Gao, Z., Hong, Y. and Wang, L. (2020) Constitutive modelling of fine-grained gassy soil: a composite approach. *International Journal for Numerical and Analytical Methods in Geomechanics*, 44(9), pp. 1350-1368. (doi: [10.1002/nag.3065](https://doi.org/10.1002/nag.3065)).

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Gao, Z., Hong, Y. and Wang, L. (2020) Constitutive modelling of fine-grained gassy soil: a composite approach. *International Journal for Numerical and Analytical Methods in Geomechanics*, 44(9), pp. 1350-1368, which has been published in final form at [10.1002/nag.3065](https://doi.org/10.1002/nag.3065). This article may be used for non-commercial purposes in accordance with [Wiley Terms and Conditions for Self-Archiving](#).

<http://eprints.gla.ac.uk/209803/>

Deposited on: 10 February 2020

General information of the article

Manuscript type: General paper

Title: Constitutive modelling of fine-grained gassy soil: a composite approach

Authors: Zhiwei GAO, Yi HONG* and Lizhong WANG

Information of the authors

Corresponding author: Dr Yi Hong*

Associate Professor, Key Laboratory of Offshore Geotechnics and Material of
Zhejiang Province, College of Civil Engineering and Architecture, Zhejiang
University, Hangzhou, Zhejiang 310058, P.R. China.

Email: yi_hong@zju.edu.cn

Co-author: Dr Zhiwei GAO

Lecturer, James Watt School of Engineering, University of Glasgow, Glasgow, G12
8QQ, UK

Email: Zhiwei.Gao@glasgow.ac.uk

Co-author: Dr Lizhong WANG

Distinguished Qiusi Professor, Key Laboratory of Offshore Geotechnics and
Material of Zhejiang Province, College of Civil Engineering and Architecture,
Zhejiang University, Hangzhou, Zhejiang 310058, P.R. China.

Email: wanglz@zju.edu.cn

ABSTRACT

Fine-grained marine sediments containing large undissolved gas bubbles are widely distributed around the world. Presence of the bubbles could degrade the undrained shear strength (s_u) of the soil, when the gas pressure u_g is relatively high as compared to the effective stress in the saturated soil matrix. Meanwhile, the addition of bubbles may also increase s_u when the difference between u_g and pore water pressure u_w becomes smaller than the water entry value, causing partial water drainage from the saturated matrix into the bubbles (bubble flooding) during globally undrained shearing. A new constitutive model for describing the two competing effects on the stress-strain relationship for fine-grained gassy soil is proposed within the framework of critical state soil mechanics. The gassy soil is considered as a three-phase composite material with compressible cavities which allows water entry from the saturated matrix. Bubble flooding is modelled by introducing an additional positive volumetric strain increment of the saturated clay matrix, which is dependent on the difference between pore gas and pore water pressure based on experimental observations. A modified hardening law based on that of the Modified Cam-Clay (MCC) model is employed, which in conjunction with the expression for bubble flooding, can describe both the detrimental and beneficial effects of gas bubbles on soil strength and plastic hardening in shear. Only two extra parameters in addition to those in the MCC model are used. It is shown that the key features of the stress-strain relationship of three fine-grained gassy soils can be reproduced satisfactorily.

Keywords: Clays, Partial saturation, Constitutive relations, Pore pressures, Shear strength, Offshore engineering

INTRODUCTION

Gas-charged marine sediments are widely distributed in the seabed throughout the world. For instance, gas-charged seabed has been found in the Bristol Channel, North Sea, Gulf of Mexico, Gulf of Guinea and Eastern China Sea. This list is far from being exhaustive (Hong *et al.*, 2017; ; Wu & Jeng, 2019). The gas mainly consists of methane, which is produced by decomposition of organic matter, thermal cracking of complex organic and inorganic compounds at great depth and gas exploration. The gas can occur in the seabed in three ways: dissolved, undissolved in the form of gas-filled voids, or gas hydrates (Sills & Wheeler, 1992). Dissolved gas has little effect on the mechanical properties of the seabed sediments unless the ambient pressure reduces or temperature increases, which creates new gas bubbles in the soil (Sultan *et al.*, 2012). But the presence of undissolved gas bubbles has been causing geo-hazards, including submarine and onshore landslide (Kvenvolden, 1988; Nisbet & Piper, 1998; Milich, 1999; Locat & Lee, 2002; Dittrich *et al.*, 2010; Rowe & Mabrouk, 2012), as well as instability of offshore foundations (Thomas *et al.*, 2011). Because they could dramatically change the mechanical behavior of soils, including shear strength, stiffness and compressibility (Sills & Wheeler, 1992; Puzrin *et al.*, 2011; Rebata-Landa & Santamarina, 2012; Sultan *et al.*, 2012).

