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Energy-Efficient Power Allocation in URLLC Enabled Wireless Control for Factory Automation Applications

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Abstract—The coming fifth-generation (5G) cellular networks encourage to support several innovations and services, some of which will demand Ultra-reliable and Low-latency Communications (URLLC). For instance, URLLC can support real-time control to facilitate several emerging applications, such as robotic arms in industrial applications, and remote surgery for healthcare applications. However, URLLC is expected to be supported without considering the resources usage efficiency in wireless control systems due to the challenging to satisfy Quality of Service (QoS) requirements at the expense of diminishing energy efficiency. In this paper, we analyze uplink energy efficiency in URLLC utilizing multiple antennas in the transmitter and the receiver as well (MIMO) in real-time wireless control systems. We firstly formulate an optimization problem to maximize energy efficiency concerning the effect of the control convergence rate constraint. Then, we develop an exhaustive search method to obtain the maximum energy efficiency. Finally, simulation results are provided to demonstrate the performance of our proposed method.

Index Terms—URLLC, Real-time wireless control, Uplink, MU-MIMO, Energy Efficiency, Joint design.

I. INTRODUCTION

Ultra-reliable and low latency communications (URLLC) is one of most important scenarios for the coming 5G cellular networks. This technique is considered as an enabler of real-time control in wireless control systems due to its high Quality of Service (QoS) requirements (i.e., the reliability should be more than 99.999 %, and End-to-End (E2E) delay should be less than 1 ms [1]), which can guarantee the control performance. It can provide an indispensable contribution to the applications of real-time wireless control such as utilizing remote surgery with high demands of accuracy to perform surgical operations in which the wireless network is expected to connect a sensor to a controller as well as a controller to an actuator [2]. However, it is extremely challenging to satisfy the URLLC QoS requirements at the expense of diminishing energy efficiency in wireless networked control systems.

The first attempt to study the design of energy efficient URLLC has been done in [3], which uses the effective bandwidth to describe queuing delay. A transmit power and bandwidth allocation policy was studied in [4] to maximize the energy efficiency under constraints on packet loss corresponding to latency components (e.g., transmission and queuing delays). Additionally, the authors in [5] studied the maximization of the energy efficient URLLC by obtaining optimal resources allocation, such as transmit power, bandwidth, and the active number of the transmit antennas. Consequently, this research found that a large amount of wireless resources is required to meet the URLLC QoS requirements. However, it is only from the communication perspective without considering the actual control requirements from control perspective.

In this paper, we use uplink multi-user multiple input multiple output antennas technique (UL-MU-MIMO) in wireless control system, and we obtain the relationship between the URLLC and control system from the perspective of communication and control joint design. We demonstrate the relationship between the control convergence rate and URLLC QoS. We maximize energy efficiency, which can be defined as the number of transmitted bits per Joule of energy through wireless network, by optimizing transmit power while maintaining the control performance. Also, the formulated optimization problem is concave based on the obtained conversion regarding to the relationship between communication and control. Therefore, we develop an exhaustive search method to obtain the maximum energy efficient URLLC by finding the optimal transmit power while maintaining the control performance.

The remainder of this paper is organized as follows. Section II presents the system model. In section III, from the perspective of communication and control co-design, the optimization problem to maximize the energy efficiency is formulated with the control performance constraint. In section IV, the relation between communication and control joint design is presented, then an exhaustive research method is used in order to obtain the maximum communication energy efficiency by determining optimal transmission power. Simulation results are provided in section V. Finally, we conclude the paper in section VI.

II. SYSTEM MODEL

As illustrated in Fig.1, it is considered a typical centralized real-time wireless control system. Sampling signals are transmitted to the remote controller in the base station (BS) by i static sensors via imperfect wireless network, then the remote controller conducts the calculations of the control command
signals in order to send them to the static plants. Since we focus on uplink transmission design, and adopt (UL-MU-MIMO) systems, we presume each plant has multiple sensors and each sensor is equipped M antennas as well as the remote controller in the BS which is the receiver side is also equipped N antennas, and \( \sum_{i=1}^{k} M \leq N \).

