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Simulations of supersonic store geometries at high incidence angles using statistical and scale-resolving models

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Predicting the flowfield around a supersonic store at a high incidence angle is challenging due to the presence of vortices and shocks that interact with each other. The complexity of the problem is further increased by the presence of wing-body and wing-tail junctions giving rise to secondary flows. Given that the flow is turbulent, linear eddy-viscosity turbulence models are unable to account for the secondary flows and are often more dissipative than their non-linear counterparts. The high incidence angle further increases the complexity. This work investigates the effect of grid refinement and turbulence modelling on three store configurations - one with wings and fins, one without fins and wings, and one with wings only. The in-house CFD solver of the University of Glasgow is used to perform simulations at different angles of incidence and roll. Grids consisting of approximately 80×10^6 cells or less were found to be inadequate to capture the flow features. This shows that even if a high-order spatial method is employed, a grid of sufficient density must be used to accurately capture the aerodynamic loads of the store. In addition, grid converged results were difficult to obtain for the full configuration due to the interaction of the wing vortices with the store's fins. Improved convergence was observed for the simplified store configurations. This further showed that the difficulty in grid convergence is related to the wing vortex interactions with the store's fins.

Nomenclature

Latin

W	=	Width of numerical domain
H	=	Height of numerical domain
k	=	Turbulent kinetic energy
L	=	Length of store
M	=	Mach number
p	=	Static pressure
$\mathbf{R}$	=	Flux residual vector
Re	=	Reynolds number
t	=	Time
$\mathbf{V} = (u, v, w)^T$	=	Velocity vector
$V = \mathbf{V} $	=	Velocity vector magnitude
$V_{i,j,k}$	=	Volume of cell i,j,k
$\mathbf{W}$	=	Vector of conservative flow variables
$\mathbf{x} = (x, y, z)^T$	=	Position vector
d	=	Diameter
S	=	Area
C_X	=	Aerodynamic coefficient in the x -direction
C_Y	=	Aerodynamic coefficient in the y -direction

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C_Z	=	Aerodynamic coefficient in the z -direction
C_l	=	Aerodynamic coefficient about the x -axis (rolling moment)
C_m	=	Aerodynamic coefficient about the y -axis (pitching moment)
C_n	=	Aerodynamic coefficient about the z -axis (yawing moment)

Greek

λ	=	Roll angle
μ	=	Dynamic viscosity
μ_t	=	Turbulent (eddy) viscosity
ρ	=	Density (content-specific)
σ	=	Total incidence angle
ω	=	Specific dissipation rate
Δ	=	Difference

Subscripts and superscripts

$(\cdot)_{ref}$	=	Reference quantity
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Acronyms

AVT	=	Applied Vehicle Technology
NATO	=	North Atlantic Treaty Organisation
EARSM	=	Explicit Algebraic Reynolds Stress Model
QCR	=	Quadratic Constitutive Relation
RANS	=	Reynolds Averaged Navier Stokes
SST	=	Shear Stress Transport
GCI	=	Grid Convergence Index
TKE	=	Turbulent Kinetic Energy
RET	=	eddy to molecular viscosity ratio

I. Introduction

IN January 2018, the NATO Science and Technology Organization (STO) Applied Vehicle Technology (AVT) I-panel established a Task Group identified as AVT-316 (Vortex Interaction Effects Relevant to Military Air Vehicle Performance). The Task Group was split into two facets: an Aircraft Facet and a Missile Facet, each focusing on the vortex interactions associated with airframes of direct interest to NATO. The Missile Facet was established to (i) Assess the current capabilities of CFD to predict missile aerodynamic characteristics for flows containing multiple vortex interactions; (ii) Share and seek to learn from comparable experience of applying CFD to other classes of NATO vehicles (combat aircraft, in particular); and (iii) Consolidate lessons learned and any attendant future requirements [1]. This paper is one of a series being presented at this conference to provide a technical overview of the activities and accomplishments of the AVT-316 Missile Facet [2–11]. The work is still ongoing: a final output, constituting a more detailed and consolidated technical record, will be published by NATO STO towards the end of 2022.

With the continued increase of computing power over the last decade, computational fluid dynamic (CFD) methods are becoming an indispensable tool during the design and development stages of air vehicles. CFD methods are especially useful in situations where the flows under investigation are difficult to measure experimentally. Many advances have been made in CFD over the past 25 years including the development of high-resolution numerical schemes and improved turbulence models capable of capturing complex flow physics. Nevertheless, there are still flows that present challenges to the state of the art CFD methods. Examples are flows around bodies of revolution and flows around delta wings at high incidence angle. Such flows are vortex dominated and, if supersonic, they contain shocks and vortex shock interactions. Historically, the difficulty in accurately predicting such flows with CFD methods stems from the fine grids

required to capture the vortex structures. In addition complex geometries featuring sharp corners further, increase the complexity of the problem. Standard linear turbulence models such as the $k-\omega$ BSL and $k-\omega$ SST are inherently unable to resolve for the secondary flows arising near leading edges. The generation of secondary corner flows is governed by the normal Reynolds stress anisotropy in the vicinity of the corner. As linear models are isotropic, they fail to generate secondary flows near corners. In the context of the current AVT-316 exercise, (see figure 1), sharp corners are present at the locations where the wings or fins meet the store body.

