Quantifying systemic risk with factor copulas

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Abstract

We propose a tail dependence based network approach to study systemic risk in a network of systemically important financial institutions (SIFIs). We utilize a flexible factor copula based method which allows to measure the level of extreme risk in a portfolio when dependence is driven by one or several factors. We identify the most "connected" SIFIs based on an eigenvector centrality approach applied to copula-implied dependence structures as "central" SIFIs. We then demonstrate that the level of systemic risk implied by such SIFIs chosen as conditioning factors in the factor copula setup exceeds that which is implied by non-central SIFIs in terms of portfolio Value-at-Risk and the portfolio return under stress. This study contributes to quantification and ranking of the systemic importance of SIFIs which is important for setting adequate capital requirements in particular and stability of financial markets in general.

Key words: factor copula, network, Value-at-Risk, tail dependence, eigenvector centrality

JEL Classification: C00, C14, C50, C58

Introduction

Systemic risk is a very important aspect of economic risk and was one of the main causes in the financial crisis of 2008. It continues to be an extremely relevant topic today. An important question is how systemic risk can be quantified. The notion of systemic risk and macroprudential regulation, relevant to global financial stability and efficient functioning of the financial markets, has gained significant attention from regulators, financial analysts, and academic researchers.

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The Financial Stability Board (FSB) together with the Basel Committee on Banking Supervision (BCBS) developed a methodology to select systemically important financial institutions (SIFIs) which divides them into categories ("buckets"). Depending on the buckets, additional common equity loss charge is prescribed in terms of a percentage of risk-weighted assets. The so-called "indicator-based measurement approach" based on a number of indicators intended to capture the level of global systemic importance encompasses indicators such as bank size, total exposure, interconnectedness, substitutability, and complexity, see Basel Committee on Banking Supervision (2013). While these indicators are important, they do not necessarily reflect the global scope of the bank's operations and may suffer from arbitrary weight assignment.

We propose an approach which is intended to better capture the complexity and interdependencies of systemic risk. Extreme loss-based risk measures such as Value-at-Risk (VaR), Expected Shortfall (ES), Marginal Expected Shortfall (MES) and Conditional VaR (CoVaR) are often used to quantify systemic risk (see Diebold and Yılmaz (2014); Girardi and Tolga Ergün (2013); Banulescu and Dumitrescu (2015); Acharya et al. (2017)). Many of these research focus on the connectedness measures based on pairwise dependencies which do not account for tail dependence in a non-Gaussian setting or do not assume a network perspective.

In this study we construct copula based networks between SIFIs which capture tail risk in a robust way as well as account for connectedness. The strength of connectedness is determined by connectivity measures based on tail dependence inferred from factor and empirical copulas. The factor copula model proposed by Krupskii and Joe (2013) is able to account for a wide range of dependence types and joint distributions. Dependence in the variables of interest can be explained by a few risk factors so that the number of parameters to fit can be dramatically reduced. Instead of directly defining the dependence structure between the variables of interest, one can project the variables into one or several factors and define such a structure through these factors.

SIFIs that contribute the most to systemic risk also potentially trigger higher connectivity which leads to broader contagion or more widespread distress; they can be viewed as "central SIFIs". Adrian and Brunnermeier (2016) points out that some institutions are *individually* systemic because of their high connectedness and ability to generate negative risk spillover effects on other

institutions. At the same time, smaller institutions can be systemic as a "herd". Given their high connectedness, the central SIFIs are natural candidates for conditioning factors in the factor copula model. We study how the central SIFIs, in comparison to non-central SIFIs, trigger higher values of systemic risk when being conditioned on. Given the theoretical tail dependence matrices implied by the factor copula model, one can identify the central nodes and quantify the VaR of the portfolio (PVaR) comprising other non-central institutions conditional on the central SIFI. Centrality analysis is capable of identifying the institutions which generate the largest PVaR. In this manner, we identify the institutions which are “the most systemic among the systemic”.

Portfolio VaR is an ideal measure of systemic risk triggered by a particular SIFI. We can rank the systemic importance of the SIFIs by comparing the magnitude of their PVaR estimates. In a factor copula framework, the systemic relevance of SIFIs can be determined by the overall tail risk they trigger in the system of the SIFIs. The central SIFIs, due to their connectedness, are more likely to spread a higher amount of distress to other institutions. As a result, the tail risk of an individual SIFI or a group of SIFIs contingent on the failure of a few major SIFIs should be more severe than the tail risk incurred by non-central institutions. Such central SIFI identification is also useful for stress-testing individual institutions. An application of the proposed framework to stress-test the fragility of the system conditional on the stress of the central SIFIs is presented.

We contribute to a growing body of research on systemic risk in several aspects. Existing studies often construct a synthetic index or system used to represent a group of institutions. Then the spillover effects become based on bivariate relations (i versus system). In contrast, the factor copula approach is able to explicitly model the joint distribution of non-i SIFIs conditional on the ith SIFI to quantify the risk impact to other institutions. Furthermore, the distributional assumption behind the CoVaR framework is Gaussian, chosen because of its analytical tractability. As pointed out by Adrian and Brunnermeier (2016), the Gaussian setting results in a neat analytical solution, but its tail properties are less desirable than those of more general distributional specifications. The factor copula model is proposed for this reason, so that the marginal distributions and copula function both can be freely chosen, resulting in a more realistic joint distribution in the end. Third, we propose three types of dependence structures, and utilise them to define the

networks and the central SIFIs. We show that the network defined by the copula-implied tail dependence matrix allows to identify central SIFIs which trigger higher tail risk and distress.

The outline of this study is as follows: In Section 1, we introduce our approach to the network analysis of SIFIs. In Section 2, we introduce the factor copula theory as well as tail dependence. In Section 3, we utilise the methods outlined in the previous two sections and introduce a factor copula-based network approach which is used to estimate the PVaR and perform a stress test conditional on the identified central SIFIs. The empirical findings and discussions are provided in Section 4.

1 Network analysis of SIFIs

1.1 The description of SIFIs and their interdependencies

Thirty global SIFIs listed and updated by FSB in November 2015 are ideal samples to study systemic risk in a network framework. For this study, we disregard two SIFIs, Agricultural Bank of China and Banque Populaire CE, due to their relatively shorter data periods, and use the remaining 28 SIFIs in the period 1 January 2007 to 31 December 2014. In Table 1, we list the names of the SIFIs with the corresponding indices and symbols assigned in this research, and summarize the bank-specific attributes such as debt ratio, firm size, country where the headquarters are located and the buckets assigned by BCBS. Debt ratio, a ratio of total debt to total assets, captures the fragility of a bank, while the size – as total assets – serves as a proxy for the bank being too big to fail. Yang and Zhou (2013) investigate the characteristics of risk transfers and risk receivers, and find leverage ratio, particularly, the short-term debt ratio as the most significant determinant. The bucket in the last column is defined in Table 2 of the Basel Committee document *Global systemically important banks: updated assessment methodology and the higher loss absorbency requirement*, July 2013, which is designed to reduce the moral hazard problems and systemic risk by requiring additional common equity loss absorbency as a percentage of risk-weighted assets from 3.5% (Bucket 5), 2.5% (Bucket 4), 2.0% (Bucket 3), 1.5% (Bucket 2) to 1%

(Bucket 1).

Among the developed tail dependence, for these 28 SIFIs we particularly display in Figure 1 the empirical lower tail dependence matrices which is defined in (15) and discussed in Section 2.2. Each panel plot in this figure depicts the empirical lower tail dependence in a particular calendar year given its daily return data collected from Datastream. Consistent with Chen et al. (2019) and Demirer et al. (2017), one can observe the tail dependence that appeared in a geographic location e.g. a cluster in the U.S., the U.K., China, Europe and Japan. The yellow squares are generated by geographic dependencies, and the tail dependencies on a geographic basis are about 0.6. During the European debt crisis in 2011-2012, the European SIFIs and the U.S. SIFIs by group exhibit stronger tail dependence with more yellow color distribution.

1.2 Adjacency matrix construction

To study systemic risk in a network framework, we need a convenient mathematical representation of a network. Graph theory is very useful to represent and visualize complexity of interactions between network elements. A graph is composed of a number of nodes/vertices and the edges between nodes. In this study, each node represents a particular SIFI, while the edge between two nodes indicates their dependence. The representation is achieved via an adjacency matrix. The adjacency matrix A with elements a_{ij} for a simple undirected graph is defined as follows:

$$A = \begin{cases} a_{ij} & \text{if there's an edge between nodes } i \text{ and } j \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

where a_{ij} determine the weights of edges between $i, j = 1, \ldots, N$. For an unweighted network (all edges bear the same weight), all $a_{ij} = 1$.

The adjacency matrix can be constructed via the aforementioned dependence matrix such as the Pearson correlation matrix, the empirical tail dependence matrix or the tail dependence matrix implied by the factor copula model. Transforming dependence matrix into a binary adjacency matrix is analogous to statistical shrinkage techniques used to select the relevant variables into

the system. The statistical rationale is that the network of SIFIs is very likely to be sparse, see Bluhm et al. (2016), which means that some edges are statistically relevant but some are not. It is not advisable to take all pairwise dependencies into account if their dependencies are not beyond a certain threshold. An observation is also made by Chen et al. (2019) and Barigozzi and Brownlees (2019). The adjacency network structure in this study is based on binary weights representing the statistically significant links between the nodes, with one (zero) used to represent a strong (weak) dependence.

The method of Ng (2006) proposes a breakpoint analysis framework to partition the order dependencies into two groups. Through a uniform spacings' analysis, the problem of testing cross-section correlation/dependency is turned into a problem of testing uniformity and non-stationarity. A subset of nonzero dependencies can be determined by minimizing a sum of square residuals.

The idea of uniform spacings can be generalized to any dependence matrix as long as its elements can be assumed $U[0,1]$-distributed. To be precise, given a $N \times N$ dependence matrix p_{ij}, $i, j = 1, \ldots, N$, breakpoint determination is achieved via several steps:

1. Sort cross-sectional dependencies into an ordered vector $\mathbf{p} = (|p_1|, |p_2|, ..., |p_n|)$, where $|p_1|$ is the smallest one and $|p_n|$ is the largest one, $n = N(N-1)/2$,

2. Perform a uniform transformation of the absolute value of $\mathbf{p}$ via the standard normal cdf:

$$\mathbf{\Phi} = \left(\Phi\left(\sqrt{T}|p_1|\right), \Phi\left(\sqrt{T}|p_2|\right), \ ... \ , \Phi\left(\sqrt{T}|p_n|\right)\right), \tag{2}$$

3. Calculate the spacings $\Delta\phi_j = \Phi\left(\sqrt{T}|p_j|\right) - \Phi\left(\sqrt{T}|p_{j-1}|\right)$,

4. Perform the optimization

$$\hat{\theta} = \arg \min_{\theta \in \mathbb{R}} f_n(\theta), \tag{3}$$

where $f_n(\theta) = \sum_{j=1}^{\lceil \theta n \rceil} (\Delta\phi_j - \mu_s)^2 + \sum_{j=\lceil \theta n \rceil + 1}^{n} (\Delta\phi_j - \mu_L)^2$ and $\mu_s = \frac{1}{\lceil \theta n \rceil} \sum_{j=1}^{\lceil \theta n \rceil} \Delta\phi_j$, $\mu_L = \frac{1}{\lceil n(1-\theta) \rceil} \sum_{j=\lceil \theta n \rceil + 1}^{n} \Delta\phi_j$, $\lceil \theta n \rceil$ is the integer part of θn. T is a time horizon.

