A Fast Blocking Matrix Generating Algorithm for Generalized Sidelobe Canceller Beamforming

Shaowei Dai*, Minghui Li, Qammer H. Abbasi and Muhammad Imran
James Watt School of Engineering, University of Glasgow, Glasgow G12 8QQ, United Kingdom
*Contact email: s.dai.2@research.gla.ac.uk

***Abstract*—A fast algorithm to generate the Blocking Matrix for Generalized Sidelobe Canceller (GSC) beamforming is proposed in this paper. The proposed algorithm uses a simplified zero placement algorithm (SZPA) to directly generate the independent null space vectors by specifying the zeros of the corresponding Z domain polynomials following the constrained signal incoming angle. Compared to the conventional methods based on Singular Value Decomposition (SVD), this method can be more than 10 times faster for scenarios with 15 constraints and will be even more advantageous for more constraints with the same converged state performance. Numerical simulation confirms the efficiency and effectiveness of the algorithm.**

I. Introduction

Adaptive Beamforming is an enabling technology for modern wireless communication standards like 5G and WiFi 6. As a widely used beamforming architecture, Generalized Sidelobe Canceller (GSC) has the benefit of converting a constrained optimization problem to an unconstrained problem [1] while maintaining the same performance as Linear Constrained Minimum Variance (LCMV) as proved in [2].

Blocking matrix is a critical component for GSC beamformer which blocks out the desired signals from being leaked to the cancellation path and thus being canceled as interference. A systematic approach for deriving the blocking matrix has been proposed in [3]. But like other implementations, it also needs to use the Singular Value Decomposition (SVD) based method to get the null basis of the constraint matrix. There is a need to improve the blocking matrix generation speed in a rapidly changing signal environment since every constraint angle change will trigger a recalculation of the blocking matrix.

In this paper, a fast algorithm working in the $\boldsymbol{Z}$ domain that combines zero placement generated polynomial and a simple linear independent sub-polynomials to derive a set of null space vectors is proposed and simulated. The novelty of this new algorithm is that it finds the null space basis vector directly in the $\boldsymbol{Z}$ domain with polynomial methods which eliminates the needs for computation demanding matrix null basis finding methods like SVD with more than 10 times faster speed and the same performance.

II. Problem Formulation

A typical GSC beamformer is illustrated in Fig. 1. It consists of a fixed beamformer $\boldsymbol{w}_f$ which controls the quiescent response for input vector $\boldsymbol{x}$ from M receivers, a blocking matrix $\boldsymbol{B}$ which projects the input vector $\boldsymbol{x}$ to the null space of the constraint matrix as vector $\boldsymbol{u}$ that is further adaptively combined to produce the estimated interference $\hat{y}$ for cancellation and an unconstrained beamformer $\boldsymbol{w}_a$ which is adaptively controlled through an adaptive algorithm block.

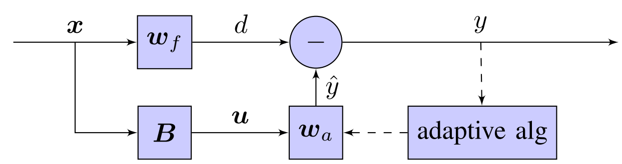

Fig. 1. Beamformer implementation with GSC structure

The required responses for signals from a set of N angles are regulated through a linear equation:

$$\boldsymbol{C}^H \boldsymbol{w} = \boldsymbol{f}^* \tag{1}$$

where $\boldsymbol{C}$ is the M by N constraint matrix with each column specified as a steering vector from a corresponding incoming angle, $\boldsymbol{w}$ is the overall equivalent weight for the GSC beamformer, $\boldsymbol{f}$ is the column vector with each element being the required response from the beamformer and $[\cdot]^*$ is the conjugate operator and $[\cdot]^H$ is the hermitian operator.

The problem for deriving the blocking matrix $\boldsymbol{B}$ in GSC could be summarized as to find vectors that satisfy:

$$\boldsymbol{C}^H \boldsymbol{w}_i = \boldsymbol{0} \tag{2}$$

where $w_i = w_1, w_2, ..., w_{M-N}$ stands for the $M-N$ degree of freedom for an M element array with N constraints.

III. Proposed Solution

Since any $M-N$ independent vectors in the null space of C span the whole null space, it might be simpler to just find $M-N$ vectors that satisfy equation (2). It turns out to be straightforward when looking from this perspective. For weight vectors $\boldsymbol{w}_i$ that satisfies (2), their $\boldsymbol{Z}$ transform $\boldsymbol{H}_B(z)$ could be separated into two parts. The first one contains the zeros for all the constraints.

$$C(z) = \prod_{i=1}^{N} (1 - z_i z^{-1}) \tag{3}$$

where N is the total number of constraints, z_i represents the zero location of the spatial frequency corresponding to the directions of interference and signals in the constraint matrix. The full $\boldsymbol{H}_B(z)$ then could be represented as

$$\boldsymbol{H}_B(z) = G(z)C(z) \tag{4}$$

where $G(z)$ represents the leftover $M-N-1$ degree of polynomial.