Fig. 1 Store geometry and sharp corners which give rise to the formation of secondary flows (contra-rotating vortices)

In addition, linear models predict excessive levels of eddy viscosity at the vortex cores and near shear layers. This is not favourable as it changes the underlying flow physics. Non-linear models and scale-resolving methods can be used to remedy this shortcoming by offering greater modelling fidelity, however, they come with their own problems. The convergence speed can decrease due to the non-linearity of the models and the well-known dependence of the DES [12] scale resolving method to the grid size. The present work investigates the effect of non-linear turbulence models and scale-resolving methods on the ability of CFD methods to accurately predict a complex vortically-dominated supersonic flow around a generic store geometry. The effect of the non-linear $k-\omega$ EARS and $k-\omega$ SST SAS models are considered. The latter model was considered due to its formulation being independent of the local grid size, in comparison to DES. First simulations of the full geometry, featuring wings and fins, are presented and the effect of turbulence modelling and grid convergence are discussed. In a second step, simplified geometries of the store were simulated to investigate the lack of grid convergence observed for the full geometry.

II. Numerical method

The Helicopter Multi-Block (HMB3) [13, 14] code is used in the present work. HMB3 solves the Unsteady Reynolds Averaged Navier-Stokes (URANS) equations in integral form using the Arbitrary Lagrangian-Eulerian (ALE) formulation for time-dependent domains, which may include moving boundaries. The Navier-Stokes equations are discretised using a cell-centred finite volume approach on a multi-block grid. The spatial discretisation of these equations leads to a set of ordinary differential equations in time

$$\frac{d}{dt} (\mathbf{W}_{i,j,k} V_{i,j,k}) = -\mathbf{R}_{i,j,k} (\mathbf{W}_{i,j,k}), \quad (1)$$

where i,j,k represent the cell index, $\mathbf{W}_{i,j,k}$ and $\mathbf{R}_{i,j,k}$ are the vector of conservative flow variables and flux residual respectively, and $V_{i,j,k}$ is the volume of the cell i,j,k . To evaluate the convective fluxes the Osher [15] approximate Riemann solver is used, while the viscous terms are discretised using a second-order central differencing spatial

discretisation. The Monotone Upstream-centered Schemes for Conservation Laws, which is referred to in the literature as the MUSCL approach and developed by Leer [16], is used to provide high-order accuracy in space. The HMB3 solver uses the alternative form of the Albada limiter [17] being activated in regions where large gradients are encountered mainly due to shock waves, avoiding the non-physical spurious oscillations. An implicit dual-time stepping method is employed to perform the temporal integration, where the solution is marching in pseudo-time iterations to achieve a fast convergence, which is solved using a first-order backward difference. The linearised system of equations is solved using the Generalised Conjugate Gradient method with a Block Incomplete Lower-Upper (BILU) factorisation as a pre-conditioner [18]. To allow an easy sharing of the calculation load for a parallel job, multi-block structured grids are used. Various turbulence models are available in the HMB3 solver, including several one-equation, two-equation, three-equation, and four-equation turbulence models. Furthermore, Large-eddy Simulation (LES), Detached-eddy Simulation (DES), and Delayed-Detached-eddy Simulation (DDES) are also available. In the present work the fully-turbulent $k - \omega$ SST by Menter [19] and $k - \omega$ EARSM by Hellsten [20] turbulence models are used.

A. Turbulence modelling

In this work, the Scale Adaptive Simulation (SAS), the $k - \omega$ SST turbulence model by Menter [19], and the $k - \omega$ EARSM turbulence model by Hellsten [20] are used. A brief description of each model follows.

Scale Adaptive Simulation (SAS)

In the original formulation of the Detached eddy Simulation proposed by Spalart [12], the turbulent length scale computed from the turbulence model is compared to the local grid spacing. The decrease of the eddy viscosity, which allows the model to switch to scale resolving mode, is controlled entirely by the destruction term ($D_k = \beta \rho \omega k F_{DES}$) for the turbulent kinetic energy. Menter [19] showed that a destruction term for the eddy viscosity can be formulated without the explicit grid dependency in his KE1E model. The destruction term is based on the von Karman length scale (which acts as an integral length scale in boundary layer regions) and is given by:

$$D = c_2 \left(\frac{\nu_t}{l_{vk}} \right)^2, \quad (2)$$

$$l_{vk} = \kappa \left| \frac{\frac{\partial \bar{u}}{\partial y}}{\frac{\partial^2 \bar{u}}{\partial y^2}} \right|,$$

where $\bar{u}$ is the velocity parallel to the wall and y is the wall-normal coordinate. For general three-dimensional calculations including modifications to avoid the singularity for vanishing mean strain-rate Menter et al. [21] used the following invariant formulation of the von Karman length scale

$$l_{vk} = \kappa \sqrt{\frac{\frac{\partial \bar{u}_i}{\partial y_j} \frac{\partial \bar{u}_i}{\partial y_j}}{\frac{\partial^2 \bar{u}_l}{\partial y_m^2} \frac{\partial^2 \bar{u}_m}{\partial y_l^2}}} \quad (3)$$

and proposed the following modification to the KE1E model:

$$\begin{aligned} \widetilde{l}_{vk} &= \max(l_{vk}, C_{SAS} \widetilde{\Delta}), \\ \widetilde{\Delta} &= \max(\Delta x, \Delta y, \Delta z), \\ C_{SAS} &= 0.6. \end{aligned} \quad (4)$$