As a consequence, $\left\lfloor \widehat{\theta} n \right\rfloor$ yields an optimal break location achieving a minimum total sum of

variances from two subgroups to make the dependencies in the given group as homogeneous as possible.

Ng (2006) discusses several relationship measures as candidates for p such as the Spearman rank correlation coefficient and Kendall's τ. As long as these relationship measures are (approximately) normal, one can apply the normal transformation and obtain (approximately) uniformly distributed values, then do the uniform spacings' analysis as described above. To arrive at approximate normality for a relationship measure p, one can also use an appropriate transformation such as the Fisher's Z-transformation for the Pearson correlation coefficient. We can apply a Box-Cox type of transformation for a more general dependence measure such as the empirical tail dependence coefficient discussed below.

1.3 Eigenvector centrality

Eigenvector centrality analysis is one of means to identify the most "important" vertices (nodes) in networks. In this study, the identified central nodes potentially contribute most to overall systemic risk.

Using the adjacency matrix of a network (graph), we can track the neighbours for each node ν_i. Let $\gamma(\nu_i)$ denote node centrality and define the centrality of a node proportional to the sum of its neighbours' centralities:

$$\gamma(\nu_i) \stackrel{\text{def}}{=} \frac{1}{\lambda} \sum_{j=1}^{N} a_{j,i} \gamma(\nu_j) \tag{4}$$

where $a_{j,i}$ are the elements of the adjacency matrix A defined in (1) and λ is a fixed constant. Letting $\Gamma = (\gamma(\nu_1), \gamma(\nu_2), ..., \gamma(\nu_N))^T$ as the centrality vectors for all nodes, we can restate the above equation as

$$\lambda \Gamma = A \Gamma \tag{5}$$

Eq. (5) indicates that Γ is an eigenvector of A, and λ is the corresponding eigenvalue. In fact, if we choose to impose a positivity constraint on the centralities' vector Γ, this is the largest eigenvalue of the adjacency matrix A, and the corresponding eigenvector is the vector of network

centralities. The central nodes can be selected by ranking the elements in the selected eigenvector.

As is intuitively seen from the definition (4), the eigenvector centrality measure assigns more importance to the nodes which have either many connections to other nodes or to the nodes which are themselves important.

2 Factor copulas

2.1 General theory

Copulas in general are flexible tools for modelling multivariate distributions which allow for the separate modelling of marginal distributions and the dependence structure. Sklar's theorem proves that every multivariate distribution can be represented via the corresponding marginal distributions and a copula. This property allows construction of a wide range of dependence structures for random variables which are converted to $U(0,1)$-uniform ones. This is done to guarantee that a copula has uniform univariate marginal distributions.

Factor copula models go a step further from other copula types to address the issue of high dimensionality and polynomial-time complexity in copula parameter estimation. Given d marginal distributions, usual copula constructions (e.g., direct multivariate copulas, vines) involve estimating $\mathcal{O}(d^2)$ parameters. Factor copulas allow for parameter estimation to be done in linear time: for instance, compared to vine pair-copula models, they reduce the number of parameters to be estimated to $\mathcal{O}(d)$, see Krupskii and Joe (2013).

A general multivariate factor copula model assumes a linear dependence structure of d observed variables Z and p common factors W:

$$Z_j = \theta_{j|1} W_1 + \ldots + \theta_{j|p} W_p + \psi_j \varepsilon_j, \quad j = 1, \ldots, d. \tag{6}$$

where $1 \leq p < d$. In a one factor case, the representation (6) assumes the form:

$$Z_j = \theta_{j|1} W + \sqrt{1 - \theta_{j|1}^2} \varepsilon_j, \quad j = 1, \ldots, d. \tag{7}$$

In the factor copula model, the copula-dependent uniform random variables $u_j \stackrel{\text{def}}{=} F_{Z_j}(z_j)$, $j = 1, \ldots, d$, obtained from the marginal transformation of Z_j in $Z \stackrel{\text{def}}{=} (Z_1, \ldots, Z_d)^T$ are assumed to be conditionally independent given variable $V \stackrel{\text{def}}{=} F_W(w)$. The factor copula expression is then derived via the *mixture families* approach. Assume $p = 1$ (one-factor case), define $U \stackrel{\text{def}}{=} (U_1, \ldots, U_d)^T$, V, all $U(0,1)$, i.i.d., then:

$$\begin{aligned} C_v(u_1, \ldots, u_d) &= F(Z_1, \ldots, Z_d) \\ &= \int_D F_{Z|V}(z|v) \mathrm{d}F_V(v) \\ &= \int_D \prod_{j=1}^{d} F_{Z_j|V}(z_j|v) \mathrm{d}F_V(v) \\ &= \int_D \prod_{j=1}^{d} C_{F_Z(Z_j)|V}(F_Z(z_j)|v) \mathrm{d}v \\ &= \int_D \prod_{j=1}^{d} C_{U_j|V}(u_j|v) \mathrm{d}v, \end{aligned} \tag{8}$$

denotes a one factor copula with conditionally independent marginals $U_1, \ldots, U_d$, given the variable V; here $D \stackrel{\text{def}}{=} [0,1]$, the first and fourth equality come from Sklar theorem and uniformity, the third one from the independence assumption. Any conditional independence model given V can be expressed in this form after uniform transformation. The dependence structure of U is then defined through conditional distributions modeled by a sequence of bivariate copulas that link variables U_j to variable V.

It is worth noting that the common factor, W, can be latent (see Krupskii and Joe (2013)) or directly observed. Although postulating the common factors as latent factors is rather popular, the economic interpretation for these factors is relatively limited. Granger et al (2006) specify the common factor in conditional distribution. They suggest that the common factor as the conditioning variable should only play a role on marginal distributions but not in their conditional

copula density function, implying a conditionally independent marginals $U_1, \ldots, U_d$ given the conditioning factor. It aligns with (8) but employs an interpretable and observable common factor. As an example, Granger et al (2006) employ a business cycle indicator as the common factor in the dependence between income and consumption.

The expression (7) allows to generate different dependence structures given the distributions of W and ε. Oh and Patton (2017) demonstrate the flexibility of the class of factor copulas by choosing marginals as normal, t and Skew-t distributions to accommodate possible dependencies. However, the copulas with asymmetric and tail dependence such as double-t and Skew-t-t factor copula normally do not have closed form. In a simple example with $W = \Phi^{-1}(v)$ and ε both being $\mathsf{N}(0,1)$, the resulting copula is Gaussian. It follows that

$$C_{U_j|V}(u_j|v) = \Phi\left(\frac{\Phi^{-1}(u_j) - \theta_{j|1}\Phi^{-1}(v)}{\sqrt{1-\theta_{j|1}^2}}\right). \tag{9}$$

The resulting expression for (8) is then

$$C_w(u_1, \ldots, u_d) = \int_D \prod_{j=1}^{d} \left\{(\Phi^{-1}(u_j) - \theta_{j|1}w)/\sqrt{1-\theta_{j|1}^2}\right\} \varphi(w)\mathrm{d}w. \tag{10}$$

In general, the conditional independence formulated by the factor model, given independent uniformly distributed random variables $V = v$, U_j, takes the form

$$C_{U_j|V}(u_j|v) = F_{\varepsilon_j}\left(\frac{F_{Z_j}^{-1}(u_j) - \theta_{j|1}F_W^{-1}(v)}{\sqrt{1-\theta_{j|1}^2}}\right), \tag{11}$$

Here W, ε_j can have arbitrary continuous distributions, the distribution F_{Z_j} is obtained from the convolution of $\theta_{j|1}W$ and $\sqrt{1-\theta_{j|1}^2}\varepsilon_j$, according to the form of (7).

2.2 Tail dependence for factor copulas

Dependence of random variables can be defined via a variety of aspects such as symmetric versus asymmetric, linear versus nonlinear or tail versus entire distribution. It can be empirically measured or model-implied. Here we discuss several prevalent methods in constructing pairwise dependence.

The Pearson correlation coefficient is a measure of monotonic linear association between random variables. Given random observations x_{it} and x_{jt}, $t = 1, \ldots, T$, T is a time horizon, the sample Pearson correlation coefficient $\widehat{\rho}_{ij}$ is defined as follows:

$$\widehat{\rho}_{ij} \stackrel{\text{def}}{=} \frac{\sum_{t=1}^{T}(x_{it} - \overline{x}_i)(x_{jt} - \overline{x}_j)}{\sqrt{\sum_{t=1}^{T}(x_{it} - \overline{x}_i)^2}\sqrt{\sum_{t=1}^{T}(x_{jt} - \overline{x}_j)^2}} \tag{12}$$

Statistical dependence is determined through joint distributions. Of particular interest are extreme or tail dependencies, because they allow measuring the level of risk in the financial markets during market crashes more efficiently than association measures. Copula functions are flexible and efficient instruments which allow setting a wide range of dependency between random variables with various marginals.

Given d dimensions, a copula is a d-dimensional joint distribution with $U[0,1]$-uniform marginals. According to the Sklar's theorem, if C is a copula and $F_{X_1}, \ldots, F_{X_d}$ are continuous marginal distributions of $X_1, \ldots, X_d$, then one can uniquely construct a joint distribution $F(x_1, \ldots, x_d) = C\{F_{X_1}(x_1), \ldots, F_{X_d}(x_d)\}$. Extreme or tail dependence can be explicitly defined given a specific copula. These measures gauge the strength of dependence in the tails of a bivariate distribution.

To be precise, the coefficients of lower and upper tail dependence are defined as follows:

$$\begin{aligned}
\Lambda_{ij}^L &\stackrel{\text{def}}{=} \lim_{q\to 0^+} \mathsf{P}(X_j \le F_j^{-1}(q) | X_i \le F_i^{-1}(q)), \\
&= \lim_{q\to 0^+} \frac{\mathsf{P}(X_i \le F_i^{-1}(q), X_j \le F_j^{-1}(q))}{q}, \qquad (13) \\
\Lambda_{ij}^U &\stackrel{\text{def}}{=} \lim_{q\to 1^-} \mathsf{P}(X_j > F_j^{-1}(q) | X_i > F_i^{-1}(q)), \\
&= \lim_{q\to 1^-} \frac{\mathsf{P}(X_i > F_i^{-1}(q), X_j > F_j^{-1}(q))}{1-q}. \qquad (14)
\end{aligned}$$

The exact result for Λ_{ij}^L for the factor copula generated by the linear factor structure (7) is presented in the Appendix 7. Closed-form solutions for Λ_{ij}^L, Λ_{ij}^U will depend on the copula parameters $\theta_{j|i}$, so the maximum likelihood estimates $\widehat{\theta}_{j|i}$ will translate into the sample estimates $\widehat{\Lambda}_{ij}^L$, $\widehat{\Lambda}_{ij}^U$.

Alternatively, as proposed by Schmidt and Stadtmüller (2006), tail dependence can be estimated by means of empirical tail copulas. This allows to estimate tail dependence coefficients in a non-parametric setting. The marginal distributions are modelled using empirical distribution functions to avoid misspecification due to possible wrong parametric fit of the marginal distributions. The non-parametric estimators for (13), (14) are written as follows:

$$\widetilde{\Lambda}_{ij}^L \approx \frac{1}{k} \sum_{t=1}^{T} \mathbf{I}_{\left\{R_i^{(t)} \le k x_i, R_j^{(t)} \le k x_j\right\}}, \qquad (15)$$

$$\widetilde{\Lambda}_{ij}^U \approx \frac{1}{k} \sum_{t=1}^{T} \mathbf{I}_{\left\{R_i^{(t)} > T - k x_i, R_j^{(t)} > T - k x_j\right\}}. \qquad (16)$$

where R_i, R_j are denoted as $T \times 1$ vectors of ranks of x_{it}, x_{jt}. The parameter $k \in \{1, \ldots, T\}$ (threshold) is chosen via a plateau-finding algorithm which corresponds to balancing bias and variance. For the asymptotic results to hold, it is assumed that $k = k(T) \to \infty$ and $k/T \to 0$ as $T \to \infty$. The estimators are shown to have asymptotically normal distribution under both known and unknown marginal distributions. The details can be found in Schmidt and Stadtmüller (2006).

