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Experimental investigation of the effect of yaw angle on the inflow of a two-bladed propeller

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Nomenclature

$a_{[a,r,\omega]}$	Induction factors [axial, radial and tangential]	$[-]$
C_{av}	Propeller average chord	$[m]$
D	Propeller diameter	$[m]$
J	Advance ratio, $J = V_\infty/nD$	$[-]$
R, r	Propeller radius, propeller radial position	$[m, -]$
$Re_{0.7}$	Reynolds number (based on advancing blade $0.7R$ position and chord)	$[-]$
$V_{axial(LDA)}$	LDA measured velocity perpendicular to the propeller plane	$[m/s]$
$V_{[\infty,R]}$	Velocity [freestream, resultant]	$[m/s]$
$V_{[n,p]}$	Freestream velocity components w.r.t the propeller disc [normal, perpendicular]	$[m/s]$
$X, Y, Z_{[D,F]}$	Axis system dimensions [disc and freestream coordinate systems].	$[-]$
$\beta_{0.7}$	Pitch angle at $0.7R$	$[\circ]$
α_R	Blade component resultant angle of attack	$[\circ]$
Γ	Circulation	$[m^2/s]$
γ	Yaw angle	$[\circ]$
φ	Blade component advance angle	$[\circ]$
ψ	Blade azimuthal position	$[\circ]$
n, ω	Propeller rotational speed	$[revolutions/s, radians/s]$
AIF	Axial Induction Factor	
CFD	Computational Fluid Dynamics	
ESDU	Engineering Sciences Data Unit	
LDA	Laser Doppler Anemometry	
N/A	Not Applicable	
WT	Wind Tunnel	

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Abstract

An experimental investigation has been performed to measure the inflow of a two-bladed propeller. This is conducted over a range of advance ratios ($J = 0.36$ to 1.54) and yaw angles ($\gamma = 0$ to 20°) at a Reynolds number of $Re \approx 111000$, based on the chord length and advancing blade resultant velocity at the 70% spanwise position. Drawing on experimental results, three major qualitative trends are observed, that characterise the axial induced inflow of inclined propellers: the formation of an approximately sinusoidal inflow trend around the azimuth featuring broad regions of increased and reduced inflow, an azimuthal phase shift of the trend from the advancing and retreating blade positions, and the dependencies of the inflow maxima, minima and phase shift on advance ratio, yaw angle and radial position. These are subsequently compared against two widely used assumptions for calculating the aerodynamic environment of inclined propellers; firstly, that inflow at each radial and azimuthal element is calculated using an axial approximation of the whole propeller disc operating within the same condition as that element, and secondly, uniform induced inflow across the entire propeller plane. The comparison demonstrates the limitations of said assumptions by highlighting their inability to account for components of the trailed and shed vortical systems of the propeller wake.

Keywords: Experimental aerodynamics, Propeller propulsion, Fluid mechanics, Yawed flow

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1. Introduction

Propellers boast a distinct performance advantage over any other form of aircraft propulsion system up to Mach 0.8 [1]. Despite this, the mid to high velocity (Mach 0.6 to 0.8) civil aviation market has been largely dominated by turbofans. This can be attributed to three main factors - the technical complexities of minimising compressibility effects on propellers, the limited regulation of emissions, and the financial availability of fossil fuel in the past. In recent years, however, the former difficulties are diminishing with high speed optimised blade designs, and the need to reduce emissions and increase cost-efficiency is rising - propeller propulsion shows promise to be more widely adopted. Research in this field is, however, somewhat fractured [2]. As a consequence, research gaps in it stand to be highlighted; a vivid example of this being the behaviour of propellers when exposed to an inclined airflow. This

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is relevant as propeller design methods use reduced-order models, which are reliant upon experimental or other data for their formulation.

The bulk of experimental data available on the topic consists of just a few studies conducted predominantly in the first half of the twentieth century. The experimental data can be divided into two categories. The first are studies interested in the effects of flow inclination angle for the aircraft as a whole, focusing on the measurement of averaged forces and moments of inclined propellers and their effect on the stability of the aircraft and propulsive efficiency. These studies were the pioneers in the topic, but only provide enough information for low order models and limited assumptions, rather than in-depth understanding (Bramwell 1914 [3], Harris 1918 [4], Durand & Lesley 1923 [5], Flachbart 1930 [6], Freeman 1932 [7], Lesley 1937 [8]).