A. Wireless Networked Control Model

The wireless control model is adopted from [6]. Constant sampling period is considered due to the communication QoS and control convergence rate are taken into our account in this work. The discrete time control model can be obtained by

\[
T_{k,l} = \overline{T_{k,l}} + d_{k,l},
\]

where \( T_{k,l} \) indicates to the sample period of \( k \)-th plant at time index \( l \), where \( l = 1,2,...,L \), and \( L \) is the maximum sampling time index. \( \overline{T_{k,l}} \) is the idle period and \( d_{k,l} \) is the transmission time delay. The discrete time control function with the transmission time delay \( d_{k,l} \) is expressed as

\[
x_{k,l+1} = \phi_{k,l} x_{k,l} + \psi_{0} u_{k,l} + \psi_{1} u_{k,l-1} + w_{k,l},
\]

where \( \phi_{k,l} = e^{A_{k,l} T_{k,l}} \), \( \psi_{0} = \int_{0}^{T_{k,l}} e^{A_{k,l} t} B_{k,l} \), and \( \psi_{1} = \int_{0}^{T_{k,l}} e^{A_{k,l} t} d_{k,l} \). By considering the packet loss and wireless network induced delay, we assume an augmented discrete time state variable \( Z_{k,l} = \begin{bmatrix} x_{k,l} & u_{k,l-1} \end{bmatrix} \), then the discrete time control function is written as

\[
Z_{k,l+1} = \phi_{k,d} Z_{k,l} + \psi_{k,d} u_{k,n} + \bar{w}_{k,l},
\]

where \( \bar{w}_{k,l} = \begin{bmatrix} w_{k,l}^T & 0 \end{bmatrix} \), \( \psi_{k,d} \) is the vector \( \begin{bmatrix} \psi_{0} \phi \end{bmatrix} \), and \( \phi_{k,d} = \phi_{k} \).

B. Wireless Communication Model

Since we focus on uplink transmission design. We use UL-MU-MIMO technique, which is adopted from [7]. The channel states information (CSI) between sensors and remote controller is predetermined through a feedback channel as in a frequency-division duplex system, and the channel state is constant during each data frame due to block fading [7], [8]. Each sensor has its CSI while the remote controller in the BS has the CSI for all sensors. We further assume flat fading channel environment. The received signal at the remote controller is given by

\[
y = H.G.P.x + n = \sum_{i=1}^{k} H_{i}G_{i}P_{i}x_{i} + n,
\]

where \( y = [y_{1}, y_{2}, \ldots, y_{N}]^{T} \), the transmitted signal of sensor \( i \) is \( x_{i} = [x_{i1}, x_{i2}, \ldots, x_{iM}]^{T} \), \( [.]^{T} \) is the vector transpose, and the power allocation of sensor \( i \) is \( p_{i} = diag(\sqrt{p_{1}}, \sqrt{p_{2}}, \ldots, \sqrt{p_{iM}}) \). \( G_{i} \) is the precoding matrix of sensor \( i \), and \( H_{i} \) is the channel matrix with size \( M \times N \) of sensor \( i \) as well as \( n \) is known as the noise vector, which is Gaussian distributed with zero mean and a covariance matrix \( \sigma^{2}I_{N} \), where \( I_{N} \) is the identity matrix of size \( N \).

\[
x = [x_{1}, x_{2}, \ldots, x_{N}]^{T},
\]

\[
P = diag\{P_{1}, P_{2}, \ldots, P_{k}\},
\]

\[
G = diag\{G_{1}, G_{2}, \ldots, G_{k}\},
\]

and

\[
H = [H_{1}, H_{2}, \ldots, H_{k}].
\]

With a linear detector, the decision vector of the transmitted signal is obtained by

\[
\hat{x} = Q.Y = Q.H.G.P.x + Q.n.
\]

The channel matrix \( H \) can be expressed by using singular value decomposition (SVD) as

\[
H_{i} = U_{i} \Lambda_{i} \tilde{V}_{i}^{H},
\]

where \( U_{i} \) and \( V_{i} \) are \( N \times N \) and \( M \times M \) unitary matrices, and \( \Lambda_{i} = diag\{\lambda_{i1}, \lambda_{i2}, \ldots, \lambda_{iM}\} \), where \( \lambda_{i} \geq 0 \). \([.]^{H}\) indicates the Hermitian transpose. \( U_{i} \) is composed of \( M \) columns of \( U_{i} \). With local knowledge \( H_{i} \), sensor \( i \) sets the precoding matrix \( G_{i} = V_{i} \). Define

\[
U = [\tilde{U}_{1}, \tilde{U}_{2}, \ldots, \tilde{U}_{k}],
\]
and
\[ \Lambda = \text{diag}\{\Lambda_1, \Lambda_2, ..., \Lambda_k\}. \]