The economic rationale behind choosing either correlation or tail dependence matrices is subject to the risk being addressed. Correlation accounts for co-variance risk while tail dependence

aims to capture tail risk. Similar reasoning can be found in Adrian and Brunnermeier (2016). Once interdependence structure between financial variables has been defined, further analysis is necessary to determine the network structure of the underlying system. Combined with the centrality approach, tail risk network analysis can provide valuable insights into extreme risk connective structure on a systemic scale.

3 A factor copula-based network approach

To quantify the systemic risk caused by SIFI i, one can estimate the tail risk in the system conditional on SIFI i being in stress. It is worth noting that the tail risk in the system is estimated through a joint distribution specified by a factor copula framework. Ranking the estimated tail risks conditional on each SIFI achieves the goal of ranking the systemic importance among SIFIs, which determines the corresponding required level of additional loss absorbency. A factor copula-based network approach is therefore proposed for this application. It is implemented via a two-stage procedure: in the first stage we perform centrality analysis to identify the SIFIs which happen to be the central nodes; at the second stage we implement the factor copula model conditional on the identified central SIFIs to estimate the tail risk in the system and perform stress tests to central SIFIs.

3.1 Identification of central SIFIs

Which nodes can be considered as central nodes is subject to the underlying network structure. The identified central nodes contribute to the tail risk in the system if the adjacency matrix is constructed from a tail dependence matrix, whereas they may account for variance risk if the correlation/covariance matrix is the underlying adjacency matrix. We perform eigenvector centrality analysis based on the adjacency matrix A obtained from the following relationship/dependence measures:

1. the sample Pearson correlation estimator $\widehat{\rho}_{ij}$ in (12),

2. $\widetilde{\Lambda}_{ij}^{L}$, the non-parametric estimator of lower tail dependence Λ_{ij}^{L} in (15),

3. $\widehat{\Lambda}_{ij}^{L}$, the estimator of Λ_{ij}^{L} implied by the double-t factor copula model, as in (24)-(25).

The first and the second investigation are based on the eigenvector centrality analysis to make use of the adjacency matrix defined by the Pearson correlation and the empirical tail dependence, respectively. Note that a breakpoint technique by Ng (2006) is applied to convert a dependence matrix into a binary one. The third investigation uses the singular value norm as a measure of systemic risk. This is motivated by the fact that the node it is conditioned upon, is omitted from the analysis; therefore, complete eigenvector centrality analysis is inapplicable.

The singular value norm of A matrix determines the "magnitude" of A; in this study it measures the degree of systemic risk caused by the degree of "connectedness" in the financial system which is generated, e.g., by extreme tail dependence or statistical association.

The (largest) singular value norm of A is defined as:

$$\begin{aligned} \|A\| &\stackrel{\text{def}}{=} \max_{x} \|Ax\|_2 \\ \text{s.t.} \quad & \|x\|_2 = 1 \end{aligned}$$

The solution can be derived as:

$$\|A\| = \sqrt{\lambda_{\max}}, \tag{17}$$

where $\lambda_{\max}$ is the largest eigenvalue of the positive semidefinite matrix $A^\top A$. The norm of matrix A is the *maximum singular value* of A, which is also the square root of the largest eigenvalue of $A^\top A$. The central SIFI can therefore be identified if the singular value norm conditional on it is the largest one in the financial system.

3.2 Central SIFI as conditioning variable

Systemic risk should be distinguished from downturns that are caused by normal market swings, although these downturns are sometimes conflated with systemic risk, they are more appropriately

labeled as *systematic risk* or market risk (Schwarcz, 2008). Explicitly, systemic risk can be described as a risk caused by an event at the *firm level* that is severe enough to cause instability in the financial system. The collapse of Lehman Brothers in 2008 is a representative event. Lehman Brothers' size and business outreach in the global level result in a domino effect that caused a major risk to the global financial system.

To quantify institution i's systemic risk for the extent to which it can endanger the system due to its tail event in its return distribution, we control for the systematic risk in the return distribution and consider only the idiosyncratic part. By doing so, we then get a clear systemic risk measure exclusively triggered by "firm-specific risk". This design aligns with the argument from Adrian and Brunnermeier (2016) and Schwarcz (2008). Furthermore, we control for the GARCH effect in the firm-specific return. To be more specific, the mean equation controls for the market effect, while the variance equation details the volatility evolution of univariate variables in a GARCH(1,1) framework. Student's t innovation is assumed for the firm-specific residual return.

$$R_{j,t} = a_j + b_j R_{M,t} + \varepsilon_{j,t}, \quad j = 1, \ldots, d. \tag{18}$$

$$h_{j,t}^2 = \omega_j + \beta_j h_{j,t-1}^2 + \alpha_j \varepsilon_{j,t-1}^2 \tag{19}$$

$$Z_{j,t} = \sqrt{\frac{\nu_j}{h_{j,t}^2(\nu_j - 2)}}\varepsilon_{j,t} \quad \sim t_{\nu_j} \tag{20}$$

where $R_{j,t}$ is the stock return series of institution j, $R_{M,t}$ is the MSCI world market index return series. All are collected from Datastream. The residual $Z_{j,t}$ is the standardized residual return series controlled for world market return and the GARCH effect. Nevertheless, the dependence among $\mathbf{Z} = (Z_1, \ldots, Z_d)$ may not be necessarily mutually independent, especially when systemic risk emerges in the system.

The central nodes obtained from section 3.1 are the financial institutions with a higher degree of connectedness to the rest of the SIFIs. In this regard, they can be perceived as factor in the factor copula model in the distributional sense. That is, if we control for the network effect of these institutions, which may induce systemic risk, we achieve approximate conditional independence in the network. The similar concept can be found in Granger et al (2006).

Then the factor representation in (7) assumes the following form:

$$Z_j = \theta_{j|i} Z_i + \sqrt{1-\theta_{j|i}^2}\varepsilon_j, \;\; j = 1, \ldots, i-1, i+1, \ldots, d, \tag{21}$$

where Z_j are residual return series of "non-central" SIFIs from (20), d is the total number of SIFIs, i is the central node index, and $j \neq i$. Like Z_j, Z_i is estimated from (20) but represents the firm level risk of "central" SIFI. The corresponding expression for the copula in (11) is then:

$$C_{U_j|Z_i}(u_j|z_i) = F_{\varepsilon_j}\left(\frac{F_{Z_j}^{-1}(u_j) - \theta_{j|i} z_i}{\sqrt{1-\theta_{j|i}^2}}\right), \tag{22}$$

3.3 Choice of distribution

We limit our attention to the double-t factor copula in this study for the following reasons: (i) it fits financial data reasonably well, see Hull and White (2004); (ii) it allows for the construction of analytical tail dependence coefficients, see Section 2.2; (iii) in Section 3.2, the systemic risk being emphasized is the risk triggered by "firm-specific risk" after controlling for market risk. The firm-specific risk is likely to be distributed as Student-t as suggested by Oh and Patton (2018), while the market factor is Skew-t distributed. Archimedean copulas, however, allow for tail dependence but usually have only one or two parameters to characterize the dependence between all variables, which presumes a relatively homogeneous dependence and is not so favorable for a high-dimension application.

We compare our proposed factor copula models with two alternative elliptical factor copulas e.g. Gaussian and skewed-t-t in terms of goodness-of-fit measured by the Akaike information criterion (AIC) which results from maximum likelihood estimation:

- Gaussian factor copula: Z_i and ε_j are chosen as $\mathsf{N}(\mu, \sigma)$ and $\mathsf{N}(0, 1)$, respectively;
- Double-t factor copula: Z_i and ε_j are chosen as $t(\mu, \sigma, \nu)$ and $t(\nu)$, respectively;
- Skewed-t-t factor copula: Z_i and ε_j are chosen as the skewed-t distribution by Hansen

(1994) and $t(\nu)$, respectively.

As shown in Table 2, these results demonstrate that, judging by average AIC values over estimates under different SIFIs as conditioning factors, the choice of double-t model is supported. It is chosen in 5 out of 8 years, including financial crisis periods 2007-2008 and 2011-2012. In other calendar years it yields the AIC values as comparable as those from the skewed-t-t model. The skewed-t-t model is selected in 2009-2010 while the Gaussian configuration can be accepted only in 2013. The choice of the double-t factor copula model is supported in terms of goodness-of-fit analysis, and the double-t specification especially possesses desirable analytical properties in the tail measures and relative parsimony. We therefore make use of it for the empirical application in the next section.

4 Empirical results

4.1 Factor loading $\theta_{j|i}$ and tail dependence coefficients

The estimation using the observations from a non-overlapping calendar year is implemented to reflect the fact that the FSB, in consultation with the Basel Committee on Banking Supervision (BCBS) and national authorities, has identified global systemically important banks (G-SIBs) since 2011 and updated the list of G-SIBs annually, in November. This discretion allows us to parallel our identifications with those reported from FSB, in the hope of offering more quantitative insights and comprehensive analysis.[1] Other possible subperiod segmentation such as the phases of economic and financial stability can be considered because the proposed approach doesn't rest on the availability of qualitative data and is rather flexible in terms of lengthening sample period.[2]

Figure 3 shows the estimates of $\theta_{j|i}$ in each calendar year. In 2007, the European SIFIs were broadly connected with each conditioning node lying on the x-axis. In 2008 and 2009, the $\theta_{j|i}$

[1]The release of the list of G-SIBs can be found under the following link: `https://www.fsb.org/work-of-the-fsb/policy-development/addressing-sifis/global-systemically-important-financial-institutions-g-sifis/`

[2]We thank this remark pointed out from the reviewer.

estimates of U.S. SIFIs conditional on the SIFIs in the U.S. ($i = 1, \ldots, 8$) or outside the U.S ($i = 9, \ldots, 28$) are generally above 0.5. Similar findings can be seen in the European debt crisis during 2011-2012. The principal investigation is to search for the node i (in x-axis) showing widespread connectedness with the remaining j nodes (in y-axis), which is observed by the greater values of $\theta_{j|i}$. Taking 2012 as an example, one can observe that the system becomes more connected (it has more yellow grids) if we set State Street (SST, node 7), Wells Fargo (WFC, node 8) or even HSBC (node 11) as the conditioning nodes. In fact, the centrality analysis through the singular value norm of copula-implied tail dependence matrix identifies these three SIFIs as central nodes which potentially trigger a system-wide tail risk and endanger the function of the banking system.

More importantly, with the $\theta_{j|i}$ estimates, the theoretical tail dependence matrix implied by the double$-t$ factor copula defined in (24), (25) and (26) is therefore derived and shown in Figure 4. As an example, in 2012, conditional on HSBC for its central role, the copula-implied tail dependencies in European (nodes 16-21) and British (nodes 9-11) SIFIs are overwhelmingly profound; however, this is not the case if conditional on an arbitrary non-central node such as Mitsubishi UFJ (MTU). In this case, one cannot observe any tail dependence between the British and European SIFIs. Similar findings can be seen in 2007 in Figure 4. Conditional on central node NDA (node 23), a strong tail dependence between the U.S. SIFIs and the remaining SIFIs is obvious, whereas it becomes invisible conditional on a non-central node such as CCB (node 15).

