Auto-calibration of Linear Array Antenna Positioning for Single Snapshot Direction of Arrival Estimation

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***Abstract*—This paper presents a novel joint auto-calibration of array positioning and single snapshot direction of arrival (DOA) estimation. This technique is useful in scenarios where the array position has uncertainties which lead to degradation in DOA estimation performance as well as in an environment where the snapshot samples are limited. Our proposed joint technique presented an improved DOA estimation performance gain of 75.22% when compared to a non-calibrated estimator. Also, our results presented similar average DOA estimation performance when compared to existing state-of-the-art joint techniques without the need for high snapshot values.**

***Keywords*—Auto-calibration, DOA (Direction of Arrival), Single Snapshot, Uniformed Linear Array (ULA)**

I. INTRODUCTION

Many algorithms have been studied and proposed during the last decade for Direction-of-Arrival (DOA) estimation [1]-[3]. However, it is known in [2]-[3] that these DOA techniques degrade the ability of the signal covariance matrix estimation in single snapshot scenarios. An alternate method has been proposed in [3] for a single snapshot scenario by exploiting the spatial properties of a Uniformed Linear Array (ULA) to estimate its covariance matrix with potential application in automotive radar where the snapshot sampling frequency is low [2] or in any low-cost situation where hardware performance is limited without compromising accuracy. The main drawback in [2] is that because it depends on the spatial properties of the ULA, precision is required when it comes to the position of the array elements. Any measurement uncertainties or miscalibration can lead to a degradation of estimation performance for the DOA estimator in [2]. Ref. [6] presented a self-calibration technique by iterating between direction estimation and gain/phase estimation via minimizing a cost function until convergence. However, this technique still requires a high number of snapshots to obtain reasonable convergence to its cost function.

To that end, the objective of this paper is to enable the spatial single snapshot technique by introducing a Sequential Quadratic Programming (SQP) optimization technique enabling auto-calibration of the antenna array positioning that allows a higher accuracy when deriving the covariance matrix.

This paper is organised as follows. Section II describes the mathematical model of a single snapshot DOA (SS-DOA) estimation and the usage of auto-calibration of antenna positioning respectively followed by a flowchart of our proposed technique. Section III presents the simulation results. Finally, we conclude this paper in section IV.

II. SIGNAL MODEL

A. Single Snapshot DOA Estimation

A typical approach for estimating the covariance matrix denoted as $\hat{R}_x$ is by using K temporal snapshots $x(k), k = 1, \dots, K$ as follows [2]-[4]]:

$$\widehat{\boldsymbol{R}}_{\boldsymbol{x}} = \frac{1}{K}\sum_{k=1}^{K}\boldsymbol{x}(k)\boldsymbol{x}^{\boldsymbol{H}}(k) = \frac{1}{K}\boldsymbol{X}\boldsymbol{X}^{\boldsymbol{H}} \tag{1}$$

where $\boldsymbol{X}$ is being composed out of K temporal snapshots. Alternatively, a single spatial snapshot approach can be realised by exploiting the spatial properties of the ULA. Thus, if we only use one spatial snapshot of a received signal vector-matrix x, the covariance matrix can be estimated as [3]:

$$\widehat{\boldsymbol{R}}_{\boldsymbol{x}(ss)} = \frac{1}{M}\boldsymbol{X}_{\boldsymbol{toep}}{}^{\boldsymbol{H}}\boldsymbol{X}_{\boldsymbol{toep}} \tag{2}$$

where $\boldsymbol{X}_{\boldsymbol{toep}}$ contains a single signal vector $x = [x_1 \quad x_2 \quad \cdots \quad x_M]^T$ in a sparse Toeplitz geometry. Thus, a finite data record of length M array elements is used to estimate the covariance matrix when compared to (1) where only 1 temporal data is acquired.

B. Auto-calibration of Antenna Positioning & Proposed Technique

We developed a cost function that measures the estimation error and the best-case relationship between (2) and the signal source. Equation (3) which is dependent on the covariance matrix $\widehat{\boldsymbol{R}}_{\boldsymbol{x}(ss)}$, the initial element position,d_{init} and estimates of the source locations,θ_{init} as an initialization criterion which is defined as follows:

$$x_{cf} = f\left(\widehat{\boldsymbol{R}}_{\boldsymbol{x}(ss)}, d_{init}, \theta_{init}\right) = \log_{10}\left\|diag\left(\boldsymbol{A}^{\boldsymbol{H}}\boldsymbol{E}_{\boldsymbol{N}}\boldsymbol{E}_{\boldsymbol{N}}{}^{\boldsymbol{H}}\boldsymbol{A}\right)\right\| \tag{3}$$

where $\boldsymbol{A}$ and $\boldsymbol{E}_{\boldsymbol{N}}$ denotes the estimated array steering vector and Eigen-decomposed noise subspace respectively from $\widehat{\boldsymbol{R}}_{\boldsymbol{x}(single)}$ which are dependent on the array positions.

