Can everyone benefit from innovation?

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Abstract

This paper investigates whether there is an allocation rule for which innovation never hurts anyone. Existing studies provide possibility characterizations together with efficiency and a natural participation constraint, assuming the domain of one input good and one output good in which nobody prefers to consume more of the input good than what she has. We show that this possibility result does not survive and we lead to impossibility either when (i) somebody wants to consume the input good more than what she has; or when (ii) there are multiple input goods.

1 Introduction

Technical innovation is widely understood as beneficial. It enlarges the possibilities of what society can achieve absent incentives. However, people are often rightfully concerned that they may lose out due to innovation. For example, technology often renders certain types of labor obsolete, leading

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to unemployment or decrease of wage income. Our aim in this note is to understand whether these concerns can be taken care of by appropriately adjusting the market mechanism; or perhaps, by considering another type of mechanism altogether.

In the following two examples, we formally demonstrate the known fact that innovation can hurt certain people in the market. The key to this observation is that innovation changes relative prices. The change in relative prices can hurt individuals through two channels:

1. If the individual prefers consuming a good rather than using it as a production input, innovation increases the factor demand for it and makes it more expensive. The associated negative welfare effect can be larger than the positive effect of making the output good cheaper.

2. If the individual relies on income from selling some of the input good, innovation which makes it dispensable decreases her income. The negative effect can be larger than the positive effect of making the output good cheaper.

We illustrate the first point with the following example.

Example 1 Suppose that there are two goods and two individuals, i and j, who have identical preferences represented by

$$u(x_1, x_2) = x_1 x_2.$$

Individual i's initial endowment is $(1, 9)$ and j's is $(9, 1)$. When there is no production, competitive equilibrium yields

$$p_1 = 1, \quad x_i = (5, 5), \quad x_j = (5, 5)$$

where the price of Good 2 is normalized to 1.

Now suppose it becomes possible to produce Good 2 from Good 1 with constant returns, and the marginal productivity is 9. Then competitive equilibrium with arbitrary profit share (profit is zero in equilibrium anyway) yields

$$p_1 = 9, \quad x_i = (1, 9), \quad x_j = \left(\frac{41}{9}, 41\right).$$

$\frac{40}{9}$ units of Good 1 are used as input and 40 units of Good 2 are produced. Individual i is made strictly worse off. Notice also that i ends up with his endowment without any exchange or production.

The second point is illustrated by the following example.

Example 2 Suppose that there are three goods and two individuals, i and j. Individual i's initial endowment is $(9, 1, 0)$ and j's is $(1, 9, 0)$. They have identical preferences represented by

$$u(x_1, x_2, x_3) = x_1 x_2 x_3$$

There is a constant returns to scale technology in which Good 3 is produced from goods 1 and 2, which is described by

$$f(z_1, z_2) = z_1^{\frac{1}{2}} z_2^{\frac{1}{2}}.$$

Then competitive equilibrium with any sharing of technology yields

$$p_1 = p_2 = \frac{1}{2}, \quad x_i = \left(\frac{10}{3}, \frac{10}{3}, \frac{5}{3}\right), \quad x_j = \left(\frac{10}{3}, \frac{10}{3}, \frac{5}{3}\right)$$

Note that here $\frac{10}{3}$ units of Good 3 are produced from $\frac{10}{3}$ units of Good 1 and $\frac{10}{3}$ units of Good 2.

Now consider the production technology given by

$$f^*(z_1, z_2) = \frac{1}{6} z_1 + \frac{3}{2} z_2$$

Note that this production possibility frontier nests the previous one.

Then competitive equilibrium yields

$$p_1 = \frac{1}{6}, \quad p_2 = \frac{3}{2}$$

and

$$x_i = \left(6, \frac{2}{3}, 1\right), \quad x_j = \left(\frac{82}{3}, \frac{82}{27}, \frac{41}{9}\right)$$

Individual i is made strictly worse off.