To visualize the outcome from uniform spacings' analysis, given the empirical tail dependence matrices in Figure 1, Figure 2 shows the resulting adjacency matrices. The adjacency matrices vary over time. In 2007 and 2008, the Chinese SIFIs (nodes 13-15) are relatively isolated from the U.S. SIFIs (nodes 1-8); however, they turn to connect with world as of 2009. We find a lower degree of adjacency between Japanese SIFIs and others, implying that weak tail dependence might be attributed to the relatively conservative lending policies launched in Japan.

4.2 Portfolio VaR, stress test and network analysis

Given a particular central SIFI Z_i, the systemic risk in the group of the non-central SIFIs Z_j is quantified by the factor-copula-based Portfolio Value-at-Risk (PVaR) and also by the portfolio return conditional on the stress of Z_i. PVaR and stress test results are estimated according to the following algorithm:

Algorithm 1 Factor copula PVaR calculation and stress test

1: Perform univariate GARCH filtering to get Z_i and Z_j.
2: Derive uniform marginals u_j for each Z_j and v for Z_i via marginal cdf transformation.
3: Estimate copula parameters $\theta_{j|i}$ in (21) by maximum likelihood.
4: Generate copula-dependent random numbers given the estimates $\widehat{\theta}_{j|i}$ (see Algorithm 2).
5: Perform GARCH simulation of dependent residuals and calculate the PVaR as 5% or 1%-quantile of the simulated portfolio returns.
6: Perform a stress test (defined below) given a stress scenario on Z_i.

Generation of copula-dependent random numbers given the estimated factor copula parameters $\widehat{\theta}_{j|i}$ is an essential step for PVaR calculation in Algorithm 1. A straightforward procedure can be applied to simulate from a one-factor copula model. Given the number of simulated samples n_{sim} and a forecast horizon H for the PVaR, we pre-allocate a $n_{sim} \cdot H \times N$ array $\mathcal{U}$ and proceed as outlined in Algorithm 2:

Algorithm 2 One-factor copula simulation

1: **for** $i \leftarrow 1, n_{sim}$ **do**
2: Simulate $v, p_1, \ldots, p_N$ as independent $U(0,1)$-distributed random numbers.
3: Compute $u_j = C^{-1}_{U_j|V}(p_j|v; \widehat{\theta}_{j|i})$, $j = 1, \ldots, N$.
4: Return $(u_1, \ldots, u_N)$.
5: Store $(u_1, \ldots, u_N)$ in the ith row of $\mathcal{U}$.
6: **end for**

The resulting row vectors $(u_1, \ldots, u_N)$ in $\mathcal{U}$ will be a sample from the distribution $C_{z_i}(u_1, \ldots, u_N; \widehat{\theta}_{j|i})$. Copula-dependent random numbers in the second step of Algorithm 2 are determined via numeric inversion of (22) as mentioned in the previous section. Given $\mathcal{U}$, in the last step of Algorithm 1 the autocorrelation and heteroscedasticity observed in the original residual returns are re-introduced back into the copula-dependent uniform random values for PVaR calculation.

A systemic crisis is caused by a failure of one institution and the subsequent spreading of the

distress to the whole system, see Brechmann et al. (2013). In our conditional factor copula framework, the distress level in the system can be measured by the expected portfolio return conditional on the stress event of the central node. Explicitly, the portfolio stress return (SR_i) conditional on a tail event happening to the institution i can be defined as follows:

$$SR_i \stackrel{\text{def}}{=} \mathsf{E}(\omega^\top Z | v_i = 0.01), \tag{23}$$

where ω is a vector of portfolio weights. The weight on each SIFI is given by its market capitalization to account for "too-big-to-fail" implications.

The 1% quantile of the return distribution of the central SIFI serves as a proxy for a stress scenario. An advantage of the factor copula framework is that we can work directly with uniformly distributed data on the quantile level. Given a stressed situation happening to the central SIFI, we simulate the resulting impact on the remaining SIFIs: a simultaneous downside shock to the returns of the remaining SIFIs is expected. The expectation in (23) is computed via Monte-Carlo simulations. Given a shock scenario for the institution i, we simulate the shock impact on the remaining SIFIs by drawing samples from the distribution of $\mathbf{U}_{-i}|U_i = 0.01$.

Our ultimate goal is to show that the central SIFIs, in comparison with non-central SIFIs, trigger higher PVaR and stress return values when being conditioned on. To this end, we seek to verify that the centrality analysis is able to identify the institutions which would generate both the largest PVaR and stress loss. We perform PVaR and stress return calculation for every year in the sample each time making an assumption that every SIFI is potentially may cause the largest amount of systemic risk. We expect that central SIFIs have the largest impact every time. Table 3 summarizes the PVaR estimates for each calendar year, while Table 4 reports the conditional stress return of the portfolio. Obviously, each SIFI triggers tail risk of different magnitude for the portfolio consisting of the SIFIs. From 2007 to 2008, the PVaR estimates, presenting the quantile value of portfolio returns controlling market risk, increase on average from 2.107% to 3.455% at the 95% level and from 3.692% to 5.648% at 99% level on a daily basis, showing an increase in systemic risk during the U.S. subprime crisis.

The PVaR estimation results based on factor copula-based estimation and simulation are shown in Table 3. These results lead to several insights showing the advantages of the network centrality approach. First of all, in 7 out of 8 times the centrality analysis based on factor copula-implied tail dependence measures has identified the SIFIs yielding the largest 99%-level PVaR when being conditioned on. On the other hand, often it is the case that the largest PVaR values are not consistently obtained for the same institution in a given year. On the opposite side, centrality-based choices for the central nodes are consistent between PVaR levels. In this case, it is possible that copula-based centrality analysis incorporates additional information about connectedness in addition to the PVaR magnitude which gives a more holistic picture of the structure and the amount of systemic risk. As to the stress return caused by the identified central nodes' stress shown in Table 4, we find, during the crisis period (2008-2012 except for 2011), that the centrality analysis based on factor copula-implied tail dependence particularly has identified the SIFIs yielding the most negative expected portfolio return conditional on the stress event of these central nodes.

The results in Tables 3 and 4 demonstrate that the choice of the central nodes by the singular value norm of the copula-implied tail dependence matrix more often coincides with the choice made by centrality analysis performed on the empirical tail dependence matrix. This is reasonable as both of these measures gauge extreme rather than volatility risk captured by the dependence matrix based on the Pearson correlation coefficients. Although they coincidentally identify the central nodes, they are not completely identical in the sense of the information content of the network (empirical tail dependence vs. copula-implied dependence), or the centrality method (eigenvector centrality vs. the singular value norm).

4.2.1 Results from empirical tail dependence-based centrality analysis

As can be seen in Tables 3 and 4, empirical tail dependence-based centrality analysis seems capable of selecting important nodes in the network over time. Note that the central node is not exclusively unique during an investigative period, it is possible that few nodes bear very comparable centrality scores. The resulting network structures are shown in Figures 5, 6. In

2008, three financial institutions - Barclays (BCS), Standard Chartered (STAN) and BNP Paribas (BNP) are chosen as central nodes. They are conveniently identified as a group or a "cluster" on the network plot in Figure 5. Two of these institutions are British SIFIs and one of them is a French SIFI. The choice is reasonable, as Barclays was the bank that might have been expected to fail. It purchased the US broker/deal operations of Lehman Brothers after the latter's bankruptcy for almost $2bn in September 2008. Furthermore, each of the three institutions chosen had wide exposure to emerging markets, including troubled assets. More specifically, BCS, STAN and BNP took over Lehman's structured products' businesses in India.

For 2012, as shown in Figure 6, STAN and HSBC are selected as central nodes, both being British SIFIs. The banks were fined $1.9 billion and $300 million, respectively, by US authorities for their role in financial transactions involving criminals and states under US sanctions. Both institutions have historically had a very large exposure to emerging markets. In the second quarter of 2015, almost 41% of HSBC's net operating income was generated in Asia. Together with Latin America and the Middle East and Africa, these markets generated 54.81% of the bank's income, as reported by **?**. At the same time, Asia, the Middle East and Africa provided 88% of STAN's operating income and 97% of profits in the first half of 2015.

In 2013, China Construction Bank (CCB) and ICBC are chosen as central nodes, as shown in Figure 6. That year, the Chinese ICBC moved to first place in the Banker's Top 1000 World Banks, see ICBC: the world's new largest bank (2013). CCB dislodged Citigroup from fifth place with a 15% increase in capital. HSBC was ranked fourth.

4.2.2 Results from factor copula-implied tail dependence-based analysis

The proposed factor copula-based network approach generates the copula-implied tail dependence. The singular value norm based on this dependence enables us to rank the connectedness scores, and identify the more relevant ones. Methodologically, we contribute to the current literature for the methods used to quantify systemic risk, and we show how it can be built in a high-dimension domain. We shed some light on this issue and report the corresponding results. Continuing

the discussion in Table 3 and 4, in 2008 we find that JP Morgan (JPM) and Citibank (CITI) are identified as central nodes, and they indeed result in higher PVaR estimates and negatively impact, measured by SR_i, to the global banking system. In 2011 and 2012, the period of the European debt crisis, HSBC is systemically very important due to the higher tail risk and more severe stress it brings into the system. The centrality analysis from both empirical and copula-implied tail dependence indicates that HSBC, as the central node, is very likely to spread a system-wide risk to other SIFIs and destabilize the system. Accordingly, this SIFI, in terms of its systemic importance, should be highly regulated by its risk exposure and charged for additional capital buffer.

It is worth noting that Wells Fargo (WFC) has been identified as a central node since 2012, which may reflect the fact that as of the third quarter of 2011 WFC has been the largest retail mortgage lender in the U.S., amounting to $1.8 trillion in home mortgages (30% market share for U.S. mortgages). In October 2012, WFC was sued by U.S. federal attorney Preet Bharara over questionable mortgage deals. The consecutive identification in the case of WFC, through the network implied by the factor copula model, warns of a possible risk propagation by WFC. Recently, FSB committed its risk potential and upgraded its bucket bracket from 1 to 2, as shown in the 2016 G-SIBs list.

The methods proposed in Section 3.1 reveal a less-convergent identification for central SIFIs shown in Table 3 and 4. Using the lists of SIFIs and the corresponding bucket level reported by FSB during 2012-2014 in Table 5, we compare the performances of three identification methods along with the bucket approach proposed by the Basel Committee. With a focus on highly important institutions, we only report the top two buckets, namely Buckets 4 (2.5% additional capital buffer) and 3 (2.0% additional capital buffer). In 2012, Deutsche Bank (DB), identified by the eigenvector centrality of the Pearson correlation matrix, is allocated in Bucket 4. However, DB generates relatively lower PVaR estimates and milder stress than HSBC (also located in Bucket 4), identified by the empirical and copula-implied tail dependence matrix. The same observation in 2014 documents that JPM (in Bucket 4), identified by the copula-implied tail dependence matrix, indeed induces higher systemic risk than the risk from the SIFIs chosen by the other two

methods. The last three rows of Table 3 report the average PVaR values from three methods (**C** for copula tail dependence; **T** for empirical tail dependence; **P** for Pearson correlation). It shows that the central node identification through copula-implied tail dependence causes higher downside risk in the system.

In summary, the ranking based on the singular value norm of a copula-implied tail dependence matrix is more capable of identifying the SIFIs with higher systemic risk as measured by the PVaR estimates and the stress returns. It also shows a certain degree of coincidence with the bucket approach, but places more emphasis on modeling the interplay among SIFIs in order to produce a system-wide quantification. The capital buffer charge calculation based on it is supposed to be reasonable. Rather naturally, the centrality analysis based on the Pearson correlation matrix performs worse as the risk being addressed is not volatility risk but tail risk.