It was found that the above length scale introduces a dynamic behaviour in the KE1E model. The SAS formulation was extended to the $k - \omega$ SST model by Menter and Egorov [22]. In the model an additional destruction term Q_{SAS} in the equation for the specific dissipation rate $-\omega$ is present.

$$Q_{SAS} = \max \left[\zeta_2 \kappa S^2 \left(\frac{l_{RANS}}{l_{vK}} \right)^2 - C \frac{2k}{\sigma_\Phi} \max \left(\frac{|\nabla \omega|^2}{\omega^2}, \frac{|\nabla k|^2}{k^2} \right), 0 \right], \quad (5)$$

where

$$\begin{aligned}
\zeta_2 &= 3.51, \\
\sigma_\Phi &= 2/3, \\
C &= 2, \\
\kappa &= 0.41, \\
l_{RANS} &= C_\mu^{-1/4} \frac{\sqrt{k}}{\omega}, \\
\widetilde{l_{vK}} &= \max\left(\frac{\kappa S}{|\nabla^2 U|}, 0.26 (\Delta x \Delta y \Delta z)^{1/3}\right), \\
S &= \sqrt{2 S_{ij} S_{ij}}.
\end{aligned} \tag{6}$$

When $Q_{SAS} = 0$ the resulting mode is RANS (attached flows). When $Q_{SAS} > 0$ the model switches to scale resolving mode providing LES-like behaviour.

Explicit Algebraic Reynolds Stress (EARS) model

The Explicit Algebraic Reynolds Stress k - ω model proposed by Hellsten [20] is derived from Menter's Baseline k - ω model [19] but uses the explicit algebraic Reynolds-stress model of Wallin and Johansson [13] as a constitutive model for the turbulent stress tensor (the constant coefficients are also re-calibrated). The turbulent stress tensor is given by:

$$\tau_{ij}^{RANS} = \tau_{ij}^{RANS} - a_{ij} \rho k, \tag{7}$$

where τ_{ij}^{RANS} is the linear stress tensor (from the Boussinesq approximation) and a_{ij} is the non-linear stress tensor (from the constitutive model). A similar approach is used by models which employ the Quadratic Constitutive Relation (QCR) proposed by Spalart [12]. Full details about the k - ω EARS model can be found on the NASA turbulence modelling resource website [23].

B. Convergence analysis

Methods for evaluating the temporal and spatial convergence of CFD simulations (based on Richardson's extrapolation) were proposed by Roache [24, 25]. The grid convergence index or GCI [24, 25] presents a simple method for uniform reporting of grid convergence studies without any restriction to grid doubling. The GCI is based on generalised Richardson extrapolation involving the comparison of discrete solutions at two different grid spacings. The error in a fine grid solution f_1 can be obtained by comparing it to the solution on a coarse grid f_2 and is defined as:

$$E^{fine} = \frac{\epsilon_{12}}{(1 - r^p)}, \tag{8}$$

where $\epsilon_{12} = f_2 - f_1$, r is the refinement ratio and p is the observed order of accuracy. A second-order numerical method will have $p = 2$, however, due to factors including but not limited to the grid the observed order of accuracy will be less. To account for the uncertainty in the generalised Richardson-based error estimates and to put the grid convergence studies on the same basis as grid doubling with a second-order numerical algorithm Roache [24, 25] incorporated a safety factor and defined the GCI as:

$$GCI^{fine} = F_s |E^{fine}|, \tag{9}$$

where F_s is a safety factor. The GCI is not an error estimator but F_s times the error estimator, representing error bands in a loose statistical sense. A value of $F_s = 3$ for two grids and $F_s = 1.25$ for three grids was recommended by Roache [25]. For three grids, having solutions f_1 , f_2 , and f_3 , where f_1 corresponds to the solution on the finest grid, the observed order of convergence, p , is given by:

$$\frac{\epsilon_{23}}{(r_{23}^p - 1)} = r_{12}^p \left[\frac{\epsilon_{12}}{(r_{12}^p - 1)} \right]. \quad (10)$$

As the above equations are transcendental in p , an iterative method is required to solve it. For well-behaved solutions simple substitution iteration with a relaxation factor of $\omega = 0.5$ works well.

$$\beta = \frac{(r_{12}^{p^i} - 1)}{(r_{23}^{p^i} - 1)} \frac{\epsilon_{23}}{\epsilon_{12}} \quad (11)$$

$$p^{i+1} = \omega p^i + (1 - \omega) \frac{\ln(\beta)}{\ln(r_{12})};$$

This approach is used here to determine whether or not grid convergence is achieved.

III. Numerical setup

In this work, three geometries are considered. The geometries were based on the geometry in the NATO AVT-316 activity [1, 26]. The first geometry represents a full store, the second a store body only and the third a store body with wings. The geometries are referred to as configuration A, B, and C. The length of the store is $L_{ref} = 3.45$ m and the diameter $d_{ref} = 0.15$ m. Figure 2 below shows the configurations. The loads are reported in a right-hand coordinate system with an origin located at $x/L_{ref} = 0.5$.