The second category consists of studies that considered more detailed aspects of inclined propellers, such as structural effects and wake aerodynamic composition. This was achieved via the employment of strain-gauges and vibrometers (Gray 1954 [9], McLemore 1954 [10], Yaggi 1960 [11]), and wake pressure rakes (Pendley 1943 [12], Russel 1953 [13]). These are the studies where defining features of the propeller in non-axial flight, such as the once per revolution oscillation of forces and the azimuthal shift of the velocity extrema, were first demonstrated.

While being informative of some basic trends, the information in those studies is constrained by a number of factors. These include, but are not limited to, restricted temporal and geometric data resolution, lack of well-populated experimental envelopes in some cases, and limited data accuracy. It should be noted that there are newer investigations in the field (e.g. [14, 15, 16]), they are, however, focused towards different aspects of inclined propeller behaviour. To summarise, while the experimental data gathered does offer some insight into the aerodynamics of inclined propellers, they offer enough information, at best, to create and validate a low order approximation of it, rather than true to life models.

The focus of the current paper is to study the inflow of the propeller. This is defined as the sum of induced and incident flows. Where the former represents the velocity increment imposed on the flow from the presence of a propeller generating thrust. The latter represents the velocity at the propeller aerodynamic environment without the presence of the propeller. Both of these are critical to the aerodynamic performance of the propeller blades, and hence - to any solvers that try to estimate it. This is of particular importance to low-order models as they often simplify the induced flow component to a first-order mathematical function. To the authors' knowledge, most of the widely used mathematical models still work under the assumptions laid out by de Young [17] in 1965, Crigler [18] in 1953, or Peters et al. [19] in 1988, reflecting their limitation in terms of both high-quality experimental data and computational power. Additionally, in the case of control models for smaller aircraft, Blade Element Theory (e.g. [20]), or functions generated from experimental data (e.g. [21]) can also be utilised in this capacity. Direct comparison to BET in this case is not possible, as the base version will not support inflow calculations outside the propeller plane. It is, however, expected to share similarities with the Crigler [18] solver presented, as it uses BET as a starting point for its calculations.

The method presented by de Young [17] in 1965 is an extension of the model initially put forward by Ribner [22] in 1943. It is forwarded for discussion in this paper because it is at the heart of the **ESDU 89047** [23] engineering data sheet, making its core a part of many consequent solvers, and by extension - one of the most widely used publicly available numerical solvers for inclined propellers. One of the assumptions it operates under is uniform induced inflow across the propeller disc. The second of the widely used models is presented by Crigler and Gilman [18] in 1952. It assumes a "steady-state" condition. This method works by calculating the aerodynamic environment for the whole propeller at a number of azimuthal positions, where each iteration uses its local azimuthal location conditions as global for the propeller.

The last of the widely utilised models is the one presented by Peters et al.[19], in 1988, which is originally developed for helicopters, but can also be adapted to propellers [24]. It is a reformulation of the original model presented by Pitts and Peters [25], that allows the inclusion of the effects of pitch and yaw among other improvements, notably the ability to solve for solutions of orders higher than one. It represents changes in induced flow as a truncated Fourier series with a prescribed radial distribution, and works under the assumption of superposition of pressure. While higher-order solvers do exist (e.g [26]), they will be omitted in this paper due to their computational expense and because the experimental result set has already been compared in detail against CFD [27].

Studying the inflow of yawed propellers is an opportunity to provide validation for more recent numerical methods, while also highlighting the limitations of previous studies performed so far.

2. Methodology

To this end, an experimental investigation has been performed, in the University of Glasgow De Havilland wind tunnel facility, to measure inflow into a two-bladed propeller, over a range of advance ratios and yaw angles as indicated in Table 1 (25 distinct cases and 50 runs). All experimental cases have a Reynolds number of 110800 ± 400 , based on the resultant velocity on the $0.7R$ position of the advancing blade. The wind tunnel has a closed-return design. The settling chamber to test section contraction ratio is 5:1, and the test section size is $2.74\text{m} \times 2.1\text{m} \times 5\text{m}$ (Width $\times$ Height $\times$ Length). It can provide a maximum freestream velocity of 70 m/s with good flow uniformity (turbulence intensity levels under 0.2 %). The measurements were conducted using a Dantec Dynamics three-component Laser Doppler Anemometry (LDA) system 3 cm upstream of the hub and propeller centre. The experimental plane consists of a coordinate grid of 360 points (36 azimuthal and 10 radial positions). Each point was measured for a duration of 3s. The distance was chosen based on the minimal distance to the propeller plane that provides laser access across the experimental grid. Axial and lateral alignment of the measurement plane to the propeller plane is illustrated in fig. 1.