Then, the decision can be easily obtained as shown in equation below
\[ \hat{x} = Q.U.\Lambda.P.x + Q.n. \] (8)

Since the receiver design is not the focus of this work, we employ the zero-forcing (ZF) receiver as \( Q_k = (U^H U)^{-1} U^H \)

It is necessary to take the restriction on \( \sum_{i=1}^{M} M \leq N \) for the ZF receiver. \( \hat{\eta} = Q.n \) is Gaussian distributed with a zero mean and a covariance matrix \( E[\hat{\eta}\hat{\eta}^H] = \sigma^2 [(U^H U)^{-1}]^H \), thus, the diagonal elements being \( \sigma^2 \) [7]. Each symbol can be detected separately by the remote controller in the BS due to the transmissions of each sensor are not coupled. The received signal-to-noise-ratio (SNR) of all symbols for sensor \( i \) transmits is independent of the radio frequency transmission, and therefore, \( P_{c_i} \) symbolizes to the circuit power consumption [7], [12]. Consequently, the overall power consumption for sensor \( i \) is demonstrated as
\[ P_i = P_{T_i} + P_{c_i}. \] (15)

The uplink energy efficiency of the K-th plant can be obtained from (14), (15), (17), (19) and (16), and it is expressed as
\[ \eta = \frac{\sum_{i=1}^{k} \xi (1 - \epsilon_i)}{\sum_{i=1}^{k} P_{c_i} + \sum_{i=1}^{k} \sum_{m=1}^{M} \frac{P_{im}}{\mu}}, \] (16)

where \( \xi \) indicates the payload information transmitted by the sensor \( i \). In this paper, we intend to obtain the maximum energy efficiency by finding the optimal transmit power for each antenna in sensor \( i \).

III. COMMUNICATION AND CONTROL JOINT DESIGN

From communication and control joint design, this subsection presents the constraints of the control system as well as the communication systems in order to formulate the joint design problem to obtain the maximum energy efficiency.

A. Communication and Control Constraints

From the communication aspect, there exist several constraints. The first constraint is the total available transmission power of sensor \( i \) is \( P_{max} \), then the sum of the transmission power of all antennas should be smaller than or equal \( P_{max} \) as expressed below
\[ \sum_{m=1}^{M} P_{im} \leq P_{max}. \] (17)

The second constraint is the reliability from the perspective of URLLC QoS, therefore, the successful transmission of the packets is expressed as
\[ P_{r,\{\alpha_{k,l} = 1\}} = P_{r,\{\sum_{m=1}^{M} p_{im} \leq P_{max}\}} = 1 - \epsilon_{k,l} \geq 1 - \epsilon_{th}. \] (18)

On the other hand, the probability of the failure transmission of the packets is described as
\[ P_{r,\{\alpha_{k,l} = 0\}} = P_{r,\{\sum_{m=1}^{M} p_{im} > P_{max}\}} = \epsilon_{k,l} < \epsilon_{th}, \] (19)

where \( \epsilon_{th} \) is the probability of the transmission packet error which is bounded by URLLC QoS requirement. Last constraint is related to communication time delay which is also bounded by the latency requirement of URLLC QoS as expressed below
\[ T_i \leq T_{th}. \] (20)
From the control aspect, it is considered the control convergence rate $\Gamma_k$ as a control constraint, in which the state sampling period $T_{k,l}$ as well as the control input $\Theta_k$ can both influence the control convergence rate $\Gamma_k$ which in turn playing a significant role in terms of the control system performance [13]. This constraint can be obtained by Lyapunov-like function for each plant as written in equation below

$$V_k(Z_k) = Z_k^TQ_kZ_k,$$

where $Q_k$ is the positive definite matrix. For provided rates $\Gamma_k < 1$, the main demand for the Lyapunov-like function is that these functions decrease with that given rate under closed loop condition during the control process [14]. Furthermore, the better control performance can be guaranteed when the plant state effortlessly updates. Thus, for any possible value of the current plant states $Z_{k,l}$, the Lyapunov-like function at the second step decreases at the desired rate $\Gamma_k < 1$ in the expectation that is

$$\mathbb{E}[V_k(Z_{k,l+1})|Z_{k,l}] \leq \Gamma_k V_k(Z_{k,l}) + Tr(Q_k \dot{R}_k),$$

where $\dot{R}_k = (R_k 0)$ which is the variance of $\bar{w}_k(t)$.