Fig. 2 Geometry of configurations A, B, and C

The right-handed coordinate system is referred to as the body-fixed coordinate system. Rotations and moments are positive clockwise when viewed along the axis of rotation in the positive sense from the origin. The moment reference point is located at 50% of the total store length. Sea-level (ISA) atmospheric conditions are used. The incidence angle σ and the roll angle λ are 15.0 deg and 2.5 deg respectively. The Mach number is 1.4 and the Reynolds number based on the store length is $Re_{L_{ref}} = 112.47 \times 10^6$. The velocity components for different σ and λ values are listed below:

$$\mathbf{V}_{ref} = (u, v, w)^T V_{ref} = \begin{cases} (0.9659, 0.2586, 0.0113)^T V_{ref} & \text{for } \sigma = 15 \text{ and } \lambda = 2.5 \text{ deg} \\ (0.9659, 0.2588, 0)^T V_{ref} & \text{for } \sigma = 15 \text{ and } \lambda = 0 \text{ deg} \\ (0.9962, 0.0872, 0)^T V_{ref} & \text{for } \sigma = 5 \text{ and } \lambda = 0 \text{ deg} \end{cases} \quad (12)$$

A. Numerical domain and grids

ICEM Hexa [27] was used to create high-quality multi-block structured grids. Two grids were created for each level of grid refinement - one for the store and one for the domain around the store. The grids were combined with the overset (Chimera) method. All grids exhibited symmetry across the x - z and x - y planes. The blocking topology of the configuration C grid is shown in figure 4.

Fig. 3 Multiblock topology used for configuration B.

The grid topology varied slightly for each configuration. Four levels of grid refinement were considered for each configuration. This resulted in 12 store grids and 12 background grids - 8 grids per configuration, 24 grids in total. Table 1 lists the grid parameters for all store configurations.

Fig. 4 Configuration B grid slices

Table 1 Grid parameters; all dimensional quantities are given in millimetres; * as a fraction of d_{ref} .

Grid	Parameter	Fine grid	Very fine grid	Extra-fine grid	Super-fine grid
Configuration A	Domain size* $W \times H \times L$	80.5x46x46	80.5x46x46	80.5x46x46	80.5x46x46
	Wake length*	4.6	4.6	4.6	4.6
	Density radius*	1.54	1.54	1.54	1.54
	Total cells* $\times 10^6$	26.8	69.2	113.5	152.3
	Total nodes* $\times 10^6$	29.9	75.0	122.0	162.5
	First layer height $\times 10^{-3}$	3.45	3.45	3.45	3.45
Configuration B	Domain size* $W \times H \times L$	80.5x46x46	80.5x46x46	80.5x46x46	80.5x46x46
	Wake length*	3.0	3.0	3.0	3.0
	Density radius*	1.8333	1.83	1.83	1.83
	Total cells* $\times 10^6$	43.1	80.0	127.8	190.7
	Total nodes* $\times 10^6$	47.2	86.4	136.0	201.5
	First layer height $\times 10^{-3}$	3.45	3.45	3.45	3.45
Configuration C	Domain size* $W \times H \times L$	80.5x46x46	80.5x46x46	80.5x46x46	-
	Wake length*	3.0	3.0	3.0	-
	Density radius*	1.83	1.83	1.83	-
	Total cells* $\times 10^6$	59.5	104.05	173.30	-
	Total nodes* $\times 10^6$	64.8	111.76	184.07	-
	First layer height $\times 10^{-3}$	3.45	3.45	3.45	-

Figure 4 shows the cell volume at $x/L_{ref} = 0.5$ of the fine, very fine, and extra-fine configuration B grids. The location of the Chimera interface was determined by the position of the vortices generated by the store. For both the $\sigma = 5$ deg and $\sigma = 15$ deg incidence angle the vortices created by the store remained within the store attached grid.

Simulations of all configurations were performed with the $k-\omega$ SST turbulence model at $\sigma = 15$ deg. In addition, the effects of the SST, $k-\omega$ EARSIM, and $k-\omega$ SST QCR non-linear turbulence models on the predicted flowfield and convergence were investigated. All simulations employed the Osher Riemann solver and used the MUSCL approach to achieve high-order accuracy in space. Solutions were performed implicitly at a CFL of 2.0 for approximately 50000 iterations. All simulations were initialised with free-stream conditions everywhere in the domain. To allow the solution to develop without any numerical instabilities, the solution was obtained explicitly for the first few hundred iterations. Such an approach is not uncommon when initialising supersonic simulations. Simulations with non-linear models were initialised with the results from the $k-\omega$ SST simulations and were run to convergence for an additional 70k iterations. The unsteady SAS simulations were also initialised with the results from the steady $k-\omega$ SST simulations. Table 2 lists the simulation parameters and the obtained force and moment coefficients.

Table 2 Simulation parameters and obtained force and moment coefficients.

