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The distributional implications of asymmetric income dynamics

Konstantinos Angelopoulos[a], Spyridon Lazarakis[b], James Malley[a,*]

[a] University of Glasgow, United Kingdom and CESifo, Munich, Germany
[b] University of Lancaster, United Kingdom

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ABSTRACT

Income dynamics differ between groups of households defined by whether the head has university education or not and have changed asymmetrically in Great Britain since 2008. Using a heterogeneous agent incomplete markets model, we examine the quantitative implications of these differences for wealth inequality and for the distribution of conditional welfare losses. Within-group wealth inequality is higher for the non-university group and has increased since 2008 for both groups, while between-group inequality has also increased. Welfare losses are significantly higher for the non-university educated since 2008, and are driven by both a greater fall in mean income and a larger rise in income risk. Non-university educated households, which had initial wealth below the median and net labour income in the lower quintiles, suffered bigger losses. Social insurance policies beyond those currently in place can mitigate such welfare losses via tax and benefit redistributive mechanisms. For the broad majority of households, social insurance is valued more when it insures against the big adverse income shocks.

1. Introduction

This paper investigates the quantitative impact of heterogeneity in income dynamics on wealth inequality and welfare. In particular, we examine the effects of this heterogeneity on wealth inequality between and within groups based on university education. We also study the implications of asymmetric changes in income dynamics, since 2008, on the distribution of conditional welfare losses for these groups. This is motivated by empirical evidence for Great Britain which shows differences in income dynamics between households that vary with respect to university education. Focusing on groups of households whose head has a university education or not, wealth data from the Wealth and Asset Survey (WAS) 2006–2016 reveals that wealth inequality is higher for the non-university educated group, and that both between and within group inequality have increased since the first wave of WAS in 2006–2008. Using income data from the British Household Panel Study (BHPS) 1991–2008 and WAS 2010–2016, we also find differences in the net labour income processes between groups and that these

* Corresponding author.
E-mail address: jim.malley@glasgow.ac.uk (J. Malley).

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have changed asymmetrically since 2008.[1] Moreover, mean income was higher for the university educated group over the entire time period.

The empirical results we present in Section 2 show that the persistence of the innovations in the idiosyncratic component to income as well as the skewness and kurtosis of these innovations were higher for the university educated group in both periods, while the variance was more similar (and lower in the BHPS period). Additionally, while both groups experience drops in mean income post 2008, and an increase in skewness and kurtosis, the drop in mean income was higher for the non-university educated group. This group also faced an increase in persistence. In contrast, for the university group, the variance of these innovations increased post 2008. These findings complement a large literature documenting differences in the labour income processes between economic agents with different education levels (for the US and the UK see e.g. Meghir and Pistaferri (2004), Blundell and Etheridge (2010), Heathcote et al. (2010a, 2010b), and Kim (2020)).

Our quantitative analysis is undertaken using a standard incomplete markets model with state-dependent (Markovian) stochastic labour processes and households that belong to one of two education groups. These groups differ in their labour processes, both in terms of mean income and in the transition matrix for idiosyncratic shocks. Recent results in theoretical research (see e.g. Acikgoz (2018)) imply that this model has a well-defined partial equilibrium with a unique invariant wealth distribution for each type of household given prices. In general equilibrium, the income process heterogeneity also affects groups' choices and outcomes via the determination of the interest rate. Since the interest rate in the UK is determined in international financial markets, we extend the closed economy incomplete markets model to allow for the interest rate to differ from a global fixed interest rate by a function of the net foreign asset position of the country (demand minus supply of assets).[2] We show that a stationary open economy general equilibrium exists, and we find that for the parameter values chosen in the calibration this is unique.

We calibrate the model using British data and estimate the labour processes using the BHPS and WAS net labour income. We discretise the idiosyncratic income processes using the method in Farmer and Toda (2017) since it allows the Markov chain approximation to capture higher moments of the shock distribution estimated in the data. This ensures that income risk differences between groups and over time are properly accounted for in the model. We find that the model predicts wealth inequality both within and between the groups that is consistent with the data. More specifically, in the stationary distribution, the university educated group has significantly lower within group wealth inequality than the non-university educated group, despite having more income risk. The model effectively matches the difference in the wealth Ginis between the two groups that are observed in reality and predicts a mean wealth ratio that is close to the data. As is commonly found using this class of models, the model under-predicts the extent of income inequality at the very top end (top 1%) of the wealth distribution. However, it produces very good predictions for the remaining distribution, especially up to the top 5%.[3]

We find that general equilibrium effects from group-level savings, via the interest rate, contribute to wealth inequality in the stationary equilibrium. Moreover, that heterogeneity in both mean income and in income risk is important in this respect. Differences in mean income and in idiosyncratic uncertainty imply different savings functions for the two cohorts. Both differences lead to higher savings for the university group. In turn, the savings of each cohort move the market interest rate away from the equilibrium level that would be consistent with mean assets of each group. Consequently, households in the non-university (university) educated groups lower (raise) their savings respectively as a result of the general equilibrium effect, leading to a rise in between group wealth inequality. Given the idiosyncratic shocks, these changes in precautionary wealth imply increased (decreased) exposure to idiosyncratic shocks, so that within group wealth inequality is increased (decreased).

We next examine the transitional dynamics of this model economy, driven by the heterogeneous changes in income processes from the pre- to the post-2008 periods observed in the BHPS and WAS data. These include an asymmetric temporary fall in mean income and an asymmetric permanent change in income risk. The model generates dynamic paths that are consistent with the empirical observation of an increase in between and within group inequality since 2008.

We find that changes in the income processes for the university and non-university educated groups since 2008 led to asymmetric welfare losses. First, the losses are higher for the non-university group, which lost an average 2.17% of lifetime consumption, compared with 0.84% for the university group. These losses are driven both by a higher drop in mean income and a sharper increase in income risk, implied by a substantial rise in precautionary savings following changes in risk only. Second, and especially for the non-university group, losses are bigger for those with low initial wealth and initial net labour income. For about 25% of households in the non-university educated group, that owned in 2008 less than median wealth and had below median net labour income, the losses averaged 2.96%. These losses, especially for the non-university group, are consistent with recent estimates of welfare losses of recessions for the US in Krueger et al. (2016b), who evaluate the

[1] While WAS reports wealth data since 2006, net labour income data is only available from 2010. To simplify the presentation, we will refer to net labour income as income below. Net labour income is measured post-policy and at the household level (see Section 2 for more details on sample selection and variable definitions).

[2] The mechanism linking the domestic interest rate to the international rate and domestic conditions to close an open economy model is motivated by Kraay and Ventura (2000) and Schmidt-Grohe and Uribe (2003).

[3] The standard incomplete markets model captures key qualitative properties of the wealth distribution, but quantitatively it under predicts the extent of inequality, especially at the top end of the wealth distribution (see e.g. De Nardi, 2015, Quadrini and Rios-Rull, 2015 and Krueger et al. (2016a) who also review extensions that can improve the model's predictions in this respect).

welfare implications of aggregate shocks and estimate this to be on average about 2.16%, and decreasing with wealth. The welfare costs we obtain are a result of a combination of an increase in idiosyncratic risk with a temporary, unexpected drop in income.

Given our focus on micro-level uncertainty, we decompose the conditional welfare losses to isolate the effect of the increase in income risk. We find that the contribution of the rise in income risk to these losses is significant for the approximately 25% of households in the non-university group that owned in 2008 less than median wealth and had below median net labour income. For this sub-group, we calculate average losses of nearly 1.3%, ranging between 0.9% and 4.2%. For the non-university cohort as a whole, the average welfare losses that are caused by the increase in income risk are 0.71%, i.e. about a third of total losses due to both changes (2.17%). On the other hand, the losses are a lot smaller for the university educated group. Given the smaller changes and shorter period we consider, the average welfare losses resulting from the increase in income risk that we find are naturally lower than the losses driven by the increase in idiosyncratic wage risk for the US in Heathcote et al. (2008, 2010b). [4] Our results underline the asymmetry of the welfare effects of the rise in idiosyncratic income risk. In particular, it is households with low wealth and low labour income in the non-university educated group that mainly drive average losses in Great Britain since 2008. The significance of the losses for those at the lower end of the distribution has also been highlighted by Krueger et al. (2016b), in terms of the effects of recessions.

Finally, motivated by the significance of the rise in net labour income risk in terms of welfare losses for poorer households, we examine additional social insurance policies that allow us to capture the effect of interventions via tax and benefits schemes. We consider state contingent benefits policies that differ in the range (i.e. breadth of eligibility) and generosity (i.e. extent of support) of coverage. Moreover, we consider the degree of progressivity associated with higher tax thresholds for the taxes required to generate the additional revenue. State-contingent benefit policy that protects households from large drops in labour income is preferred, for the majority of households, to intervention that provides milder support for a wider range of negative labour income shocks. Under incomplete insurance markets and asymmetric shock distributions, social insurance is valued more when it focuses on the left tail of earnings shocks. On the other hand, a higher (lower) threshold for the additional taxes is preferred for non-university (university) educated households.

The rest of the paper is organised as follows. We first discuss data and income dynamics in Section 2, followed by a presentation of the model in Section 3. In Section 4, we evaluate the predictions of the model with respect to between and within group wealth inequality, studying the importance of income process heterogeneity and general equilibrium effects. We then present the transitional dynamics and conditional welfare implications as well as the effects from fiscal policy interventions in Sections 5 and 6. Finally, we present our conclusions in Section 7.

2. Income and wealth heterogeneity in the data

In this section, we describe the data and analysis to empirically investigate heterogeneity in income dynamics and wealth inequality. More specifically, we consider two groups of households, those whose head has university education, and those whose head does not.[5] At the age of 25, which is the minimum age for heads of households in our post-selection sample, the education level is predetermined for the households. Hence, all households belong to one of the two types.

We estimate the parameters relating to the Markov processes for the idiosyncratic shocks for both cohorts of households using data on net labour income from both BHPS (1991–2008) and WAS (2010–2016). We use household-level income measures since wealth is also measured at the household-level. We use net labour income, capturing the part of labour income that is disposable to households for consumption or saving. This quantity is relevant for the model that we employ for our analysis. We estimate the parameters and approximate the processes pertaining to net labour income and its idiosyncratic component separately for both groups of households. Moreover, we use these to calibrate the model in Section 3. We also employ the wealth inequality estimates from data from WAS (2006–2016) that we discuss here to evaluate the predictions of the model regarding wealth inequality in Section 4.

2.1. BHPS

The BHPS (University of Essex, 2010) is a comprehensive longitudinal study for GB, covering 1991 to 2008.[6] It includes information for up to 5000 households on earnings and other sources of income for individuals and households, as well as on socio-economic characteristics of the respondents. We also make use of the auxiliary dataset Derived Current and Annual Net Household Income Variables (DCANHIV), compiled by Bardasi et al. (2012), which contains derived data on household disposable income. The Bardasi et al. (2012) dataset tracks the same individuals/households for the same time periods as the BHPS i.e. 1991–2008. The BHPS dataset has been widely used to measure and estimate income inequality and risk in the literature (see e.g. Blundell and Etheridge, 2010, Etheridge, 2015 and Cappellari and Jenkins, 2014).

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[4] Our results are compared in more detail with the literature in Section 5.

[5] See also (Blundell et al., 2008) for a similar classification of households into two groups. Note that we also control below for the educational level of the spouse as part of potential observable variation of income within the groups of “university” and “non-university” households.

[6] Data on Northern Ireland are available from 1997 via the additional BHPS sub-sample European Community Household Panel Survey. Moreover, boost samples for Scotland and Wales are available after 1999. However, we focus on Great Britain since the WAS data refers to Great Britain only.

We define net labour income as gross household annual labour income from employment or self employment net of taxes and national insurance contributions and private pension contributions, plus social benefits and private transfers. Households are defined as the family or group of individuals who live in the same residence. The head is defined as the member of the household in whose name the accommodation is owned or rented, or is otherwise responsible for the accommodation. We focus on households whose head is between 25–59 years. To avoid cases with income very close to zero, for each year we order the households according to their net labour income and discard the observations in the bottom 1% (for similar treatment see e.g. Heathcote et al., 2010a). We further only keep households who are in the sample for at least three consecutive periods. Finally, following Storesletten et al. (2004) we trim extreme growth rates. In particular, we trim the top and bottom 0.25% of observations in the distributions of net labour income growth rates. The final BHPS sample consists of 39,896 observations from 4363 unique households. In Appendix A, we report more information on the net labour income series and the sample selection process.

2.2. WAS

The WAS (Office for National Statistics (ONS), 2018) is a longitudinal survey for GB reporting information on earnings, income, the ownership of assets (financial assets, physical assets and property), pensions, savings and debt, as well as on socio-economic characteristics of the respondents over five waves between 2006 and 2016, including data on net labour income since wave 3, starting in 2010. The sample corresponds to the households included in the wave, but the interviews in each wave are carried over a two year period, with the respondents providing information for the year of the interview.

WAS uses a ‘probability proportional to size’ method of sampling cases. This means that the probability of an address being selected is proportional to the number of addresses within a given geographic area, with a higher number of addresses being selected from densely populated areas. The design of WAS recognizes the fact that wealth is highly skewed, with a small proportion of households owning a large share of the wealth. Thus, WAS over-samples addresses likely to be in the wealthiest 10 percent of households at a rate three times the average. Moreover, the large overall sample size (around 20,000 households) provides robust cross-sectional estimates. These features ensure both good coverage of the wealthy and more precise estimates of overall household wealth. However, as in similar surveys, the very rich (e.g. Forbes 400) are not typically included and this can affect the estimates of the top 1 percent.

We harmonise the definition of the household and of the head of household as it is defined in the previous section. We select households of whose the heads are between 25–59 years of age and we discard those with missing educational information for the head. We use household net worth, i.e. the sum of assets minus debt for all household members, as our measure for wealth.[7] Net worth also admits a substantial proportion of the population which have negative current wealth. Details on the wealth data are in Appendix A, which includes key statistics summarising the wealth distributions for all five waves in Table A.2.

We also use the WAS dataset to obtain measures of net labour income processes for the two groups. Measures of net labour income are available only since wave 3 (i.e. 2010), so for this exercise we only use waves 3–5. We further restrict the sample to the households who exist for at least two consecutive waves in the sample. For the rest we follow similar steps as in the sample selection for the BHPS (see above). We define net labour income as household annual labour income from employment or self employment net of any deductions, plus social benefits and private transfers.

As noted above, WAS oversamples from wealthier households, so that given a positive relationship between earnings and wealth (a correlation of about 0.45 on average in the WAS data), high earnings could be over sampled in WAS relative to BHPS. To make WAS more consistent with BHPS in this respect, we order the households according to their gross labour income and discard the top 0.4% of the WAS gross labour income distribution to ensure that the percent ownership in earnings of the top 1% and top 5% is similar across the pooled WAS and BHPS samples, i.e. about 5% and 16% respectively (see also e.g. Heathcote et al., 2010a and Krebs et al., 2017 for examples of trimming to harmonise samples).

2.2.1. Wealth inequality in WAS

We report results for wealth inequality from both the untrimmed WAS and trimmed (harmonised) WAS sample in Appendix A (see Table A.3), and from the untrimmed sample in the main body of the paper below (see Table 5). Overall, within group inequality is higher for the group of non-university educated, while mean wealth is lower for this group. Over the time observed, mean wealth has dropped for both groups, but more for the non-university educated cohort, leading to increasing between group wealth inequality, while within group wealth inequality has increased. Naturally, trimming reduces the extent of wealth inequality observed in the data, although the effect is very small. However, it does not affect the qualitative properties of between and within group wealth inequality that we are mainly interested in.

2.3. Income dynamics

To focus on the idiosyncratic component of income, we follow the literature (see e.g. Meghir and Pistaferri, 2004, Blundell et al., 2008, and Blundell and Etheridge, 2010) and assume that household net labour income is composed of three

[7] We do not add pension wealth to our measure of net-worth. This allows us to maintain comparability with the infinite horizon incomplete markets literature that generally excludes pension wealth. Further note that pension wealth is highly imputed in WAS.

Table 1
The idiosyncratic component of net labour income.

	BHPS sample, 1991–2008		WAS waves 3–5 (annual)	
	Uni	Non-Uni	Uni	Non-Uni
ρ	0.827	0.768	0.821	0.797
CI_{90}	[0.814,0.839]	[0.762,0.775]	[0.803,0.838]*	[0.785,0.809]*
variance	0.071	0.079	0.077	0.076
CI_{90}	[0.069,0.073]	[0.078,0.080]	[0.074,0.080]	[0.072,0.079]
skewness	-1.046	-0.658	-1.127	-0.798
CI_{90}	[-1.098,-0.994]	[-0.682,-0.634]	[-1.194,-1.059]	[-0.845,-0.752]
kurtosis	9.626	6.918	10.850	7.067
CI_{90}	[9.523,9.730]	[6.870,6.965]	[10.715,10.985]	[6.974,7.161]
n	6,920	32,976	6,052	12,643

Notes: Persistence, ρ, is estimated from (2) and (3) by OLS. Variance, skewness and kurtosis are calculated using the OLS residuals $\widehat{\mu}_{i,t}^{h}$. We calculate the confidence intervals by assuming normality and using the theoretical standard errors (see e.g. Cramer, 1997 for the appropriate formulas). *For the WAS, the standard errors of ρ are calculated by applying a block bootstrap procedure (2000 samples).

components, an element capturing aggregate conditions common to all households, a deterministic part depending on observable characteristics and the idiosyncratic component. By denoting the natural logarithm of the measure of income in period t as $y_{i,t}^{h}$, for $h = u, b$, we assume that it follows the process:

$$y_{i,t}^{h} = D_t^{h} + b^{h} x_{i,t} + f^{h}(age_i) + \varepsilon_{i,t}^{h}, \tag{1}$$

where the vector of parameters for each h is given by b^h and $x_{i,t}$ is a set of dummy variables for region of residence, gender of the head of household, marital status, number of adults, number of children and the educational level of the spouse (if married). We also control for experience (approximated by age) with a cubic polynomial in age_i of the head, denoted as $f^h(AGE_i)$. Note that the educational level of the spouse is defined in a similar way to the head's, i.e. University educated and below University educated. The function D_t^h captures the aggregate conditions common to all households and is specified as calendar year time effects, i.e. $D_t^h = \sum_{t=1991}^{2008} 1_t d_t^h$ for the BHPS and $D_t^h = \sum_{t=2010}^{2016} 1_t d_t^h$ for the WAS, where 1_t is an indicator function which is one when a household i is present at time t and zero otherwise.

For the region dummies we use the UK Government Office Regions classification which corresponds with the highest tier of sub-national division in England, Scotland and Wales. Furthermore, following Meghir and Pistaferri (2004) and to be consistent with our model, we estimate (1) separately for the households whose head has University education and those households whose head does not. Finally, since in our econometric analysis we employ household quantities for the arguments in (1), we define all the variables, apart from the spouse's educational level, in terms of the head of the household.