B. Problem Statement

The original communication and control joint design can be expressed as

$$\max_{p_{im},I_k} \eta = \frac{\sum_{i=1}^{k} \xi(1 - \varepsilon_i)}{\sum_{i=1}^{k} P_i c_i + \sum_{i=1}^{k} \sum_{m=1}^{M} \frac{p_{im}}{\mu}},$$

s.t. $\sum_{m=1}^{M} p_{im} \leq P_{max}$, $T_i \leq T_{th}$, $\varepsilon_i \leq \varepsilon_{th}$

$$\mathbb{E}[V_k(Z_{k,l+1})|Z_{k,l}] \leq \Gamma_k V_k(Z_{k,l}) + Tr(Q_k \dot{R}_k).$$

As demonstrated in the above optimization problem, based on the control-communication co-design, it is required to study the impacts of control convergence rate constraint on the communication performance by converting this constraint into communication constraint.

IV. THE PROPOSED METHOD FOR MAXIMIZING UPLINK ENERGY-EFFICIENT URLLC IN WIRELESS CONTROL

In this section, we present how to convert the previous optimization problem into solvable problem regarding to the relationship between communication and control models.

A. Problem conversion based on the relationship between communication and control

From the equation (5), the relationship between communication with respect to reliability constraint, and the Lyapunov-like constraint from the control system as [14]

$$\mathbb{E}[V_k(Z_{k,l+1})|Z_{k,l}] = P_r \{\alpha_{k,l} = 1\} Z_{k,l}^T \phi_1^T Q_k \phi_1 Z_{k,l} + Pr\{\gamma_{k,n} = 0\} Z_{k,l}^T \phi_1^T Q_k \phi_0 Z_{k,l} + Tr(Q_k \dot{R}_k).$$

When $Z_{k,l} \neq 0$, plugging (28) at the left hand side of the constraints in (26), thus, we can obtain

$$Pr\{\alpha_{k,l} = 1\} \geq \frac{Z_{k,l}^T (\phi_0^T Q_k \phi_0 - \Gamma_k Q_k) Z_{k,l}}{Z_{k,l}^T (\phi_0^T Q_k \phi_0 - \phi_1^T Q_k \phi_1) Z_{k,l}} - (25)$$

By using the method in [13], we can easily obtain the optimal $c_k^*$. **Lemma 1**: The control convergence rate $\Gamma_k$ is bounded by $\varepsilon_{th}$, and therefore, the communication reliability in real-time wireless networked control system can be determined by this bounded convergence rate $\Gamma_k$. Consequently, the relationship between the communication reliability and the control convergence rate $\Gamma_k$ [14], [15] is expressed as

$$Pr\{\alpha_{k,l} = 1\} \geq c_k^*.$$ (27)

Based on Lemma 1, it is required to convert the optimization problem into a solvable problem, and an algorithm is developed in order to find the maximum uplink energy efficiency. Due to the energy efficiency is independent over time, the time index is dropped and decomposed by $k$ subproblems. Additionally, the communication reliability in (27) is replaced the relationship as mentioned in Lemma 1. Therefore, (27) is expressed as

$$\max_{p_{im},I_k} \eta = \frac{\sum_{i=1}^{k} \xi(1 - \varepsilon_i)}{\sum_{i=1}^{k} P_i c_i + \sum_{i=1}^{k} \sum_{m=1}^{M} \frac{p_{im}}{\mu}},$$

s.t. $\sum_{m=1}^{M} p_{im} \leq P_{max}$, $T_i \leq T_{th}$, $\varepsilon_i \leq 1 - c_k^*$. Consequently, the control convergence rate $\Gamma_k$ is determined by $\varepsilon_i \leq 1 - c_k^*$. The large time delay is required in order to reduce some resources consumption [15], i.e., $T_i = T_{th}$. 
B. Problem Solution

(23) is concave function with respect to $p_{im}$. By obtaining $f_Q(.)$ which decreases with optimal transmission power of each antenna. Thus, $\varepsilon_i$ increases as well as the uplink energy efficiency increases. Based on that, we propose an exhaustive search method to obtain the solution of the optimization problem in (28). The proposed algorithm is described in detail in Algorithm 1 for case I, and same algorithm is straightforwardly applied for case II.