Suppose instead that we do not take the market mechanism as given, but ask whether there are other methods of allocating resources which avoid the problems as in the previous examples. We propose to study this question axiomatically. To this end, we study *social choice functions.* These objects map triples of preference profiles, endowment profiles and production technologies into feasible allocations.

We suppose that technologies exhibit *constant-returns-to-scale.* This restrictive hypothesis should, if anything, make positive results easier to obtain.

We consider three axioms. First is Technology Monotonicity, the primary requirement that innovation should not hurt anybody. Second is Efficiency, requiring that any selected allocation must be Pareto-efficient. Third is Free Access Lower Bound, which requires that nobody should receive a worse consumption bundle than what she could obtain by accessing the technology alone. Since in a production economy with constant returns the technology is replicable, the assumption that everybody can/should be able to freely access the technology is reasonable. Hence the lower bound condition is understood as a natural participation constraint. Alternatively, we may simply assume that each individual has the economy-wide production possibility set as her own production possibility set, and full ownership of (a) firm with this set. While we could generalize the model to allow individuals to possess firms with different technologies, this is not needed in order to demonstrate our impossibility.

In existing studies assuming the domain of one input good and one output good in which no individual prefers to consume more of the input good than she initially has (Moulin [10, 6], Fleurbaey and Maniquet [3], Maniquet [5]), it has been shown that *the constant-returns-to-scale-equivalence solution* satisfies the three axioms and is indeed characterized by them.

We show that this possibility result does not extend. We are led to impossibility when either (1) some individual wants to consume more of the input than she has; or (2) there are multiple input goods, each of which corresponds to the case illustrated above. In the real world, these two cases are rather generic.

Related Literature

Technology Monotonicity was introduced by Roemer [13]. He investigated this as one of the axioms in his characterization of welfare egalitarianism.

As mentioned above, Moulin [10, 6] considered a production economy in which there is one input and one output good, where technology exhibits either decreasing returns to scale or increasing returns to scale. In each setting, he characterized a solution, *constant-returns-to-scale-equivalence*, which satisfies Technology Monotonicity, Efficiency and an axiom stating that nobody should receive a better (resp. worse) consumption bundle than he can get by freely accessing the technology where endowments are equally divided. The lower bound condition there is essentially equivalent to our Free Access Lower Bound. See also Fleurbaey and Maniquet [3] and Maniquet [5] for alternative characterizations, and Moulin [7, 8] for results in the setting of public good provision.

We obtain impossibilities with the three axioms of Technology Monotonicity, Efficiency and Free Access Lower Bound. On the other hand, the above-noted papers show that the constant-returns-to-scale-equivalence solution satisfies the three axioms and is indeed characterized by them, in the domain of one input and one output in which nobody wants to consume more of the input good than she initially has. Thus our result shows that the existing possibility characterizations are indeed tight, in the sense that they do not extend to a larger domain in which somebody wants to consume the input good that she has or there are multiple input goods.

There are axiomatic studies of solidarity conditions with respect to other kinds of economic changes. They show that it is hard to reconcile the idea of solidarity with efficiency of allocations when we also require a natural condition on distributive justice, participation or operationality, such as welfare lower bound, informational efficiency or path independence.

Moulin and Thomson [12] considered an exchange economy starting with aggregate endowments, and asked if everyone can benefit from an increase in the aggregate endowment vector. The property was called Resource Monotonicity. They showed that there is no allocation rule which satisfies Resource Monotonicity, Efficiency and Equal-Division Lower Bound: the requirement that nobody should be worse off than at equal division. They further established an impossibility when Equal-Division Lower Bound is replaced by the requirement that nobody's consumption bundle should be physically dominated by anybody else's.

Chambers and Hayashi [1] considered exchange economies, in which the set of tradable goods varies. They asked if everyone can benefit from opening up markets for goods which had not been tradable. This requirement was termed No Loss from Trade. They showed that No Loss from Trade, Efficiency for any given set of tradable goods and Independence of Untraded Commodities, imply that only one person can gain from trade at early steps of trade liberalization.