Conclusion

We propose a factor copula-based network approach to quantify and rank the systemic importance of 28 SIFIs selected by the Financial Stability Board and the Basel Committee of Banking Supervision. In this framework, we construct copula-implied network structures and identify the central SIFIs using centrality analysis. We use the joint distribution defined by the factor copula model to quantify the tail risk of a portfolio of the SIFIs conditioned on a predefined central SIFI. We then verify that the centrality analysis is able to identify the institutions which would generate both the largest portfolio Value-at-Risk and stress loss.

We visualize the connectedness and network structure between SIFIs resulting from dependencies defined by the Pearson correlation matrix, the empirical and the copula-implied tail dependence. The network structure resulting from the Pearson correlation matrix is able to account for variance risk and risk connectedness but is not suitable for measuring tail risk. The central SIFI selected based on this type of network is less likely to trigger tail risk contagion in the system.

The network structure implied by the factor copula model accounts for more realistic non-linear

and non-Gaussian tail dependence. It is able to do that in a tractable and computationally efficient way due to its "conditional" nature. The networks conditional on the central SIFIs appear to be more dense compared to those constructed conditioned on the non-central SIFIs.

Using the singular-value matrix norm of the copula-implied tail dependence matrix, we show that the identified central SIFI induces the largest amount of systemic tail risk and the most severe stress in the system. Accordingly, such central SIFIs, due to their systemic importance and ability to create large spillovers of tail risk, should be more closely regulated with respect to their risk exposure and capital buffers. The factor copula-based network approach can be therefore useful for regulators to quantify the connectedness of networks of financial institutions and overall tail risk conditional on specific SIFIs.

5 Tables

Table 1: Summary information on SIFIs

Index	SIFI	Firm Size	Debt Ratio	Bucket	Country
1	JP MORGAN CHASE (JPM)	21.506	0.261	4	U.S.
2	BANK OF AMERICA (BAC)	21.446	0.302	2	U.S.
3	BANK OF NEW YORK MELLON (BKM)	19.499	0.095	1	U.S.
4	CITIGROUP (CITI)	21.359	0.300	3	U.S.
5	GOLDMAN SACHS (GS)	20.624	0.509	2	U.S.
6	MORGAN STANLEY (MS)	20.501	0.417	2	U.S.
7	STATE STREET (SST)	19.106	0.153	1	U.S.
8	WELLS FARGO (WFC)	20.980	0.183	1	U.S.
9	ROYAL BANK OF SCTL (RBC)	21.588	0.252	1	U.K.
10	BARCLAYS (BCS)	21.604	0.286	3	U.K.
11	HSBC (HSBC)	21.682	0.127	4	U.K.
12	STANDARD CHARTERED (STAN)	20.136	0.187	1	U.K.
13	BANK OF CHINA (BOC)	21.200	0.160	1	China
14	ICBC (ICBC)	21.508	0.089	1	China
15	CHINA CON.BANK (CCB)	21.281	0.092	1	China
16	BNP PARIBAS (BNP)	21.684	0.136	3	France
17	CREDIT AGRICOLE (ACA)	21.489	0.211	1	France
18	SOCIETE GENERALE (GLE)	21.184	0.139	1	France
19	DEUTSCHE BANK (DB)	21.630	0.200	3	Germany
20	UNICREDIT (UCG)	20.929	0.360	1	Italy
21	ING GROEP (ING)	21.156	0.103	1	Netherlands
22	SANTANDER (SAN)	21.158	0.368	1	Spain
23	NORDEA BANK (NDA)	20.476	0.326	1	Sweden
24	CREDIT SUISSE GROUP (CS)	20.744	0.339	2	Switzerland
25	UBS GROUP (UBS)	21.008	0.251	1	Switzerland
26	MITSUBISHI UFJ (MTU)	21.533	0.159	2	Japan
27	MIZUHO (MFG)	21.247	0.233	1	Japan
28	SUMITOMO.MITSUI (SMFG)	21.044	0.125	1	Japan

* Debt ratio is defined as the ratio of total debt to total assets of a bank; and bank size is the log value of total assets; denominated in US dollars.

** Mean values during the sample period (2007-2014) are shown. The buckets assigned by BCBS correspond to required levels of additional common equity loss absorbency as percentage of risk-weighted assets from 3.5% (Bucket 5), 2.5%(Bucket 4), 2.0%(Bucket 3), 1.5%(Bucket 2) to 1%(Bucket 1)

Table 2: Computed AIC values for the double-t factor copula for SIFIs as conditioning factors

	2007	2008	2009	2010	2011	2012	2013	2014
JPM	1651.28	2045.88	1141.86	1357.16	2028.69	1513.45	1640.60	1614.30
BAC	1706.51	2031.10	1174.55	1441.01	2055.94	1580.20	1629.78	1630.56
BKM	1636.44	2074.27	1194.11	1397.11	2072.05	1523.37	860.99	1645.34
CITI	1634.90	2063.41	1325.78	1394.93	2046.94	1556.02	1636.46	1592.45
GS	1715.00	2042.24	1284.64	1382.44	2050.40	1577.14	1638.96	1554.63
MS	1684.45	2057.85	1328.22	1444.16	2029.10	1564.48	1628.97	1681.20
SST	1686.35	2087.27	1307.09	1428.15	2069.61	1463.50	1570.56	1680.90
WFC	1651.84	2028.51	1167.38	1437.59	2066.66	1533.89	1641.15	1674.39
RBC	1595.68	1849.94	1484.51	1365.52	1853.41	1463.53	845.95	1569.47
BCS	1451.79	1877.24	1498.32	1292.15	1879.69	1446.24	854.39	1484.49
HSBC	1596.56	1945.31	1512.65	1381.76	2019.95	1524.02	852.89	1657.91
STAN	1608.93	1992.46	1525.08	1384.64	2039.55	1514.48	855.66	1631.04
BOC	1640.55	2076.90	1494.07	1398.34	2071.61	1570.16	1630.85	1640.68
ICBC	1703.39	2084.07	1174.55	1427.86	2077.28	1574.48	1628.97	1635.90
CCB	1707.12	2086.74	1178.36	1402.34	2076.61	1576.92	1614.87	1668.66
BNP	1377.52	1739.24	1497.51	1337.11	1753.46	1317.70	1345.96	1440.48
ACA	1483.89	1751.81	1514.19	1346.18	1869.66	1243.30	1418.62	1428.57
GLE	1441.02	1842.23	1506.49	1341.52	1783.64	1435.82	909.70	1375.61
DB	1574.20	1892.67	1483.93	1342.50	1879.15	1459.55	890.65	1549.03
UCG	1523.50	1875.63	1496.74	1364.46	1869.26	1371.54	1447.63	1460.33
ING	1583.65	1900.56	1478.99	1363.44	1923.17	1409.35	1053.89	1493.61
SAN	1611.87	1876.42	1508.17	1356.46	1905.50	1452.59	1047.23	1455.44
NDA	1642.83	1911.01	1511.90	1352.62	2004.55	1537.59	858.64	1615.30
CS	1544.85	1838.39	1493.00	1331.33	1934.72	1373.82	1506.82	1561.10
UBS	1552.63	1833.23	1521.49	1322.45	1948.43	1424.10	846.26	1561.43
MTU	1696.56	2055.61	1496.29	1455.03	2069.47	1559.77	1632.48	1602.83
MFG	1675.29	1960.57	1475.75	1327.70	2089.85	1546.52	1618.28	1679.91
SMFG	1702.30	2085.83	1461.46	1349.79	2070.68	1550.42	1644.11	1683.69
Average double-t	1610.03	1960.94	1401.32	1375.92	1983.54	1488.00	1312.55	1581.04
Average Gaussian	1497.87	1840.38	1385.13	1253.23	1789.66	1413.78	1462.90	1484.61
Average skewed-t-t	1608.48	1938.29	1407.89	1377.42	1969.90	1481.97	1301.56	1576.41

* The last three rows show the average AIC values across SIFIs generated by double-t, Gaussian and skewed-t-t factor copulas, respectively.

** In the upper panel, AIC values estimated for copula fits given a particular SIFI as a conditioning factor are shown only for the double-t case due to space constraints.

*** The minus sign in front of the AIC values is omitted.

Table 3: Portfolio VaR estimates in double-t copula model at 99% and 95% level conditional on each SIFI

