotor, the scissor angle $\Delta\psi$ is defined as the angle between two adjacent blades (less than 90°),

as shown in Figure 1. The azimuths of the four blades can be written as

$$\begin{cases} \psi_1 = \psi \\ \psi_2 = \psi + \Delta\psi \\ \psi_3 = \psi + \pi \\ \psi_4 = \psi + \pi + \Delta\psi \end{cases} \quad (3)$$

where, ψ is the azimuth of blade 1.

To simply Equation 2, the sum of the cosine harmonics can be written as follows:

$$\sum_{k=1}^4 \cos(n\psi_k) = 4 \cos \frac{n\pi}{2} \cos \frac{n\Delta\psi}{2} \cos \left(n\psi + \frac{n\pi + n\Delta\psi}{2} \right) \quad (4)$$

Similarly, the sum of the sine harmonics is

$$\sum_{k=1}^4 \sin(n\psi_k) = 4 \cos \frac{n\pi}{2} \cos \frac{n\Delta\psi}{2} \sin \left(n\psi + \frac{n\pi + n\Delta\psi}{2} \right) \quad (5)$$

Using Equations 4 and 5, Equation 2 can be simplified to

$$\begin{aligned} Z_s = & 4Z_0 + 4 \sum_{n=1}^{\infty} Z_{nc} \left[\cos \frac{n\pi}{2} \cos \frac{n\Delta\psi}{2} \cos \left(n\psi + \frac{n\pi + n\Delta\psi}{2} \right) \right] \\ & + 4 \sum_{n=1}^{\infty} Z_{ns} \left[\cos \frac{n\pi}{2} \cos \frac{n\Delta\psi}{2} \sin \left(n\psi + \frac{n\pi + n\Delta\psi}{2} \right) \right] \end{aligned} \quad (6)$$

For the odd harmonic components, the sum of the harmonics is zero, since $\cos(n\pi/2)$ is zero. It can then be concluded that the odd harmonic blade loads cannot be transferred to the rotor hub. The even harmonic components can be transferred to the rotor hub with the same harmonic order. However, the magnitude is amplified by $\cos(n\Delta\psi/2)$, and this value is determined by the scissors angle. Its absolute value is equal or less than 1.0. Figure 2 shows the value of $\cos(n\Delta\psi/2)$ with the scissor angle $\Delta\psi$. The scissors angles corresponding to the minimum (0.0) and maximum (± 1.0) magnitudes can be determined. For different harmonic orders, the scissors angles corresponding to the minimum and

maximum magnitudes of the loads transferred to the rotor hub are listed in Table 1. For the second harmonic, increasing the scissors angle can decrease the load transferred to the rotor hub. When the scissors angle is 90°, the resultant force is zero, which is corresponding to the rotor with evenly distributed four blades. For the fourth harmonic, adjusting the scissors angle around 45° can decrease the loads transferred to the rotor hub.

Figure 2 The value of $\cos(n\Delta\psi/2)$ with the scissor angle $\Delta\psi$.

Table 1 Scissors angles corresponding to the minimum and maximum magnitudes.

Harmonic Number	Angles (minimum)	Angles (maximum)
2	90°	0°
4	45°	0°, 90°
6	30°, 90°	0°, 60°
8	22.5°, 67.5°	0°, 45°, 90°

For evenly distributed blades, **along the axis of the rotor**, only the integer harmonic forces of rotor blades can be transferred to the rotor hub. However, this principle does not apply to the four-bladed scissors rotor. The even harmonic components can all be transferred to the rotor hub with the same harmonic orders. The 2nd, 6th, 10th, ..., remained, and

the amplification of each harmonic is determined by the scissors angle.