Configuration	Grid	σ	λ	C_X	C_Y	C_Z	C_l	C_m	C_n	Model
A	Fine	15.0	2.5	-0.9352	-0.7033	-6.1863	-0.0298	-4.5706	2.1727	k- ω SST
	Very fine	15.0	2.5	-0.9220	-0.7169	-6.2043	-0.0203	-4.8504	2.1703	
	Extra-fine	15.0	2.5	-0.9232	-0.7340	-6.1096	-0.0160	-4.1133	2.3312	
	Super-fine	15.0	2.5	-0.9169	-0.7400	-6.1096	-0.0146	-4.1280	2.3572	
A	Fine	15.0	2.5	-0.8051	-0.7722	-5.9478	0.3485	-2.3008	2.4724	k- ω SST SAS
	Very fine	15.0	2.5	-0.8319	-0.7854	-6.0457	0.4094	-3.2320	2.4820	
	Extra-fine	15.0	2.5	-0.8418	-0.7843	-5.9473	0.5308	-2.4383	1.9510	
	Super-fine	15.0	2.5	-0.8493	-0.7222	-5.9473	0.5308	-2.4383	1.9510	
B	Fine	5.0	0	-0.6086	0.0003	-0.3519	0.0000	2.1209	0.0023	k- ω SST
	Very fine	5.0	0	-0.6076	-0.0000	-0.3369	0.0000	2.0864	-0.0005	
	Extra-fine	5.0	0	-0.6065	-0.0000	-0.3268	-0.0000	2.0784	-0.0004	
	Super-fine	5.0	0	-0.6071	0.0001	-0.3244	-0.0000	2.0570	0.0001	
B	Very fine	15.0	0	-0.6924	-0.0000	-1.6731	0.0000	7.5781	-0.0004	k- ω SST
	Extra-fine	15.0	0	-0.6896	-0.0001	-1.6517	-0.0000	7.5627	0.0006	
	Super-fine	15.0	0	-0.6888	-0.0008	-1.6401	-0.0000	7.5152	-0.0020	
B	Very fine	15.0	0	-0.5973	-0.0002	-1.7324	-0.0000	7.0560	-0.0014	k- ω EARSM
	Extra-fine	15.0	0	-0.5957	0.0002	-1.7265	-0.0000	7.0934	-0.0005	
	Super-fine	15.0	0	-0.5947	0.0005	-1.7258	-0.0000	7.0925	-0.0045	
B	Extra-fine	15.0	0	-0.6849	-0.0009	-1.6986	-0.0000	7.4905	-0.0027	k- ω SST QCR
C	Fine	15.0	2.5	-0.7831	-0.5884	-4.9706	-0.0188	8.0600	0.9271	k- ω SST
	Very fine	15.0	2.5	-0.7805	-0.6092	-4.9826	-0.0163	7.9106	1.0714	
	Extra-fine	15.0	2.5	-0.7770	-0.6194	-4.9438	-0.0113	7.9829	1.2198	

IV. Results and discussion

A. Configuration A

First simulations of configuration A were performed on all grids with the k- ω SST and SAS turbulence models. Here, only results from the k- ω SST SAS simulations will be discussed as turbulence model investigations showed that the k- ω SST simulations result in large values of μ_t/μ which suppresses the formation of vortices around the missile. Figure 5 shows the stagnation pressure contours p_0/p_{ref} at two streamwise locations ($x/d_{ref} = 14$ and $x/d_{ref} = 21.5$) for the very fine, extra-fine, and super-fine grids. The contours qualitatively agree, however, there are noticeable differences in the location of the vortices at $x/d_{ref} = 21.5$. The port and starboard vortices are located closer to the surface of fins 3 and 4 on the super-fine grid. The movement of the vortices was observed to affect the pressure distribution and consequently the loads and the convergence. From the component loads breakdown, it was observed that the largest differences in the component loads come from fins 3 and 4. This shows that the position of the vortices near fins 3 and 4 and their proximity to the surface of the fins can affect the loads significantly. Noticeable differences in the component loads between grids were also observed for wings 1 and 2. The differences were caused by differences in the location of the same vortices that interact with fins 3 and 4. These vortices are created by the leading edges of wings 1 and 2 which are positioned on the windward side of the missile. The variation of the forces for each component between grids was observed to be considerably smaller than the variation of the moments.

Effect of turbulence models

The k- ω SST, k- ω EARSM, and k- ω SST SAS turbulence models were compared on the fine grid. The stagnation pressure, p_0/p_{ref} , and eddy viscosity to molecular viscosity ratio μ_t/μ at $x/d_{ref} = 21.5$ were compared. Figure 6 shows the comparison of the stagnation pressure obtained with the three models. The stagnation pressure contours are saturated to 0.5 and 1.0 for better visualisation of the vortex cores. The k- ω SST predicts fewer vortex structures than the k- ω SST SAS and the k- ω EARSM models. The vortical structures are not as pronounced as the ones predicted by

Fig. 5 Stagnation pressure for the $k-\omega$ SST SAS* model on the very fine (top), extra-fine (middle), and super-fine (bottom) grids at $x/d_{ref} = 14$ (left) and $x/d_{ref} = 21.5$ (right); * - instantaneous quantities.

the latter turbulence models. Both the $k-\omega$ SAS and the $k-\omega$ EARSMS predict two strong vortices at the vicinity of fins 3 and 4. The presence of these strong vortices affects the pressure distribution over the tail which consequently affects the loads. The rolling moment predicted by the $k-\omega$ SST is very small compared to the one predicted by the $k-\omega$ SAS and $k-\omega$ EARSMS models due to the inability of the model to predict these vortices. Figure 7 shows the comparison of the modelled eddy to molecular viscosity ratio (RET). The contours are saturated to 0 and 2000 for better visualisation. At $x/d_{ref} = 21.5$ location the $k-\omega$ SST model predicts high values for the μ_t/μ ratio.