The LDA system consists of three diode pumped solid state 1W lasers of frequencies 488 nm, 514 nm and 532 nm. These are fed into two LDA probes of diameter 160 mm and focal length of 2 m. The 488nm and 514 nm beams are fed into a two-component probe, and the 532nm into a single component probe. The two probes

Figure 1: Measurement plane diagram

were inclined to align the beams at a point to form a LDA measurement volume of a $2.62\text{mm} \times 0.12\text{mm} \times 0.12\text{mm}$ ellipsoid, with notional separation angles between the probes of 28 up to 48° for cases of 0 up to 20° yaw. Both beam expanders are mounted on a Dantec 9041T3332 3D traverse system with a $1\text{m} \times 1\text{m} \times 1\text{m}$ motion range and accuracy of $\pm 0.01\text{mm}$. The basic setup and notation is illustrated in figs. 2, 5 and 6. Figure 2 shows a schematic diagram of the test rig mounted on the turntable and the location of the LDA probes. Figure 5 shows a diagram of a generic propeller with the sense of the propeller yaw angle γ relative to the wind tunnel axis, wind tunnel speed V_∞ and its components normal and perpendicular to the propeller disc, V_n and V_p . A view along the blade axis is shown in fig. 6, which illustrates the axial freestream component V_n , resultant aerodynamic velocity V_R , advance angle φ , aerodynamic angle of attack α_R , pitch angle $\beta(r)$ and the axial and tangential induction factors a_a and a_ω .

Figure 2: Experimental setup diagram

Data from the LDA probe are consequently fed to a Dantec BSA F60 Burst Spectrum Analyser Flow processor. Particle seeding is provided by a PivTec aerosol particle generator, providing particle sizes of $1\mu\text{m}$ at an expected particle supply rate of 10^8 per second. Olive oil is used as particle substrate. Propeller blade specifications are

illustrated in fig. 3. Freestream velocity in the facility is measured via a Pitot-static probe, on the port side of the

Figure 3: Propeller specification

working section entry and a secondary system using pressure measurement locations in the settling chamber and working section entry, which in turn have been calibrated with the Pitot-static probe. These measurements have, furthermore, been validated via LDA as consistent with the central section of the empty wind tunnel. The hub is rigid and non-articulated, with a total diameter including the propellers of 0.499m. The motor powering the rig is a Panasonic MSMA082A1C 750W AC servo motor paired with a programmable driver. The rated maximum continuous operation frequency is 3000 revolutions per minute. The rated and peak torque are 2.4 and 7.15 Nm. Motor rotational speed values were varied between 40 revolutions/s for the highest J values and 50 revolutions/s for the lowest. To diminish any wind tunnel interaction and installation effects each set of data is taken twice, once without the propellers installed on the hub, and once with them, where the prior matches all of the latter's case conditions, while having the blade installation cavities plugged. Additionally, each pair of data is examined using Fourier transform to ensure the said influences are minimised. The axial induction factor a_a is hence defined as follows:

$$a_a = \frac{V_{axial(LDA,Prop)} - V_{axial(LDA,Installation)}}{V_\infty} \quad (1)$$

Following the acquisition of the results, the temporal resolution of the data is checked against Nyquist criteria for the expected frequency of the signal (*Motor rotational speed* $\times$ *Number of blades* $\times$ *Confidence factor of 2*) and provided it is found to be satisfactory - it is forwarded for processing. Signal processing consists of two steps: outlier reduction and band-pass filtering, with boundaries set based on multiples of the expected driving frequency of the flow (between 0.6 and 16 times the once per revolution signal). This is illustrated in fig. 4.

Results are consequently converted into radial, axial and tangential components of the propeller axis system,

Figure 4: Single point signal processing example
($V_{\infty} = 20\text{m/s}$, $J = 0.87$, $n = 45.9\text{ rev/s}$, $\gamma = 0^\circ$)

and are available in both phase averaged and fully averaged formats.

