



Tong, X., Li, L., Zhao, G., Meng, Z., Chen, Z. and Yang, G. (2020) Packet Management for Optimizing Control Performance in Real-Time Feedback Control Systems. In: 46th Annual Conference of the IEEE Industrial Electronics Society (IECON 2020), Online Only, 19-21 Oct 2020, ISBN 9781728154152.

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# Maximizing Control Performance Via Packet Management in Real-Time Feedback Control Systems

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**Abstract**—In real-time feedback control systems, control performance, e.g., control cost and tracking error, is severely affected by information freshness, which in turn relies heavily on the design of the feedback update policy. Therefore, the feedback update policy is of vital importance. In this paper, we first present that minimizing *age of information* (AOI) is not always equivalent to maximizing control performance, where AOI represents the level of “dissatisfaction” for information staleness. Then, a new metric, called *value of information* (VOI), is designed to evaluate timeliness of system update by linking AOI to the decay rate of the control system. Finally, simulation results demonstrates that the proposed metric has superiorities in improving both control performance and communication cost.

## I. INTRODUCTION

In recent years, the rapidly deployment of real-time feedback control applications, e.g., remote surgery and vehicle tracking in smart transportation systems [1][2][3], has stimulated active research on information freshness. A key aspect of these research is that the environment information should be timely updated, which effectively improves the control performance. Therefore, how to keep the information fresh is crucial to real-time control systems.

Traditionally, a metric, called the *age of information* (AOI), is developed to describe the freshness of information [4]. It is defined as

$$\Delta(t) = t - U(t), \quad (1)$$

where  $t$  is the current time instant, and  $U(t)$  is the timestamp when the current information was generated, i.e., AOI indicates the “need” for new information update. Based on this, researchers have proposed many packet update polices to keep information fresh [4]-[12].

A typical information update process is shown in Fig. 1, where the information in data center is updated via a communication network. In Fig. 1, delivering both the packet 2 and the packet 3 improves the information freshness. However, the packet 4 is useless since it doesn’t bring any new information compared to the previous packet. Based on this opinion, Yin Sun proposed a novel packet update policy, called  $\epsilon - wait$ , which suggested that the sensor should

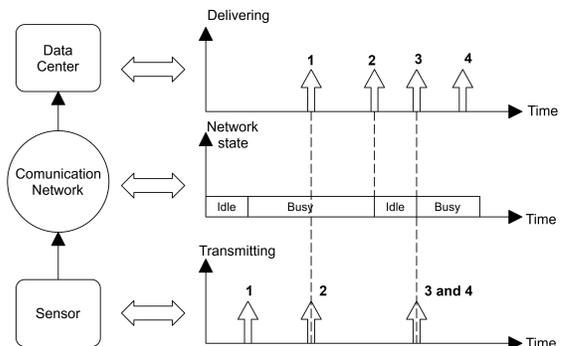


Fig. 1. A typical information update process.

wait a certain time before submitting a new packet [7][8].  $\epsilon - wait$  improves the overall freshness by minimizing AOI. Additionally, similar methods for nonlinear cases have been discussed in [12]. Because control commands are generated based on the information in data center, control performance is improved by minimizing AOI.

However, there is a question: Is minimizing AOI equivalent to maximizing the control performance?

In this paper, we consider a stable control system with bounded time delays, and try to answer the above question by investigating the relationship between control performance and AOI. In particular, a new metric, called the *value of information* (VOI), is defined to link the decay rate of the control system to AOI. Based on this, we propose a new packet update policy to maximize VOI, where the control performance can be maximized.

The rest of this paper is organized as follows: In Section II, we provide the system model. Then, we discuss the relationship between control performance and AOI. In Section III, we propose a novel metric VOI and then formulate a optimization problem. In Section IV, a new packet update policy, called the  $\alpha - wait$ , is proposed to solve the optimization problem. In Section V, simulation results are provided. Finally, Section VI concludes the paper.