Chambers and Hayashi [2] considered exchange economies with variable populations, and asked if everyone can benefit from integrating economies.

This requirement was termed Integration Monotonicity. An idea of path independence is built in the setting, as a larger economy might have come from many different histories of economic integration. They showed that Integration Monotonicity and Efficiency imply that the solution must select a core allocation in any economy. Because of the core convergence theorem, any such sequence of allocations must converge to a competitive allocation after replications.

In the class of transferable utility games, Integration Monotonicity is equivalent to Population Monotonicity introduced by Sprumont [14]. He characterized the class of TU games which admit population-monotonic payoff configurations, and showed that impossibility is obtained with a smaller number of individuals when core is small.

Population Monotonicity, a different axiom, was introduced by Thomson [17], who considered allocating a fixed amount of resources among variable numbers of individuals. It is a solidarity requirement that everybody should lose together when there are new participants. See Sprumont [15] for a detailed survey. In the setting of allocating private goods with fixed social endowments, Thomson [18] showed that there is a population-monotonic and efficient allocation rule, while Moulin [9] suggested that we reach impossibility if we additionally impose envy-freeness. Kim [4] gave a formal proof. In the setting of allocating fixed amounts of private goods and a fixed amount of numeraire good, where preferences are linear in the numeraire good, Moulin [11] showed that in general there is no population monotonic and efficient allocation rule. He showed that when preferences exhibit substitutability, the Shapley value applied to this setting is population-monotonic.

2 Setting and axioms

Let $I = \{1, \cdots, n\}$ be the set of individuals. The number of goods is denoted by l.

Let $\mathcal{R}$ be the set of preference orderings over $\mathbb{R}^l_+$, which are complete, transitive, continuous on $\mathbb{R}^l_+$ and strictly convex, strongly monotone on $\mathbb{R}^l_{++}$. Mostly, we are interested in differentiable preferences satisfying a boundary condition. This condition works as follows: given R_i for individual i, let u_i denote a utility representation which is differentiable on $\mathbb{R}^l_{++}$, let $MRS_i^{k,h}(x_i)$ denote the marginal rate of substitution of Good h for Good k for i at $x_i \in \mathbb{R}^l_{++}$. It is given by

$$MRS_i^{k,h}(x_i) = \frac{\frac{\partial u_i(x_i)}{\partial x_{ik}}}{\frac{\partial u_i(x_i)}{\partial x_{ih}}}.$$

Then the boundary condition is:

$$\lim_{x_{ik}\to 0} MRS_i^{k,h}(x_i) = \infty, \quad \lim_{x_{ih}\to 0} MRS_i^{k,h}(x_i) = 0$$

for all $k \neq h$.

Cobb-Douglas preference and CES preference are the typical examples of preferences satisfying the above assumptions.

Let $\mathcal{Y}$ be the set of *constant-returns-to-scale* technologies. That is, $Y \in \mathcal{Y}$ if and only if

1. $Y \subset \mathbb{R}^l$;
2. $Y \cap \mathbb{R}^l_+ = \{\mathbf{0}\}$;
3. $Y \supset -\mathbb{R}^l_+$;
4. Y is closed and convex;
5. for all $y \in Y$ and $\lambda \geq 0$ it holds $\lambda y \in Y$.

For each $i \in I$, the initial endowment is denoted by $\omega_i \in \mathbb{R}^l_+$. A list of endowments is denoted by $\omega = (\omega_1, \cdots, \omega_n) \in \mathbb{R}^{nl}_+$.

An *economy* is a triple $(R, \omega, Y) \in \mathcal{R}^I \times \mathbb{R}^{nl}_+ \times \mathcal{Y}$, which consists of a preference profile $R = (R_1, \cdots, R_n)$, an endowment profile $\omega = (\omega_1, \cdots, \omega_n)$ and a production set Y.