Table 1: Experimental conditions envelope (25 cases & 50 runs)

Freestream velocity V_{∞} [m/s]	Yaw angle γ [°]	Advance ratio J	Blade pitch $\beta_{0.7}$ [°]
9	0,5,10,15,20	0.36	12
15		0.62	20
20		0.87	28
27		1.30	42
30		1.54	49

Accuracy-wise, a number of considerations are taken into account for LDA. A correction for velocity biasing[28], or the correlation of data rate and velocity, is employed via signal temporal interpolation [29, 30]. Additionally, a calculation is carried out to check the impact of performing off-axis LDA measurements through more than one medium due to refraction effects. Using procedures described by Zhang [30, 31], the displacement difference between the two beams was estimated at 0.68 mm and the interception angle of the two beams would not change, hence the raw data require no correction for these effects. Lastly, the possibility of simultaneously detecting multiple particles [29, 32, 33] is corrected via elimination of clear outliers to the data sets, which can be observed in fig. 4, where a few outliers are exhibited in the Raw data prior to processing.

Beyond the aforementioned errors, LDA is expected to produce an ideal result accuracy fluctuation equal to or worse than $\pm 0.00004\text{ m/s}$ [34]; however, adopting the uncertainty calculation scheme proposed by Orloff and Snider [35] an expected averaged uncertainty of $\pm 0.019\text{ m/s}$ (or $0.0009V_{\infty}$) is expected. For ease of use, all accuracy-related parameters are summarised in table 2, which represents the experimental accuracy envelope. In terms of deviation, the most notable ones would be the wind tunnel velocity fluctuation, propeller rotational speed and the blade pitch. These are all, however, consequently reduced via averaging techniques. Based on corrections, accuracy-related parameters presented (table 2) and baseline cases across the experimental envelope, it can be considered that the cumulative LDA uncertainty is negligible in the context of the experimental aims.

Figure 5: Propeller environment diagram

3. Experimental results

3.1. Baseline

Illustrated in fig. 7 is the baseline experimental propeller inflow corresponding to the zero yaw case. Firstly, it can be observed that the contours of a_a show nearly concentric circles, with the exception of two points at $\psi = 200^\circ$, $r = 0.52R$ and $\psi = 250^\circ$, $r = 0.76R$. These points were caused by reflections of the LDA lasers, hence azimuthal uniformity can reasonably be assumed for the baseline case. Radially, the inflow demonstrates an increase from the blade root up to a maxima region between approximately $r = 0.6R$ and $r = 0.8R$. This is followed by a decrease towards the tip sections. In physical terms, the increment in the region outboard of the root corresponds to the radial increase of azimuthal velocity. The decrease further outboard is predominantly attributed to the influence of the tip vortices generated by the propeller, which generate downwash on the outer sections and tend to press the inflow radially inwards. Both of the aforementioned effects influence the induced flow by changing the lift generation of the blade. These factors are also influenced by the geometry of the propeller. Figure 8 shows the effect of advance ratio J upon the inflow a_a around the propeller azimuth at the $r = 0.68R$ position for zero yaw angle. The maximum deviation exhibited from the trendlines is $\Delta a_a = \pm 0.0049$. The solid lines in the following figures, unless otherwise stated, represent a Piecewise Cubic Hermite Interpolating Polynomial (PCHIP) of the experimental data points to emphasise trend clarity.

Figure 6: Blade environment diagram, advancing blade ($\psi = 180^\circ$)

Table 2: Table of experimental envelope maximum accuracy deviation

Source of deviation	maxima	Accounted for by
Propeller rotational speed	$\pm 0.05 \text{ rev/s}$	Temporal averaging
Wind tunnel speed fluctuation	$\pm 0.03 \text{ m/s}$	Temporal averaging
Wind tunnel turbulence intensity	$0.2 \% \leq$	N/A
Measurement plane lateral deviation	$\pm 0.001 \text{ m}$	N/A
Measurement plane axial deviation across 1m lateral line	$\pm 0.001 \text{ m}$	N/A
Measurement plane axial deviation across 1m horizontal line	$\pm 0.001 \text{ m}$	N/A
Blade pitch, all blades, 0.7R	$\pm 0.0122 \text{ rad}$	Blade averaging
Averaged blade pitch across all blades, 0.7R	$\pm 0.0035 \text{ rad}$	N/A
LDA minimal uncertainty (ideal case)	$\pm 0.00004225 \text{ m/s}$	N/A
LDA expected uncertainty	$\pm 0.019 \text{ m/s}$	N/A

As observed from fig. 8, the azimuthal uniformity remains constant and the only major effect on the azimuthal distribution by varying J is a change in the magnitude of a_a . This is expected because the freestream starts to dominate the induced flow, and the a_a factor is normalised by the freestream velocity V_∞ .