We next retain the residuals $\widehat{\varepsilon}_{i,t}^{h}$ for each t as a proxy for the unobserved, idiosyncratic component of $y_{i,t}^{h}$. We assume that this idiosyncratic component is determined by an $AR(1)$ process:

$$\varepsilon_{i,t+1}^{h} = \rho^{h}\varepsilon_{i,t}^{h} + \mu_{i,t+1}^{h}, \tag{2}$$

where the shocks $\mu_{i,t}^{h}$ are *i.i.d.* over time and determined by a distribution $\mathcal{F}$ and $|\rho^h| < 1$. Note that the higher moments (variance, skewness and kurtosis) of $\mu_{i,t}^{h}$ are, in effect, the conditional higher moments of $\varepsilon_{i,t}^{h}$. These processes can in turn be estimated for the British economy up to 2008 using the BHPS data, and from 2010 using the WAS data. However, recall that in WAS, annual net labour income is observed every second year. Therefore, applying (2) with an annual time step on the observed data implies that for WAS, at an annual frequency, we observe (see also e.g. Krueger et al. (2016a, 2016b) using PSID data):

$$\varepsilon_{i,t+1}^{h} = \left(\rho^{h}\right)^{2}\varepsilon_{i,t-1}^{h} + \rho^{h}\mu_{i,t}^{h} + \mu_{i,t+1}^{h}. \tag{3}$$

Therefore, for WAS, we link the variance, skewness and kurtosis of $\mu_{i,t}^{h}$ to those of $\tilde{\mu}_{i,t}^{h} \equiv \rho^{h}\mu_{i,t-1}^{h} + \mu_{i,t}^{h}$ that we can estimate from the data (see Appendix A for the expressions).

We report the estimated persistence, ρ^h, and the higher moments of $\mu_{i,t}^{h}$ (hence the conditional higher moments of $\varepsilon_{i,t}^{h}$), obtained using the empirical distributions of the residuals from the four cases given by (2) and (3), in Table 1. Comparing the two groups, the results suggest that skewness, kurtosis and persistence is higher for the university educated group, compared with the non-university educated, while variance is higher for the Non-Uni in the BHPS period and similar in the recent decade. Therefore, there are asymmetries in risk between the two groups.[8] Comparing the two periods, while there has been an increase in skewness and kurtosis for both groups, these are of different magnitude. Moreover, the variance

[8] Using PSID data for the US, Kim (2020) finds that on average university educated have lower persistence of idiosyncratic wage shocks, but generally higher variance.

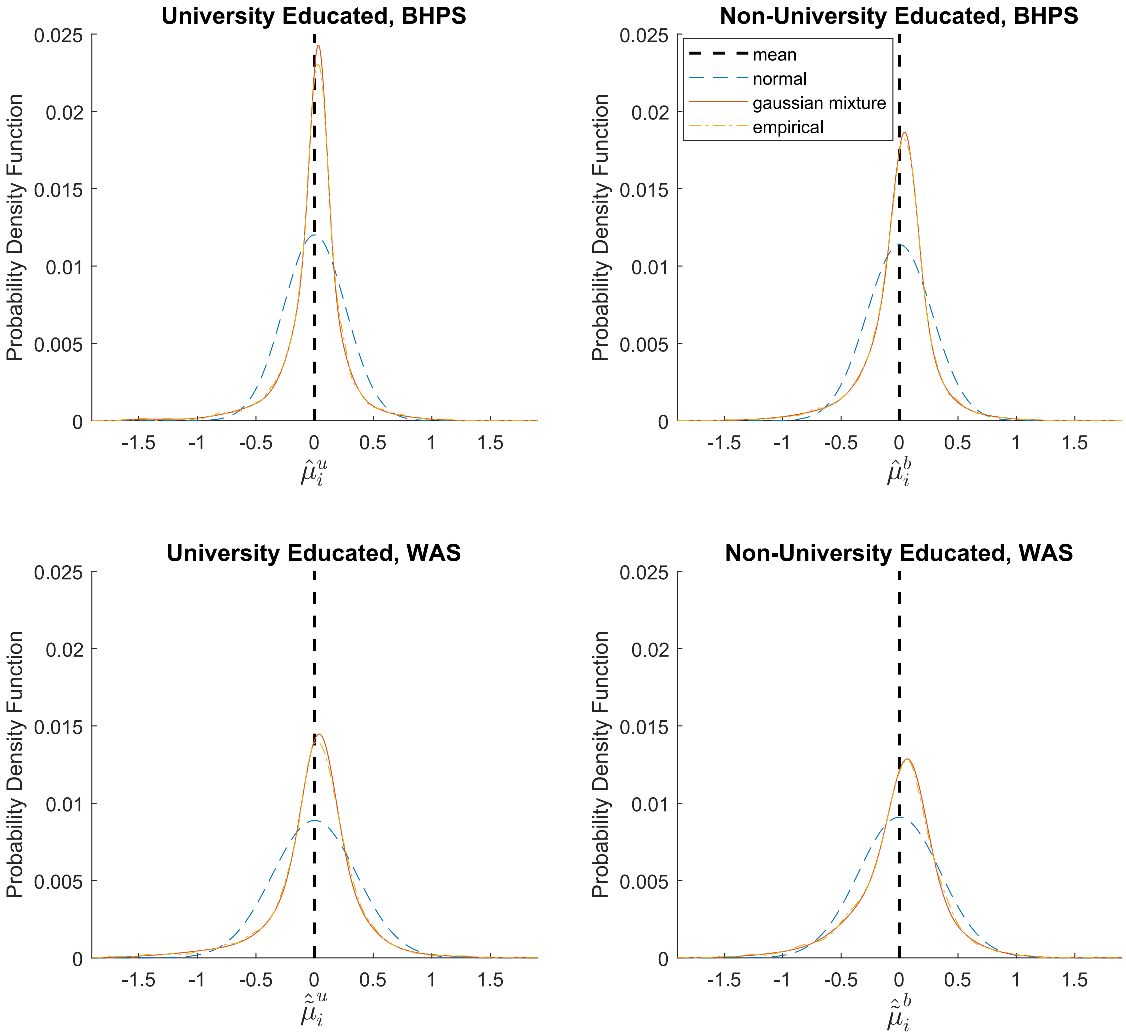

Fig. 1. Densities fitted to the AR(1) residuals.

has increased only for the Uni group and persistence only for the Non-Uni group.[9] Therefore, the changes over time have been asymmetric for the two groups. The findings imply changes to idiosyncratic income risk in ways that are not fully summarised by examining only the persistence and variance of the shocks (see also e.g. Angelopoulos et al., 2019a on the time variation of earnings risk in the UK), suggesting that an approximation to income risk must take higher moments into consideration.

The importance of higher moments in describing risk is reinforced by more closely investigating the distributions of the pooled residuals $\widehat{\mu}_{i,t}^h$ from (2), for BHPS, and for the residuals $\widehat{\tilde{\mu}}_{i,t}^h$ from (3) for WAS, which are plotted in Fig. 1. As can be seen, a normal distribution assumption for $\mathcal{F}$ does not fit the empirical distribution of the shocks well, especially with respect to the negative skewness and thickness of tails. However, the empirical distribution can be well approximated by letting $\mathcal{F}$ be determined as a mixture of four Gaussians, which allows the analytic derivation of the expressions for the first four moments (see also Farmer and Toda, 2017 for an application of a Gaussian mixture distribution to shocks to an AR(1) process). We fit a mixture of four Gaussians to the empirical distributions, by maximising the Gaussian mixture model likelihood using the iterative expectation-maximization (EM) algorithm (see e.g. Mc Lachlan and Peel, 2004).[10] As can be seen, the fitted Gaussian mixture captures the shape of the empirical distributions.

[9] It is useful to note that we compare net labour income risk in two periods that are separated by the 2008 recession. This implies that the direct short-run impact of the largest macroeconomic shock of recent decades is not included in the estimates of idiosyncratic risk. Therefore, the differences in Table 1 should not simply reflect short-run fluctuations.

[10] To fit the distribution $\mathcal{F}$ to the empirical distributions, and discretise (below) the process in (2), we apply Matlab code made available by Farmer and Toda (2017).

Table 2
Markov Chain Approximation.

	BHPS sample, 1991–2008		WAS waves 3–5 (annual)	
	Uni	Non-Uni	Uni	Non-Uni
ρ	0.827	0.768	0.821	0.797
variance	0.071	0.079	0.077	0.076
skewness	-1.061	-0.663	-1.127	-0.798
kurtosis	9.681	6.908	10.850	7.067

Notes: The moments shown are the averages across the 27 conditional distributions for $\varepsilon^h_{i,t}$, as implied by the Markov chain approximation obtained by applying the discretisation method in Farmer and Toda (2017).

Table 3
The idiosyncratic component of net household labour income.

	BHPS sample, 1991–2008			
	Data (pooled annual)		Model (annual)	
	Uni	Non-Uni	Uni	Non-Uni
Gini	0.233	0.231	0.245	0.231
sdlog	0.457	0.439	0.473	0.439
cv	0.531	0.474	0.483	0.452
	WAS waves 3–5			
	Data (pooled bi-annual)		Model (annual)	
	Uni	Non-Uni	Uni	Non-Uni
Gini	0.251	0.237	0.249	0.238
sdlog	0.482	0.463	0.487	0.455
cv	0.495	0.449	0.517	0.464

To capture the idiosyncratic risk implied by the stochastic environment where higher-order moments matter, we discretise the income process following the method in Farmer and Toda (2017). This method chooses the conditional probabilities in the transition matrix of the Markov chain so that the conditional moments of the Markov-chain generated conditional distributions, target the empirical moments in each of the four cases. By choosing: (i) $m = 27$, where m is the number of states in the Markov chain; (ii) an evenly spaced grid over $[-3, 3]$; and (iii) an initial guess for the conditional probabilities based on a quadrature approximation of the Gaussian mixture obtained from the empirical distribution in each of the four cases;[11] the Markov chain implies conditional distributions that accurately approximate the empirical higher moments in Table 1. Indeed, the implied persistence and conditional variance, skewness and kurtosis for $\varepsilon^h_{i,t}$ averaged across the 27 conditional distributions implied by the Markov chain approximation, as summarised in Table 2, closely track their empirical counterparts in Table 1.

The model predictions regarding the distribution of the stochastic component of net labour income, as implied by the stationary distribution resulting from this approximation, are summarised in Table 3. This table reports the Gini coefficient, the coefficient of variation (CV) and variance of logarithms of the invariant distribution predicted by the Markov Chains with their empirical counterparts, calculated using the distributions of residual net household labour income.[12]

The invariant distribution of the Markov chain approximation captures well the quantitative differences in the idiosyncratic component of net household labour income distributions between the groups, as well as the overall level of dispersion in each group. Moreover, consistent with the data in Table 3, it predicts a mild increase in the cross-household variation of idiosyncratic net household labour income shocks in the recent decade. In Fig. 2, we plot the invariant distributions implied by the Markov chains and the empirical distributions, to confirm a good fit.

In both periods, net household labour income is higher for university educated relative to non-university educated. Moreover, the heterogeneous changes in the idiosyncratic component of net labour income post 2008 are accompanied by heterogeneous changes in its predicted component, as obtained from (1). Compared with 2008 (BHPS), mean predicted net labour income was on average 6% lower for the group of non-university educated and 2.7% lower for the group of university educated in 2010–2013 (WAS), before signs of recovery post 2013.

[11] To obtain the quadrature approximations to the mixtures of four Gaussians based on the WAS data, we express the four Gaussian mixture estimated using the residuals from (3) into their annual time-step equivalent, by ensuring that they capture the higher moments at the annual frequency in Table 1. The grid over $[-3, 3]$ captures all observed residuals, except for one observation (dropping this observation does not affect our results).

[12] In particular, in Table 3 we report statistical measures of $\exp\left(\varepsilon^h_{i,t}\right)$ and of the invariant distribution of the Markov chain approximated as above, for the state space $[\exp(-3), \exp(3)]$. The plots in Fig. 2 are based on the same distributions.

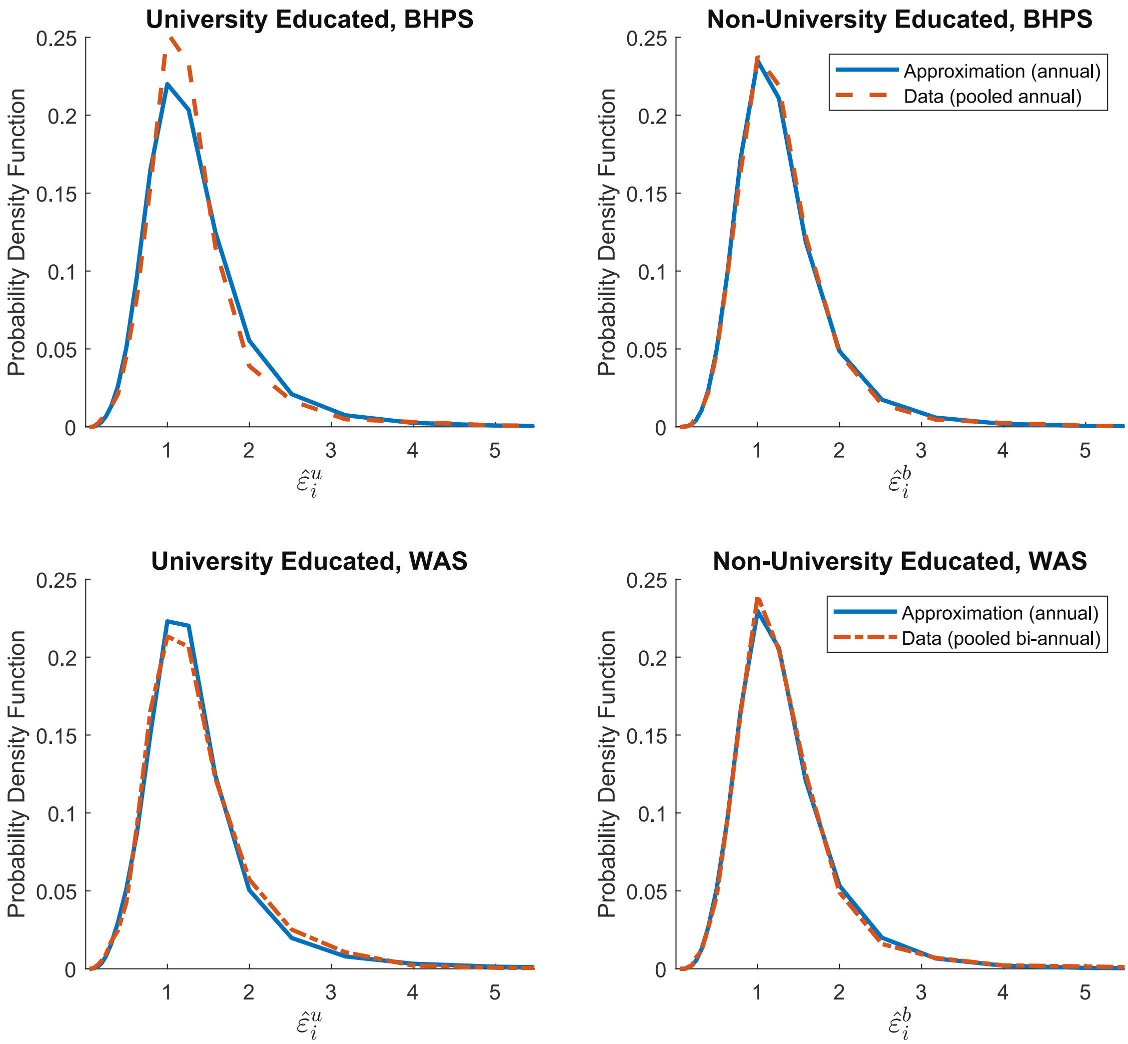

Fig. 2. Invariant distributions implied by the Markov chains vs the data.

3. Model

We consider a standard incomplete markets model where agents differ in their labour income processes. The economy is populated by a continuum of infinitely lived agents (households) distributed on the interval $I = [0, 1]$. Time is discrete and denoted by $t = 0, 1, 2, \ldots$. All households have exogenous labour supply and derive utility from consuming one good that can be acquired by spending either labour income or accumulated savings. Households are identical in their preferences and can invest in a single asset, implying that they cannot fully insure themselves against shocks to labour income.

There are two labour income processes, corresponding to university educated (u) households, which belong to a set $I^u \cup I$, and below university educated (b) households, which belong to a set $I^b \subset I$, such that $I^u \cup I^b = I$ and $I^u \cap I^b = \varnothing$. The proportions of these types of households are given respectively by n^u and $n^b = 1 - n^u$. Households are *ex ante* and permanently allocated to one of the two groups. Therefore, households draw idiosyncratic shocks independently from a Markov chain which differs for the two types of households.

In this Section, we examine and calibrate a stationary equilibrium, in which aggregate quantities are constant. This is the equilibrium that we use for our first set of results in Section 4, to investigate the importance of income process heterogeneity, and of the general equilibrium effects that it creates, for between and within group wealth inequality. It is also the basis for the analysis of transitional dynamics in Sections 5 and 6, where we revisit this stationary model to allow for deterministic time paths to exogenous quantities and social insurance mechanisms.

3.1. Households

Denote the idiosyncratic component of labour income of a typical household $h = u, b$ at time t by s_t^h. Labour income is given by $w\zeta^h s_t^h$, where w is a common wage rate and ζ^h reflects differences in mean productivity between the two types of households due to non-stochastic, predetermined differences. Therefore, s_t^h contains shocks that may affect work hours in a time period and/or household labour productivity.[13] The idiosyncratic shock follows a first-order Markov chain. In particular, we assume that the process s_t^h is an m-state Markov chain with state space S^h and transition matrix Q^h. The state space $S^h = [\bar{s}_1^h, \bar{s}_2^h, \ldots, \bar{s}_m^h]$ is ordered according to $\bar{s}_1^h > 0,\ \bar{s}_{j+1}^h > \bar{s}_j^h,\ j = 1, \ldots, m-1$ and has the natural σ-algebra $\mathcal{S}^h$ made up of all subsets of S^h. The elements of the transition matrix Q^h are denoted $\pi^h\left(s_{t+1}^h | s_t^h\right) = Pr(s_{t+1}^h = \bar{s}_{j'}^h | s_t^h = \bar{s}_j^h)$. We follow Acikgoz (2018) and assume that $\pi^h\left(\bar{s}_1^h | \bar{s}_1^h\right) > 0$ and that the Markov chain is *irreducible* and *aperiodic*, i.e. there exists a $k_0 \in \mathbb{N}$ such that $\left[\pi^h\left(s_{t+1}^h | s_t^h\right)\right]^{(k)} > 0$ for all $\left(s_{t+1}^h, s_t^h\right) \in S^h$ and $k > k_0$. This implies that the Markov chain has a unique invariant distribution, with probability measure that we denote by ξ^h.