	2007		2008		2009		2010		2011		2012		2013		2014	
	PVaR95	PVaR99	PVaR95	PVaR99	PVaR95	PVaR99	PVaR95	PVaR99	PVaR95	PVaR99	PVaR95	PVaR99	PVaR95	PVaR99	PVaR95	PVaR99
JPM	1.972	3.570	3.922^{C}	6.813^{C}	4.155	7.028	2.498	3.296	2.614	4.387	2.755	5.428	2.114	3.827	3.068^{C}	4.825^{C}
BAC	1.786	3.621	3.581	5.755	3.567	6.825	2.218	4.956	3.452	5.629	3.120	5.270	2.667	3.986	2.087	3.956
BKM	1.802	3.822	3.448	6.225	3.363	5.254	1.950	4.014	2.821	4.569	2.742	5.088	1.923	3.848	1.831	3.970
CITI	2.122	3.408	4.016^{C}	7.212^{C}	4.491	7.136	2.943	4.962	2.481	3.600	2.514	3.943	2.508	4.734	2.539^{P}	4.147^{P}
GS	2.366	4.521	3.431	6.296	3.861	5.439	2.621	4.332	2.028	4.417	2.084	5.309	2.587	4.230	1.873	3.348
MS	2.549	5.095	4.260	6.206	3.790	5.801	2.440	3.743	2.375	4.207	2.477	3.953	2.307	4.591	2.446	3.912
SST	1.783	3.829	3.639	5.270	4.008	6.046	2.776	4.787	2.095	4.833	3.161^{C}	6.088^{C}	3.268	5.394	1.812	4.015
WFC	2.689	4.870	3.893	6.603	4.868	6.853	2.747	4.836	2.297	4.672	3.477^{C}	6.255^{C}	3.331^{C}	5.880^{C}	3.218^{C}	4.900^{C}
RBS	2.226	4.031	3.229	5.052	3.903	5.791	2.558	4.215	3.106	4.783	3.170	4.653	2.608	4.462	2.316	4.287
BCS	$2.256^{P,T}$	$3.616^{P,T}$	2.781^{T}	4.317^{T}	4.891	6.436	2.704	4.718	2.540	3.885	2.238	3.406	2.088^{P}	3.461^{P}	2.618	4.385
HSBC	1.347	3.211	2.765	4.165	3.414	5.422	1.821	3.312	$2.654^{T,C}$	$5.747^{T,C}$	$3.707^{T,C}$	$5.859^{T,C}$	2.258^{T}	4.040^{T}	2.217	3.878
STAN	2.352	3.742	3.852^{T}	6.195^{T}	2.719	5.026	2.434	4.003	2.114	3.919	1.806^{T}	3.641^{T}	2.824	5.028	2.366	4.237
BOC	2.455	5.029	3.327	6.344	2.470	3.499	2.645	4.272	2.430	4.710	2.053	4.143	3.107	5.151	2.051	4.075
ICBC	2.212	3.820	3.235	5.307	2.411	3.986	2.413	3.761	2.287	5.170	2.913	4.303	1.683^{T}	3.580^{T}	2.114	3.791
CCB	1.849	3.126	3.226	5.374	2.856	4.353	2.296	4.215	2.207	4.888	2.606	4.487	3.133^{T}	5.213^{T}	2.278	4.173
BNP	2.185	3.502	$4.185^{T,C}$	$6.614^{T,C}$	3.456	5.807	$3.107^{T,C}$	$5.024^{T,C}$	3.619	5.298	2.816	4.549	2.721^{P}	3.877^{P}	2.605	4.598
ACA	2.173	4.562	3.060	6.123	3.311	5.382	2.495	4.432	2.688	4.061	3.110	4.017	2.291	3.601	1.992	4.383
GLE	2.124	3.277	3.628	4.929	3.107	5.246	2.899	4.325	2.488	5.115	2.322	5.417	2.388	3.239	2.852	3.857
DB	1.651^{P}	4.352^{P}	3.863	6.567	3.069^{P}	4.363^{P}	2.527^{P}	3.704^{P}	3.563^{P}	4.767^{P}	3.255^{P}	4.589^{P}	2.631	3.880	2.800	4.247
UCG	1.741	2.568	3.280	4.894	4.014	5.934	2.594	4.167	2.978	4.378	1.678	3.053	2.752	3.917	2.216	3.938
ING	2.521	3.866	4.128	5.816	3.754	5.345	2.569	4.704	3.076	5.209	2.988	4.856	2.497	4.149	1.805	4.358
SAN	1.374	2.981	3.255	5.232	3.045^{C}	5.043^{C}	3.107	4.754	2.881	4.780	3.054	5.414	2.239	3.920	2.360^{T}	4.741^{T}
NDA	2.608^{C}	5.114^{C}	3.233	5.349	3.556	5.043	3.254	5.011	1.729	3.403	2.231	3.625	3.276	5.226	2.200	3.910
CS	1.258	2.706	3.976	5.883	3.108	5.119	2.281	3.652	1.980	5.121	2.209	5.165	2.450	4.494	2.610	4.120
UBS	1.630	3.559	2.656^{P}	4.683^{P}	3.373^{T}	4.990^{T}	2.611	4.861	2.862	4.739	3.250	4.311	2.966	4.399	2.193	4.059
MTU	2.637	2.340	2.645	4.540	2.482	4.166	2.219	4.152	1.816	2.798	2.249	3.374	2.944	4.515	2.936	4.527
MFG	2.699	2.872	3.192	4.426	2.523	4.436	3.086	4.579	3.322	4.906	3.416	5.190	3.074	4.536	2.213	3.953
SMFG	2.575	2.357	3.027	5.961	2.990	4.215	2.623	4.270	2.024	2.726	2.359	5.969	2.288	4.439	2.911	4.601
Avg of all	2.107	3.692	3.455	5.648	3.484	5.435	2.587	4.324	2.590	4.525	2.706	4.584	2.604	4.343	2.377	4.186
Avg of C	2.608	5.114	4.041	6.880	3.045	5.043	3.107	5.024	2.654	5.747	3.448	6.067	3.133	5.880	3.143	4.862
Avg of T	2.256	3.616	3.606	5.708	3.373	4.990	3.107	5.024	2.654	5.747	2.756	4.700	2.358	4.277	2.360	4.741
Avg of P	1.953	3.984	2.656	4.683	3.069	4.363	2.527	3.704	3.563	4.767	3.255	4.589	2.404	3.668	2.539	4.147

* PVaR is calculated via factor copula estimation and simulation when each time one SIFI is assumed to be the driving factor

** Superscripts P, T and C represent the central nodes identified through the Pearson correlation (P), empirical tail dependence (T) and tail matrices implied by factor copula (C), respectively.

*** For each year, the largest PVaR is marked in red

Table 4: Stress testing conditional on each SIFI

	2007	2008	2009	2010	2011	2012	2013	2014
JPM	-0.21	-1.78^{C}	0.16	-2.20	-0.12	-0.72	-0.93	-0.50^{C}
BAC	-0.32	-0.51	-0.97	-1.14	0.19	-0.24	-0.26	-0.11
BKM	-0.05	-0.63	-1.51	-0.73	0.00	-0.59	-0.67	-0.43
CITI	-0.45	-1.84^{C}	-1.08	0.91	-0.27	-0.65	-0.17	-0.34^{P}
GS	-0.75	-0.68	-0.74	-1.38	-0.22	-0.50	-0.94	-0.13
MS	-0.97	-0.51	0.34	-1.64	-0.59	-0.48	-0.62	-0.72
SST	-0.20	-0.95	-0.46	-0.94	-0.78	-0.67^{C}	-0.14	0.32
WFC	-0.64	-1.02	-0.41	-1.81	-0.56	-0.53^{C}	-0.40^{C}	-0.58^{C}
RBC	-1.35	-0.39	-1.18	-0.64	-0.79	-0.45	-0.06	-0.62
BCS	$-0.57^{P,T}$	-0.24^{T}	-2.00	2.03	-1.82	-0.57	0.10	-1.02
HSBC	-0.24	-0.38	-0.53	-1.12	$-1.79^{T,C}$	$-0.78^{T,C}$	-0.63^{T}	-0.75
STAN	-0.06	-1.52^{T}	-0.20	0.04	-0.52	0.07^{T}	-0.28	0.07
BOC	-0.80	-0.90	0.05	0.14	-0.79	-0.39	-0.21	-0.16
ICBC	-0.41	-1.11	0.27	0.13	-0.41	0.15	-0.21^{T}	-0.33
CCB	-0.82	-0.64	-0.25	-0.08	-1.01	-0.24	0.12^{T}	-0.01
BNP	0.08	$-1.99^{T,C}$	-1.41	$-2.39^{T,C}$	0.02	-0.33	-0.19^{P}	-0.50
ACA	-0.64	-1.00	-0.68	-0.44	-0.23	-0.03	-0.71	-0.74
GLE	-0.37	-1.25	-1.05	-1.72	-1.36	-0.30	-1.04	-0.34
DB	-0.47^{P}	-1.27	-0.40^{P}	-0.22^{P}	-0.73^{P}	-0.22^{P}	-0.54	-0.52
UCG	-0.18	-0.56	-0.48	-0.81	-0.32	-0.44	0.10	-0.42
ING	-1.28	-0.91	-0.68	-1.84	-0.05	-0.20	-0.74	-0.16
SAN	-0.16	-0.49	-2.49^{C}	-1.02	-0.71	-0.56	0.05	-0.36^{T}
NDA	-1.11^{C}	-0.37	-0.22	-1.85	-0.18	-0.20	-1.04	0.01
CS	-0.12	-1.89	-0.74	-1.22	-0.38	-0.12	-0.69	0.08
UBS	-0.74	-0.36^{P}	-0.79^{T}	-0.59	-1.19	-0.61	0.17	0.13
MTU	-0.84	-0.24	-0.36	0.78	0.04	0.03	-0.94	-0.06
MFG	-0.59	-0.27	0.52	0.52	-0.73	0.20	-0.99	-0.13
SMFG	-0.61	-0.84	-0.34	-0.08	-0.80	-0.04	-0.71	-0.30

* Superscript P, T and C represent the central nodes identified through the Pearson correlation matrix (P), the empirical tail dependence matrix (T) and the tail matrix implied by factor copula (C), respectively

** The expected portfolio return conditional on the stress of given SIFI is estimated through (23)

Table 5: List of SIFIs/G-SIBs from 2012 to 2014

Bucket	2012	2013	2014
4 (2.5%)	JPM (1) HSBC (11) CITI (4) DB (19)	JPM (1) HSBC (11)	JPM (1) HSBC (11)
3 (2.0%)	BCS (10) BNP (16)	BCS (10) BNP (16) CITI (4) DB (19)	BCS (10) BNP (16) CITI (4) DB (19)

* The buckets assigned by the BCBS correspond to required levels of additional common equity loss absorbency as percentage of risk-weighted assets from 3.5% (Bucket 5), 2.5%(Bucket 4), 2.0%(Bucket 3), 1.5%(Bucket 2) to 1%(Bucket 1)