An allocation $x = (x_1, \cdots, x_n) \in \mathbb{R}^{nl}_+$ is *feasible* in economy (R, ω, Y) if

$$\sum_{i \in I} x_i - \sum_{i \in I} \omega_i \in Y.$$

A feasible allocation x in economy (R, ω, Y) is *Pareto-efficient* if there is no feasible allocation x' in (R, ω, Y) such that

$$x'_i P_i x_i$$

for all $i \in I$, where P_i denotes the strict counterpart of R_i.[1]

A *social choice function* $\varphi : \mathcal{R}^I \times \mathbb{R}^{nl}_+ \times \mathcal{Y} \to \mathbb{R}^{nl}_+$ is a mapping such that $\varphi(R, \omega, Y)$ is feasible for all $(R, \omega, Y) \in \mathcal{R}^I \times \mathbb{R}^{nl}_+ \times \mathcal{Y}$.

We impose the following three axioms.

Technology Monotonicity: For all $R \in \mathcal{R}^I$, $\omega \in \mathbb{R}^{nl}_+$ and $Y, Y' \in \mathcal{Y}$ with $Y \subset Y'$, it holds

$$\varphi_i(R, \omega, Y') \; R_i \; \varphi_i(R, \omega, Y)$$

for all $i \in I$.

Efficiency: For all $R \in \mathcal{R}^I$, $\omega \in \mathbb{R}^{nl}_+$ and $Y \in \mathcal{Y}$, $\varphi(R, \omega, Y)$ is Pareto-efficient.

[1]This is a weaker definition of Pareto-efficiency, but in the current domain it is equivalent to the stronger definition: there is no feasible allocation x' in (R, ω, Y) such that $x'_i R_i x_i$ for all $i \in I$ and $x'_i P_i x_i$ for some $i \in I$.

Free Access Lower Bound: For all $\omega \in \mathbb{R}^{nl}_+$, for all $R \in \mathcal{R}^I$ and $Y \in \mathcal{Y}$ it holds

$$\varphi_i(R, \omega, Y) \; R_i \; (\omega_i + y_i)$$

for all $y_i \in Y$ with $\omega_i + y_i \in \mathbb{R}^l_+$, for all $i \in I$.

There are two motivations for Free Access Lower Bound. One is normative. Because of constant-returns-to-scale, somebody's access to a technology does not interfere another's. Thus it is reasonable that everybody is allowed to access the technology. The second is descriptive. Free Access Lower Bound can be interpreted as a participation constraint, given that nobody can be excluded from accessing the technology.

Here is a prominent example of a solution satisfying Free Access Lower Bound, in which each individual simply maximizes her utility using the technology and her endowment.

Example 3 The *free access solution* FA is defined as follows. Given any $\omega \in \mathbb{R}^{nl}_+$, for every $R \in \mathcal{R}^I$ and $Y \in \mathcal{Y}$ let $FA_i(R, \omega, Y) = \omega_i + y_i$ with $y_i \in Y$, $\omega_i + y_i \in \mathbb{R}^l_+$, for each $i \in I$, which satisfies

$$(\omega_i + y_i) \; R_i \; (\omega_i + y_i')$$

for all $y_i' \in Y$ with $\omega_i + y_i' \in \mathbb{R}^l_+$.

The free access solution satisfies Technology Monotonicity and Free Access Lower Bound but fails Efficiency.

The solution is inefficient because it

1. lacks exchange; and

2. lacks social coordination of production.

The first problem arises even in exchange economies. Note that when there is no production (i.e., when $Y = -\mathbb{R}^l_+$) the free access solution simply gives each individual his or her initial endowment. To understand the second problem, imagine that under the free access solution i inputs more of Good 1 and less amount of Good 2 in order to produce Good 3, and that the opposite is true for Individual j. Then, technical rates of substitution fail to equalize across their uses of the technology, which means there is inefficiency.

There is a solution which satisfies Technology Monotonicity and Efficiency, and has appeared in the context of axiomatic bargaining (see Thomson and Myerson [16] for example).