Algorithm 1: The proposed exhaustive search method to obtain the maximum uplink energy efficiency

**Input:** $T_{th}$, $B_0$, $\mu$, $\xi$, $N_0$, $P^{max}$, $N$

**for** $i=1$:length $(P^{max})$ **do**

$$p_{im} = \frac{P^{max}(i)}{N} ; \gamma_m = \frac{P^{max}}{N_0}$$

**for** $i=1$:length $(P^{im})$ **do**

$$\varepsilon_i = f_Q\left(\frac{T_{th}B_0}{\sqrt{\gamma_m}}\right)$$

To obtain $p_{im}^*$, it is required to solve;

$$1 - c_k^* = f_Q\left(\frac{T_{th}B_0}{\sqrt{\gamma_m}}\right)$$

**end**

**find** ($\varepsilon_i < \varepsilon_{th}$)

**end**

**for** $i = 1$:length ($\varepsilon_i$) **do**

$$\eta = \frac{(1 - \varepsilon(i))}{\sum_{i=1}^{k} P_c + \sum_{i=1}^{k} \sum_{m=1}^{M} \frac{p_{im}^*}{\mu}}$$

**end**

**Output:** Maximum $\eta$

$i = i + 1$

**end**

V. SIMULATION RESULTS AND DISCUSSION

This subsection provides the simulation results. Table 1 shows the simulation parameters. We assume there exist two sensors for each plant. Each sensor has two antennas and the remote controller has four antennas in the BS. We further assume the identical dynamics are considered for the plant. Therefore, we propose $\phi_0 = 1.5$, and $\phi_1 = 0.5$. The positive definite matrix $Q_k$ is 1, and $B_k$ is equal 1. We consider the sample period is 150 ms. Because of using the exhaustive search method, we choose $N = 1000$ in order to reduce the complexity of this method and obtain accurate results of the maximum energy efficiency. When we choose, for example, $N = 100$ the results are not accurate as well as when $N = 10000$, the complexity will be increasing.

![Graph showing the relationship between the control convergence rate and the energy efficiency.](https://via.placeholder.com/150)

Fig. 2. The relationship between the control convergence rate and the energy efficiency.

Fig.2 shows that the energy efficiency in bits per Joule increases linearly with the control convergence rate $\Gamma_k$. That means small value of $\Gamma_k$ can reduce the communication reliability when the value of $c_k^*$ increases. This is reasonable due to high reliability of the communication system can guarantee high control performance with respect to small decrease of the plant state due to the small value of $c_k^*$. Consequently, based on different values of $\Gamma_k$ we can obtain

- Case I: $\Gamma_k < \Gamma_{th}$, then $c_k^* > 1 - \varepsilon_{th}$. The energy consumption in this case is high due to high communication reliability can be guaranteed by high transmit power. Thus, more power allocation is needed to guarantee the required control performance.
- Case II: $\Gamma_k > \Gamma_{th}$, then $c_k^* < 1 - \varepsilon_{th}$. The energy consumption in this case is lower than the previous case because low communication reliability can be obtained by low transmit power. Thus, the required control performance is guaranteed by lower power allocation than URLLC.

![Graph illustrating the energy efficiency, regarding to mentioned in case I and case II, with the total transmit power in dBm.](https://via.placeholder.com/150)

Fig. 3 illustrates that the energy efficiency, regarding to mentioned in case I and case II, with the total transmit power in dBm. When the transmit power of each antenna in sensor $i$ increases, the communication reliability increases as well. Thus, when the actual system requirement is larger than the communication reliability threshold, the energy consump-
As mentioned previously, high communication reliability consumes high wireless resources. Consequently, more wireless resources are required in order to achieve the needed control performance. Additionally, the required control performance can be obtained by lower power allocation than what is required in URLLC.

Fig. 3. The energy efficiency with different transmit power in dBm.

Fig. 4 shows that the energy efficiency increases with different payload information in the proposed method compared to the traditional method in this paper which is considered as each sensor is equipped with single antenna. Consequently, the energy efficiency in the proposed method with optimal power allocation increased by 100% when payload information increased as well.

VI. CONCLUSIONS

In this paper, we proposed the energy efficient power allocation method in URLLC enabled real time wireless control for industrial automation applications. From the perspective of communication and control joint design, the control convergence rate constraint from the control perspective was taken as a control performance constraint, and was converted into communication reliability problem. To make this problem solvable, we defined the relationship between communication and control systems, and we developed exhaustive search algorithm to obtain the maximum energy efficiency with optimal transmit power. The simulation results in this paper illustrated that the proposed method can increase the energy efficiency by 100% than the traditional method while maintaining the control performance. Additionally, lower power allocation can guarantee the required control performance compared to URLLC which requires higher power allocation to achieve better QoS.

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