Households' shock s_t^h is observed at the beginning of period t. They also receive interest income from accumulated assets ra_t^h, and use their income for consumption and to invest in future assets, subject to the budget constraint for each $h = u, b$:

$$c_t^h + a_{t+1}^h = (1 + r)a_t^h + w\zeta^h s_t^h, \tag{4}$$

where $c_t^h \geq 0,\ a_t^h \geq -\phi^h$ and $-\phi^h < 0$ denotes a borrowing limit on the household. The set comprising a_t^h is defined as $\mathcal{A}^h = [-\phi^h, +\infty)$. The factor prices (interest rate r and wage rate w) are assumed to be fixed and non-random quantities. This holds if the household's actions take place in a stationary equilibrium, which is defined below. Households's preferences are given by a per period utility function $u(c_t^h)$ and an intertemporal discount factor $\beta \in (0, 1)$. The utility function $u : [0, +\infty) \to \mathbb{R}$ is bounded, twice continuously differentiable, strictly increasing and strictly concave.[14] Furthermore, it satisfies the conditions $\lim_{c\to 0} u_c(c) = +\infty,\ \lim_{c\to\infty} u_c(c) = 0$ and $\lim_{c\to\infty} \inf -\frac{u_{cc}(c)}{u_c(c)} = 0$. These assumptions are typically employed in the literature of partial equilibrium income fluctuation problems (see e.g. Miao, 2014, ch. 8) and in the literature relating to incomplete markets with heterogeneous agents in general equilibrium (see e.g. Aiyagari, 1994 and Acikgoz, 2018) to ensure a well-defined stationary equilibrium. The assumption that $\lim_{c\to\infty} \inf -\frac{u_{cc}(c)}{u_c(c)} = 0$ implies that the degree of absolute risk aversion tends to zero as consumption tends to infinity.

Households take the interest rate and wage rate as given and we assume that $r > -1$ and $w > 0$. Moreover, as has been shown (see e.g. Aiyagari, 1994, Miao, 2014, ch. 8 and Acikgoz, 2018), a necessary condition for an equilibrium with finite assets at the household level in this class of models is that $\beta(1 + r) < 1$. Borrowing limits are imposed following e.g. Aiyagari (1994), i.e. assets must satisfy:

$$\begin{aligned} &a_t^h \geq -\phi^h, \text{ where} \\ &\phi^h = \min\left[\gamma, \frac{\bar{s}_1^h \zeta^h w}{r}\right], \text{ if } r > 0 \text{ or} \\ &\phi^h = \gamma, \text{ if } r \leq 0, \end{aligned} \tag{5}$$

and $\gamma > 0$ is arbitrary parameter, capturing an *ad hoc* debt limit.

The problem of the typical household $h = u, b$ is summarised as follows. For given values of (w, r) and given initial values $(a_0^h, s_0^h) \in \mathcal{A}^h \times S^h$, the household chooses plans $\left(c_t^h\right)_{t=0}^{\infty}$ and $\left(a_{t+1}^h\right)_{t=0}^{\infty}$ that solve the maximisation problem:

$$V^h(a_0, s_0) = \max_{\left(c_t^h, a_{t+1}^h\right)_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^h), \tag{6}$$

subject to (5), where $\beta \in (0, 1)$, and $c_t^h \geq 0$ is given by (4). To obtain the dynamic programming formulation of the household's problem, let $v^h\left(a_t^h, s_t^h\right)$ denote the optimal value of the objective function starting from asset-income state $\left(a_t^h, s_t^h\right)$ and given the interest and wage rate. The Bellman equation is:

$$v^h\left(a_t^h, s_t^h\right) = \max_{\substack{a_{t+1}^h \geq -\phi^h \\ c_t^h \geq 0}} \left\{u\left(c_t^h\right) + \beta E\left[v^h\left(a_{t+1}^h, s_{t+1}^h\right) \mid s_t^h\right]\right\} \tag{7}$$

In this case, we aim to find the value function $v^h\left(a_t^h, s_t^h\right)$ and the policy functions $a_{t+1}^h = g^h\left(a_t^h, s_t^h\right)$ and $c_t^h = q^h\left(a_t^h, s_t^h\right)$, which generate the optimal sequences $\left(a_{t+1}^{*h}\right)_{t=0}^{\infty}$ and $\left(c_t^{*h}\right)_{t=0}^{\infty}$ that solve (6). Standard dynamic programming results imply that the policy functions exist, are unique and continuous.

[13] Examples include the quality of the match between employer and employee, health shocks, or changes in personal circumstances.

[14] Boundedness is not needed for equilibrium (see Acikgoz, 2018). In the calibration and computation below we will use a CRRA utility function which is not bounded below. However, we will work there with a compact set for assets, needed for computation, which, given the continuity of the utility function, implies boundedness.

Following e.g. Stokey et al. (1989, ch. 9), we define $\Lambda^h[(a,s), A\times B] : \left(\mathcal{A}^h \times S^h\right) \times \left(\mathcal{B}\left(\mathcal{A}^h\right) \times \mathcal{S}^h\right) \to [0,1]$, for all $(a,s) \in \mathcal{A}^h \times S^h$, $A\times B \in \mathcal{B}\left(\mathcal{A}^h\right) \times \mathcal{S}^h$, to be the transition functions on $\left(\mathcal{A}^h \times S^h\right)$, induced by the Markov process $\left(s_t^h\right)_{t=0}^{\infty}$ and the optimal policy $g^h\left(a_t^h, s_t^h\right)$.[15] The transition function is given by:

$$\Lambda^h[(a,s), A\times B] = \left\{\begin{matrix} \Pr\left(s_{t+1}^h \in B | s_t^h = s\right), \text{ if } g^h(a,s) \in A \\ 0, \text{ if } g^h(a,s) \notin A \end{matrix}\right\}. \tag{8}$$

In this setup, Proposition 5 in Acikgoz (2018) implies that the Markov process on the joint state-space $\left(\mathcal{A}^h \times S^h\right)$ with transition matrix Λ^h has, for each $h = u, b$, a unique invariant distribution denoted by $\lambda^h(A\times B)$. Furthermore, Proposition 6 in Acikgoz (2018) implies that assets for the typical household tend to infinity when $\beta(1+r) \to 1$. Moreover, Theorem 1 in Acikgoz (2018) implies that the expected value of assets using the invariant distribution is continuous in the interest rate, r.

3.2. General equilibrium in an open economy

We analyse the general equilibrium in an open economy, following Angelopoulos et al. (2019b) in modelling the latter within a heterogeneous agent model.

3.2.1. Firm

A representative firm operates the technology to transform borrowed assets from the financial market to capital to be used in production, and an aggregate constant returns to scale production function, using as inputs the average (per capita) levels of capital K and employment L. The production function is given by $Y = F(K,L)$ and is assumed to satisfy the usual Inada conditions. More specifically, F is continuously differentiable in the interior of its domain, strictly increasing, strictly concave and satisfies: $F(0,L) = 0$, $F_{KL} > 0$, $\lim_{K\to 0} F_K(K,L) \to +\infty$ and $\lim_{K\to\infty} F_K(K,L) \to 0$. The capital stock depreciates at a constant rate $\delta \in (0,1)$. The firm takes the interest and wage rate as given and chooses capital and employment to maximise profits, which gives the standard first order conditions, defining factor input prices equal to the relevant marginal products:

$$w = \partial F(K,L)/\partial L, \tag{9}$$

$$r = \partial F(K,L)/\partial K - \delta. \tag{10}$$

3.2.2. General equilibrium

The economy trades in global financial markets taking the interest rate as given, which implies that aggregate household savings, A^s, can be above or below the capital demanded by firms, K. The difference between domestic savings and domestic capital determines the net foreign asset position, which is given in each period by $NFA \equiv K - A^s$ for the domestic economy, implying interest payments to foreign households equal to $rNFA$. We assume that the interest rate r incorporates a premium on top of a risk-free international interest rate $\bar{r}$. The premium is increasing in NFA (see e.g. Kraay and Ventura (2000) or Schmidt-Grohe and Uribe (2003) for interest rate premia that increase with international borrowing). In particular, we assume that the premium is positively correlated with the NFA over output:

$$r = \bar{r} + \psi\left[\exp\left(\frac{NFA}{Y}\right) - 1\right], \tag{11}$$

which is well defined for $r > \bar{r} - \psi$, for $0 < \psi < \bar{r} + \delta$[16]. The parameter ψ measures the elasticity of the country specific interest rate premium relative to its position in the international financial market. Household optimisation and (11) jointly define a constraint set for the interest rate in general equilibrium, R^{ge}, given by $r \in R^{ge} = \left(\bar{r} - \psi, \frac{1}{\beta} - 1\right)$.

In Appendix B we define formally the stationary general equilibrium in the open economy and show existence. We also present the computational algorithm. Note that while uniqueness of general equilibrium cannot be guaranteed in general, as is commonly the case in this class of models (see e.g. Aiyagari, 1994 and Acikgoz, 2018), we confirm uniqueness for the calibration. The equilibrium is represented graphically in Fig. 4 below.

3.3. Calibration

We calibrate the model to an annual frequency. We use the Markov chains constructed in Section 2 from the BHPS data for the base calibration of the stationary equilibrium and for the results we analyse in Section 4. In turn, this provides a natural base to examine in Section 5 the effect of changes in income risk associated with a transition from the Markov chains computed from the BHPS data to those computed from the WAS data. This is explained in more detail in Section 5.

Table 4
Model Parameters.

β	σ	δ	A	α	n^u	γ	ψ	$\bar{r}$	ζ^u/ζ^b
0.968879	1.50	0.0983	1.00	0.30	0.30	1.016055	0.0027997	0.0215	1.4195

Table 5
Wealth distributions by group.

	WAS Data		Model	
	Uni	Non-Uni	Uni	Non-Uni
Q1 share	-0.006	-0.015	-0.010	-0.043
Q2 share	0.038	0.003	0.057	0.044
Q3 share	0.101	0.076	0.144	0.132
Q4 share	0.206	0.226	0.258	0.274
Q5 share	0.661	0.710	0.551	0.593
T 90–95%	0.137	0.152	0.135	0.153
T 95–99%	0.192	0.205	0.152	0.158
T 1%	0.152	0.148	0.059	0.062
Gini	0.660	0.730	0.567	0.643
a_u/a_b	2.244		2.094	
Gini Total	0.718		0.634	

Note: "WAS Data" refers to the average statistics over waves 1–5.

The model parameters that do not relate to the Markov chains are summarised in Table 4. Regarding preferences, following the literature we use a CRRA utility function:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \tag{12}$$

and set $\sigma = 1.5$, which is the mid-point of values typically employed in calibration studies for the UK (see also Harrison and Oomen, 2010 who econometrically estimate $\sigma = 1.52$).

The annual depreciation rate is set to $\delta = 0.0983$ which implies that the capital over income ratio, given the interest rate (see below), is 2.5 at the equilibrium.[17] We use a Cobb-Douglas production function with constant returns to scale with respect to its inputs:

$$Y = AK^{\alpha}L^{1-\alpha}. \tag{13}$$

We normalise $A = 1$ and set α to 0.3 (see, e.g. Faccini et al., 2013 and Harrison and Oomen, 2010). The value of n_u is set to 0.3 based on information on the percentage of university educated households in the WAS dataset. To calibrate ζ^u and ζ^b we make use of the ratio of the predicted net labour income components using the BHPS dataset from 1991 to 2008. For each period we calculate the mean predicted component of each group and calculate the time average of the ζ^u/ζ^b ratios. We then choose ζ^u and ζ^b to ensure that ζ^u/ζ^b matches the data and that expected labour supply is equal to one. This implies that $\zeta^u = 1.143$ and $\zeta^b = 0.805$. Moreover, we set the international interest rate, $\bar{r}$, to 0.0215 which is the average value of the real short-term yields in the data for 17 countries for the period 1990–2013 (see Carvalho et al., 2016).

Conditional of the above parameters, we calibrate β, γ (the borrowing limit) and ψ to match the following data: (i) the average value of foreign liabilities minus assets over GDP for the UK for the period 1990–2013, equal to 6.9%, in the extended External Wealth of Nations Mark II database (see also Lane and Milesi-Ferretti, 2007); (ii) the percentage of indebted households (i.e. those with zero or negative net-worth) in the WAS data, which is 18.35%; and (iii) the interest rate in equilibrium, $r = 0.0217$, which is the average value of the real short-term yields in the data for UK for the period 1990–2013 (see Carvalho et al., 2016). Note that given $\frac{K_t - A_t}{Y_t} = 6.9\%$, $r = 2.17\%$ and $\bar{r} = 2.15\%$, ψ is determined by $\psi = \frac{r-\bar{r}}{[\exp(\frac{NFA}{Y})-1]}$. Therefore, in effect we calibrate γ and β to match $\frac{K_t - A_t}{Y_t}$ and the percentage of indebted households.

4. Heterogeneity and wealth inequality

We next investigate the channels by which *ex ante* heterogeneity in mean income and inthe stochastic component of income contribute to between and within group wealth inequality in the stationary general equilibrium. We first examine the model's predictions regarding wealth inequality within and between the groups of university and non-university educated, by comparing model predictions to the data for Great Britain.

[15] For any set D in some n-dimensional Euclidean space $\mathbb{R}^n$, $\mathcal{B}(D)$ denotes the Borel $\sigma-$algebra of D.

[16] Note that $\psi < \bar{r} + \delta$, implying $\bar{r} - \psi > -\delta$, guarantees that $r > -1$; and that firm's demand is finite, irrespective of A^s. Also note from (10) that $\bar{r} + \psi\left[\exp\left(\frac{NFA}{Y}\right) - 1\right] + \delta > 0$; and that $\left[\exp\left(\frac{NFA}{Y}\right) - 1\right] > -1$.

[17] This is also very close to the values in Faccini et al. (2013) and Harrison and Oomen (2010).

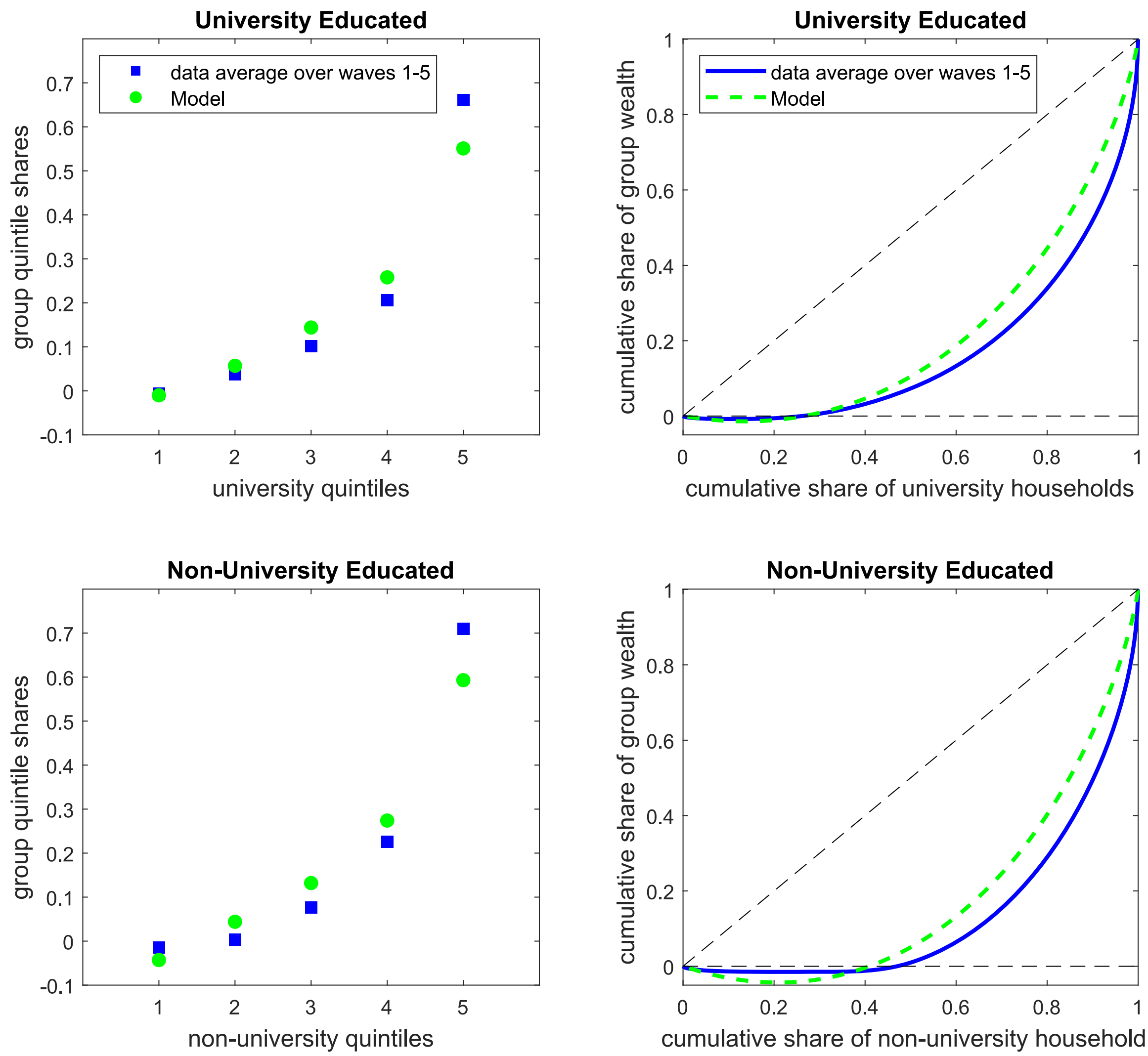

Fig. 3. Quintile Shares and Lorenz Curves of the Wealth Distribution by Group.

4.1. Model predictions and data

We summarise the data and base calibration model predictions for key statistics of wealth inequality in Table 5, following standard practice in the choice of these statistics, see e.g. Quadrini and Rios-Rull (2015) and Krueger et al. (2016a). The first two columns in Table 5 summarise wealth distributions in the data, by presenting the averages of the relevant quantities across the five waves of WAS. These statistics are very similar for the first wave, which overlaps with the end of the BHPS period, that is used to provide the income risk estimates used in the model calibration generating the predictions in the last two columns of Table 5.[18] We complement Table 5 with Fig. 3, which provides a graphical representation of the wealth distributions using the quintile measures of the proportion of total wealth owned by households in the relevant quintile (the 1st column) and the Lorenz curves (the 2nd column).

The main observation from the data in Table 5 and Fig. 3 is that households whose head is university educated (denoted as Uni) have lower wealth inequality than households whose head is not university educated (non-Uni). This can be seen in Table 5 by comparing the wealth distributions (approximated by the quintile statistics), wealth ownership at the upper tail and the Gini indices.

The quintile shares suggest a relatively smaller concentration of wealth in the lower three quintiles and a relatively higher concentration of wealth in the upper two quintiles for the non-university educated. Given the implied spread between the lower and upper parts of the wealth distributions, all of these observations suggest that wealth inequality is higher for the

[18] See Appendix A for summary statistics for all five waves in WAS.