6 Figures

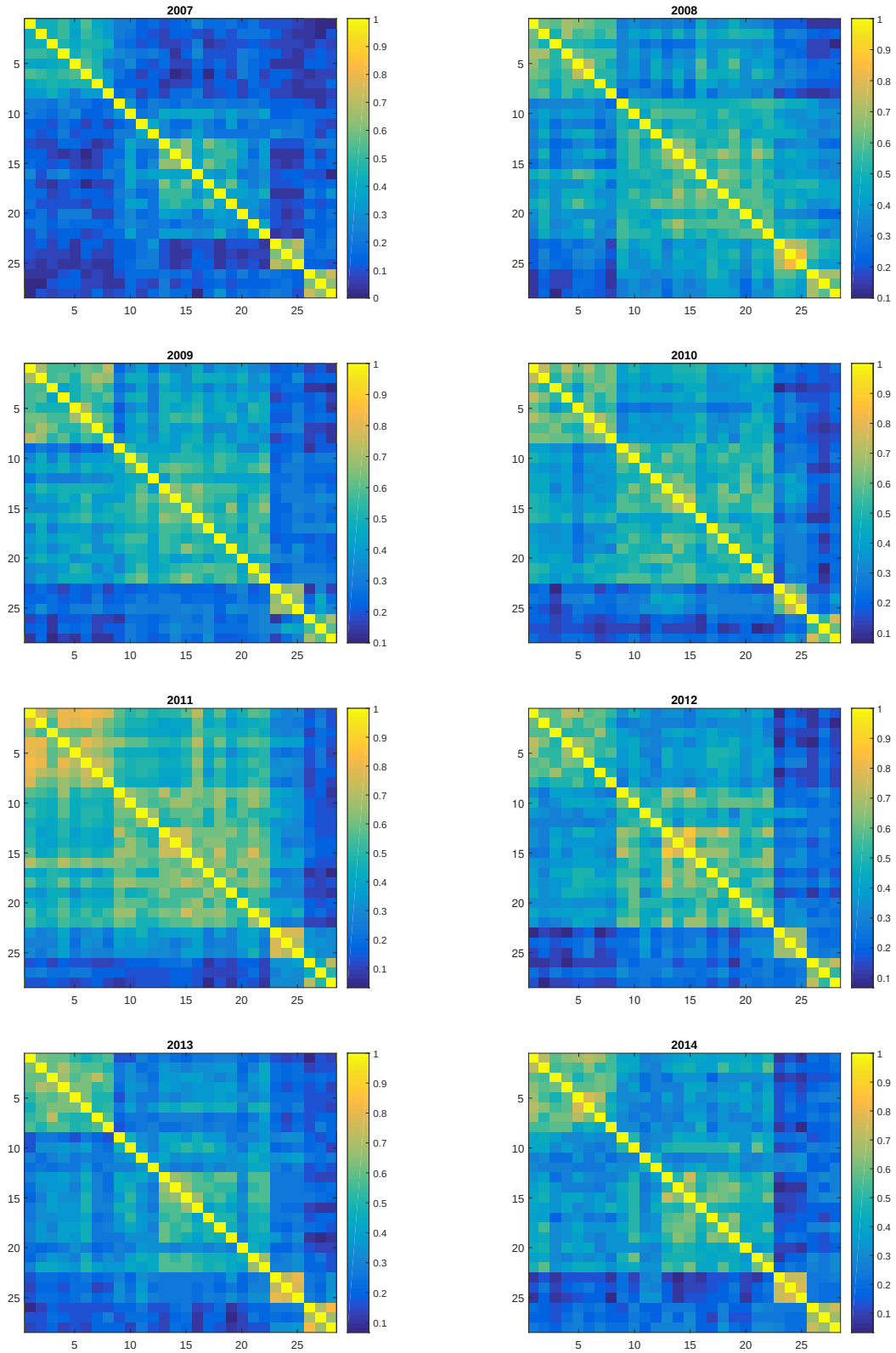

Figure 1: Empirical tail dependence matrices for 28 SIFIs

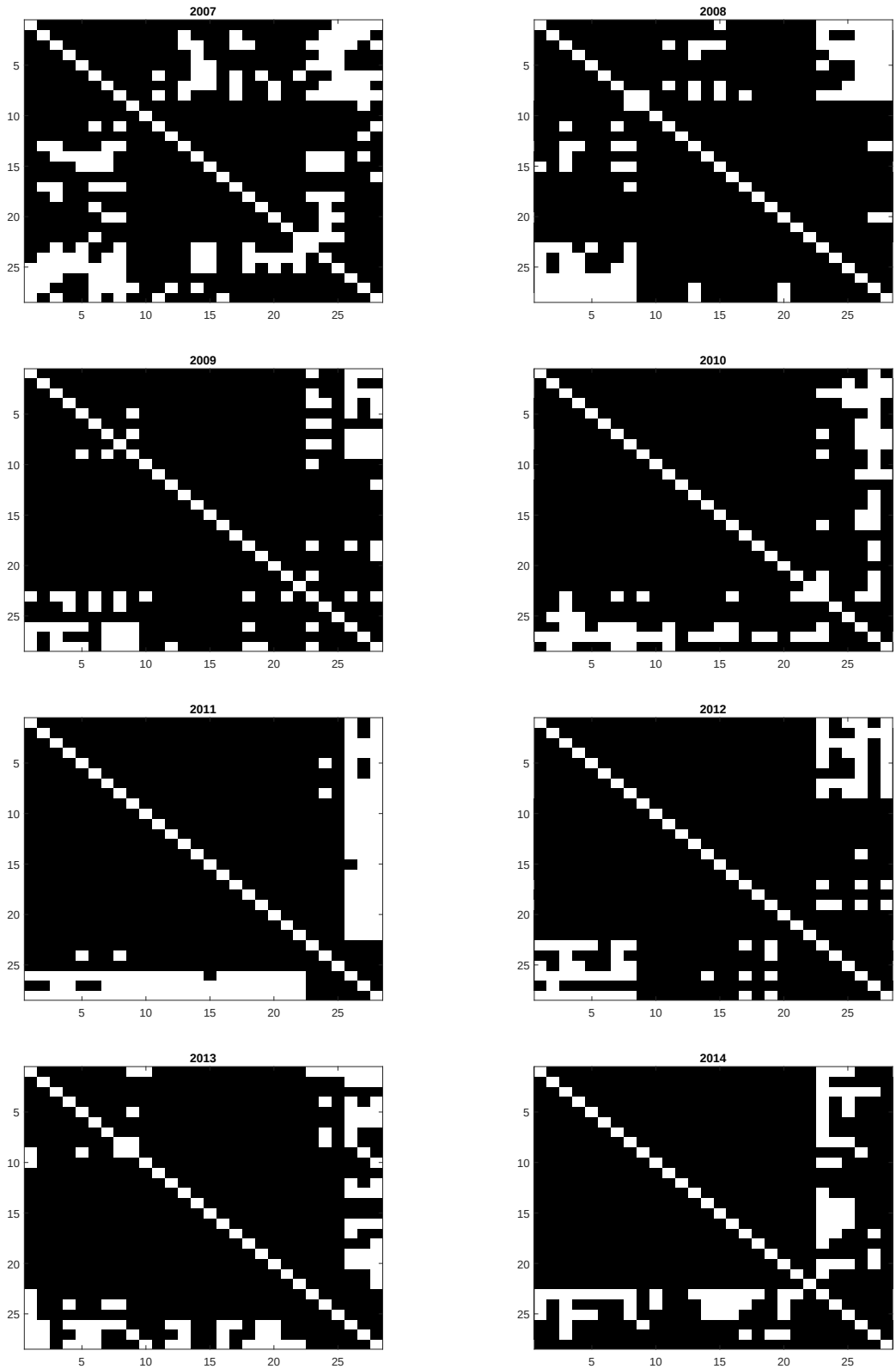

Figure 2: Binary adjacency matrices for 28 SIFIs obtained from empirical tail dependence in Figure 1 (black: 1, white: 0)

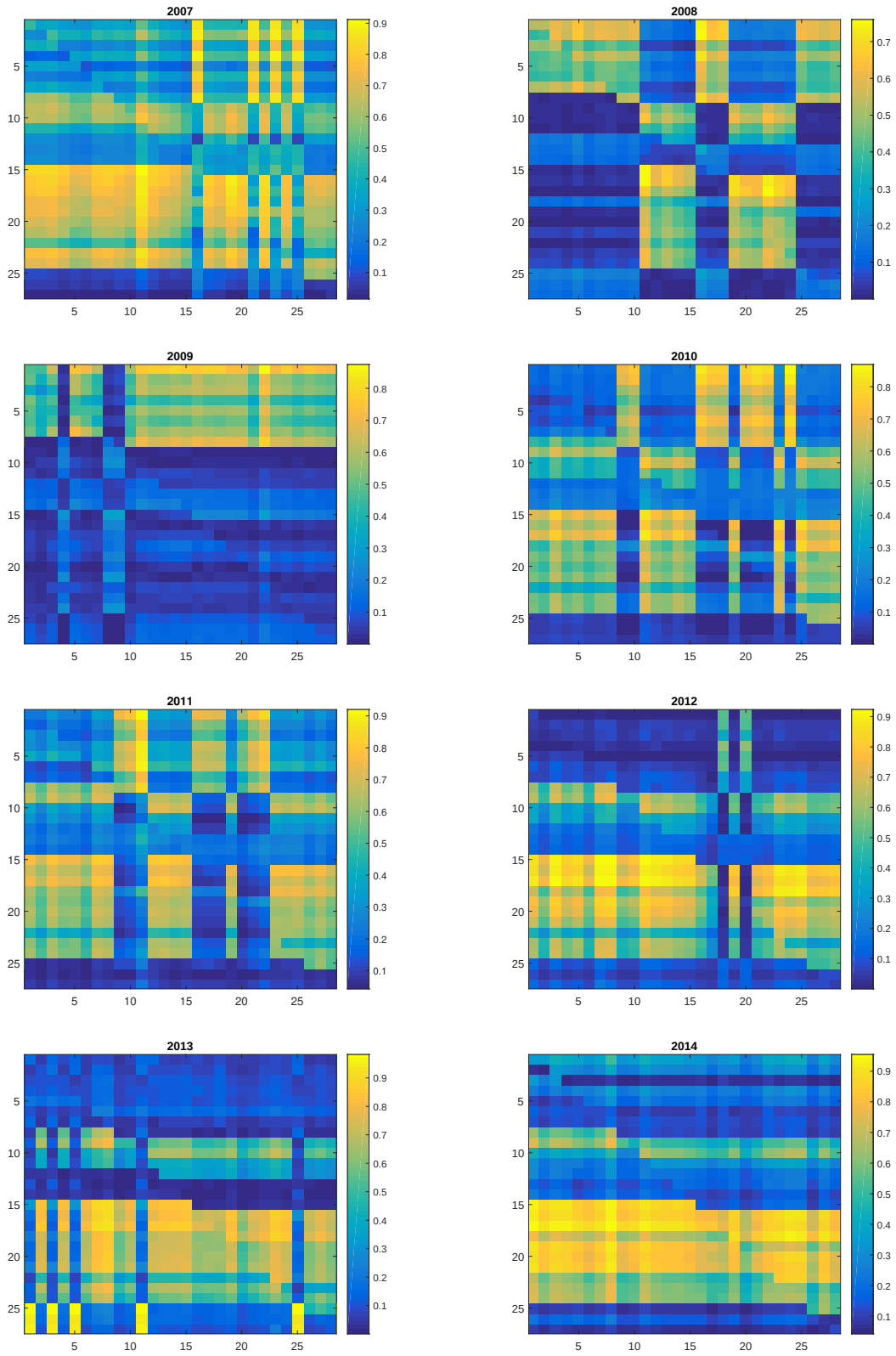

Figure 3: The estimates of $\theta_{j|i}$ in Eq. 21 (x-axis : i, y-axis : j)

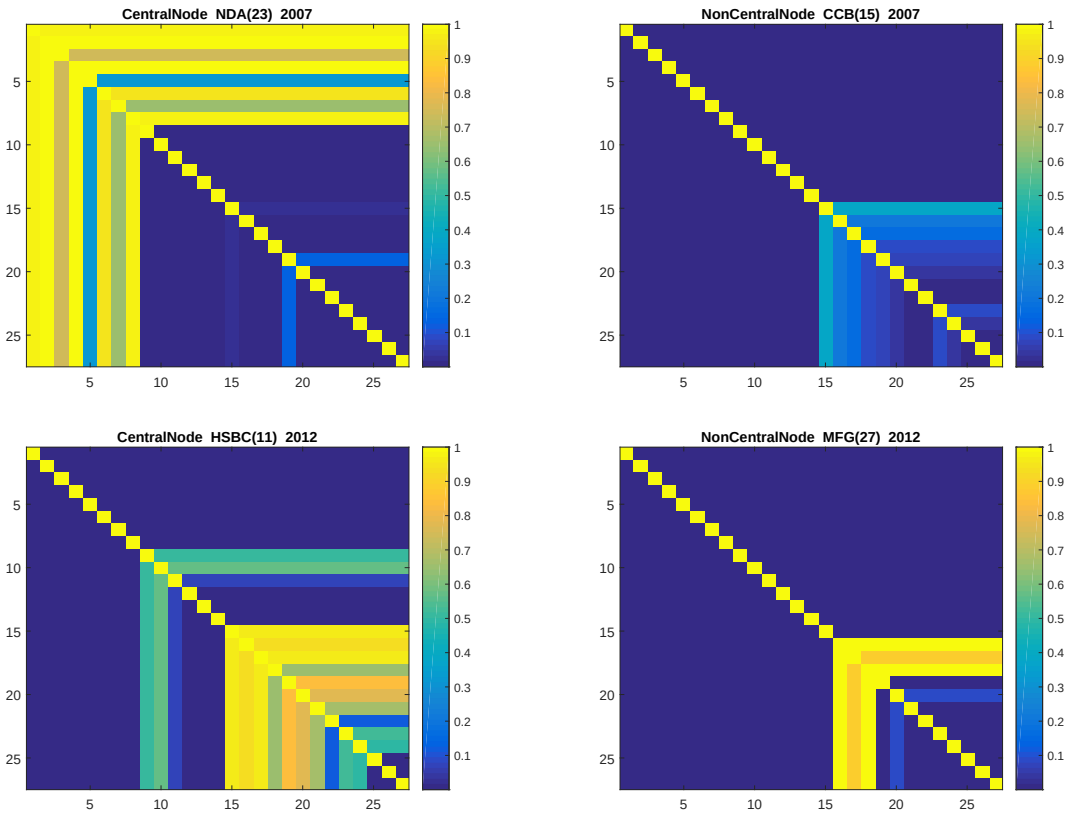

Figure 4: Factor copula implied tail dependence derived through Eqs. 24, 25 and 26