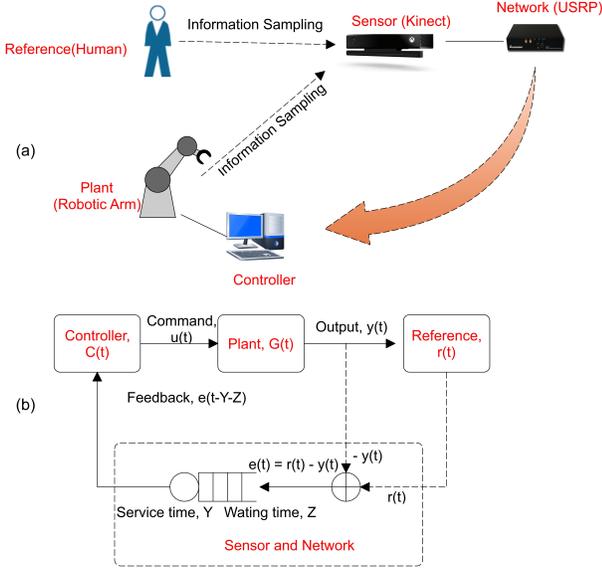


Fig. 2. A typical networked feedback control system with bounded time delays.

## II. SYSTEM MODEL

### A. Feedback Control System

A motion-tracking system<sup>1</sup> is considered in this paper, which is shown in Fig. 2 (a). In this scenario, motion information is sampled and transmitted by a camera and a network, respectively, which helps the robotic arm to reproduce the human's movement. This can be simplified as a typical networked feedback control model, which is shown in Fig. 2 (b). Firstly, information of both the plant output  $y(t)$  and the reference  $r(t)$  is sampled by the sensor. Then, the tracking error information  $e(t) = r(t) - y(t)$  is transmitted to the controller via the network. After receiving the error information  $e(t)$ , a command  $u(t)$  is generated based on a specific control law  $c(t)$ . Then, the command  $u(t)$  is going to be executed at the plant  $G(t)$ . Finally, we obtain the system output  $y(t)$  which is a function of  $G(t)$  and  $u(t)$ . The above control process can be expressed by

$$(R(s) - Y(s))C(s)G(s) = Y(s), \quad (2)$$

where  $E(s)$ ,  $G(s)$ ,  $C(s)$ , and  $R(s)$  is the Laplace transforms of  $e(t)$ ,  $g(t)$ ,  $c(t)$ , and  $r(t)$ . Additionally, since the information becomes more and more stale before next packet updating, the above equation is developed as

$$(R(s) - Y(s))e^{-s\Delta(t)}C(s)G(s) = Y(s), \quad (3)$$

which shows how AOI plays a role in feedback control systems. In this paper, we consider a first-order linear feedback control system with a simple proportional controller, i.e.,  $G(s) = 1/(s + a_0)$  and  $C(s) = K$ , where  $a_0$  and  $K$  are known to the system.

<sup>1</sup>The detailed prototype demonstration video can be found at <https://www.youtube.com/watch?v=zxDhQEcu4Vkk&feature=youtu.be>.

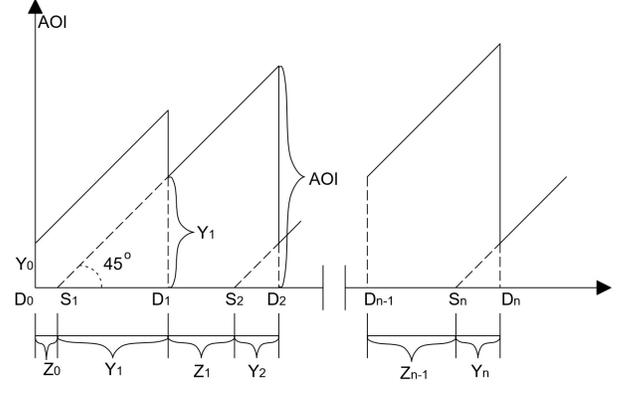


Fig. 3. A typical evolution of AOI, where  $Z$  is the waiting time, and  $Y$  is the serving time.

### B. Network and AOI

Again in the Fig. 2 (b), the closed-loop process is supported by two communication links, the Plant-Controller link and the Controller-Plant link. In this paper, we assume that the Controller-Plant link is perfect with no time delay [1][10][15], while the time delays in Plant-Controller link follows uniform distribution with parameter  $u$ .