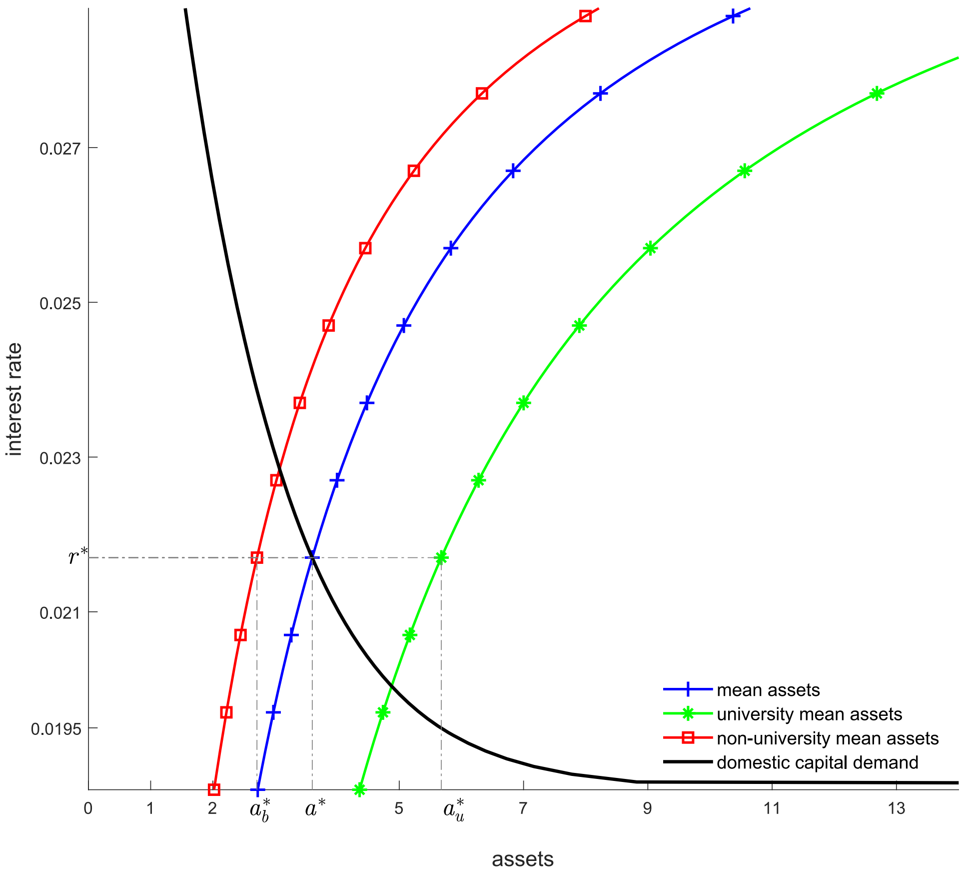

Fig. 4. General Equilibrium.

non-university than for the university educated groups, which is confirmed by the summary Gini measures. Further note that the group of university educated has higher wealth on average, compared with the non-university educated, i.e. the relative wealth ratio, a_u/a_b, is at 2.24 on average across the five waves of data.

The next two columns in Table 5 summarise the predictions of the model. The calibration implies an average wealth ratio of Uni to Non-Uni households predicted by the model of about 2.1, which is consistent with (but lower than) between group wealth inequality in the data. Importantly, the model coheres with key properties of within group wealth inequality for the two groups, i.e. higher wealth inequality for the Non-Uni group relative to the Uni group. This result can be seen by comparing the Gini indices, but is more comprehensively demonstrated by examining the relative rankings of the measures of wealth ownership for the two groups. The model predictions track those in the data. When the quintile shares are higher in the data for the Uni group (the Q1, Q2 and Q3 shares), they are also higher in the model. Whereas, when the quintile measures are higher in the data for the Non-Uni group they are also higher in the model. Overall, the model predicts a Gini index for the non-university educated that is significantly higher than the respective index for the university educated.

The model's predictions regarding the extent of wealth inequality relative to the data are close for both groups. The exceptions are for the top 5 percent and especially the top 1 percent, where the model significantly underestimates wealth inequality, consistent with other models of this class in the literature. The 1^{st} column in Fig. 3 shows the wealth distribution approximated by the quintile shares for the base calibration in Table 5. Both show that the model magnitudes are similar to the data for both groups. The 2^{nd} column of Fig. 3 suggests that the level of predicted inequality within each group is lower compared with the data, reflecting that overall the model quantitatively under-predicts the extent of wealth inequality. This can also been seen by referring to the Gini index implied by the model for the aggregate economy in the last row of Table 5.

The model's predictions regarding the relative ranking of the group wealth concentrations in the top percentiles below the top 1 percent (i.e. the shares owned by the top 90–95 percent and 95–99 percent) are very similar to the data. In contrast to the data in Table 5, the model does not predict a higher wealth concentration for the top 1 percent of the Non-Uni relative to the Uni groups. However, a closer look at each of the WAS waves shows that the wealth concentration ranking for the top 1 percent is not consistent over all the waves (see Appendix A). For example, in the first three waves, wealth ownership by the top 1 percent is higher for the Non-Uni while it is higher for the Uni in the last two waves. In contrast, the ranking of the remaining statistics between the two groups in Table 1 does not change over the waves.

Table 6
General equilibrium effects and inequality per group.

	Base	NI_u	NI_b
r^*	0.0217	0.0209	0.0223
a^*	3.600		
a_u^*	5.676	5.246	
a_b^*	2.710		2.904
Wealth Gini Uni	0.567	0.581	
Wealth Gini Non-Uni	0.643		0.626

Notes: (i) the NI_hmodels are based on the same income processes as in the Base model; (ii)$\frac{a_u}{a_b}$=2.24, Gini=0.718 in the data; (iii) $\frac{a_u^*}{a_b^*}$=2.10, Gini=0.634 for the model; and (iii) $\frac{a_u^*}{a_b^*}$=1.81 for NI.

Overall, the model's predictions regarding wealth inequality capture the main differences between the two groups and the overall extent of inequality, for the majority of the distribution. As is well known in the literature, this class of standard incomplete markets models does not match quantitatively the extent of wealth inequality that we observe in the data with respect to wealth ownership at the very top end.

4.2. General equilibrium effects

We next examine the channels by which *ex ante* heterogeneity in mean income and in the stochastic component of income contribute to between (i.e. relative asset ratio) and within (i.e. Gini) group wealth inequality, and the importance of general equilibrium effects which work via each groups' savings on the interest rate. We compare inequality and key aggregate quantities for the base model developed above with those obtained in artificial economies. In these economies the two types of households do not interact via the financial market, thus eliminating the general equilibrium effect of heterogenous savings on the interest rate.[19]

We start with the model analysed above and in Fig. 4 we plot mean assets as a functions of the interest rate for a typical household in both groups of university and non-university educated, as well as mean assets and domestic capital demand for the aggregate economy (see also Appendix B). We summarise key quantitative information relating to this Figure in Table 6 under the column "Base". In addition, we add in Table 6 key statistics that capture model predicted wealth inequality. The general equilibrium is obtained at the intersection point of the mean assets curve at the aggregate-level with domestic capital demand, giving an interest rate of $r^* = 2.17\%$ and capital stock of $a^* = 3.6$.

In Fig. 5, we again plot the mean assets and domestic capital demand curves for this model, which provide the equilibrium (already shown in Fig. 4) when the two groups interact via the market in a single economy. We complement this by plotting the mean asset curves for a typical household in each group. Also plotted are the mean domestic capital demand curves that would apply if these two groups were separate economies, each populated with the *ex ante* identical university or non-university educated agents. To obtain these hypothetical economies, denoted by NI_u and NI_b, we set (respectively) $n^b = 0$ ($n^u = 0$), leaving n^b (n^u) and all other parameters to be the same as in the base calibration. In each case, we derive the NI (non-interaction) mean assets and domestic capital demand. The intersection points of the respective curves represent the equilibrium interest rate and assets in the absence of group interaction, which are reported in Table 6 under the NI_h, $h = u, b$ columns.

The mean asset curves for a typical household in each group in the Base model encapsulate their optimal policy functions and thus choices for savings given aggregate outcomes under market incompleteness. Therefore, from Fig. 5 and Table 6, we can see that compared with the case where the groups' savings do not affect each other, the savings of the other group in the general equilibrium of Base economy, work to lower (increase) the interest rate for the Non-Uni (Uni) groups.

Viewed from the perspective of the Non-Uni (Uni) group, the reduction (increase) in the interest rate in the general equilibrium of the Base economy reduces (increases) their respective incentives to save.[20] Hence, mean assets are reduced (increased) for the Non-Uni (Uni) group, leading to an increase in the ratio of mean wealth by about 16% percent. In turn, this under-accumulation (over-accumulation) of assets works to increase (decrease) wealth inequality in each group, by increasing (decreasing) the exposure to income variability. To illustrate the effect of the change in the interest rate on asset accumulation and inequality for a given group (in partial equilibrium), we plot in Fig. 6 mean assets and the within group Gini index for wealth inequality for a range of interest rates, holding income risk and all other parameters fixed, for the non-Uni group. As can be seen, an increase in the interest rate, *ceteris paribus*, increases mean group savings and decreases within group inequality.

[19] Strictly speaking, the economies without market interaction also shut down general equilibrium effects on the wage rate. To control for this, we have repeated the experiments in this section by adjusting the wage rate for each group to be the same as in the baseline economy. We find that our results are very similar quantitatively, suggesting that the general equilibrium effects work predominantly via the interest rate.

[20] Note that the (fall) rise in the interest rate also creates income effects, in addition to substitution, effects. In this case, the substitution effects dominate in terms of mean savings.

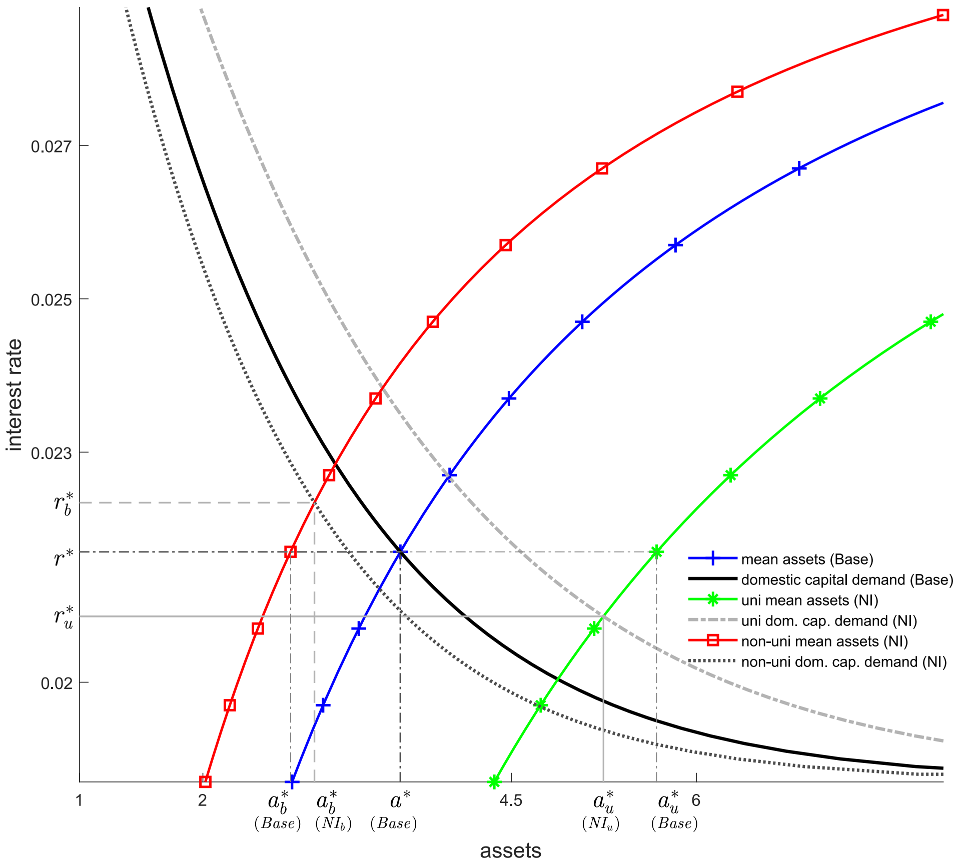

Fig. 5. General Equilibrium Effects from Income Process Heterogeneity.

Therefore, the savings of each group affects inequality in the other group via the general equilibrium determination of the interest rate. Indeed, comparing the NI_h equilibria to the Base model equilibrium in Table 6, we see that the latter implies higher wealth inequality within the non-university educated, and lower wealth inequality within the university educated.

4.2.1. Heterogeneity in mean income and risk

The general equilibrium effects analysed above arise as a result of *ex ante* heterogeneity between the university and non-university educated groups in terms of their stochastic processes for net labour income. These processes differ because they have different means and different higher moments that imply different risk. We next evaluate the contribution of these two forms of *ex ante* heterogeneity to between and within group inequality via the general equilibrium determination of the interest rate.

4.2.1.1. Differences in income risk. To first investigate the importance of differences in income risk, we conduct a counterfactual analysis that repeats the experiment leading to the results in Table 6, by considering a fictional base economy where the two groups have the same mean income, so that only differences in income risk, associated with differences in the transition matrices of the income processes, remain.[21] The results are summarised in the first three columns of Table 7.

In this case, wealth inequality in the base economy (denoted as EHM in Table 7) is overall lower compared to that in Table 6, and the difference in the wealth Gini indices between the two groups is smaller in Table 7. There is still between group wealth inequality, as the university group faces higher risk and thus requires higher precautionary savings. However, this is naturally smaller compared with Table 6, given that levels of income are the same for the results in Table 7.

The main result from the counter factual in Table 7 is obtained by comparing between and within group inequality for the base economy with that under no interaction. The general equilibrium effects via the interest rate work, as in Table 6, to increase between group wealth inequality, increase within group inequality for the non-university group, and decrease it for the university group. The changes in within group inequality are smaller (about half) compared to those in Table 6 for the non-university group and of a similar magnitude for the university educated group. Between group inequality increases

[21] To obtain the relevant results in Table 7, we standardise ζ^u/ζ^b relative to the base calibration so that mean income for each group is the same between groups, while mean labour supply also remains equal to unity.

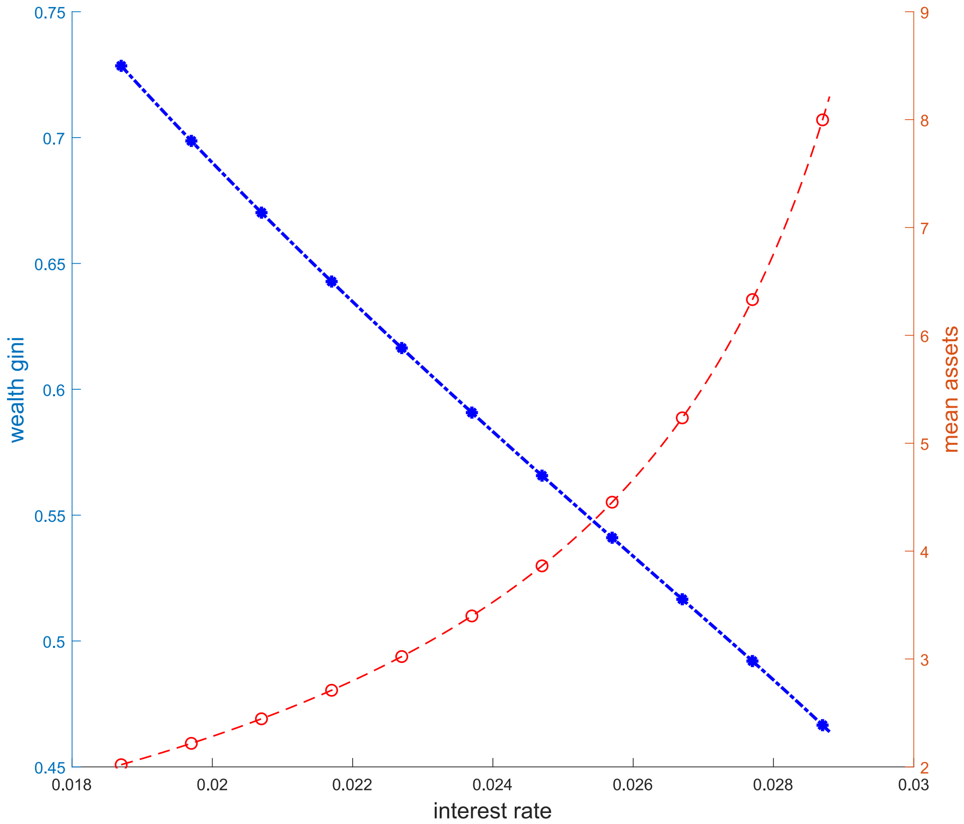

Fig. 6. Interest Rate Comparative Statics (non-Uni Group).

Table 7
The importance of heterogeneity in mean income and risk.

	Same mean income			Same risk		
	EHM	NI_u	NI_b	EHM	NI_u	NI_b
r^*	0.0218	0.0210	0.0222	0.0219	0.0216	0.0220
a^*	3.549			3.517		
a_u^*	4.323	4.005		4.714	4.617	
a_b^*	3.218		3.353	3.004		3.048
Wealth Gini Uni	0.592	0.607		0.573	0.577	
Wealth Gini Non-Uni	0.615		0.606	0.629		0.625

Notes: (i) Under same mean income: $\frac{a_u^*}{a_b^*}$=1.34, Gini=0.611 for EHM and $\frac{a_u^*}{a_b^*}$=1.20 for NI; (ii) Under same risk: $\frac{a_u^*}{a_b^*}$=1.57, Gini=0.616 for EHM and $\frac{a_u^*}{a_b^*}$=1.52 for NI.

by about 12% (compared to 16% in Table 6). Therefore, risk heterogeneity is an important factor in determining the general equilibrium effects on between and within group inequality.

4.2.1.2. Differences in mean income. To further contextualise the importance of heterogeneity in mean income and in income risk for wealth inequality via the general equilibrium determination of the interest rate, we repeat the above counterfactual analysis by considering the situation where the two groups differ in their mean income, as in Table 6, but their idiosyncratic risk is determined by a common Markov chain. To obtain the latter, we use a Markov chain which is the linear combination of the Markov chains of the two groups, with n^u and n^b being the respective weights. The results for the base and the two non-interaction economies are also summarised in Table 7. As can be seen, the general equilibrium effects are similar qualitatively to those under risk heterogeneity in the first part of Table 7, but of smaller magnitude, both for within and between group inequality.

Overall, as can be see by comparing Tables 6 and 7, the Gini index is reduced when we shut down any of the two forms of heterogeneity, in either mean income or income risk, suggesting that income process heterogeneity works to increase overall wealth inequality. Similar results in this respect are obtained for the US in Kim (2020), who shows that wage risk heterogeneity between skilled and unskilled increases wealth inequality for the aggregate economy, in a life-cycle model that endogenises the education decision.

5. Implications of changes in income processes

Motivated by the importance of the differences in the net labour income processes for wealth inequality, in this section we examine the distributional implications from heterogenous changes in these processes since 2008. We analyse the inequality and welfare implications of ceteris paribus changes in the predicted and in the stochastic component of net labour income. As analysed in Section 2, the changes differ between the university and non-university educated groups. We examine deterministic transition paths following unanticipated changes in the level of income and in income uncertainty, associated with changes in the stochastic processes for the two groups. The analysis uses the stationary distribution in 2008, based on BHPS calibration, as the base year, and the WAS estimates to inform the transition. We partial out other changes in the British economy, to evaluate the significance of changes in mean income and in household-level uncertainty, in terms of wealth inequality and welfare gains/losses across the assets and income distributions.