Fig. 6 Stagnation pressure for the $k-\omega$ SST (left), the $k-\omega$ EARSM (middle), and the $k-\omega$ SST SAS* (right) models on the very fine grid at $x/d_{ref} = 21.5$; * - instantaneous quantities.

High values of μ_t/μ prevent the development of instabilities in the flow, hence less vortical structures are observed in the $k-\omega$ SST solution. The modelled turbulent eddy viscosity μ_t is reduced in the $k-\omega$ SAS simulations due to the increase of the Q_{SAS} term in the ω -equation. Due to the lower eddy viscosity flow structures can develop and this can result in a complex flowfield. The $k-\omega$ EARSM model also predicts lower values for μ_t which results in more vortical structures in agreement with the $k-\omega$ SAS solution.

Fig. 7 RET for the $k-\omega$ SST (left), the $k-\omega$ EARSM (middle), and the $k-\omega$ SST SAS* (right) models on the very fine grid at $x/d_{ref} = 21.5$; * - instantaneous quantities.

Figure 8 compares the trajectory of the wing 1 vortex for the $k-\omega$ SST, $k-\omega$ EARSM, and the $k-\omega$ SST SAS models. The vortex trajectories were obtained with the λ_2 method. The dependency of the vortex trajectory on the turbulence model is clearly visible. Both the $k-\omega$ EARSM and the $k-\omega$ SST SAS models predict the wing 1 vortex to pass below fin 4. In addition, the difference between the $k-\omega$ EARSM and the $k-\omega$ SST SAS vortex trajectories in the vicinity of fin 4 contributes to the difficulty in obtaining grid converged results.

Fig. 8 Trajectory of wing 1 vortex for the $k-\omega$ SST (left), the $k-\omega$ EARSIM (middle), and the $k-\omega$ SST SAS (right) models on the very fine grid.

B. Configuration B

To investigate the the lack of mesh convergence of configuration A, simulations of configuration B at an incidence angle of $\sigma = 5$ deg were performed. Figure 9 shows the iteration history of the aerodynamic loads. The large difference in C_Z between the fine, very fine, and extra-fine grid shows that grids of the same size as the very fine grid or larger must be used to accurately predict C_Z . A further increase of the grid size by 65.32×10^6 points reduced the difference in C_Z significantly. The C_X , C_Z , and C_m aerodynamic coefficients were observed to settle after approximately 30000 iterations and exhibited monotonicity i.e. all values were increasing or decreasing as the grid was refined. A total of 50k iterations were performed for the C_X , C_Z , and C_m to show iterative convergence.

Fig. 9 Aerodynamic loads for configuration B resulting from pressure and viscous contributions at $\sigma = 5$ deg

The viscous contributions to the C_m and C_Z aerodynamic loads exhibited a non-monotonic behaviour. The value of C_m on the very fine grid was less than the value on the extra-fine grid, which was greater than the value on the super-fine grid. Even though the viscous values for C_Z and C_m were non-monotonic the total values were, due to the small contribution of the viscous values. The difference in the total C_m value between the super-fine and extra-fine grid was greater than between the extra-fine and very fine grids. One would expect the difference in all aerodynamic coefficients to decrease as the grid is refined (C_Z is such an example). Further analysis of the C_m values per component (store body and base) and per contribution (pressure and viscous) showed that the difference in C_m originates from the store body. Figure 10 shows the stagnation pressure contours p_0/p_{ref} and the TKE contours k/V_{ref}^2 at the $x/d_{ref} = 21.5$

streamwise location (base). Only the very fine, extra-fine, and super-fine clean grids are shown. The fine grid is omitted due to the large difference in the predicted C_Z value.

Fig. 10 Stagnation pressure (left) and TKE (right) for the $k-\omega$ SST model on the very fine (top), extra-fine (middle), and super-fine (bottom) grids at $x/d_{ref} = 21.5$.

No significant differences in the stagnation pressure and TKE contours were observed between the three grids. It is, therefore, hard to identify the cause for the differences in the aerodynamic loads solely from the stagnation or TKE contour plots. A better comparison is shown in figure 11. The figure shows the difference in pressure coefficient between the four grids.

Fig. 11 Difference in pressure coefficient ΔC_p between the fine and very fine (left), the very fine and extra-fine (middle) and extra-fine and super-fine (right).