This time delay in delivering one packet is also called serving time, denoted as  $Y_0, Y_1, \dots, Y_n$ . Before each packet submitting, the sensor introduces a waiting time, denoted as  $Z_0, Z_1, \dots, Z_n$ . Then, we can obtain the same AOI evolution model as [7][8], which is shown in Fig. 3. At first, the sensor generates and submits a new update at the time slot instant  $S_0, S_1, \dots, S_n$ . After packet  $i$  is delivered at time  $D_i$ , the sensor introduces a waiting time  $Z_i$  before submitting a new packet. As a result, AOI grows from  $Y_{i-1} + Z_{i-1}$  to  $Y_{i-1} + Z_{i-1} + Y_i$ .

### C. Control Performance

In this paper, tracking error  $e(t)$  is introduced to represent the control performance, where  $e(t)$  indicates the difference between the plant's output  $y(t)$  and the reference  $r(t)$  [10], i.e.,

$$e(t) = r(t) - y(t). \quad (4)$$

By combining the Equ. (3) and the Equ. (4), the Laplace form of the tracking error can be expressed as

$$E(s) = R(s) - Y(s) = \frac{1}{1 + C(s)G(s)e^{-\Delta(t)s}}R(s). \quad (5)$$

In general, performance with a unit impulse response is always used to evaluate the control design [10], i.e.,

$$R(s) = \frac{1}{s}. \quad (6)$$

## III. PROBLEM FORMULATION

In this section, we first define a novel metric VOI to link the decay rate and AOI. Then, we formulate a optimization problem to maximize the average VOI, where the system can achieve the maximum control performance.

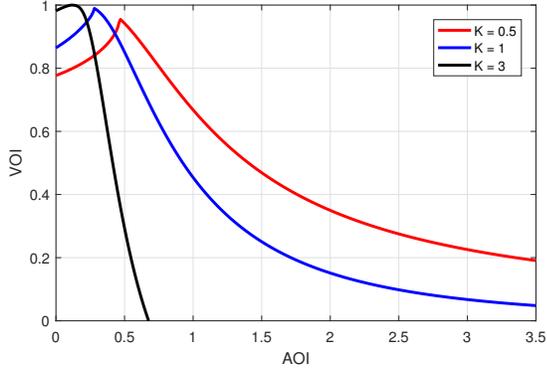


Fig. 4. The relationship between VOI and AOI.

### A. Decay Rate and VOI

The property of the control system can be captured by discussing the *characteristic equation* that is obtained in Equ. (5) and expressed as

$$1 + C(s)G(s)e^{-\Delta(t)s} = 0. \quad (7)$$

The roots of the above equation is called poles, denoted as  $X = \{x_1, x_2, \dots, x_i\}$ . Then, by properly designing the coefficients  $\{A_1, A_2, \dots, A_i\}$ , we have

$$E(s) = \frac{A_1}{s - x_1} + \frac{A_2}{s - x_2} + \dots + \frac{A_i}{s - x_i}, \quad (8)$$

and

$$e(t) = A_1 e^{x_1 t} + A_2 e^{x_2 t} + \dots + A_i e^{x_i t}. \quad (9)$$

It is well known that the rightmost pole  $x_r$ , called the *dominant pole*, is most important to control performance. Then, Equ. (9) is simplified as

$$e(t) \approx A_r e^{x_r t}, \quad (10)$$

which shows that the tracking error  $e(t)$  reduces with time. The convergence rate, also known as the decay rate, is proportional to  $e^{x_r t}$ . Then, maximizing the decay rate can effectively minimize the tracking error.

Here, a novel metric VOI is defined to represent the normalized decay rate, which is expressed as

$$V(\Delta(t)) = 1 - \exp\{\mathbb{R}\{x_r\}\}, \quad (11)$$

where  $\mathbb{R}\{\cdot\}$  indicates the real part. To better demonstrate the relationship between VOI and AOI in the above equation, an example with  $G(s) = \frac{1}{1+s}$  and  $C(s) = K = \{0.5, 1, 3\}$  is shown in Fig. 4. The VOI increases and then decreases with AOI, which verifies our opinion that there exists a “best” AOI. In particular, an interesting result is that the “best” AOI moves right as  $K$  decreases, i.e., traditional AOI becomes more and more inaccurate for control performance. In other words, traditional AOI is a special case of the proposed VOI.