5.1. Transition dynamics

To obtain the dynamic paths we work as follows. Until period zero, the economy is assumed to be in the stationary equilibrium that is described by the parameter values for the baseline results in the previous section. Then, we impose an unanticipated drop in mean income for each group in period zero, lasting for a five year period.[22] The drop is 2.7% and 6% for the Uni and Non-Uni groups respectively, determined by the fall in mean predicted component going from the BHPS in 2008 to WAS in wave 4 (2013). After wave 4, this component increases for both groups. Thus, we assume a gradual return, over ten years, to the 2008 values. At the same time, idiosyncratic income uncertainty increases over an eight year transition period, to reach the uncertainty that was estimated using the WAS data in Section 3 for 2010–2016. After that, it remains at this level forever. This transition in income risk is implemented by constructing a time series of matrices for the transition period, obtained by interpolating between the matrices from BHPS to WAS data.[23] We assume that at time period T the economy has converged to the new stationary equilibrium.

We assume that all dynamic paths for exogenous and aggregate quantities for $t = 1, \dots, T$ are deterministic and common knowledge; they are taken into account by households as given sequences. The problem of the typical household $h = u, b$ is now given by:

$$v_t^h\left(a_t^h, s_t^h\right) = \max_{\substack{a_{t+1}^h \geq -\phi^h \\ c_t^h \geq 0}} \left\{u\left(c_t^h\right) + \beta E\left[v_{t+1}^h\left(a_{t+1}^h, s_{t+1}^h\right) \mid s_t^h\right]\right\},$$

where

$$c_t^h + a_{t+1}^h = (1 + r_t)a_t^h + w_t \zeta_t^h s_t^h.$$

In this case, we aim to find the sequences of value functions $\left(v_t^h\left(a_t^h, s_t^h\right)\right)_{t=0}^{\infty}$ and policy functions $\left(g_t^h\left(a_t^h, s_t^h\right), q_t^h\left(a_t^h, s_t^h\right)\right)_{t=0}^{\infty}$. The rest of the economy remains as in Section 3.

To solve for the transition paths, we follow Boppart et al. (2018) and use a shooting-algorithm to iterate on the path of prices, updating the prices by using a constant weight. In particular, we: (i) solve for the final stationary equilibrium; (ii) guess on the path of prices (interest rate) for $t = 0, \dots T$, $T = 200$; (iii) solve the household problem backward in time from T, where v_T equals the value function in the new stationary equilibrium to obtain a sequence of policy and value functions; (iv) construct a sequence of transition functions $\left(\Lambda_t^h\right)_{t=0}^{T}$ on (a^h, s^h) using the transition matrix for idiosyncratic shocks and policy functions for each $t = 0, \dots T$ and, starting from the original stationary distributions, use $\left(\Lambda_t^h\right)_{t=0}^{T}$ to simulate the distributions forward; (v) use the sequences of policy functions and distributions to aggregate and calculate prices; and (vi) if the maximum difference between guessed and solved interest rate along the path is greater than 10^{-6}, update the guess. The economy effectively converges to a new stationary equilibrium after roughly 100 years.

We calculate the conditional welfare implications of this transition for all households on the cross-sectional distribution associated with the initial stationary economy, as well as average conditional welfare gains/losses for each type of household.[24] Welfare changes for each household are based on the consumption equivalent variation, conditional on initial assets and income. This is computed as the percentage change in consumption required to be taken from the household under the initial stationary equilibrium so that it is indifferent between remaining in this economy compared with the economy under the dynamic transition. Further details on the welfare calculations are in Appendix C.

[22] Note also that in Krueger et al. (2016b) a large recession lasts 22 quarters, i.e. 5.5 years.

[23] To ensure that we capture changes in risk only, associated with the changes in the transition matrices, we standardise idiosyncratic income post 2008 to remove mean effects implied by changes in the transition matrices. Thus, the mean stochastic income for each group is maintained at the level of the base year.

[24] For studies calculating conditional welfare gains/losses across the distribution following changes in the economic environment, see e.g. Domeij and Heathcote (2004) and Kitao (2008); for a comparison of average welfare gains, including between households of different types, see e.g. Heathcote et al. (2008, 2010b).

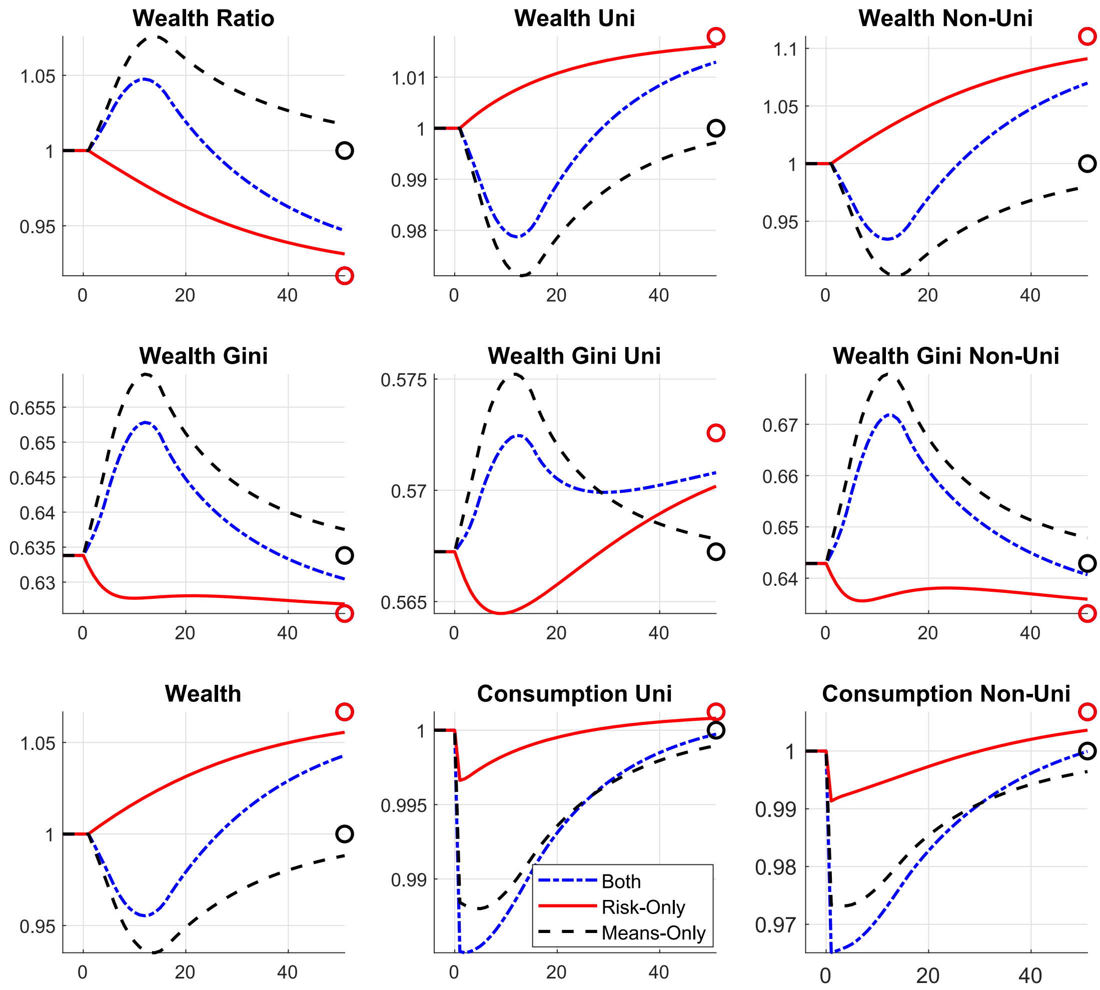

Fig. 7. Transitional dynamics, change in mean income and risk.

5.2. *Wealth inequality implications*

We next summarise dynamic paths for within group wealth inequality (the Gini index) and mean wealth and consumption in Fig. 7. We plot the first 50 years of the transition following the temporary reduction in mean net labour income and the increase in income risk. We denote the values associated with the new stationary distribution, implied by the stochastic process estimated from WAS, by the circles at the end of each plot.[25] To study the relative importance of the two drivers, we also plot the dynamic paths obtained when we implement one change at a time, i.e. either the temporary drop in mean income only, or the permanent increase in income risk only (see also e.g. Heathcote et al., 2010b for a similar decomposition of the sources of variation).

5.2.1. *Wealth inequality in general equilibrium*

The first observation in Fig. 7 is that under both changes in income processes, between group wealth inequality has increased, while mean wealth and consumption for each group has decreased in the short- and medium-run. It is the reduction in mean net labour income that leads to a fall in mean wealth and consumption for both groups over the short- to medium-run, evident in the means-only experiment. Instead, the increase in income risk tends, ceteris paribus, to increase mean wealth via standard precautionary savings motives (see the risk-only experiment). Indeed, the changes in income uncertainty post 2008 are associated with more income risk for the Non-Uni group, reflected in the sharper rise in precautionary savings for this cohort. Together, the changes in mean income and income risk lead to a drop in mean wealth that

[25] Note that after the temporary means-only changes the economy returns to the original stationary distribution. The new stationary equilibrium is identical for the risk-only and for the joint mean-earnings and risk changes.

Table 8
Welfare implications (consumption equivalent variation).

	Both	Risk-Only	Means-Only
		University	
average losses	0.84%	0.21%	0.63%
av. losses, < p50 of wealth and > p80 of income	1.03%	0.31%	0.72%
av. losses, < p50 of wealth and > p50 of income	0.98%	0.25%	0.73%
% of hh's with losses > 0%	100.00%	95.41%	100.00%
% of hh's with losses > 1%	20.77%	0.00%	0.04%
% of hh's with losses > 2%	0.00%	0.00%	0.00%
% of hh's with losses > 3%	0.00%	0.00%	0.00%
		Non-university	
average losses	2.17%	0.71%	1.46%
av. losses, < p50 of wealth and < p20 of income	3.18%	1.48%	1.73%
av. losses, < p50 of wealth and < p50 of income	2.96%	1.27%	1.71%
% of hh's with losses > 0%	100.00%	97.07%	100.00%
% of hh's with losses > 1%	99.19%	23.90%	96.11%
% of hh's with losses > 2%	54.73%	1.10%	0.77%
% of hh's with losses > 3%	8.81%	0.01%	0.00%

is bigger for the Non-Uni group, implying that between group wealth inequality increases in the ten year period following 2008, consistent with the wealth statistics in WAS (see also Appendix A).

The second observation from Fig. 7 is that, in the short- and medium-run, within group wealth inequality increases for both groups, as well as for the aggregate economy, following both changes in the income processes, which is also consistent with the data in WAS. The decline in mean wealth, together with the increase in income risk, imply that households are more exposed to net labour income shocks, leading to greater wealth inequality. Indeed, the increase in income risk on its own does not generate an increase in wealth inequality (see the risk-only experiment), since the increase in precautionary wealth that it encourages insulates the effects of higher risk on wealth. On the other hand, the drop in mean income (and by implication in mean wealth) raises exposure to idiosyncratic labour income shocks. Thus, wealth inequality increases.

Note also that the dynamic effects of the two changes are in effect additive, a property of the transition paths that is emphasised in Boppart et al. (2018). In this context, this also implies later in our welfare analysis that the average welfare gains/losses from the two changes are additive.

5.2.2. Wealth inequality in partial equilibrium

We can further decompose the effects of the changes in the income processes on wealth inequality to partial out the general equilibrium effects. To this end, we repeat the previous analysis in a partial equilibrium version of the model where prices remain constant at their initial levels (see also e.g. Storesletten et al., 2001 for a similar decomposition). We summarise the dynamic paths resulting from this counterfactual in Appendix C, Fig. C.1, which is the counterpart of Fig. 7.

As can be seen there are some differences in the new stationary equilibria. When the general equilibrium effects are shut down, wealth accumulation is higher for both groups while wealth inequality is lower for both groups in the new steady-state. However, the transitions are very similar between Figs. 7 and Fig. C.1, suggesting that the general equilibrium effects impact primarily the stationary distribution, when the effects of the additional incentives or disincentives for wealth accumulation that they create have cumulated over time.

5.3. Welfare implications

We next plot the distribution of welfare losses, in terms of consumption equivalent units, conditional on initial wealth and stochastic net labour income in 2008, in the two plots of the 1^{st} column of Fig. 8, when both changes are considered.[26] We show results for three levels of initial stochastic net labour income. These correspond to the lowest, the middle and the highest state. We then show the same distributions, in the following two columns of the Figure, for the means-only and the risk-only experiments, to study the relative importance of the two sources of changes in the income processes for the distribution of welfare losses.[27]

We complement Fig. 8 with Table 8, where we summarise relevant statistics that correspond to the distribution of gains/losses for the two groups implied by the Figure. For each household type, we report the average welfare losses across

[26] The scale of the x axis in Fig. 8 is obtained by transforming assets in £, by multiplying by mean net labour income in 2008.

[27] See also e.g. Storesletten et al. (2001), Heathcote et al. (2010b) and Krueger et al. (2016b) for a decomposition of welfare losses to the sources that gave rise to these losses.

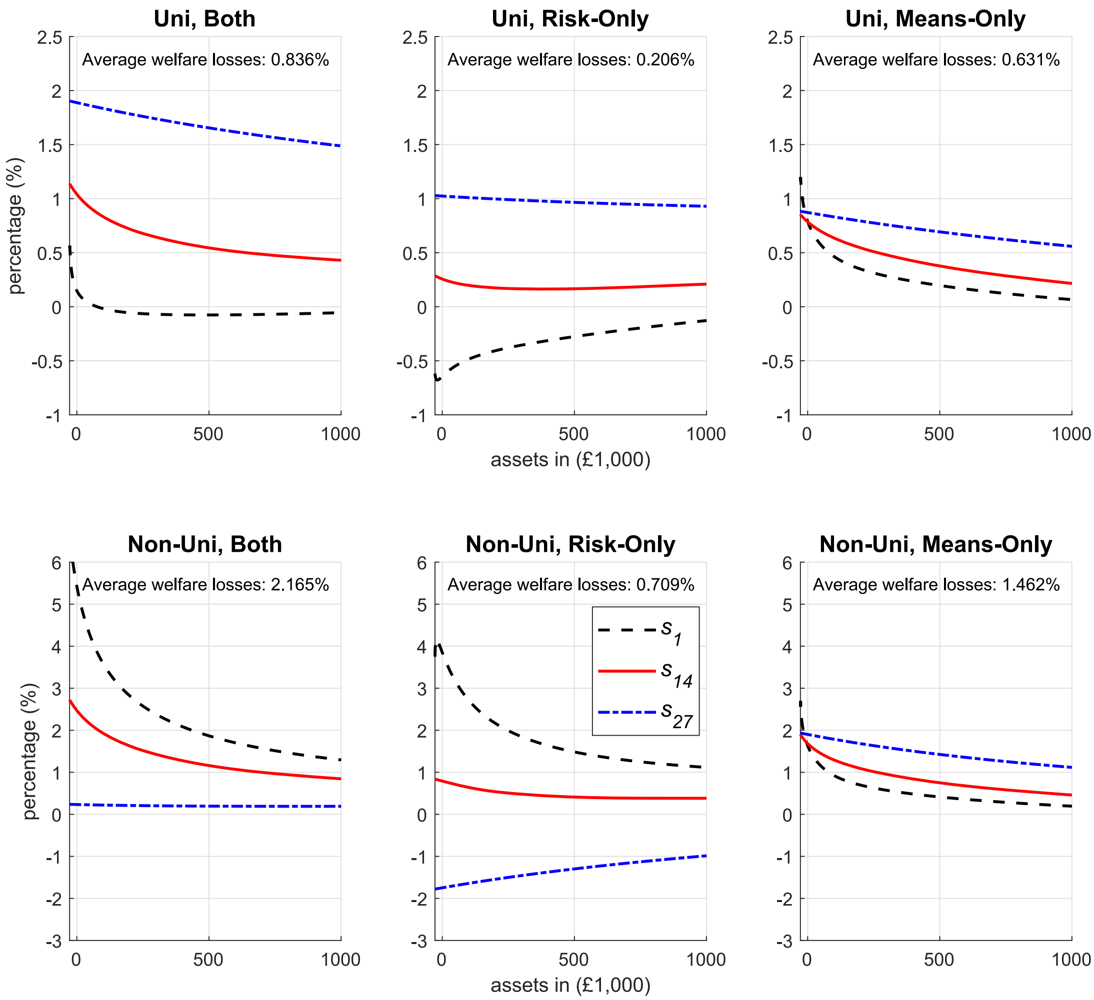

Fig. 8. Conditional welfare losses per experiment.

the whole distribution, average welfare losses for parts of the distribution that show the higher losses in Fig. 8, and the percentage of households that report high welfare losses. The findings show a significant variation in welfare losses both between the groups and within them, depending on the initial asset/labour income position of the household, which we analyse in turn.

5.3.1. Between group welfare changes

Referring to the 1st columns of Figure 8 and Table 8, we see that the changes in income processes overall imply that the non-university educated have suffered higher welfare losses than the university educated. On average across households, the losses are nearly three times as large. These bigger welfare losses are driven both by a higher drop in mean income (3rd columns) and a larger increase in income risk (2nd columns). At the average level, and for both groups, the largest component of the welfare losses was the macroeconomic effect post 2008, namely the reduction in mean net labour income. The average welfare loss of about 1.77% post 2008[28] (note that this is 2.17% for the Non-Uni group), including both changes in the income process for Great Britain, is comparable with (but smaller than) the average welfare loss of 2.16% of Great Recessions calculated for the US economy in Krueger et al. (2016b). Similarly, Storesletten et al. (2001) report average welfare gains of 2.49% of eliminating aggregate fluctuations in the US. Compared with these studies, our interest is in the effects of changes in uncertainty at the household level, as opposed to aggregate fluctuations. However, it is interesting to note that the temporary, unanticipated income shock in the last recession in Great Britain, combined with the increase in idiosyncratic risk, generates comparable welfare losses.

[28] Average welfare losses across the population are obtained as the weighted average of the group-level averaged losses, where the weights are the population shares.

Focusing on micro-level uncertainty (2^{nd} columns), we see that especially for the Non-Uni group the effects of the change in household-level risk are not trivial, and up to 0.71% (and 0.56% on average across the population), as a result of the increase in risk post 2008. These numbers are smaller than magnitudes of average welfare losses following increases in the variance of shocks for the US economy reported in Heathcote et al. (2008, 2010b). However, they are comparable to these once we account for the relatively smaller changes in higher moments that we examine, over the shorter horizon that the effects apply to in our analysis. In particular, Heathcote et al. (2008) calculate average welfare losses of 2.77% from an increase in wage dispersion by about 40% in the US, using a Bewley-type economy. In our case, even for the Non-Uni group for which we observe the greater changes, these are about 2–4% for persistence and kurtosis, and only for skewness it is about 20%. Heathcote et al. (2010b), using a model that allows for more opportunities for household-level insurance than we do, report average welfare losses of about 4% between cohorts entering the labour market in 1960 and those in 2000, due to the increase in idiosyncratic wage risk. When assessing the effect of changes over decade-long periods, losses are about 1% of lifetime consumption.