The largest ΔC_p was observed near the nose of the store, downstream differences in C_p were insignificant. As the grid was refined, ΔC_p reduced, however, it was still identifiable. In addition, pressure contours between the three grids were compared directly by overlaying them on top of each other. In the area of the nose, the pressure contours showed good agreement between the three grids. Downstream the nose, however, noticeable differences were observed. The low-pressure region on the leeward side of the store body, extending to the port and starboard sides changed its shape slightly as the grid was refined. A change in the pressure region on the windward side was also observed. Since the pressure distribution on the windward and leeward sides is the main contributor to the pitching moment, differences in the pressure distribution in these areas between grids are expected to affect the pitching moment. The pressure contours on the base showed no significant differences, further supporting the component analysis which showed that the base is not contributing to the differences in C_m . Comparing the windward and leeward streamwise C_p distribution showed small changes in C_p as the grid was refined. At $x/L_{ref} = -0.4$ the super-fine grid gave a slightly higher C_p than the extra-fine grid. This difference may be the cause for the difference in the pitching moment coefficient C_m between the extra-fine and super-fine grids.

Configuration B at high incidence angle

Similar observations for configuration B were made at $\sigma = 15$ deg incidence angle. The aerodynamic loads exhibited monotonic behaviour. The change in the pitching moment coefficient between the extra-fine and super-fine grids was again slightly larger than the change between the very fine and extra-fine grids. The surface pressure contours and the vortex trajectories agreed well between the grids. Figure 12 shows the stagnation pressure p_0/p_{ref} and the turbulent kinetic energy k/V_{ref}^2 contours at the $x/d_{ref} = 21.5$ streamwise location (the store base).

Non-linear models on configuration B

The effect of the $k-\omega$ EARSIM and the $k-\omega$ SST QCR V1 models on the loads was investigated. The $k-\omega$ EARSIM simulation was restarted from the $k-\omega$ SST simulation and run for additional 70000 iterations. A large number of iterations was necessary for the loads to settle. The largest difference was in the C_X coefficient - approximately 13.4% relative to the $k-\omega$ SST simulation. The differences in C_Z and C_m were approximately 4.6% and 6.3%. Figure 13 shows a side-by-side comparison of the stagnation pressure contours predicted by the $k-\omega$ SST and $k-\omega$ EARSIM models. The linear $k-\omega$ SST model predicts higher values for μ_t/μ which reduces unsteadiness in the flow and results in fewer vortical structures. The $k-\omega$ EARSIM model, which considers higher-order strain and rotation terms, reduces the μ_t/μ values and results in more pronounced vortical structures with more laminar cores. Unlike the $k-\omega$ SST model, the values of k/V_{ref}^2 at the vortex cores are small. The $k-\omega$ SST QCR V1 was tested only on the extra-fine grid. No significant differences in the loads or in the flow topology were observed when compared to the $k-\omega$ SST. Differences between the two models are expected to occur with the inclusion of wings and/or fins as the right angles between the fins and/or wings and the body give rise to secondary flows which will be accounted for by the $k-\omega$ SST QCR V1 model only.

Unsteady simulations of configuration B

Unsteady RANS simulations of the body at $\sigma = 15$ degrees were also performed to identify unsteadiness if any. A non-dimensional time step of $\Delta t = 0.01$ was used which corresponded to a dimensional time step of 7.24×10^{-5} s based on L_{ref} and 3.15×10^{-6} s based on d_{ref} . A total of 6 flow travel times were simulated where 1 flow travel

Fig. 12 Stagnation pressure (left) and TKE (right) for the $k-\omega$ SST model on the very fine (top), extra-fine (middle), and super-fine (bottom) grids at $x/d_{ref} = 21.5$.

time was the time it took the flow to travel the length of the body. Averaging was performed over different intervals, however, no significant differences in the aerodynamic coefficients were observed as long as the first travel time was discarded during the averaging. No significant differences in the aerodynamic coefficients were observed. Table 3 lists the averaged aerodynamic coefficients.

Fig. 13 Stagnation pressure contours for the $k-\omega$ SST model (left) and the $k-\omega$ SST model (right) at $x/d_{ref} = 7.5$, $x/d_{ref} = 12.5$, and $x/d_{ref} = 19$.

C. Configuration C

Figure 14 shows the iteration history of the aerodynamic loads. The total loads, including pressure and viscous contributions, and the pressure and viscous only loads were considered. For all grids the loads begin to stabilise after the 30000th iteration, however, results are compared at the final - 50000th iteration. All of the loads apart from C_Z and C_m exhibit monotonic behaviour. The largest difference is observed in the C_Z aerodynamic coefficient. By examining the pressure and viscous loads separately it was observed that the non-monotonicity originates from the pressure loads.

Fig. 14 Aerodynamic loads for configuration C resulting from pressure and viscous contributions at $\sigma = 15$ deg