Example 4 *Monotone path solutions* are defined as follows. Given a preference profile $R \in \mathcal{R}^I$, fix a profile of continuous utility representations $u[R] = (u_1[R_1] \cdots, u_n[R_n])$.

Given $u[R]$ and $\omega \in \mathbb{R}^{nl}_{++}$, fix a weakly monotone and continuous path $\Phi(u[R], \omega)$ in $\mathbb{R}^n$.

For every $Y \in \mathcal{Y}$ define the utility possibility set

$$U(Y; u[R], \omega) = \left\{ u[R](x) = (u_1[R_1](x_1), \cdots, u_n[R_n](x_n)) \in \mathbb{R}^n : \sum_{i \in I} x_i - \sum_{i \in I} \omega_i \in Y \right\}$$

Then $\Phi(u[R], \omega) \cap U(Y; u[R], \omega)$ has a unique largest vector element, which is denoted by $\max \Phi(u[R], \omega) \cap U(Y; u[R], \omega)$. Then one can define φ by taking $\varphi(R, \omega, Y)$ as an allocation satisfying

$$u[R](\varphi(R, \omega, Y)) = \max \Phi(u[R], \omega) \cap U(Y; u[R], \omega)$$

where such an allocation is unique up to Pareto indifference, and unique when preferences are strictly convex.

The monotone path solution satisfies Efficiency and Technology Monotonicity. The converse statement that Efficiency and Technology Monotonicity uniquely characterize a monotone path solution is not true, because in general how we select an allocation can depend on the technology, whereas the monotone path solution only considers utility profiles. See Roemer [13] for a related point.

3 Two difficulties

3.1 Impossibility when somebody prefers to consume more of an input than she has

Proposition 1 Assume that there are two goods and two individuals. Assume that $\mathcal{Y}$ includes two classes of constant returns to scale technologies, one in which Good 1 is produced from Good 2, the other in which Good 1 is produced from Good 2.

Then there is no allocation rule which satisfies Efficiency, Technology Monotonicity and Free Access Lower Bound.

Proof. The proof is by example, while the argument applies to any generic profile of preferences and endowments.

Go back to Example 1. Assume that there are two goods and two individuals, i and j, who have identical preferences represented by

$$u(x_1, x_2) = x_1 x_2.$$

Individual i's initial endowment is $(1, 9)$ and j's is $(9, 1)$. Note that at the endowment point Individual i's MRS is $\frac{1}{9}$ and Individual j's is 9.

Let Y_0 denote the technology in which no production is possible. Let Y_1 denote the technology in which 9 units of Good 2 is produced from 1 unit of Good 1. Let Y_2 denote the technology in which 9 units of Good 1 is produced from 1 unit of Good 2.

Under Y_1, the free access solution delivers an efficient allocation $x_i = (1, 9)$ and $x_j = \left(\frac{41}{9}, 41\right)$. Hence it is the only allocation which meets Efficiency and

Free Access Lower Bound. In order that Technology Monotonicity is not violated between Y_0 and Y_1, we must have $(1,9)\, R_i\, \varphi_i(R,\omega,Y_0)$. On the other hand, in order that Free Access Lower Bound is met at Y_0, we must have $\varphi_i(R,\omega,Y_0)\ R_i\ (1,9)$. Thus we obtain $\varphi_i(R,\omega,Y_0)\ I_i\ (1,9)$.

By the same argument applied to Y_2, we obtain $\varphi_j(R,\omega,Y_0)\ I_j\ (9,1)$.

Thus we have $\varphi_i(R,\omega,Y_0)\ I_i\ (1,9)$ and $\varphi_j(R,\omega,Y_0)\ I_j\ (9,1)$, but this is a violation of Efficiency. ■

To understand the domain assumption that $\mathcal{Y}$ includes two classes of technologies, one in which Good 1 is produced from Good 2, the other in which Good 1 is produced from Good 2, imagine that Good 1 is fuel and Good 2 is food. Although the traditional direction is to use fuels to produce foods (through using agricultural machineries), a recent innovation happened in the way that fuels can be produced from foods (e.g., corn). It is widely recognized that the second type of innovation has led to a hike of food price in markets.