5.3.2. Within non-Uni welfare changes

Regarding the distribution of welfare losses within the Non-Uni group, we find that losses are decreasing with initial wealth (for example, Krueger et al. (2016b) also find that the welfare costs of great recessions decrease with initial wealth). For the majority of the households losses were substantial, especially for households who also had low net labour income prior to the changes. To illustrate this, and focusing on the results under both changes (1^{st} columns), we calculated the welfare losses for those below median wealth and in the bottom quintile of net labour income for the Non-Uni group (about 10% of households in this group), and found that these are on average about 3.2% and range from 6.6% to 2.5%. Even for those below median net labour income and median wealth (about 25% of households), welfare losses are on average approximately 3.0% and range from 6.6% to 2.3%. Moreover, nearly all households in that group suffer losses of more than 1%, more than half suffer losses greater than 2% and about 9% suffer losses greater than 3%, suggesting that overall losses are widespread. As the decomposition in the last two columns of Table 8 shows, this characteristic is driven by the common-to-all means-only effect, which creates a relatively moderate (less than 1.5%) drop, to which the increase in risk adds more heterogeneous losses. We next turn to these.

The welfare losses for those with low initial wealth and net labour income for the Non-Uni group are to a large extent driven by the increase in risk. Indeed, as can be seen by comparing the 2^{nd} and 3^{rd} columns in Fig. 8 and Table 8, the welfare losses due to the drop in mean income are more evenly distributed across the distribution, compared with the losses due to the increase in risk, which have a big impact at the lower end of the initial distribution. For example, for those households below median wealth and in the bottom quintile of net labour income for the Non-Uni group, the losses due to the increase in risk relative to those from the drop in mean income are about 85%. Note that this ratio is less than 50%, on average, for all non-university educated. Moreover, the effects were broadly similar even for those Non-Uni households below median net labour income and median wealth.

The increase in risk affected more those households with low initial wealth, and those households who were in a poor net labour income position. Households with low initial wealth lacked the asset buffer required to smooth stochastic drops in income made more likely by the increase in income risk. Hence, income shocks passed through to consumption. Households with low initial income were further affected by the increase in the persistence of income shocks post 2008, which implied an increased probability of remaining in a poor net labour income position over time.[29]

5.3.3. Within uni welfare changes

Regarding the distribution of welfare losses within the Uni group, the losses are also decreasing with initial wealth, but are larger for those who had higher labour income prior to 2008. The effects from the drop in mean net labour income are symmetric to those for the Non-Uni group, but smaller. However, for the Uni group, the change in the process for idiosyncratic income implied a fall in the probability of remaining in the current labour income state (recall that the persistence of income shocks was reduced slightly between the two periods in the BHPS and WAS data). Hence, as a result of the change in risk, the high earners with a university degree have suffered greater losses, compared to remaining in the 2008 economy, than low earners with university degree. Together, the combined changes imply that those who were affected the most in this group were households with little wealth and good jobs. For example, the losses of those in the top quintile of net labour income and below the median in wealth for the university educated group (10% of households in this group) range between 1.9% and 0.9%, and are about 1% on average.

5.3.4. Partial equilibrium welfare changes

We also examined the relative contribution of the general equilibrium effects to the welfare losses documented in this section. In Appendix C Table C.1, we reproduce the results in Table 8 for the partial equilibrium version of the dynamic analysis, where the prices are fixed to those in 2008. Moreover, in Fig. C.2 we plot the partial equilibrium counterpart of Fig. 8. As can be seen, the results are very similar to those reported in this section, suggesting that the welfare implications of the

[29] On the other hand, the drop in mean net labour income affected more those households with higher initial net labour income, for whom labour income was relatively more important.

changes in prices in this case are small. This is consistent with the findings in the previous section that the transition paths, which are critical for lifetime utility, are not much changed by the general equilibrium effects. Storesletten et al. (2001) also report small general equilibrium effects contributing to the welfare cost of business cycles.

5.3.5. Summary

The welfare analysis in this section suggests that an important source of welfare losses since 2008 in Great Britain has been the increase in income risk over the recent decade. For average welfare losses, the main driver was the macroeconomic events leading to the reduction in mean income. However, for the lower part of the distribution, and especially for households whose head does not hold a university degree, the increase in income risk was a significant cause of losses in welfare. In turn, this implies that there is potential for social insurance policies, in addition to those that were already in place, to deliver sizeable welfare benefits to a large share of the population.

6. Insurance policy

Motivated by the welfare losses as a result of the increase in income risk, we next use the model to conduct counterfactual policy analysis to investigate the distribution of gains/losses from different social insurance policies. We consider policies that differ in the extent of coverage (i.e. breadth of eligibility) and generosity (i.e. degree of support) of state contingent benefits, as well as in the labour income threshold for the taxes required to generate the additional tax revenue. For the same fiscal size, an intervention with broader eligibility implies more emphasis on smoothing income variation across a larger segment of the population, while an intervention that conditions on significant negative shocks implies more emphasis on insurance provision in adverse situations. We examine policies that subsidise households with low net labour income proportionately to the distance of their net labour income to a target level of net labour income, while taxing those above median net labour income proportionately to the distance of their net labour income to a tax threshold.[30] Because they are applied to net labour income, the tax/benefit policies we consider imply additional redistribution and insurance to that already in place.

To operationalise the counterfactual analysis, consider first the stochastic process e_t with the state space of net labour income $E_t = w_t \zeta_t^u S^u \cup w_t \zeta_t^b S^b$ and distribution $\xi_t^e = \left[n^u \xi_t^u; n^b \xi_t^b\right]$. The insurance policy then pays $\tau^s(e_t)$ as determined by:

$$\tau^s(e_t) = \begin{Bmatrix} \omega_t(\bar{e}_t - e_t), \text{ if } e_t < \bar{e}_t \\ 0, \quad \text{otherwise} \end{Bmatrix}, \tag{14}$$

where a higher $0 \leq \omega_t \leq 1$ implies more generous support and a higher $\bar{e}_t \leq \max(E_t)$ implies a wider coverage. The interest lies in evaluating the welfare gains, across the distribution, under different insurance policies that imply more extended coverage (i.e. a higher $\bar{e}$) versus more intensive support (i.e. a higher ω_t), consistent with a given government spending Ω_t:

$$\Omega_t = \int_{j \in E_t} \tau^s\left(e_t^j\right) \xi_t^e(dj). \tag{15}$$

We assume that the social benefits policies are financed by a labour income tax that is contingent on earnings, $\tau^\tau(e_t)$. For an arbitrary threshold $\tilde{e}_t$, the tax schedule is given by:

$$\tau^\tau(e_t) = \begin{Bmatrix} \tilde{\omega}_t(e_t - \tilde{e}_t), \text{ if } e_t > \tilde{e}_t \\ 0, \text{ otherwise} \end{Bmatrix}. \tag{16}$$

We choose $\tilde{\omega}_t$ conditional on $\tilde{e}_t$, so that, in each time period, the collected tax revenue equals Ω_t, i.e.:

$$\int_{j \in E_t} \tau^\tau\left(e_t^j\right) \xi_t^e(dj) = \Omega_t. \tag{17}$$

The households' budget constraint becomes:

$$c_t^h + a_{t+1}^h = a_t^h + r_t a_t^h + w_t \zeta_t^h s_t^h + \tau^s(e_t) - \tau^\tau(e_t). \tag{18}$$

To focus on marginal effects of additional social insurance policies, we take Ω_t to be set exogenously so that, for each policy and for each t, $\Omega_t = 0.1\% \times Y_t$. We consider two benefits schemes, one that offers a relatively extensive cover, obtained by setting $\bar{e}_t$ to be the 20^{th} percentile of the distribution of net labour income, and the other that targets the lower tail of the income distribution, obtained by setting $\bar{e}_t$ to be the net labour income of the 5^{th} percentile. Since ω_t follows residually to satisfy (15), it is higher in the second case, implying that the focused policy provides more support for a smaller target group. We also consider two tax schemes, one where the tax threshold is obtained by setting $\tilde{e}_t$ to be the median net labour income, and the other that targets the upper deciles of the income distribution to generate the additional tax revenue, obtained by setting $\tilde{e}_t$ to be the net labour income of the 80^{th} percentile of the distribution. We plot, in Appendix Fig. C.3, the implied tax/benefit functions under each policy and for each group separately.

In Fig. 9, we plot the welfare losses relative to the base welfare of households under the "Risk Only" changes in Fig. 8 and Table 8, for the alternative tax/benefit policies, across the initial wealth distribution. We show these for households initially

[30] We focus on taxes above median labour income, since in the UK income tax revenue comes predominantly from the above median income households (see e.g. Adam, 2019).

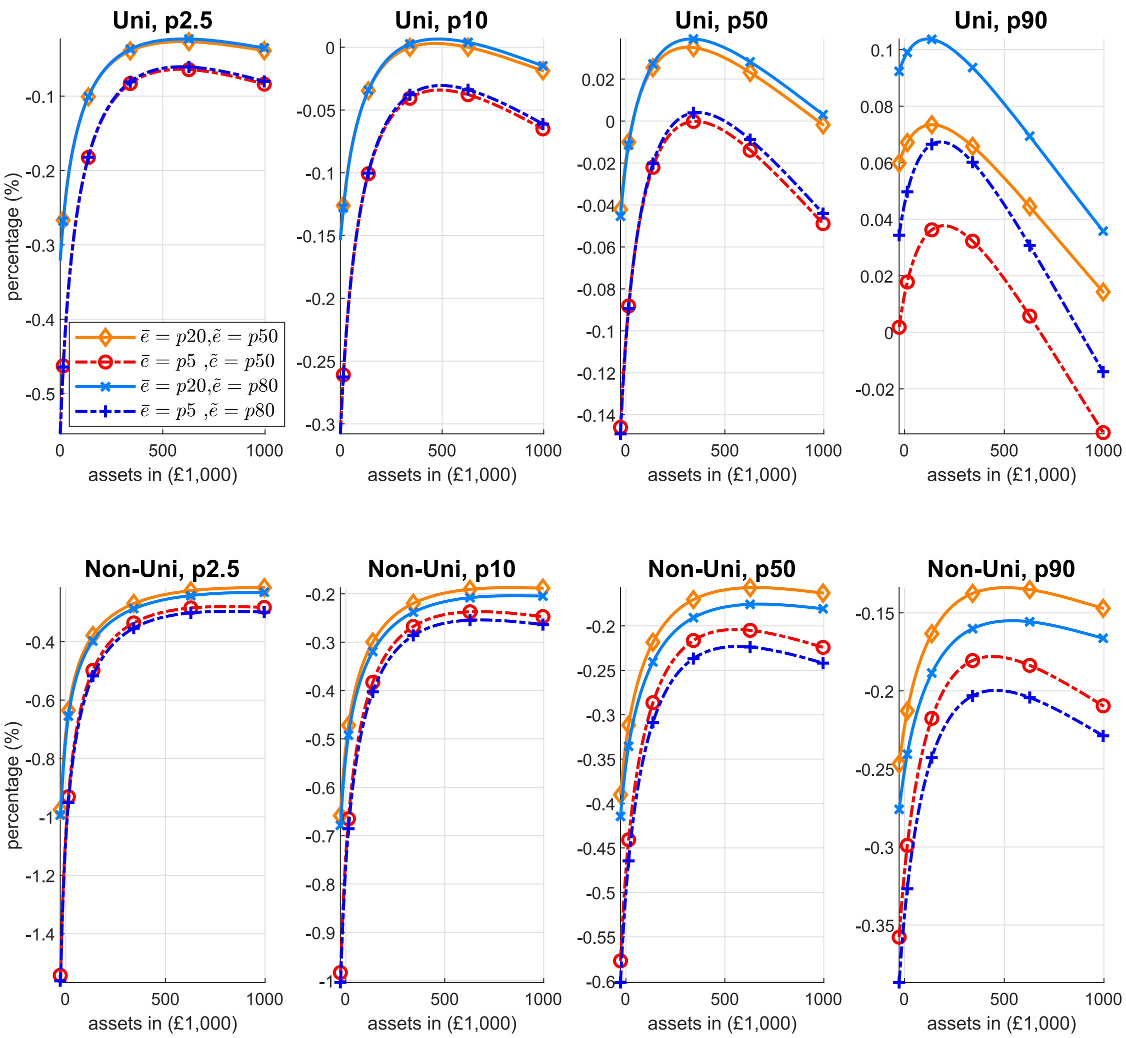

Fig. 9. Conditional welfare losses relative to Risk-Only.

on the 2.5^{th}, 10^{th}, 50^{th} and 90^{th} decile of the net labour income distribution. We choose the 2.5^{th} (10^{th}) decile as the median of the households to the left of the lower (upper) benefits threshold, and the 90^{th} as the median of the households to the right of the higher tax threshold. In Appendix Table C.2, we also summarise the welfare gains/losses for the same groups we examined in Table 8, under the same increase in income risk, for the same policies we consider in Fig. 9. Moreover, in Appendix Fig. C.4, we plot the welfare gains and losses for all policies, as well as for the Risk-only case, relative to the 2008 economy, to illustrate graphically the extent to which the interventions mitigate the welfare losses implied under Risk-only.

The first observation in Fig. 9 is that the additional social insurance interventions we consider and the implied between group redistribution (Fig. C.3) imply that gains and losses are not distributed uniformly. While the majority of non-university educated gain from the additional social insurance schemes, gains for the university educated materialise for those with lower net labour incomes (and, as discussed below, the proportion of the university educated households that have gains depends on the policy). More generally, non-university educated have higher gains than university educated (see also Table C.2). This result is an implication of the higher labour incomes of the university educated, and of the smaller increase in risk that they faced relative to the non-university educated. The relationship between gains/losses from the tax/benefit policies and wealth is not linear, especially for higher earnings groups. On one hand, gains from the insurance value of the policies tend to decrease with wealth, because additional insurance mainly matters for households with low wealth. At the same time, the losses from labour income taxation are also decreasing in wealth. Hence, as wealth increases, both the insurance gains and the tax losses fall. Because for low wealth the insurance gains are more important, it is their fall that gives rise to the upward part of the curves. On the other hand, because for high wealth insurance gains are little, it is the fall in tax losses with wealth that explains the downward slopping part of the curves. Overall, the higher welfare gains from such policies are for the households with low net labour income and low wealth. Conversely, the bigger losses or lower gains are for the households with higher labour income and lower wealth. The wealth dependence of insurance gains further underlies the magnitude of the welfare gains for non-university educated households, which, on average, have lower wealth than the university educated households.

An emerging result from Fig. 9 is that there is greater consensus regarding the targeting of the subsidies than the targeting of the taxes. There are differences between households regarding the desirability of higher tax thresholds, as university educated, generally with higher labour income, would prefer lower tax thresholds, and vice versa for non-university educated households. However, there are wider gains from insurance policies that support more those who receive large negative income shocks (i.e. the below policies), relative to those aiming to smooth income variation below the 20^{th} percentile (i.e. the $\bar{e} = p20$ policies). This is true even for households at the upper earnings quintiles, which are more likely to need the $\bar{e} = p20$ compared with the $\bar{e} = p5$, policy, highlighting the value that households attach to insurance against big income drops. Indeed, as can be seen in Fig. 9, the $\bar{e} = p5$ policies imply gains for the majority of the university educated households, in addition to the non-university educated. Overall, the results suggest that policy interventions to insure against bad shocks create wider benefits than policy to smooth income variation more broadly.

7. Conclusions

Focusing on groups of households defined according to whether the head of the household is university educated or not, this paper analysed the implications of heterogeneity in household income processes for the distributions of wealth and conditional welfare losses post 2008. Our analysis was motivated by empirical evidence documenting differences in wealth inequality and in income dynamics, both between university and non-university educated households and since 2008.

University educated households have higher mean net labour income, mean wealth and lower wealth inequality. Since 2008, mean income and wealth dropped more for the non-university educated households, wealth inequality increased for both groups, and income risk as well as precautionary savings increased more for non-university educated. Our analysis highlights the asymmetry of conditional welfare losses, between and within the groups. In particular, there are significantly higher welfare losses for the non-university educated, which are decreasing with initial wealth and labour income. We further find that the contribution of the rise in income risk to these welfare losses is sizeable.

The importance of income risk and the asymmetry of welfare losses can motivate social insurance intervention, in addition to policies already in place. Social insurance via tax/benefit policies creates welfare gains for the broad majority of non-university educated households, and for university educated households with lower net labour income. A lower benefits threshold increases gains whilst reducing losses, and is preferred by the broad majority of households. Social insurance is thus valued more when it insures against big adverse income shocks.

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Appendix A

The WAS started in July 2006 with a first wave of interviews carried out over two years to June 2008. The WAS interviewed approximately 30,500 households including 53,300 adult household members in Wave 1. The same households were approached again for a Wave 2 interview between July 2008 and June 2010. In this wave 20,170 households responded (around 70 percent success) including 35,000 adult household members. Waves 3–5 covered the periods between July and June for the years 2010-12, 2012-14 and 2014-16 respectively. After Wave 2, due to sample attrition, the WAS started implementing boost samples in each wave to keep the number of interviewed households around 20,000 and 35,000-40,000 adult household members.[31]

The BHPS is a longitudinal study for the UK running from 1991 to 2008. In the first wave, the BHPS achieved a sample size of around 5000 households (10,000 adult interviews) or a 65% response rate. After the first wave, due to sample attrition, the sample size shrank slightly. For example, in 2000 it achieved around 4200 complete interviews or a 75% response rate (see Taylor et al., 2010). The DCANHIV is a supplement to the set of derived income variables in the official BHPS release which focus on gross income (see Bardasi et al., 2012).

[31] The WAS and BHPS datasets employed in this paper refer to the free "End User Licence" versions of the datasets (i.e. WAS: SN-7215 and BHPS: SN-5151). Additionally, the BHPS Derived Current and Annual Net Household Income Variables dataset (DCANHIV) that we use is DCANHIV: SN-3909.

A1. Demographics (BHPS)

1. **Head of the Household**: We use the BHPS definition of the head of household. The head of household is defined as the principal owner or renter of the property, and, where there is more than one head, the eldest takes precedence. (wHOH)
2. **Education level**: BHPS includes very detailed information on educational attainment. We have used the variable wQFEDHI (where the prefix w denotes wave). To examine the potential heterogeneity of income risk in the main text, the sample is split into degree holders and non-degree holders. The former are the individuals who hold either a Higher Degree or 1st Degree, while the latter are the individuals who hold either Higher National Certificate/Diploma or teaching qualifications or A-levels/AS level/Highers or GCSE/O level/other qualification or they have no qualifications.
3. **Age**: The BHPS provides the age variable consistently in all waves. (wAGE)
4. **Year**: Note that for the BHPS measures, the period of observation refers to an annual cycle starting in September.

A2. Definition of net labour income (BHPS)

Household net labour income:[32] is obtained from the DCANHIV dataset (Bardasi et al., 2012) and is defined as household net labour earnings plus benefits, plus private transfers. It is equal to household total annual gross labour income, plus annual social benefits, plus annual transfers income minus taxes, NI contributions and private pension contributions. Private transfers income totals all receipts from other transfers (including education grants, sickness insurance, maintenance, foster allowance and payments from TU/Friendly societies, from absent family members). Social benefits income totals all receipts from state benefits including national insurance retirement pensions. Household (Annual) Net Labour Income=Net Labour Income (whhyrln) + Private Transfers (whhyrt) + Public Benefits (whhyrb).