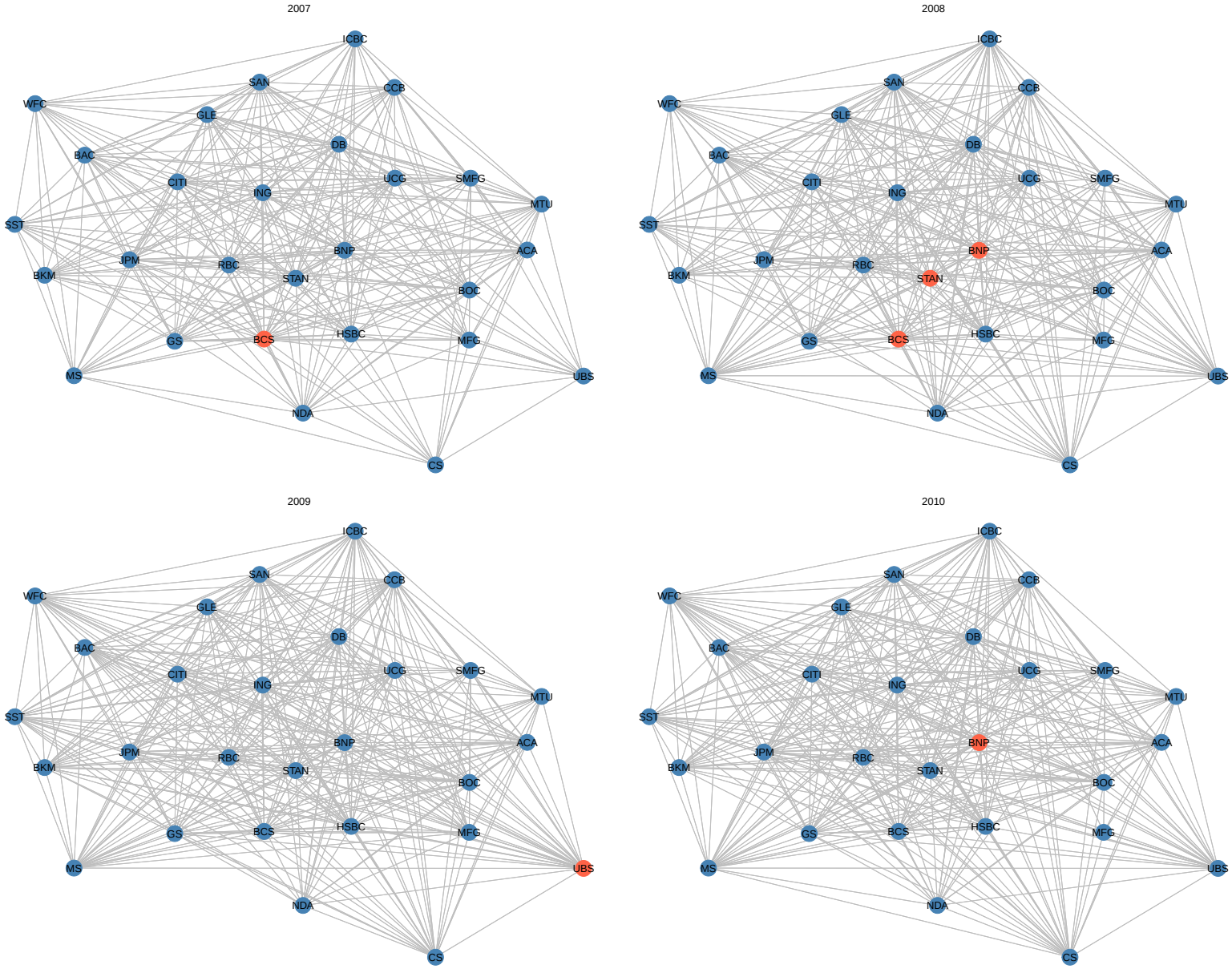

Figure 5: SIFI network structures produced by adjacency analysis on the empirical tail dependence matrix

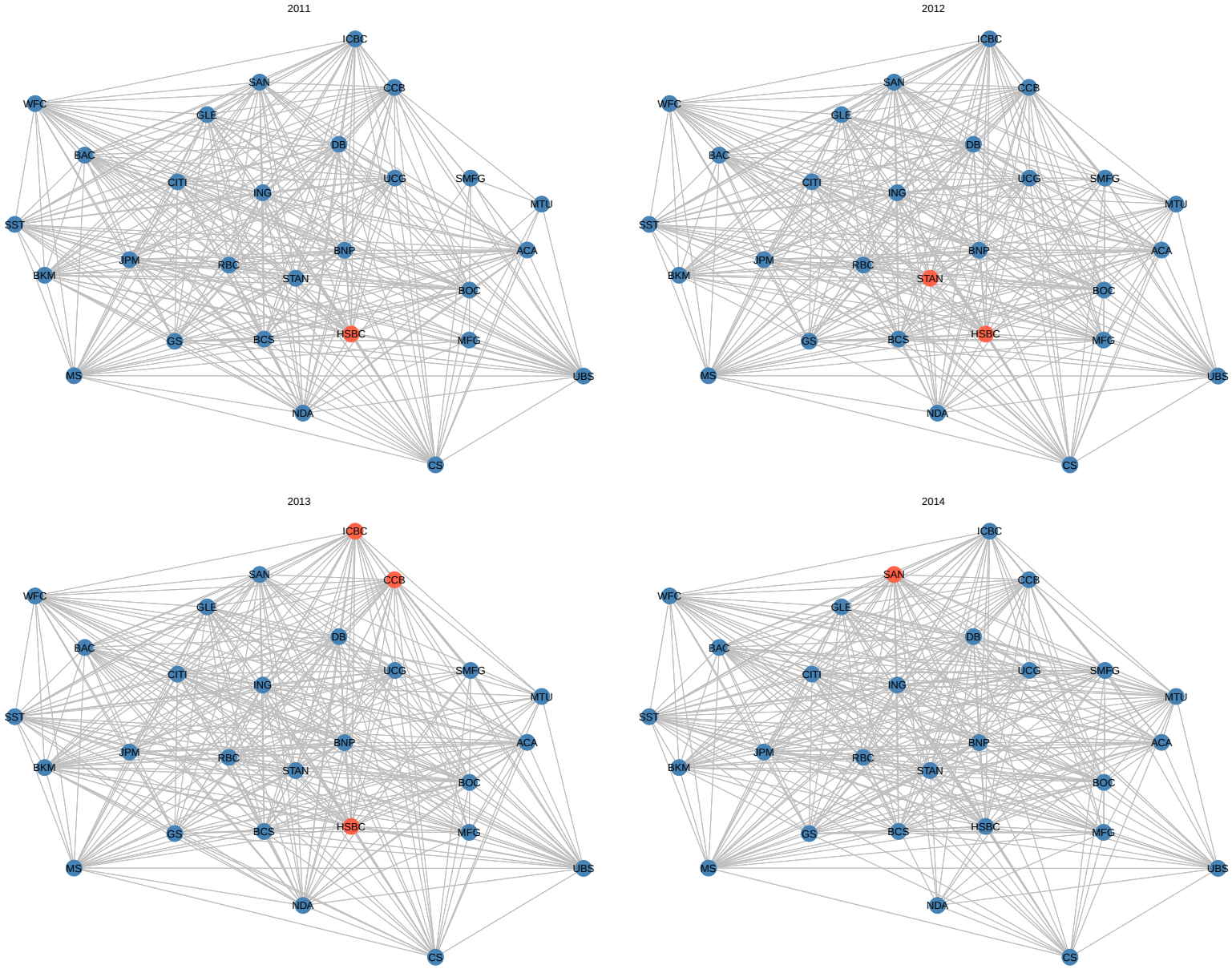

Figure 6: SIFI network structures produced by adjacency analysis on the empirical tail dependence matrix

7 Appendix

For a factor copula represented by a linear structure (7) the tail dependence coefficients in (13) and (14) can be derived in explicit form. Although factor copulas generally lack a closed-form density, using extreme value theory the analytical expression for the implied tail dependence can be therefore achieved. The implied tail dependence from factor copulas is the "conditional tail dependence", that is, it is derived given the factor W. Conditioning on the chosen factor, we can define a d-dimensional tail dependence matrix in a conditional fashion and compare it with the unconditional one. The choice of factor together with the selected copula distribution determine the tail dependence matrix.

Proposition 7.0.1. Let the factor copula be generated by the linear factor structure (7). Also let F_W and F_{ε_j} have regularly varying tails with a common tail index $\alpha > 0$ so that $\mathsf{P}(W < -s) = \mathsf{P}(W > s) = A_W s^{-\alpha}$, $\mathsf{P}(\varepsilon_j < -s) = \mathsf{P}(\varepsilon_j > s) = A_\varepsilon s^{-\alpha}$ as $s \to \infty$, $A_W > 0$, $A_\varepsilon > 0$.

Then it follows that

$$\Lambda_{ij}^L = \frac{A_W \theta_{i|1}^\alpha}{A_W \theta_{i|1}^\alpha + A_\varepsilon (1 - \theta_{i|1}^2)^{\alpha/2}} \tag{24}$$

if the following conditions hold: $A_W \theta_{i|1}^\alpha \theta_{j|1}^\alpha + A_\varepsilon (1 - \theta_{i|1}^2)^{\alpha/2} \theta_{j|1}^\alpha > A_W \theta_{j|1}^\alpha \theta_{i|1}^\alpha + A_\varepsilon (1 - \theta_{j|1}^2)^{\alpha/2} \theta_{i|1}^\alpha$ and simultaneously $\theta_{i|1} < \theta_{j|1}$ or $A_W \theta_{i|1}^\alpha \theta_{j|1}^\alpha + A_\varepsilon (1 - \theta_{i|1}^2)^{\alpha/2} \theta_{j|1}^\alpha < A_W \theta_{j|1}^\alpha \theta_{i|1}^\alpha + A_\varepsilon (1 - \theta_{j|1}^2)^{\alpha/2} \theta_{i|1}^\alpha$ and simultaneously $\theta_{i|1} > \theta_{j|1}$. On the other hand, it holds that

$$\Lambda_{ij}^L = \frac{A_W \theta_{j|1}^\alpha}{A_W \theta_{j|1}^\alpha + A_\varepsilon (1 - \theta_{j|1}^2)^{\alpha/2}} \tag{25}$$

if the following conditions hold: $A_W \theta_{i|1}^\alpha \theta_{j|1}^\alpha + A_\varepsilon (1 - \theta_{i|1}^2)^{\alpha/2} \theta_{j|1}^\alpha < A_W \theta_{j|1}^\alpha \theta_{i|1}^\alpha + A_\varepsilon (1 - \theta_{j|1}^2)^{\alpha/2} \theta_{i|1}^\alpha$ and simultaneously $\theta_{i|1} < \theta_{j|1}$ or $A_W \theta_{i|1}^\alpha \theta_{j|1}^\alpha + A_\varepsilon (1 - \theta_{i|1}^2)^{\alpha/2} \theta_{j|1}^\alpha > A_W \theta_{j|1}^\alpha \theta_{i|1}^\alpha + A_\varepsilon (1 - \theta_{j|1}^2)^{\alpha/2} \theta_{i|1}^\alpha$ and simultaneously $\theta_{i|1} > \theta_{j|1}$.

Proposition 7.0.2. Let the factor copula be generated by the linear factor structure (7). Also let F_W and F_{ε_j} be $t(\mu, \sigma, \nu)$ and $t(\nu)$, then

$$A_W = \frac{(\nu\sigma^2)^{\frac{\nu+1}{2}}}{\nu^{3/2} \sigma B(\nu/2, 1/2)}, \tag{26}$$

where $B(\cdot, \cdot)$ is the beta function, ν is degree of freedom.

Through Eqs.(24), (25)and (26), one can derive the resulting theoretical tail dependence matrix conditional on W in an application of double-t factor copula.

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