Since the signals from the N constrained direction corresponds to the N embedded z_i in (4), any signal from those directions will result in 0 output. So any valid $G(z)$ would make $\boldsymbol{H}_B(z)$ a valid transform for vectors in null space of constraint matrix.

So the task is simplified to just choose $M-N-1$ linearly independent polynomials to make up the null basis vectors.

And we can choose the simplest ones which is easy for hardware implementation and guaranteed to be independent.

$$G(z) = \{1, z^{-1}, z^{-2}, \dots, z^{-(M-N-2)}\} \tag{5}$$

Since the z^{-1} is just a delay operator, the $M-N-1$ vectors that form the null basis is just the $M-N-1$ shifted version of the core vector that produces the zero response for all the constraints. The algorithm is described as follows.

Algorithm 1 SZPA: Calculate zero location for $\boldsymbol{B}$ based on constraint matrix $\boldsymbol{C}_{MN}$

Require: $M \geq N$
1: **for** i in 1 to N **do**
2: $\quad \xi_i \Leftarrow \frac{d sin(\theta_i)}{\lambda_i}$ //Convert to spatial frequency
3: $\quad z_i \Leftarrow e^{j2\pi\xi_i}$
4: **end for**
5: $C(z) \Leftarrow \prod_{i=1}^{N}(1 - z_i z^{-1})$
6: **for** i in 1 to N **do**
7: $\quad \boldsymbol{h}(i) \Leftarrow$ coefficient of z^{-i}
8: **end for**
9: **for** i in 1 to M-1-N **do**
10: $\quad \boldsymbol{B}(:, i) = [zeros(i-1, 1); \boldsymbol{h}; zeros(M-1-N-i, 1)]$
11: **end for**

The shifting operation is equivalent to adding additional zeros in the origin for the $\boldsymbol{Z}$ transform. So effectively in step 10 we are getting all the required independent vectors in null space by just shifting. It is a very efficient and light operation. To our best knowledge, it has not been reported in the literature before.

IV. Numerical Simulation and Analysis

The blocking matrix $\boldsymbol{B}$ calculation time comparison is shown in Fig. 2. An Uniform Lineary Array (ULA) with 20 antenna elements spaced at half carrier wavelength is simulated in a MATLAB running on a Dell laptop with Intel i7 CPU. The angle of constraints increased from 1 to 15 for this antenna array which could happen in multiple user scenarios. The calculation time is averaged over 20 times of running.

It clearly shows that the proposed algorithm has very stable performance over the different sizes of constraints. In comparison, the calculation time for normal GSC increases linearly following the increased number of constraints partly

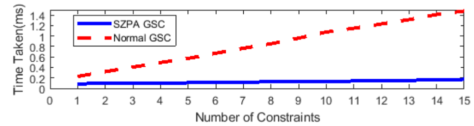

Fig. 2. Blocking Matrix Calculation Time Comparison

due to its requirement to directly build the constraint matrix from the specified angles and do matrix inversion and SVD operation which is known to be expensive. For 15 constraints, the speed to calculate the blocking matrix using SZPA is more than 10 times faster than Normal GSC (0.1ms vs 1.4ms).

The beam pattern performance is illustrated in Fig. 3. For clarity, the scenario with two angles constraints is depicted for an 8 element ULA where the desired signal is from 20°, a constrained interference is from 40° and an unconstrained interference is from 50°.

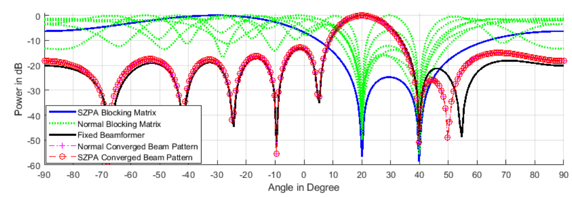

Fig. 3. Beam Pattern Performance Comparison

The blocking matrix prevents the constrained signal from being leaked to the cancellation path thus there are nulls formed at 20° and 40°. The overall converged beam patterns confirm that the main beam is untouched and nulls are formed at the constrained interference direction 40° and unconstrained direction 50°. The green dotted lines show patterns for each vector of the normal blocking matrix. But vectors of the SZPA blocking matrix are just shifted versions of each other, their amplitude response is the same thus the blue pattern appears as only one track. The overall converged state beam pattern shows that both algorithms achieve exactly the same performance.

V. Conclusion

A simplified zero placement algorithm (SZPA) to generate the blocking matrix is proposed and simulated. The simulation results confirm the effectiveness of the proposed method and it could be more than 10 times faster than the conventional SVD based method for scenarios with 15 constraints and even more advantageous for more constraints and provides the same converged state performance.

References

[1] S. Dai, M. Li, Q. H. Abbasi, and M. Imran, "Improve tracking speed of beamformer with simplified zero placement algorithm," in *2019 IEEE MTT-S International Wireless Symposium (IWS)*, May 2019, pp. 1–3.

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