### B. Problem Formulation

In this subsection, we formulate an optimization problem to maximize the average VOI. To analyze the average VOI, we decompose the sum of VOI into a sum of disjoint components:

Consider the time interval  $[0, T_n]$ , where  $T_n = \sum_{i=0}^{n-1} (Z_i + Y_{i+1})$ .

In the interval  $[T_{i-1}, T_i]$ , AOI  $\Delta(t)$  increases from  $Y_{i-1}$  to  $Y_{i-1} + Z_{i-1} + Y_i$  with time [7][8]. Meanwhile, the VOI changes from  $V(Y_{i-1})$  to  $V(Y_{i-1} + Z_{i-1} + Y_i)$ . The total VOI in the interval is expressed as  $P_i$ . Then, we have

$$P_i = \int_{T_{i-1}}^{T_i} V(\Delta(t)) dt. \quad (12)$$

Our goal is to minimize the average VOI, denoted as  $V_a$ , by controlling the waiting time  $(Z_0, Z_1, \dots)$ . Let  $h \triangleq (Z_0, Z_1, \dots)$  denotes the updating policy. Then, the optimization problem can be formulated as

$$\max_h : V_a = \lim_{n \rightarrow \infty} \frac{\sum_{i=0}^{n-1} P_i}{\sum_{i=0}^{n-1} (T_i - T_{i-1})}, \quad (13)$$

In the above optimization problem, by designing the optimal update policy  $h$  to influence the distribution  $\Delta(t)$ , we try to maximize the average VOI, which can minimize the tracking error.

## IV. SOLUTION

In this section, we firstly simplify the optimization problem in Equ. (13). Then, we solve it based on the standard water-filling method. As a result, a new packet update policy is obtained, called the  $\alpha$ -wait.

### A. Linear Fitting

Since  $G(s) = \frac{1}{s+a_0}$  and  $C(s) = K$ , the characteristic equation can be expressed as

$$s + K e^{-s\Delta(t)} = 0. \quad (14)$$

Let  $s = a + bi$ , where  $a$  is the real part of  $s$ , and  $b$  is the imaginary part of  $s$ . Let

$$F_{real} = a + a_0 + K e^{-a\Delta(t)} \cos(b\Delta(t)) = 0, \quad (15)$$

$$F_{imag} = b - K e^{-a\Delta(t)} \sin(b\Delta(t)) = 0. \quad (16)$$

Based on traditional methods [13], we have that

$$\Delta_2 = \frac{\text{acos}(-a_0/K)}{K^2 - a_0^2}, \quad (17)$$

and  $\Delta_1$  is the solution of

$$K e^{\Delta_1} e^{a_0 \Delta_1} - 1 = 0, \quad (18)$$

which can be simply solved by binary search. In Equ. (17),  $\text{acos}(\cdot)$  is the arc-cosine function. Additionally,  $s = -K - a_0 + 0i$  is obviously the root of the characteristic equation. Thus, three points  $(0, 1 - e^{-K-a_0})$ ,  $(\Delta_1, 1 - e^{-\Delta_1-a_0})$ ,  $(\Delta_2, 0)$  are

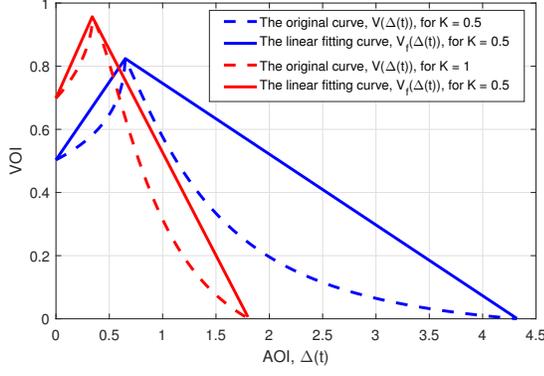


Fig. 5. An example of the linear fitting method based on Equ. (19).