It should be noted that the aerodynamic interference between the different blade pairs should have some effect on the aerodynamics of the rotors. This can change the scissors angles corresponding to the minimum and maximum magnitudes of the harmonic loads. The design of scissors rotor [2], the lower rotor leading the upper rotor with a vertical separation is used to reduce the aerodynamic interference between blades and the rotor noise. In hover, the rotor thrust is reported to be increased by about 10% compared with a normal configuration (scissors angle equals to 90° , and no vertical separation) [2]. **It can be concluded that the aerodynamic effect on the rotor thrust is small. When analyzing the vibratory loads of scissors rotors, the degree of influence level of the aerodynamic interference on the rotor thrust is used as a reference for the degree of influence level of it on the vibratory load. Specific analysis is required for specific rotors. The results given here are theoretical.**

III. In-plane Forces

The rotor drag from the blades can be written as

$$\begin{aligned}
X_s &= \sum_{k=1}^{N_b} (X_{bk} \cos \psi_k - Y_{bk} \sin \psi_k) \\
&= \sum_{k=1}^{N_b} (X_0 \cos \psi_k - Y_0 \sin \psi_k) \\
&\quad + \sum_{n=1}^{\infty} \sum_{k=1}^{N_b} (X_{nc} \cos n\psi_k \cos \psi_k + X_{ns} \sin n\psi_k \cos \psi_k - Y_{nc} \cos n\psi_k \sin \psi_k - Y_{ns} \sin n\psi_k \sin \psi_k) \\
&= \sum_{k=1}^{N_b} (X_0 \cos \psi_k - Y_0 \sin \psi_k) \\
&\quad + \frac{1}{2} \sum_{n=1}^{\infty} \sum_{k=1}^{N_b} (X_{nc} + Y_{ns}) \cos(n+1)\psi_k + \frac{1}{2} \sum_{n=1}^{\infty} \sum_{k=1}^{N_b} (X_{nc} - Y_{ns}) \cos(n-1)\psi_k \\
&\quad + \frac{1}{2} \sum_{n=1}^{\infty} \sum_{k=1}^{N_b} (X_{ns} - Y_{nc}) \sin(n+1)\psi_k + \frac{1}{2} \sum_{n=1}^{\infty} \sum_{k=1}^{N_b} (X_{ns} + Y_{nc}) \sin(n-1)\psi_k
\end{aligned} \tag{7}$$

To simplify Equation 7, the following expressions are used

$$\left\{ \begin{array}{l} \sum_{k=1}^4 \cos(n+1)\psi_k = 4 \cos \frac{(n+1)\pi}{2} \cos \frac{(n+1)\Delta\psi}{2} \cos \left[(n+1)\psi + \frac{(n+1)(\pi + \Delta\psi)}{2} \right] \\ \sum_{k=1}^4 \cos(n-1)\psi_k = 4 \cos \frac{(n-1)\pi}{2} \cos \frac{(n-1)\Delta\psi}{2} \cos \left[(n-1)\psi + \frac{(n-1)(\pi + \Delta\psi)}{2} \right] \\ \sum_{k=1}^4 \sin(n+1)\psi_k = 4 \cos \frac{(n+1)\pi}{2} \cos \frac{(n+1)\Delta\psi}{2} \sin \left[(n+1)\psi + \frac{(n+1)(\pi + \Delta\psi)}{2} \right] \\ \sum_{k=1}^4 \sin(n-1)\psi_k = 4 \cos \frac{(n-1)\pi}{2} \cos \frac{(n-1)\Delta\psi}{2} \sin \left[(n-1)\psi + \frac{(n-1)(\pi + \Delta\psi)}{2} \right] \end{array} \right. \quad (8)$$

Using Equation 8, Equation 7 can be simplified as

$$\begin{aligned} X_s = & 2 \sum_{n=1}^{\infty} (X_{nc} + Y_{ns}) \left[\cos \frac{(n+1)\pi}{2} \cos \frac{(n+1)\Delta\psi}{2} \cos \left[(n+1)\psi + \frac{(n+1)(\pi + \Delta\psi)}{2} \right] \right] \\ & + 2 \sum_{n=1}^{\infty} (X_{nc} - Y_{ns}) \left[\cos \frac{(n-1)\pi}{2} \cos \frac{(n-1)\Delta\psi}{2} \cos \left[(n-1)\psi + \frac{(n-1)(\pi + \Delta\psi)}{2} \right] \right] \\ & + 2 \sum_{n=1}^{\infty} (X_{ns} - Y_{nc}) \left[\cos \frac{(n+1)\pi}{2} \cos \frac{(n+1)\Delta\psi}{2} \sin \left[(n+1)\psi + \frac{(n+1)(\pi + \Delta\psi)}{2} \right] \right] \\ & + 2 \sum_{n=1}^{\infty} (X_{ns} + Y_{nc}) \left[\cos \frac{(n-1)\pi}{2} \cos \frac{(n-1)\Delta\psi}{2} \sin \left[(n-1)\psi + \frac{(n-1)(\pi + \Delta\psi)}{2} \right] \right] \end{aligned} \quad (9)$$