A3. Sample selection (BHPS)

Our sample selection for BHPS, reported in Table A.1. The household heads must be between 25–59 years of age and must not have missing values for region and educational attainment. First, we keep the households with positive net labour income. Then, for each group and for each year, we drop the observations at the bottom 1% of the net labour income distribution. Then, we only keep households who are in the sample for at least three consecutive periods. In addition, we further trim the top and bottom 0.25 percent of observations in the distributions of net labour income growth rates (see Storesletten et al., 2004 for similar trimming of the growth rates). As in the WAS, we exclude Northern Ireland.

Table A.1
Households observations per selection step, BHPS.

selection step	Uni	Non-Uni	Total
1. Whole sample			130,974
2. Drop proxy & non-full interviews			128,348
3. Original sample			82,355
4. Full interview of all members in household			74,602
5. Keep if heads' age ≥ 25, ≤ 59			46,850
6. Drop if no head's educational info			46,443
7. Drop if head's region missing			46,409
8. Drop if head's marital status missing			46,406
9. Drop if net labour income is zero	7,910	38,185	46,095
10. Drop the bottom 1% of income distribution (per year for each group)	7,842	37,810	45,652
11. Keep if present at least at 3 consecutive waves	6,988	33,120	40,108
12. Drop observations when the head changes educational group	6,950	33,120	40,070
13. Drop the top and bottom 0.25% of first differences distribution	6,920	32,976	39,896
Average net labour income obs per year	384	1,832	2,216

A4. Demographics (WAS)

1. **Head of the Household**: We define the head of household as the principal owner or renter of the property, and, when there is more than one head, the eldest takes precedence. This follows the reference person definition in BHPS. We use of the following variables: (HhldrW), (HiHNumW), (DVAGEw) and/or (DVAge17w).
2. **Education level**: There are two educational attainment variables in the WAS. The first is the TEAw, which is the age that the individual completed education. The second is the EdLevelw which is a derived variable of the education level

[32] All monetary values are expressed in 2008 prices as measured by CPI.

Table A.2
Wealth Inequality in Great Britain.

	mean	Gini	$\frac{sd}{mean}$	$\frac{mean}{median}$	top 10%	$\frac{a_u}{a_b}$
			WAS (wave 1)			
Uni	258.3k	0.644	1.948	1.846	0.460	
Non-Uni	123.9k	0.702	1.972	2.073	0.480	2.085
Total	159.7k	0.696	2.121	2.000	0.492	
			WAS (wave 2)			
Uni	224.4k	0.632	1.697	1.800	0.442	
Non-Uni	104.5k	0.714	1.983	2.408	0.481	2.148
Total	137.5k	0.699	1.977	2.141	0.487	
			WAS (wave 3)			
Uni	224.0k	0.650	1.962	1.963	0.470	
Non-Uni	100.9k	0.728	2.447	2.515	0.502	2.220
Total	136.6k	0.714	2.343	2.273	0.511	
			WAS (wave 4)			
Uni	233.1k	0.690	2.835	2.274	0.522	
Non-Uni	93.2k	0.747	2.300	3.344	0.530	2.502
Total	136.3k	0.741	3.030	2.732	0.555	
			WAS (wave 5)			
Uni	232.4k	0.682	2.278	2.254	0.510	
Non-Uni	100.0k	0.758	2.356	3.710	0.538	2.325
Total	145.9k	0.739	2.541	2.782	0.546	

Table A.3
Comparison between trimmed and untrimmed sample (waves 3–5).

	Untrimmed WAS waves 3–5		Trimmed WAS waves 3–5	
	Uni	Non-Uni	Uni	Non-Uni
Q1 share	-0.005	-0.015	-0.005	-0.016
Q2 share	0.035	0.002	0.036	0.002
Q3 share	0.095	0.067	0.098	0.068
Q4 share	0.196	0.218	0.202	0.222
Q5 share	0.679	0.728	0.669	0.723
T 90–95%	0.136	0.155	0.136	0.157
T 95–99%	0.197	0.213	0.189	0.211
T 1%	0.168	0.155	0.161	0.147
Gini	0.674	0.744	0.666	0.740
a_u/a_b	2.344		2.267	
Gini Total	0.731		0.725	

and represents the highest educational level that respondent has achieved. EdLevelw provides three categories: (i) degree level or above; (ii) below degree qualifications (iii) no qualifications. The TEAw has the disadvantage that it cannot distinguish the type of qualification that the respondent had achieved. Moreover, 33 percent of the TEAw observations of working-age adults have either missing values or partial answers. Thus, we choose to work with the EdLevelw which is a derived variable and has only 2,942 missing values, i.e. around 2.7 percent of working-age adult observations. However, using EdLevelw, we note that there are respondents for whom educational attainment changes in a way that indicates misreporting. For example, for some respondents, there is an increase of educational attainment just for one wave and then a return back to the previous level of education in subsequent waves. Thus, we have chosen to make some corrections to the educational level when a respondent's educational attainment changes. If we observe a respondent for all the 5 waves, we replace her educational attainment with the level that was reported the most times across the 5 waves. We follow a similar procedure if a respondent changes her educational attainment just once. More specifically, we require the respondents being present in the sample for at least 3 waves and we use the most commonly recorded education level across waves. These corrections were applied to 4,873 observations out of 107,320 total amount observations of adult respondents (around 4.5 percent) and only half of these 4873 observations correspond to a head of a household. Despite these corrections, the results are very similar when they are not made.

A5. Definition of wealth (WAS)

1. **Net property wealth**: is the sum of all property values less the value of all mortgages and amounts owed as a result of equity release. (HPROPWW).
2. **Net financial wealth**: is the sum of the values of formal and informal financial assets, plus the value of certain assets held in the names of children, plus the value of endowments purchased to repay mortgages, less the value of non-mortgage debt. The informal financial assets exclude very small amounts (less than £250) and the financial liabilities are the sum of current account overdrafts plus amounts owed on credit cards, store cards, mail order, hire purchase

and loans plus amounts owed in arrears. Finally, money held in Trusts, other than Child Trust Funds, is not included. (HFINWNTW_sum)

3. **Net Worth:** is the sum of the net property wealth and net financial wealth.

A6. Definition of net labour income (WAS)

Household net labour income: is defined as household net labour earnings plus benefits, plus private transfers. It is equal to household total annual gross labour income, plus social benefits, plus annual transfers income minus taxes, NI contributions and private pension contributions. Household (Annual) Net Labour Income=Net self-empl. income (DVNISEw_aggr) + Net empl. income (DVNIEMPw_aggr) + total benefit income (DVTotAllBenAnnualw3_aggr and DVBenefitAnnualW_aggr) + Net outside income (DVoiNfrAnnualw_aggr) + Net educgrant income (DVoiNegAnnualw_aggr) + Net govtrain income (DVoiNgtAnnualw_aggr) + Net redundancy income (DVoiNrrAnnualw_aggr).

A7. Sample selection for wealth statistics (WAS)

Table A.4 shows the various sample selection steps. The household heads must be between 25–59 years of age and have full information for the relevant demographic details. The remaining sample selection decisions follow those made for the BHPS.

Table A.4
WAS Sample selection, household observations per selection step.

selection step	Uni	Non-Uni	Total
1. Whole sample of households			110,963
2. Drop households with misreported age variable			110,937
3. Drop households with duplicate hh grid numbers			110,910
4. Keep if heads' age ≥ 25, ≤ 59			59,457
5. Drop if no or misreported head's educational info	17,490	41,056	58,546
Average net worth obs per wave	3,498	8,211	11,709

A8. Sample selection for net labour income risk (WAS)

Table A.5 shows the various sample selection steps for WAS regarding the calculation of net labour income risk. The household heads must be between 25-59 years of age, have full information for the relevant demographic details and be in the sample for at least two waves. The remaining sample selection decisions follow those made for the BHPS.

Table A.5
WAS Sample selection, household observations per selection step.

selection step	Uni	Non-Uni	Total
1. Whole sample of households			110,963
2. Drop households with misreported age variable			110,937
3. Drop households with duplicate hh grid numbers			110,910
4. Keep if heads' age ≥ 25, ≤ 59			59,457
5. Drop if no or misreported head's educational info	17,490	41,056	58,546
6. Drop waves 1 and 2	9,584	20,177	29,761
7. Drop if net labour income is zero	9,418	19,879	29,297
8. Drop the obs at the top 0.4% of net labour income distribution	9,320	19,860	29,180
9. Drop the bottom 1% of income distribution (per wave for each group)	9,228	19,663	28,891
10. Drop obs when they change educational groups	8,854	19,264	28,118
11. Drop if less than one obs in 3 waves	6,068	12,679	18,747
12. Drop the top and bottom 0.25% of first differences distribution	6,052	12,643	18,695
Average net labour income obs per wave	2,017	4,214	6,231

A9. From biannual to annual higher moments

Let the moments with tilde denote the moments in *biannual* frequency. Then for each group we calculate the higher moments in *annual* frequency by using the following transformations[33]:

$$m_2^{\mu} = \frac{\tilde{m}_2^{\mu}}{(1+\rho^2)}, \tag{19}$$

[33] We suppress the subscript h to make the notation simpler.

$$m_3^\mu = \frac{\tilde{m}_3^\mu}{(1+\rho^3)}, \tag{20}$$

$$m_4^\mu = \frac{\tilde{m}_4^\mu - 6\rho^2 \left(m_2^\mu\right)^2}{(1+\rho^4)}, \tag{21}$$

$$skewness_\mu = \frac{m_3^\mu}{\left(m_2^\mu\right)^{3/2}}, \tag{22}$$

$$kurtosis_\mu = \frac{m_4^\mu}{\left(m_2^\mu\right)^2}. \tag{23}$$

Appendix B

We define a stationary recursive equilibrium following e.g. Miao (2014, ch. 17) and Acikgoz (2018).[34]

`Stationary Recursive General Equilibrium`

For $h = u, b,$ a *Stationary Recursive Equilibrium* is stationary distributions $\lambda^h(A \times B)$, policy functions $a_{t+1}^h = g^h\left(a_t^h, s_t^h\right) : \mathcal{A}^h \times S^h \to \mathcal{A}^h,$ $c_t^h = q^h\left(a_t^h, s_t^h\right) : \mathcal{A}^h \times S^h \to \mathbb{R}_+,$ value functions $v^h\left(a_t^h, s_t^h\right) : \mathcal{A}^h \times S^h \to \mathbb{R},$ and positive real numbers K, $w(K)$, $r(K)$ such that:

1. The firm maximises its profits given prices, so that the latter satisfy (9) and (10).
2. The policy functions $a_{t+1}^h = g^h\left(a_t^h, s_t^h\right)$ and $c_t^h = q^h\left(a_t^h, s_t^h\right)$ solve the households' optimum problems in (7) given prices and aggregate quantities, and the value functions $v^h\left(a_t^h, s_t^h\right)$ solve equations (7).
3. Each $\lambda^h(A \times B)$ is a stationary distribution:

$$\lambda^h(A \times B) = \int_{\mathcal{A}^h \times S^h} \Lambda^h[(a,s), A \times B] \lambda^h(da, ds),$$

for all $A \times B \in \mathcal{B}\left(\mathcal{A}^h\right) \times \mathcal{S}^h,$ where $\Lambda^h[(a,s), A \times B] : \left(\mathcal{A}^h \times S^h\right) \times \left(\mathcal{B}\left(\mathcal{A}^h\right) \times \mathcal{S}^h\right) \to [0,1]$ are transition functions on $\left(\mathcal{A}^h \times S^h\right)$ induced by the Markov process $\left(s_t^h\right)_{t=0}^\infty$ and the optimal policy $g^h\left(a_t^h, s_t^h\right)$.

4. When $\lambda^h(A \times B)$ describe the cross-section of households at each date, markets clear. The labour market clears, i.e. $L = L^s = 1,$ where:

$$L^s = n^u \zeta^u \sum_{j \in S^u} \bar{s}_j^u \xi^u\left(\bar{s}_j^u\right) + n^b \zeta^b \sum_{j \in S^b} \bar{s}_j^b \xi^b\left(\bar{s}_j^b\right),$$

the international market clears, i.e.

$$r = \bar{r} + \psi\left[\exp\left(\frac{K - A^s}{F(K,L)}\right) - 1\right],$$

where

$$A^s = n^u \int_{\mathcal{A}^u \times S^u} g^u(a,s) \lambda^u(da, ds) + n^b \int_{\mathcal{A}^b \times S^b} g^b(a,s) \lambda^b(da, ds),$$

and the goods market clears, which, using factor input market clearing, implies:

$$F(K,1) - \delta K - r(K - A)$$
$$= n^u \int_{\mathcal{A}^u \times S^u} q^u(a,s) \lambda^u(da, ds) + n^b \int_{\mathcal{A}^b \times S^b} q^b(a,s) \lambda^b(da, ds).$$

Results in Acikgoz (2018) imply that there is a unique stationary distribution at the household level, which also determines mean assets. Following standard arguments (commonly used in this class of models since Aiyagari (1994)), it can be shown that a general equilibrium exists.[35] In particular, using results in Acikgoz (2018) and adapting arguments from Angelopoulos et al. (2019b), we can show the existence of a general equilibrium in the open economy.

Proposition 1. *For ψ sufficiently large, satisfying $\frac{K}{Y}(r) > \ln\left(\frac{r - \bar{r} + \psi}{\psi}\right)$, a stationary recursive general equilibrium exists.*

Proof. The properties of the production function imply that the wage rate is a monotonic function of the interest rate, and, given that $L = 1,$ K is a decreasing function of r, as is Y and the ratio $\frac{K}{Y}$. Firm's profit maximisation implies a demand for

[34] Aggregation over the households can be obtained by using the methods discussed e.g. in Uhlig (1996), Al-Najjar (2004) and Acemoglu and Jensen (2015).

[35] A general proof of existence of equilibrium for this class of models can be found in Acemoglu and Jensen (2015).

domestic capital via (11), given by:

$$A^d = \left[\left(\frac{K}{Y}\right) - \ln\left(\frac{r - \bar{r} + \psi}{\psi}\right)\right] Y,$$

which is a continuous function in r. When $\frac{r-\bar{r}+\psi}{\psi}$ is small enough such that $\frac{K}{Y} > \ln\left(\frac{r-\bar{r}+\psi}{\psi}\right)$, $\frac{dA^d}{dr} < 0$. Moreover, when $r \to \frac{1}{\beta} - 1$, $A^d \to A^{\min} < +\infty$, whereas when $r \to \bar{r} - \psi$, $A^d \to +\infty$. As shown in Acikgoz (2018), mean assets, A^s, are a continuous function of r and when $r \to \frac{1}{\beta} - 1$, $A^s \to +\infty$.[36] Moreover, when $r \to -1$, $A^s \to 0$. Therefore, an intersection point of the mean assets and domestic capital demand curves, A^s and A^d respectively, exists. □

Regarding the sufficient condition, $\frac{K}{Y}(r) > \ln\left(\frac{r-\bar{r}+\psi}{\psi}\right)$ note that the left hand side is decreasing in r, whereas the right hand side is increasing in r, meaning that it is more difficult to satisfy this condition for higher interest rates. The maximum value of the interest rate considered for capital demand, to permit existence of equilibrium, is $r^{\max} = \frac{1}{\beta} - 1$. Hence, when $\psi > \psi^{\min}$ such that $\psi^{\min}$ satisfies $\frac{K}{Y}(r^{\max}) = \ln\left(\frac{r^{\max}-\bar{r}+\psi^{\min}}{\psi^{\min}}\right)$, the sufficient condition is satisfied for all permissible values of the interest rate. Our calibration implies $\psi^{\min} = 0.0012$ and $\psi = 0.0028$.

The mean assets and domestic capital demand curves, A^s and A^d respectively, for our calibration, are shown in Fig. 4. As can be been, there is a single intersection point.

Computation

To compute the stationary general equilibrium, we implement the following algorithm:

1. Guess a value for r^n, which, given the first-order conditions (9) and (10) implies a value for K^n, Y^n and w^n.
2. Calculate the demand for domestic assets implied by the international financial markets via (11), given by

$$A^n = K^n - [\ln(r^n - \bar{r} + \psi) - \ln\psi]Y^n.$$

3. Given r^n and w^n, solve the "typical" households' problem to obtain $g^h\left(a_t^h, s_t^h\right)$, for $h = u, b$.
4. Use $g^h\left(a_t^h, s_t^h\right)$ and the properties of the Markov processes $\left(s_t^h\right)$ to construct the transition functions $\Lambda_{K_j}^h$. Using $\Lambda_{K_j}^h$, calculate the stationary distributions λ^h.
5. Using λ^h, compute the aggregate values of $A^s(r^n)$ that is supplied by the domestic economy and the updated value of

$$r^{n^*} = \bar{r} + \psi\left[\exp\left(\frac{K^n - A^s(r^n)}{Y^n}\right) - 1\right].$$

6. If $\left|r^{n^*} - r^n\right| < \varepsilon$, where ε is a pre-specified tolerance level, a stationary open economy general equilibrium has been found. If not, go back to step 1, and update $r^{n+1} = (1 - \varsigma)r^n + \varsigma r^{n^*}$ with $0 < \varsigma \leq 1$.

To solve the household problem we use the Endogenous Grid Method (Carroll, 2006). To implement this algorithm we first choose $a^{\min} = -\phi$. We then let $a^{\max} = 150$, which implies that, in the solution, the probability of asset holdings greater than 150 is less than $5 * 10^{-10}$. Following Maliar et al. (2010) we discretise the space of household assets $[a^{\min}, a^{\max}]$ by allowing for 500 points with the following formula:

$$a^i = a^{\min} + (a^{\max} - a^{\min})\left(\frac{i-1}{500-1}\right)^{\varkappa}, \forall i = 1, \ldots, 500$$

where $\varkappa = 2$. We have found that the obtained wealth distribution is robust to increasing $a^{\max}$ up to 200 and to decreasing it down to 100.