Observing the auto-calibration as a constrained optimization problem, we employ a state-of-the-art constraint optimizer SQP coupled with an SS-DOA estimation cost function. The SQP algorithm can be referred to in [6]. Fig.1 presents a flowchart of our proposed technique. The cost function in (3) alongside an SQP optimizer is used. The iteration ends when the step size goes below the step, angle and location tolerance which are the convergence properties. The cost function in (3) is minimised when (2) corresponds to its smallest eigenvalue. The auto-calibration will iterate between estimating the source position and updating the calibration parameters within the user-defined tolerance.

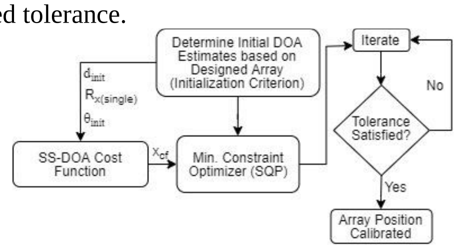

Fig. 1: Auto-Calibrating SS-DOA Estimator Flowchart

III. Simulation Results

Consider a 4-element ULA operating at 5500 MHz centre frequency spaced at half-wavelength. Next, assume the perturbed first reference sensor and the direction to the second sensor is known [5] with 2 narrowband signal sources impinging on the array at 10° and 35° with a Signal-to-Noise Ratio of -30dB. We ran 300 independent simulation runs to obtain the root-mean-squared error (RMSE). In this simulation, we compared against a non-calibrated SS-DOA derived in (2) [3], against the proposed auto-calibrating SS-DOA technique to demonstrate single snapshot performance. Table. 1 presents the array coordinates measured from geometric center for the designed, perturbed, calibrated and the difference in position between perturbed and calibrated positions. From Δd, we can see that the calibrated array closely resembles the perturbed array after auto-calibration with an average standard deviation of 21mm. The calibrated positions will have an estimation impact as demonstrated next in Fig. 3.

Array Position:	1 (ref)	2	3	4
Designed/Perfect (mm):	-40.9	-13.6	13.6	40.9
Perturbed (mm):	-40.9	-12.8	12.3	41.2
Calibrated (mm +/- 5%):	-40.9	-3.6	21.0	30.9
Δd of Perturbed & Calibrated (mm +/- 5%):	0	-9.2	8.7	10.3

Table 1: Averaged Array Sensor Coordinates

Fig. 3 presents a normalised polar plot of the auto-calibrating and non-calibrated SS-DOA respectively where the magnitude of each result has been normalized to unity for comparison. We observe that the calibrated estimator achieves a closer estimate to the true directions when compared to the non-calibrated results. The calibrated and non-calibrated DOA estimators presented an averaged RMSE of 2.915° and 11.767° respectively. This is a 75.22% average DOA estimation performance gain for the auto-calibrating SS-DOA. As the steering vector and Eigen-decomposed subspaces are dependent variables of the array positions, a calibrated ULA would provide an accurate estimation as opposed to a non-calibrated SS-DOA. In addition, our proposed technique has RMSE DOA estimation performance that is similarly comparable when compared to [6] without the need for high snapshot values.

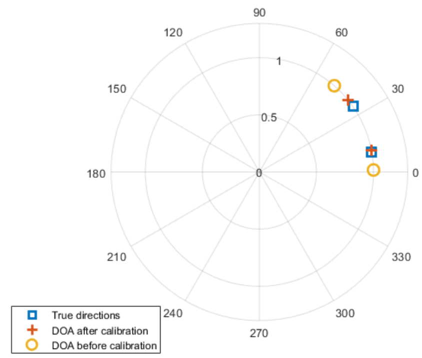

Fig. 2: Averaged DOA Estimation Results in Normalized Polar Plot

IV. Conclusion

This paper presented a preliminary study of a novel integration of auto-calibration of array position and SS-DOA estimation. From the simulation, we observed that our proposed auto-calibration SS-DOA technique has a DOA estimation performance gain of 75.22% and a calibrated array position standard deviation of 21mm against non-calibrated algorithms. When compared to other state-of-the-art techniques, we obtain similar averaged DOA estimation performance without the need for high snapshot values.

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References

[1] M. Muhammad, M. Li, Q. H. Abbasi, C. Goh and M. Imran, "Performance Evaluation for Direction of Arrival Estimation Using 4-Element Linear Array," 2019 13th European Conference on Antennas and Propagation (EuCAP), Krakow, Poland, 2019, pp. 1-5.

[2] C. Degen, "On single snapshot direction-of-arrival estimation," 2017 IEEE International Conference on Wireless for Space and Extreme Environments (WiSEE), Montreal, QC, 2017, pp. 92-97.

[3] D. Rajeswaran, "Single Snapshot Direction of Arrival(DoA) Estimation Using Variant Min-Norm Method," 2018 International Conference on Communication and Signal Processing (ICCSP), Chennai, 2018, pp. 0468-0471.

[4] Y. Rockah and P. M. Schultheiss. Array shape calibration using sources in unknown locations- Part I: Farfield sources. IEEE Trans. ASSP, 35:286-299, 1987.

[5] S. Rao, *Engineering Optimization: Theory and Practice*, 4th ed. Hoboken, NJ: Wiley, 2009, pp. 222-230.

[6] B. Friedlander, "A self-calibration technique for direction estimation with diversely polarized arrays," 2013 Asilomar Conference on Signals, Systems and Computers, Pacific Grove, CA, 2013, pp. 2145-2149.