on the VOI curve  $V(\Delta(t))$ . After some algebraic manipulations, the linear fitting expression  $V_f(\Delta(t))$  is designed as

$$V_f(\Delta(t)) = \begin{cases} a_1\Delta(t) + b_1, & \text{for } 0 < \Delta(t) \leq \Delta_1, \\ a_2\Delta(t) + b_2, & \text{for } \Delta_1 < \Delta(t) \leq \Delta_2, \end{cases} \quad (19)$$

where

$$a_1 = \frac{e^{-K-a_0} - e^{-\frac{1}{\Delta_1}-a_0}}{\Delta_1}, \quad (20)$$

$$b_1 = 1 - e^{-K-a_0}, \quad (21)$$

$$a_2 = \frac{e^{-\frac{1}{\Delta_1}-a_0} - 1}{\Delta_2 - \Delta_1}, \quad (22)$$

$$b_2 = -a_2\Delta_2. \quad (23)$$

For example, an example is shown in Fig. 5, where  $C(s) = \frac{1}{s+0.3}$  and  $G(s) = \{-0.5, -1\}$ . The dotted curves indicate the original expression of  $V(\Delta(t))$ , while the solid lines indicate the linear fitting expression  $V_f(\Delta(t))$ .

### B. Water-Filling

In [7][8], it proved that there exists a *stationary deterministic* policy for this kind of problem. A stationary deterministic policy means that a waiting time  $Z_i$  can be optimized only based on the value of  $Y_{i-1}$ , i.e., the optimization process is memoryless. This kind of optimization problem can be solved by the water-filling method, i.e., there exists a water level  $\theta$  and  $Z_i = \max\{\theta - Y_i, 0\}$ . In this way, the original optimization problem can be simplified as

$$\max_{\theta} : V_a = \lim_{n \rightarrow \infty} \frac{\sum_{i=0}^{n-1} P_{i_f}}{\sum_{i=0}^{n-1} (T_i - T_{i-1})}, \quad (24)$$

where

$$P_{i_f} = \int_{T_{i-1}}^{T_i} V_f(\Delta(t)) dt, \quad (25)$$

$$Z_i = \max\{\theta - Y_i, 0\}. \quad (26)$$

To optimize  $\theta$ , we need to derive the expectation of  $P_{i_f}$  and  $\{T_i - T_{i-1}\}$ . The expectation of  $P_i$  can be calculated by

$$\mathbb{E}\{P_{i_f}\} = \int_0^u \frac{1}{u} \int_0^u \frac{1}{u} \int_{Y_{i-1}+Z_{i-1}+Y_i}^{Y_{i-1}} V_f(x) dx dY_i dY_{i-1}. \quad (27)$$

The expectation of  $T_i - T_{i-1}$  can be calculated by

$$\mathbb{E}\{T_i - T_{i-1}\} = \int_0^u \frac{1}{u} \int_0^u \frac{1}{u} (Z_{i-1} + Y_i) dY_i dY_{i-1}. \quad (28)$$

Then, after some algebraic manipulations, a closed-form expression of  $V_a(\theta)$  is obtained as

$$V_a(\theta) = \frac{\mathbb{E}\{P_i\}}{\mathbb{E}\{T_i - T_{i-1}\}} = \begin{cases} F_1(\theta), & \text{for } \theta \leq \Delta_1, \\ F_2(\theta), & \text{for } \theta > \Delta_1. \end{cases} \quad (29)$$

where  $F_1(\theta)$  and  $F_2(\theta)$  are shown at the bottom of this page. Then,  $\theta^*$  can be easily obtained by maximizing Equ. (29).