Appendix C

C1. The distribution of welfare gains/losses

To examine the welfare gains/losses we calculate the conditional welfare change for each type separately (on the cross-sectional distribution associated with the initial stationary economy), resulting from the transition that follows the change in mean incomes and income risk. We define the expected lifetime utility associated with the decision rule, $c_t^* = q_t^*(a_t, s_t)$, under the initial stationary equilibrium as:

$$V^*(a, s) = E_0 \sum_{t=0}^{\infty} \beta^t u(q_t^*(a_t, s_t) \mid a_0 = a, s_0 = s),$$

[36] For details see Acikgoz (2018), Theorem 1. Further note that continuity of mean assets with respect to the interest rate, for each type of household, also implies continuity for the weighted average between households.

and the expected lifetime utility associated with the sequence of decision rules, $\tilde{c}_t = \tilde{q}_t(a_t, s_t)$, along the transition path following the dynamics for mean incomes and income risk as:

$$\tilde{V}(a,s) = E_0 \sum_{t=0}^{\infty} \beta^t u(\tilde{q}_t(a_t, s_t) \mid a_0 = a, s_0 = s).$$

We then define the consumption equivalent variation, conditional on initial assets and income, $\upsilon(a,s)$, as the percentage change in consumption required to be taken from the household under the initial stationary equilibrium, so that it is indifferent between remaining in this economy as opposed to the economy under the dynamic transition. In particular, $\upsilon(a,s)$ is defined as the quantity that solves:

$$E_0 \sum_{t=0}^{\infty} \beta^t u([1-\upsilon(a,s)]q_t^*(a_t, s_t)) = E_0 \sum_{t=0}^{\infty} \beta^t u(\tilde{q}_t(a_t, s_t)).$$

For each type, the average gain/losses under the dynamic transition is calculated as follows:

$$e\upsilon^u = \int_{\mathcal{A}^u \times S^u} \upsilon^u(a,s)\lambda^u(da, ds),$$
$$e\upsilon^b = \int_{\mathcal{A}^b \times S^b} \upsilon^b(a,s)\lambda^b(da, ds),$$

and for the whole economy as:

$$e\upsilon = n^u e\upsilon^u + n^b e\upsilon,$$

where λ^u and λ^b are the initial unique invariant distributions for each type of household.

C2. Further results

This section contains further results relating to the welfare analysis in partial equilibrium and insurance policy when only risk changes are considered.

C2.1. Partial equilibrium

Table C.1
Partial equilibrium.

	Both	Risk-Only	Means-Only
		University	
average losses	0.84%	0.21%	0.63%
av. losses, < p50 of wealth and > p80 of income	1.04%	0.33%	0.71%
av. losses, < p50 of wealth and > p50 of income	0.99%	0.28%	0.71%
% of hh's with losses > 0%	100.00%	96.45%	100.00%
% of hh's with losses > 1%	21.31%	0.00%	0.02%
% of hh's with losses > 2%	0.00%	0.00%	0.00%
% of hh's with losses > 3%	0.00%	0.00%	0.00%
		Non-university	
average losses	2.17%	0.73%	1.44%
av. losses, < p50 of wealth and < p20 of income	3.20%	1.53%	1.69%
av. losses, < p50 of wealth and < p50 of income	2.97%	1.32%	1.67%
% of hh's with losses > 0%	100.00%	96.08%	100.00%
% of hh's with losses > 1%	99.08%	25.63%	96.56%
% of hh's with losses > 2%	54.37%	1.19%	0.60%
% of hh's with losses > 3%	9.11%	0.02%	0.00%

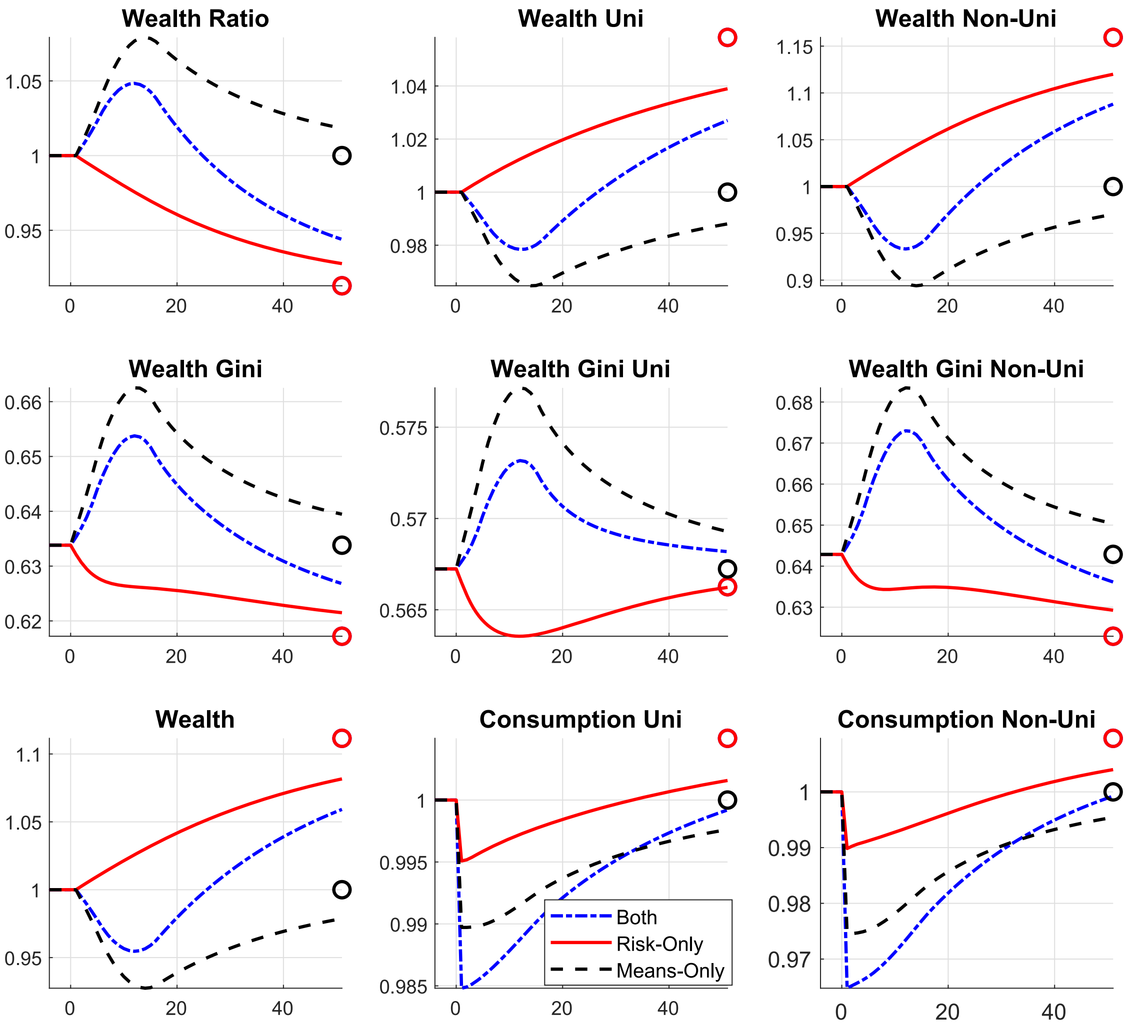

Fig. C.1. Partial equilibrium transition dynamics.

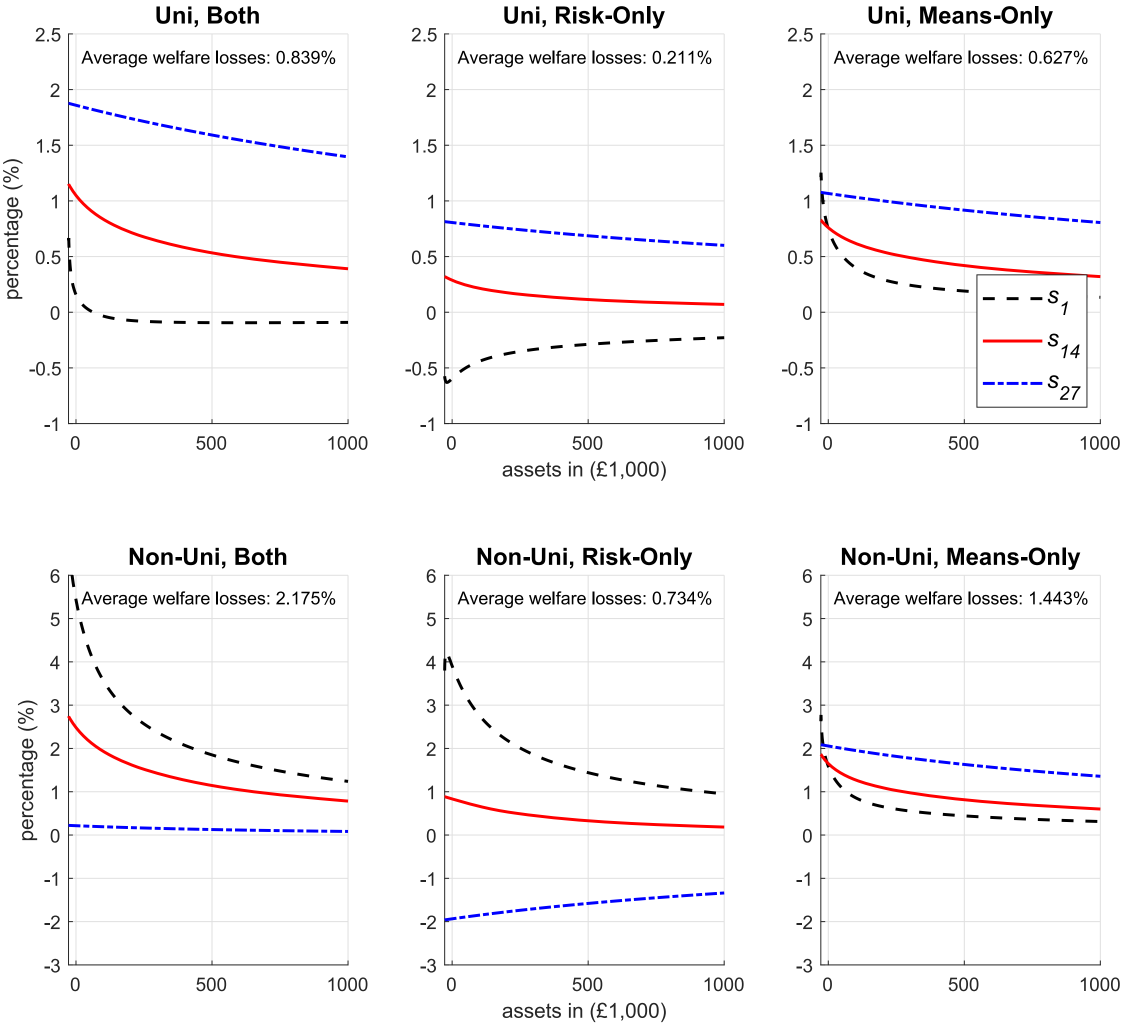

Fig. C.2. Conditional welfare losses per experiment, partial eq.

C2.2. Insurance policy

Table C.2
Welfare consequences of the insurance policy (Risk-Only).

	risk only	$\bar{e} = p5$ $\tilde{e} = p50$	$\bar{e} = p20$ $\tilde{e} = p50$	$\bar{e} = p5$ $\tilde{e} = p80$	$\bar{e} = p20$ $\tilde{e} = p80$
			University		
average losses	0.21%	0.13%	0.20%	0.14%	0.21%
av. losses, < p50 of wealth and > p80 of income	0.07%	-0.28%	-0.10%	-0.28%	-0.10%
av. losses, < p50 of wealth and > p50 of income	1.27%	-0.14%	0.00%	-0.14%	0.00%
% of hh's with losses > 0%	95.41%	82.93%	89.76%	82.69%	89.81%
% of hh's with losses > 1%	0.00%	0.00%	0.00%	0.02%	0.02%
% of hh's with losses > 2%	0.00%	0.00%	0.00%	0.00%	0.00%
% of hh's with losses > 3%	0.00%	0.00%	0.00%	0.00%	0.00%
			Non-university		
average losses	0.71%	0.29%	0.41%	0.27%	0.39%
av. losses, < p50 of wealth and < p20 of income	1.48%	0.62%	0.91%	0.60%	0.89%
av. losses, < p50 of wealth and < p50 of income	1.27%	0.55%	0.78%	0.53%	0.76%
% of hh's with losses > 0%	97.07%	86.32%	91.74%	79.40%	91.74%
% of hh's with losses > 1%	23.90%	0.00%	3.15%	0.00%	2.94%
% of hh's with losses > 2%	1.10%	0.00%	0.00%	0.00%	0.00%
% of hh's with losses > 3%	0.01%	0.00%	0.00%	0.00%	0.00%
ω(stationary eq.)		0.3655	0.0500	0.3655	0.0500
$\tilde{\omega}$(stationary eq.)		0.0062	0.0062	0.0155	0.0155

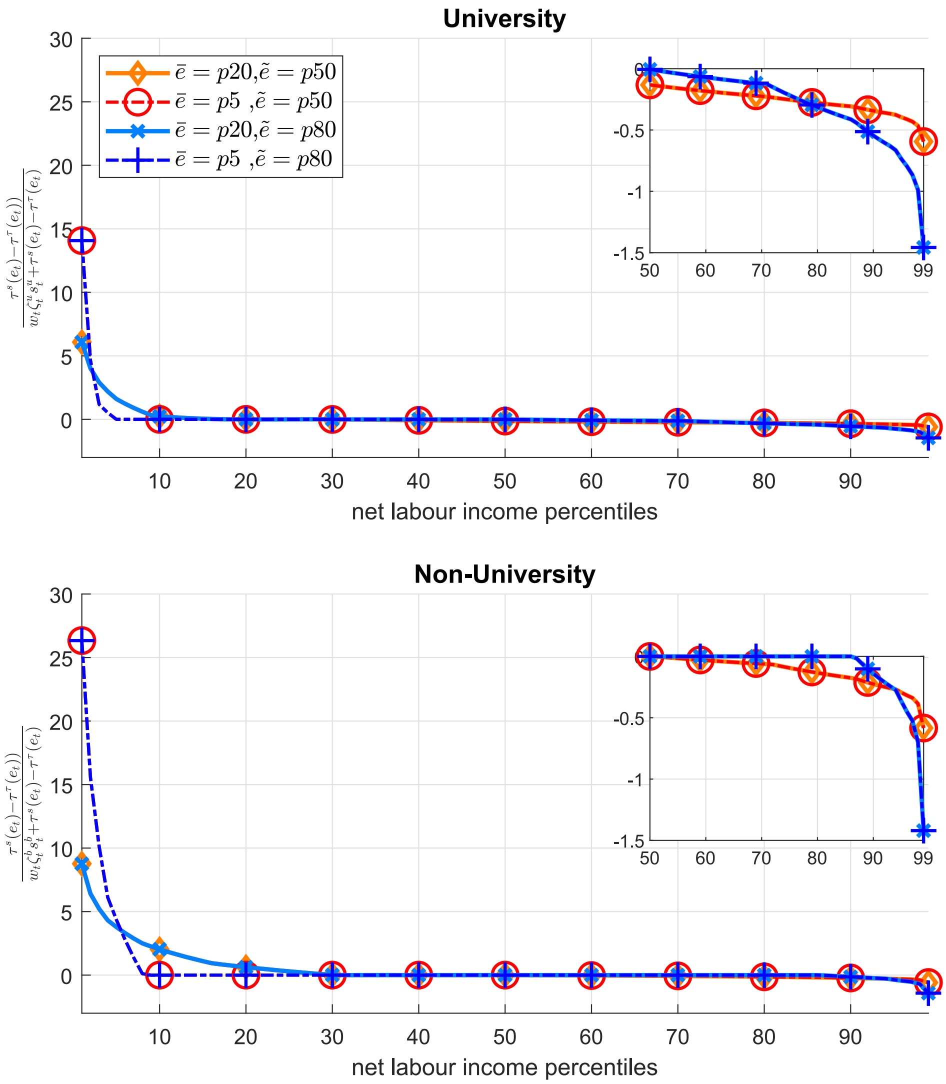

Fig. C.3. Effects of taxes and benefits by net labour income percentile.

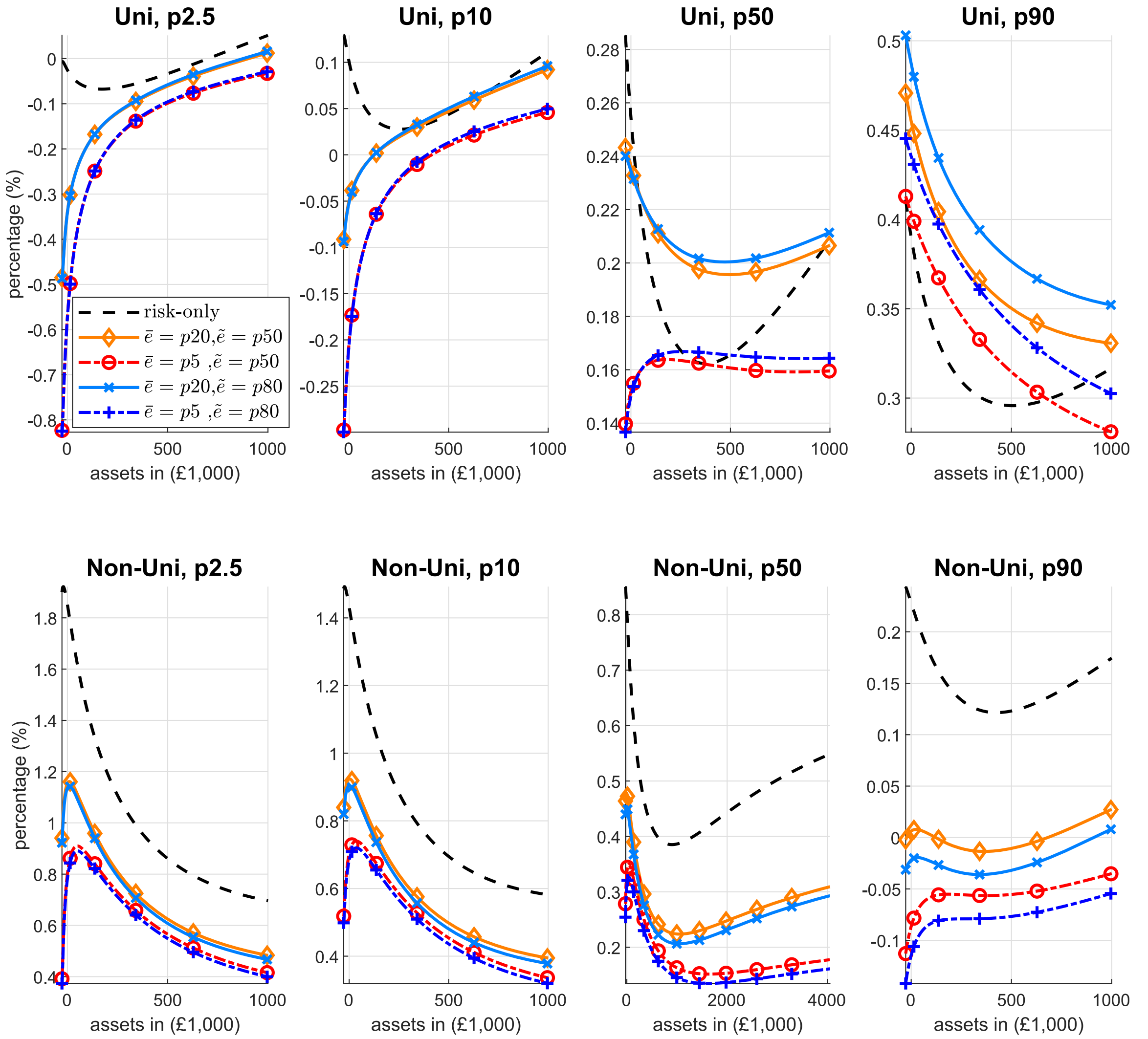

Fig. C.4. Conditional welfare losses.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.euroecorev.2020.103502

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