3.2 Impossibility when there are multiple input goods

Proposition 2 Assume that there are three goods and two individuals. Assume that $\mathcal{Y}$ includes the class of constant returns to scale technologies, in which one fixed good (say Good 3) is produced from the other two (goods 1 and 2).

Then there is no allocation rule which satisfies Efficiency, Technology Monotonicity and Free Access Lower Bound.

Proof. As before, the proof demonstrates an example, while the argument applies to any generic profile of preferences and endowments.

Go back to Example 2. Suppose that there are three goods and two individuals, i and j. Individual i's initial endowment is $(9,1,0)$ and j's is

$(1, 9, 0)$. They have identical preferences represented by

$$u(x_1, x_2, x_3) = x_1 x_2 x_3$$

Let Y_0 be the constant returns to scale technology in which Good 3 is produced from goods 1 and 2, which is described by

$$f_0(z_1, z_2) = z_1^{\frac{1}{2}} z_2^{\frac{1}{2}}.$$

Note that i's free access to Y_0 yields consumption bundle $x_i = \left(6, \frac{2}{3}, 1\right)$ with input vector $z_i = \left(3, \frac{1}{3}\right)$, and j's free access to Y_0 yields consumption bundle $x_j = \left(\frac{2}{3}, 6, 1\right)$ with input vector $z_i = \left(\frac{1}{3}, 3\right)$.

Let Y_1 be the constant returns to scale technology in which Good 3 is produced from goods 1 and 2, which is described by

$$f_1(z_1, z_2) = \frac{1}{6} z_1 + \frac{3}{2} z_2.$$

Note that $Y_0 \subset Y_1$.

Let Y_2 be the constant returns to scale technology in which Good 3 is produced from goods 1 and 2, which is described by

$$f_2(z_1, z_2) = \frac{3}{2} z_1 + \frac{1}{6} z_2.$$

Note that $Y_0 \subset Y_2$.

Under technology Y_1, the free access solution delivers an efficient allocation in which i receives $\left(6, \frac{2}{3}, 1\right)$. Hence it is the only allocation which meets Efficiency and Free Access Lower Bound. In order that Technology Monotonicity is not violated between Y_0 and Y_1, we must have $\left(6, \frac{2}{3}, 1\right) R_i$ $\varphi_i(R, \omega, Y_0)$. On the other hand, in order that Free Access Lower Bound is met we must have $\varphi_i(R, \omega, Y_0) R_i \left(6, \frac{2}{3}, 1\right)$. Hence we obtain $\varphi_i(R, \omega, Y_0) I_i$ $\left(6, \frac{2}{3}, 1\right)$.

By the same argument applied to Y_2, we obtain $\varphi_j(R, \omega, Y_0) I_j \left(\frac{2}{3}, 6, 1\right)$.

Thus we have $\varphi_i(R, \omega, Y_0) I_j \left(6, \frac{2}{3}, 1\right)$ and $\varphi_j(R, \omega, Y_0) I_j \left(\frac{2}{3}, 6, 1\right)$, but this is a violation of Efficiency. ■

4 Conclusion

We have studied a resource allocation problem with variable technology and asked if there is an allocation rule under which innovation never hurts anyone. The requirement is presented as an axiom called Technology Monotonicity.

We showed that the competitive solution fails Technology Monotonicity, because of one of the two possible phenomena. One is that innovation renders an input more expensive, hurting individuals who want to consume it. The other is that innovation makes some input goods dispensable and hurts an individual who relies on income from selling it.

Then we considered a social choice problem, without taking the market solution as given, and considered two additional axioms, Efficiency and Free Access Lower Bound.

We showed that the existing possibility result, which is obtained in the domain of one input good and one output good in which nobody wants to consume more of the input than she has, does not survive. There is impossibility either when somebody wants to consume more of the input good than she has; or when there are multiple input goods.

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