## V. SIMULATION RESULTS

### A. Parameters Setting

In this section, *mean square error* (MSE) is introduced to evaluate the tracking error, where it is defined as

$$E_{mse} = \frac{1}{1 + T_n/T_s} \sum_{t=\{0, T_s, 2T_s, \dots, T_n\}} \{(y(t) - r(t))^2\}. \quad (32)$$

In the above equation, we set the total time of the control process with 1KHz sampling rate is  $T_n = 100$  seconds. Additionally, we use the channel occupation rate to measure

$$F_1(\theta) = \frac{1}{12u(\theta^2 + u^2)} \left\{ \frac{(b_1 - b_2 + (a_1 - a_2)\theta)^3 (b_1 - b_2 + 3(-a_1 + a_2)\theta)}{(a_1 - a_2)^3} + 4 \left( \frac{(b_1 - b_2)^3}{(a_1 - a_2)^2} + 3b_2\theta^2 + 2a_2\theta^3 \right) u + 6a_2\theta^2 u^2 + 12b_2 u^3 + 10a_2 u^4 \right\} \quad (30)$$

$$F_2(\theta) = \frac{1}{6(a_1 - a_2)^2(\theta^2 + u^2)} \left\{ 2b_1^3 - 6b_1^2 b_2 + 6b_1 b_2^2 - 2b_2^3 + 6(a_1 - a_2)^2 b_2(\theta^2 + u^2) + (a_1 - a_2)^2 a_2(4\theta^3 + 3\theta^2 u + 5u^3) \right\} \quad (31)$$

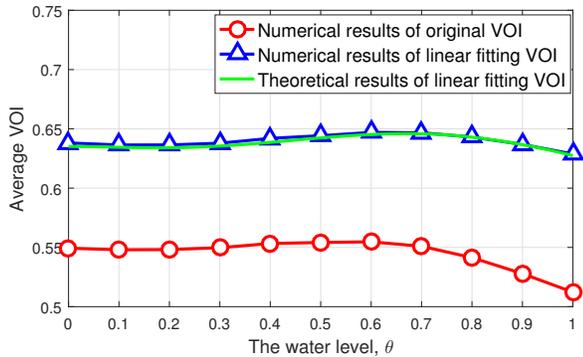


Fig. 6. The numerical and theoretical results of VOI.

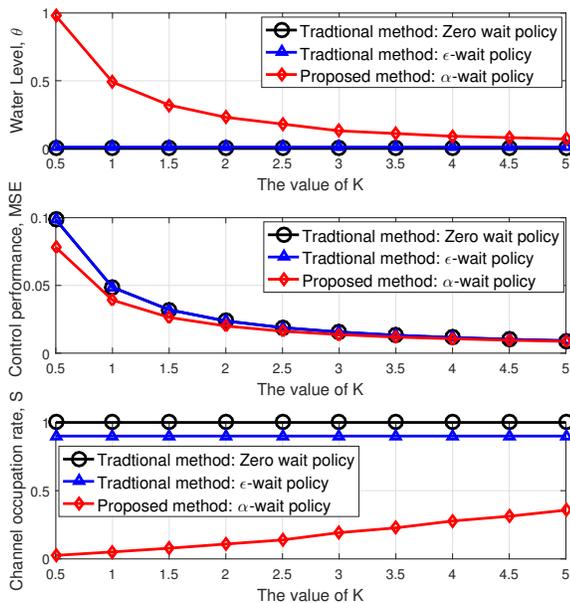


Fig. 7. The numerical results obtained by zero-wait policy, minimizing AOI, and maximizing VOI, where the maximal wireless transmission time  $u = 0.05s$ .

the communication cost, where it is defined as

$$S = \mathbb{E}\left\{\frac{Y_i}{Y_i + Z_i}\right\}. \quad (33)$$

### B. Numerical Results for Completely Stable Systems

In Fig. 6, the linear fitting method in Section IV is verified. Firstly, the same trend can be observed on the original  $V(\Delta(t))$  (red curve) and the linear fitting  $V_f(\Delta(t))$  (blue curve). Secondly, the theoretical results (green curve) obtained by Equ. (29) matches the numerical results very well.