To further investigate the origin of the non-monotonicity in C_Z and C_m caused by the pressure loads, figure 15 shows the stagnation pressure p_0/p_{ref} and the TKE contours k/V_{ref}^2 at the $x/d_{ref} = 12.5$ streamwise location. The cores of the vortices generated by the windward wings were stronger on the very fine and extra-fine grids, however, no significant differences in the vortex topology from the p_0/p_{ref} contours were observed between the grids. Similarly, the TKE contours show little difference between the three grids. It is generally hard to determine the origin of the differences in the aerodynamic coefficients from the stagnation or TKE contour plots alone. A better comparison is shown in figure 16. The figure compares the difference in pressure coefficient between the three grids. Red corresponds to areas of large negative change and blue corresponds to areas of large positive change in C_p . Such comparison often allows the origin of the differences in the aerodynamic coefficients to be determined. Large differences in C_p between the grids were observed near the leading edges and between the wings. Refining the grid reduced the differences, however, the locations of the largest differences remained the same. Since the difference in C_p between the grids was obtained with an interpolation method, the method itself might have introduced additional unwanted uncertainty. To reduce the uncertainty, the surface pressure contours were compared between the three grids directly. The pressure contours agreed well between grids. Only slight differences were observed in the extra-fine grid. To further investigate the origin of the differences in C_Z and C_m between the grids, the streamwise distribution of the aerodynamic coefficients was considered. The distribution was obtained by taking approximately 225 equally spaced slices. The aerodynamic coefficients for each slice were obtained from the pressure distribution by integration. Figure 17 shows the streamwise variation of the aerodynamic coefficients. No significant differences were observed in the streamwise variation of the C_X , C_m , C_Z aerodynamic coefficients. More significant differences were observed in the C_Y and C_n aerodynamic coefficients at $x/L_{ref} = 0.9$. There was also a significant difference in the C_l aerodynamic coefficient at $x/L_{ref} = 0.61$. The difference in C_Z and C_m observed previously in figure 14 was not present in the streamwise variation of C_Z and C_m . Due to the equal spacing of the slices, the resolution near the leading edge of the wings is low. Larger differences in C_Z and C_m may be observed in this area if the number of slices (resolution) is increased. The λ_2 method was used to identify the vortex cores. Overall the locations of the vortex cores agreed well between the three grids.

Fig. 15 Stagnation pressure (left) and TKE (right) for the $k-\omega$ SST model on the fine (top), very fine (middle), and extra-fine (bottom) grids at $x/d_{ref} = 21.5$.

Slight differences were observed in the location where the vortices created from the leeward wings interacted with the vortices created from the body. Figure 18 shows the stagnation pressure contour slices along the store. Values of p_0 greater than 0.9 are clipped.

Fig. 16 Difference in pressure coefficient ΔC_p between the fine and very fine (left), the very fine and extra-fine (right) grids.

Fig. 17 Streamwise variation of the aerodynamic coefficients for the $k-\omega$ SST model on the fine, very fine, and extra-fine grids.

Fig. 18 Stagnation pressure contours for the $k-\omega$ SST model on the extra-fine grid.

Table 3 Unsteady simulations parameters; t is the time it takes for the flow to travel one store body length

C_X	C_Z	C_m	Averaging interval
-0.690	-1.651	7.574	2t-6t
-0.690	-1.651	7.574	3t-6t
-0.690	-1.651	7.574	4t-6t
-0.690	-1.651	7.574	5t-6t
-0.690	-1.652	7.563	RANS

V. Conclusions

The flow around a generic store geometry at a high angle of incidence was investigated. The flow features vortices that originate from the fore-body and windward wings of the store and interact with the downstream fins on the store. Predicting the interaction of these vortices with the downstream fins of the store is key to predicting accurately the aerodynamic loads acting on the store. The $k-\omega$ SST SAS simulation showed that grid convergence is difficult to obtain due to changes in the location of the vortices with respect to the store fins. Even if the location change is not very significant, the aerodynamic forces and moments of the fin are affected. The forces of each component were less sensitive to changes in the location of the vortices than the moments. Comparison of the different turbulence models and methods (statistical and scale-resolving) showed that the $k-\omega$ SST model predicts higher levels of eddy viscosity which inhibits the formation of instabilities in the flow and results in less vortical structures which leads to considerably smaller rolling moments. The $k-\omega$ EARSIM model predicted significantly smaller values of eddy viscosity and the predicted location and size of the vortices compared favourably with the scale-resolving $k-\omega$ SST SAS method. As the $k-\omega$ EARSIM model and the $k-\omega$ SST SAS method reduced the eddy viscosity, which improved the prediction of the aerodynamic loads, especially of the rolling moment, they are put forward as a suitable method for simulating the vortical flows around store configurations. The non-linear $k-\omega$ EARSIM can offer a significant reduction in computational resources compared to a scale resolving method such as the $k-\omega$ SST SAS. The favourable agreement between the statistical non-linear model and the scale resolving method showed that the prediction of the loads is not driven by an unsteadiness, but from the position of the vortices with respect to the fins. Additional simulations of simplified store geometries showed that statistical models can be used to obtain grid converged results. Some of the predicted aerodynamic loads for configurations B and C exhibited monotonic behaviour. For the latter, the differences between the aerodynamic loads were larger, compared to the geometry without wings or fins. The improved convergence for the simplified geometries showed that the difficulty in obtaining grid converged results for configuration A comes from the interaction of the wing 1 vortex with fin 4 and from the interaction of the body and wing vortices. The location of the vortices past the interaction or their merging location needs to be resolved better. Further work will focus exclusively on RANS and scale-resolving simulations of the store body with wings. Detailed investigation of the iterative convergence and the integrated loads will be performed component by component to further support that the lack of convergence for the full store is caused by the complex interaction between the vortices and the fins. This work is carried out as a part of the NATO AVT-316 activity [1, 26].

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