3.2. Experimental data trends

In terms of characterising the effects of yaw, the following major trends can be observed within the confines of the experimental envelope. These were observed across all 25 experimental cases. An example of these inflow conditions is illustrated in fig. 9.

1. As the propeller is yawed, propeller axial induction factor minima and maxima develop, and are shifted in azimuth from the retreating and advancing peak locations. This is illustrated with a dashed line in fig. 9. The magnitude of the shift depends upon radial position, advance ratio and angle of incidence, leaving the extrema across the entire blade in the regions of $\psi = 100 - 150^\circ$ and $\psi = 300 - 340^\circ$ for the maxima and minima. At any fixed radial location the axial induction factor has a nearly sinusoidal variation around the azimuth, as shown in

Figure 7: Propeller induced mean a_a for a non-yawed propeller
($V_\infty = 20\text{m/s}$, $J = 0.87$, $n = 45.9\text{ rev/s}$, $\gamma = 0^\circ$)

fig. 10; which is useful as it allows the representation of data in terms of the amplitude and phase of the waveform, as illustrated in Figure 11. In terms of parameter influence, a steady increase of the azimuthal lagging can be observed when moving radially towards the tip of the blade, as can clearly be seen in fig. 11 and the surface plots presented in fig. 9. Advance ratio increments tend to increase the magnitude of the maxima lag as an offset, as demonstrated in fig. 11. The advance ratio also influences sensitivity to the effects of radial position on the magnitude of the lag. The latter can be observed when comparing peak positions in fig. 10, where the lower J cases exhibit a significantly higher change with radial position across the three figures, as opposed to higher J cases. This trend is demonstrated by the light grey highlight of the shift in maxima position in fig. 10. Beyond causing the shift, yaw angle does not appear to cause any consistent change to the phase shift.

2. As the propeller becomes more inclined, the difference between the velocities at the advancing and retreating sides becomes larger. This results in a correspondingly larger discrepancy of axial induction factor at those azimuthal locations. Axial induction factor amplitude against radial position is plotted in fig. 12 for $J = 0.89$ and increasing incidence γ . There is a clear increase in amplitude with incidence, with the magnitude maximum increasing up to 194% at the $r = 0.6R$ position. Radially, across the experimental cases, the amplitude maxima are observed to be located between the $0.3R$ and $0.7R$ radial positions.

Figure 8: Propeller induced axial induction factor vs azimuth
(all $\gamma = 0^\circ$ cases, $r = 0.68R$ position)

Figure 9: Propeller induced mean a_a at a yaw angle as seen from front
Dashed line indicates approximate a_a radial extrema
($V_\infty = 27\text{m/s}$, $J = 1.33$, $n = 41.6\text{ rev/s}$)

(a) $r = 0.36R$ position

(b) $r = 0.52R$ position

(c) $r = 0.92R$ position

Figure 10: Propeller induced axial induction factor vs azimuth
(all $\gamma = 20^\circ$ cases)

Figure 11: a_a maxima phase deviation from advancing peak
($\gamma = 20^\circ$ cases)

Figure 12: Axial induction factor amplitude vs radial position
($J = 0.89$ cases)

3. As the advance ratio increases, the axial induction factor decreases. These effects are shown in fig. 13 for the $r = 0.68R$ radial position of all $\gamma = 20^\circ$ cases. As illustrated, the magnitude of the propeller induction factor decreases by 75% across the envelope. It should be noted, however, that the induction factor is scaled with respect to the freestream velocity (V_∞) by definition, and the axial velocity would only suffer a 17% reduction. The dimensionalised values exhibit nearly a linear correlation with the changes in rotational speed within the experimental envelope (22%).