In Fig. 7, the control performance and communication cost are provided based on different information update policy, where random communication time delays with  $u = 0.05s$  is adopted. The zero-wait policy and  $\epsilon$ -wait policy are set as traditional methods. Firstly, since AOI is a metric that is

only related to the freshness of information, the optimal water level  $\theta^*$  based on minimizing AOI is a constant as  $K$  grows. Then, the wireless channel occupation rates (communication cost) of both traditional methods are fixed. Secondly, the optimal water level  $\theta^*$  based on maximizing AOI (proposed method) decreases as  $K$  grows. The reason is that proper-aged information raises the decay rate. Thirdly, when  $K$  is small, i.e., a slow control process, the proposed method can not only improve the control performance but also significantly reduce the communication resource consumption. As  $K$  grows, the proposed method still reduces the communication cost, while it achieves the same control performance as the traditional methods.

## VI. CONCLUSION

In this paper, we studied how to design the information update method for real-time control system. By investigating the relationship between AOI and control performance, we found that minimizing AOI is not always equivalent to maximizing control performance. Thus, we defined VOI to link AOI to the decay rate. By maximizing VOI, the proposed method maximized the control performance, and significantly reduced the communication cost as well.

## REFERENCES

- [1] P. Park, S. Coleri Ergen, C. Fischione, C. Lu, and K. H. Johansson, "Wireless network design for control systems: A survey," *IEEE Communications Surveys & Tutorials*, vol. 20, no. 2, pp. 978-1013, Second quarter 2018.
- [2] H. Laaki, Y. Miche, and K. Tammi, "Prototyping a digital twin for real time remote control over mobile networks: Application of remote surgery," *IEEE Access*, vol. 7, pp. 20325-20336, 2019.
- [3] M. Li, Z. Li, and A. V. Vasilakos, "A survey on topology control in wireless sensor networks: Taxonomy, comparative study, and open issues," *Proceedings of the IEEE*, vol. 101, no. 12, pp. 2538-2557, Dec. 2013.
- [4] S. Kaul, R. Yates, and M. Gruteser, "Real-time status: How often should one update?" in *Proc. IEEE INFOCOM*, Apr. 2012, pp. 2731-2735.
- [5] R. Yates and S. Kaul, "Real-time status updating: Multiple sources," in *Proc. IEEE ISIT*, Jul. 2012.
- [6] R. Yates, "Lazy is timely: Status updates by an energy harvesting source," in *Proc. IEEE ISIT*, 2015.
- [7] Y. Sun, E. Uysal-Biyikoglu, R. Yates, C. E. Koksal, and N. B. Shroff, "Update or wait: How to keep your data fresh," in *Proc. IEEE INFOCOM 2016*, San Francisco, CA, 2016, pp. 1-9.
- [8] Y. Sun, E. Uysal-Biyikoglu, R. D. Yates, C. E. Koksal, and N. B. Shroff, "Update or wait: How to keep your data fresh," *IEEE Transactions on Information Theory*, vol. 63, no. 11, pp. 7492-7508, Nov. 2017.
- [9] J. P. Champati, M. H. Mamduhi, K. H. Johansson, and J. Gross, "Performance characterization using AoI in a single-loop networked control system," in *Proc. IEEE INFOCOM*, Paris, France, 2019, pp. 197-203.
- [10] R. C. Dorf and R. H. Bishop, *Modern Control Systems*, 13th ed, Boston, 2016.
- [11] C. Y. Kao and B. Lincoln, "Simple stability criteria for systems with time-varying delays," *Automatica*, vol. 40, no. 8, pp. 1429-1434, 2004.
- [12] A. Kosta, N. Pappas, A. Ephremides, and V. Angelakis, "Age and value of information: Non-linear age case," in *Proc. IEEE ISIT*, Aachen, 2017, pp. 326-330.
- [13] Sipahi, Rifaf, et al, "Stability and stabilization of systems with time delay," *IEEE Control Systems Magazine*, vol. 31, no. 1, pp. 38-65, 2011.
- [14] Feyzmahdavian, Hamid Reza, Themistoklis Charalambous, and Mikael Johansson, "Asymptotic stability and decay rates of homogeneous positive systems with bounded and unbounded delays," *Journal of Control and Optimization*, vol. 52, no. 4, pp. 2623-2650, 2014.
- [15] A. Ahlen, J. Akerberg, M. Eriksson, et al, "Toward wireless control in industrial process automation: A case study at a paper mill," *IEEE Control Systems Magazine*, vol. 39, no. 5, pp. 36-57, 2019.