The even harmonic components cannot be transferred to the rotor hub, since $\cos[(n-1)\pi/2]$ and $\cos[(n+1)\pi/2]$ are zeros. The odd harmonic components can be transferred to the rotor hub with the harmonic order increased or decreased by one. Different from the evenly distributed blades, more harmonic in-plane components of the scissors rotor can be transferred to the rotor hub. For this four-bladed scissors rotor, the rotor drag contain 2nd, 6th, 10th, ..., harmonic components, which can be filtered by the four-bladed rotor with evenly distributed blades. The amplifications of these harmonics are determined by the scissors angle. Increasing the scissors angle can reduce the second harmonic rotor drag. For the fourth harmonic hub load from the third and fifth harmonic blade load, adjusting the scissors angle around 45° can reduce the loads transferred to the rotor hub.

Similarly, the rotor side force can be written as

$$\begin{aligned}
Y_s &= \sum_{n=1}^{\infty} \sum_{k=1}^{N_b} (X_{nc} \cos n\psi_k \sin \psi_k + X_{ns} \sin n\psi_k \sin \psi_k + Y_{nc} \cos n\psi_k \cos \psi_k + Y_{ns} \sin n\psi_k \cos \psi_k) \\
&= 2 \sum_{n=1}^{\infty} (X_{nc} + Y_{ns}) \left[\cos \frac{(n+1)\pi}{2} \cos \frac{(n+1)\Delta\psi}{2} \sin \left[(n+1)\psi + \frac{(n+1)(\pi + \Delta\psi)}{2} \right] \right] \\
&\quad - 2 \sum_{n=1}^{\infty} (X_{nc} - Y_{ns}) \left[\cos \frac{(n-1)\pi}{2} \cos \frac{(n-1)\Delta\psi}{2} \sin \left[(n-1)\psi + \frac{(n-1)(\pi + \Delta\psi)}{2} \right] \right] \\
&\quad + 2 \sum_{n=1}^{\infty} (X_{ns} + Y_{nc}) \left[\cos \frac{(n-1)\pi}{2} \cos \frac{(n-1)\Delta\psi}{2} \cos \left[(n-1)\psi + \frac{(n-1)(\pi + \Delta\psi)}{2} \right] \right] \\
&\quad - 2 \sum_{n=1}^{\infty} (X_{ns} - Y_{nc}) \left[\cos \frac{(n+1)\pi}{2} \cos \frac{(n+1)\Delta\psi}{2} \cos \left[(n+1)\psi + \frac{(n+1)(\pi + \Delta\psi)}{2} \right] \right]
\end{aligned} \tag{10}$$

The loads transfer principle of the rotor side force is the same as the rotor drag. Similarly, the loads transfer principle of the rotor torque is the same as the rotor thrust, and the principles of rotor pitching and rolling moments are the same as the rotor drag and rotor side force.

From the above analyses, the loads transfer principles of the four-bladed scissors rotor are more similar to two-bladed conventional rotors. However, the relative magnitudes of the harmonics, such as 2/rev, 4/rev, 6/rev, etc., decrease due to the scissors angle. These principles are **different** from conventional four-bladed conventional rotors, due to the appearance of the harmonic components, such as 2/rev, 6/rev, 10/rev, etc..

IV. Conclusions

The loads transfer of a four-bladed scissors rotor is investigated. The theoretical expressions of rotor hub forces are derived by summing the Fourier series of the blade root forces from the four blades. The analytical relations between the blade loads and rotor hub loads indicate that the loads transfer of scissors rotors different from the rotors with evenly distributed blades. The rotor forces contain all even harmonic components not just the integer harmonics of rotor blades. This phenomenon is similar to a conventional two-bladed rotor. The loads transferred to the non-rotating frame can be decreased by a factor determined by the scissors angle. Increasing the scissors angle can reduce the second harmonic rotor forces and moments. For the fourth harmonics, adjusting the scissors angle around 45° can decrease **the** loads

transferred to the rotor hub.

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