Figure 13: Propeller induced axial induction factor vs azimuth
(all $\gamma = 20^\circ$ cases, $r = 0.68R$ position)

3.3. Physical observations

The formation of an advancing and retreating side will result in a corresponding increase and decrease in lift and an approximately sinusoidal lift profile around the azimuth. As a consequence, the tip vortices will have varying strength, with stronger circulation occurring around the advancing peak and the weaker around the retreating peak. This will result in an increase of axial induction factor, negative radial velocity (towards the root) and imposed downwash on the advancing side. The opposite behaviour, due to the same factors, will be exhibited on the retreating side. Due to this imbalance, a skew of the wake in the direction of the advancing peak can be expected. Additionally, as the tip vortices are convected downstream, from the propeller plane perspective, the wake structure will constantly be shifting towards the direction of the yaw. This movement of the trailed vortical wake will play an important role in the azimuthal displacement of the maxima observed across all results, especially for larger angles of incidence and lower advance ratios. A second result of the sinusoidal variation of lift would be the shed vorticity in the wake. In order to satisfy constant total circulation, the propeller will constantly be shedding vorticity that will be opposing the changes in bound circulation, as illustrated in fig. 14. Shed vortices corresponding to increases in Γ will impose a downwash on the blade element, effectively reducing some of the lift, but also increasing the axial induction factor. The opposite is true for shed vortices corresponding to decreases in Γ . Because the changes in velocity are largest between the two peaks, the shed vorticity and its effects would

be strongest in these azimuthal areas. This makes the shed vorticity an element in the azimuthal displacement of axial flow extrema that is exhibited within the experimental data. This effect is expected to be around one order of magnitude lower than that of the tip vortical structure depending on the propeller loading and the relative velocity amplitude between the advancing and retreating blades. All effects of vortical structures on the propeller plane are a function of distance, as such, at higher advance ratios the vortices will be transported downstream faster and influence the inflow less. Additionally, all vortical structures will be subject to viscous dissipation as they mature, reducing their effects (Effect can be considered small, but can be enhanced by the proximity of vortical structures). Because the measurements are conducted 30 mm upstream of the propeller plane - a small decrease in axial induction factor and a radial contraction of the inflow pattern can be expected. Unsteady wake interactions between all of the aforementioned wake components and the aerodynamic environment will take place but are considered negligible compared to the quasi-steady effects captured in the measurement plane; discrepancies (not caused by laser reflections) between quasi-steady and fully unsteady signals were observed to be at the minima of the obtainable accuracy spectrum.

To summarise the main points, the resultant inflow velocity is sinusoidal, causing the lift and bound circulation variation to be sinusoidal as well. The change of bound circulation will cause variation in tip vortex magnitude, and shed vortex sheet magnitude and direction. Tip vortices are a function of lift, and will be stronger where the lift is at its maxima around the azimuth, and vice-versa. Tip vortex variation will create variation in imposed downwash, induced inflow and skew the wake towards the azimuthal locations where vortex circulation is stronger. Shed vortical sheets are a function of the rate of change of lift, hence their maxima will be in between the extrema of lift around the azimuth. Since the sheet changes both magnitude and direction it can impose both upwash and downwash on the propeller as a way to oppose increase and decrease of lift respectively. The unsteady contribution at the measurement plane is much smaller than the quasi-steady one.

4. Model comparison and discussion

4.1. Model of de Young

The main assumptions this method operates under is uniform induced inflow across the propeller disc. Drawing on this and comparing to fig. 9 it can immediately be discerned that this assumption cannot replicate any of the experimental data trends and, therefore, holds little physical justification. Solvers based on this assumption cannot calculate fluctuating loads acting on a propeller as a consequence of yaw angle.

4.2. Model of Crigler

To assess the viability of this method, the figures used to illustrate the experimental data trends will be recreated using a generic steady-state model, as proposed by Smith [36]. The model utilises blade element theory and lifting line theory in order to establish blade circulation strength Γ . This is consequently used to create an axial rigid

(a) Shed vortex formation (Gamma increase example)

(b) Shed vortex magnitude and direction as a function of propeller lift

Figure 14: 2D Shed wake diagram for fixed radial position

Figure 15: Propeller induced mean a_a as seen from front
Dashed line indicates approximate a_a radial extrema
($V_\infty = 27\text{m/s}$, $J = 1.33$, $n = 41.6 \text{ rev/s}$, $\gamma = 20^\circ$)

wake, which in conjunction with Biot-Savart law iteratively reconstructs the aerodynamic environment of the blade. A detailed account of the solver can be found in [37].

4.2.1. Axial induction factor phase shift vs radial position

Upon comparison between the illustrated experimental data and calculated data (fig. 15), it can be observed that the first stated trend cannot be satisfied by the provided solver in any of its stated parts. No counterparts are presented to fig. 11 as it would demonstrate a flat line across all cases. It is speculated that the reason for this is the convection of the wake axially from the propeller plane and the lack of any shed wake implementation.

4.2.2. Axial induction factor vs yaw angle

Studying the data produced by the steady-state solver, illustrated in fig. 16, it can be ascertained that the main definition of the second trend is satisfied, i.e. the AIF a_a amplitude increases with yaw angle. The experimental data (fig. 12) tends to form peak functions with maxima between the $0.4R$ and $0.7R$ radial positions, which are relatively well kept by the presented solver data. However, the peak regions of the solver functions are shifted towards the root of the blade, by approximately $0.1R$, and tend to be significantly narrower, by approximately 50%. It should furthermore be noted that the amplitudes for the calculated data are significantly higher than the ones presented by the experimental data, by up to 33% on the maxima of both figures.

Figure 16: Propeller induced axial induction factor maxima vs radial position ($J = 0.89$ cases, Crigler's model vs experimental data)

4.2.3. Axial induction vs advance ratio

When comparing experimental data to the results of the steady-state solver (fig. 17) a few discrepancies become immediately noticeable. Looking at the overall magnitude of the cases within the envelope, it can be observed that the solver is under predicting the a_a values for the three lower J values (by up to 36%), while over-predicting the higher cases (by up to 89%). This trend suggests that larger discrepancies could be exhibited outside of the experimental data envelope. Additionally, for all cases, the amplitudes presented by the solver data over-predict the results (by up to 60% for the high J cases and 40% for the lower J cases). The final discrepancy between the two is the already discussed shift in peak azimuthal position, which is disregarded by the solver.

Drawing on the presented data comparison, a possible way to improve on the solver is to include the effects of shed vorticity, which would naturally attenuate the effects of the changing blade conditions and introduce a part of the expected extrema azimuthal shift. Additionally, the introduction of a skew angle to the wake structure would allow for a correction of the effects of the wake vortical structure on the inflow. This can mitigate some of the overestimation presented by the solver at low advance ratio values due to stacking the tip vortex axially downstream of the propeller. It is furthermore expected to somewhat attenuate the discrepancy increase with yaw angle and introduce an improvement to the axial inflow azimuthal difference. Lastly, it is recommended that the wake geometry is modified to an alternative model that would reflect the cross-interaction of the vortical structures within the wake, which are of particular importance at low advance ratio cases. The latter is expected to have a large impact on the solver's ability to estimate both the induction factor magnitude and azimuthal pattern shift in the aforementioned case.

Figure 17: Propeller induced axial induction factor vs azimuth
($\gamma = 20^\circ$ cases, $r = 0.68R$ position, Crigler's model vs experimental data)

5. Conclusions

Drawing on all of the above, the following qualitative trends can be considered to characterise propeller inflow behaviour at yaw angles within the confines of the experimental envelope. Firstly, as the flow transitions from axial to yawed the inflow loses its azimuthal uniformity and does not possess a line of symmetry. Second, as yaw angle is introduced the propeller axial induction factor a_a minima and maxima exhibit a lagging azimuthal shift from the retreating and advancing peak locations. Third, increasing the yaw angle has a proportional relationship with propeller induction factor amplitudes. Fourth, as the advance ratio increases the propeller axial induction factor decreases.

Additionally, experimental data trends were compared against widely used assumptions for calculating loads on propellers at yaw angle, i.e. de Young's uniform induced flow and Crigler's steady-state aerodynamic environment assumptions. Comparing the results of these methods with the experimentally obtained data demonstrates that they fail to correctly predict the inflow distribution over the propeller plane. For perspective, the maximum discrepancy exhibited when comparing a case from the experimental study with CFD data was over an order of magnitude smaller [27]. As propeller inflow is tied to thrust generation this puts in question the ability of said methods to determine forces over the propeller plane itself. Lastly, as low-order methods solve the entire inflow based on the magnitude and location of the wake vortical structures, it is crucial that all assumptions made to simplify the solver would allow for the accurate calculation of both. For more detailed quantitative trends and comparisons, the experimental data will be made available under the following reference [38].

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