Influence of undissolved gas on the mechanical behavior of soil is dependent on the grain size. For fine-grained soils, the gas bubbles are typically much bigger than the soil particles, and therefore, they fit in the soil matrix rather than the pore water, as characterized microscopically by Hong *et al.* (2019a) (see Fig. 1). This indicates that

fine-grained gassy soil is not a soil with compressible pore fluid. Instead, it is a composite material with saturated clay matrix and compressible cavities (Wheeler, 1988a, 1988b; Sills *et al.*, 1991; Hong *et al.*, 2017).

Fig. 1 Micro-structure of the fine-grained gassy soil: (a) a typical X-ray microtomography (μ CT) images; (b) a typical scanning electron microscopy (SEM) image (Hong *et al.*, 2019a).

There have been extensive experimental studies on the compressibility and undrained shear behavior of fine-grained gassy soil (Nageswaran, 1983; Sham, 1989; Thomas, 1987; Wheeler, 1986; Sills *et al.*, 1991; Lunne *et al.*, 2001; Hight & Leroueil, 2003; Sultan *et al.*, 2012; Hong *et al.*, 2017). It is revealed that the presence of gas bubbles could increase the undrained shear strength s_u (i.e., beneficial effect), when the difference between pore gas pressure u_g and pore water pressure u_w becomes smaller than the water entry value. Under this circumstance, the pore water in the saturated soil matrix can drain into the gas cavities (i.e., known as “bubble flooding”) in a globally undrained shear test, which reduces the void ratio of the saturated matrix, and thus

increases s_u (Wheeler, 1986; Wheeler, 1988b; Hong *et al.*, 2017). On the other hand, the inclusion of gas bubbles degrades the undrained shear strength of the soil (i.e., damaging effect), particularly when the gas pressure u_g is relatively high in reference to the effective stress in the saturated matrix. This is mainly associated with additional yielding of the soil caused by the stress concentrations surrounding the highly pressurized bubbles.

Various attempts have been made in modelling the mechanical behavior of gassy clay. But the existing models cannot capture both the detrimental and beneficial effect of gas bubbles on the soil response satisfactorily. Thomas (1987) was the first to model the mechanical behaviour of gassy clay as a composite material with compressible bubbles. This method can give good prediction for the soil response in compression but fails to capture the effect of potential bubble flooding in shear. Some models have been developed by considering gassy clay as a soil with compressible fluid (Grozic *et al.*, 2005). This is physically unreasonable, because the gas bubbles fit in the saturated clay matrix, rather than pore water (Fig. 1). Since compressible fluid reduces the positive pore pressure for normally consolidated clay in undrained tests, these models are only able to describe the beneficial effect of gas bubbles on the undrained shear strength. Pietruszczak & Pande (1996) have realized that the gas bubbles are larger than the particle size for gassy clays but their method still cannot capture the detrimental effect of gas bubbles on the undrained shear strength. Sultan & Garziglia (2014) proposed a constitutive model which specifically describes the structural damage caused by gas exsolution and expansion due to reduction of pore water pressure (i.e., sampling from

deep waters). This is achieved by relating the pre-consolidation pressure to a damage parameter dependent on the gas content.

In this study, a new critical-state-based constitutive model is proposed, which simultaneously captures the two competing effects (i.e., both damaging and beneficial effects) of gas bubbles on the mechanical behavior for fine-grained gassy soil. An important feature of the model is that the gassy soil is considered as a three-phase composite material with compressible cavities allowing for potential entry of water (bubble flooding) from the saturated matrix. The bubble flooding is modelled by introducing an additional positive volumetric strain increment of the saturated matrix. A new hardening law based on that of the Cam-Clay models is proposed to capture both the detrimental and beneficial effects of gas bubbles on the plastic hardening and shear strength of the soil. The predictive capabilities of the model are validated against experimental data of three types of fine-grained gassy soils under various initial conditions. To facilitate the discussion, only the soil response in triaxial compression and isotropic compression will be considered. Two stress quantities, the mean effective stress p $[(= \sigma_a + 2\sigma_r)/3]$ and deviator stress q $(= \sigma_a - \sigma_r)$, and two strain quantities, volumetric strain ε_v $(= \varepsilon_a + 2\varepsilon_r)$ and shear strain ε_q $[= \frac{2}{3}(\varepsilon_a - \varepsilon_r)]$ are used, where σ_a is the total axial stress, σ_r is the total radial stress, ε_a is the axial strain and ε_r is the radial strain. The stress quantities with a prime symbol ($'$) are effective ones.

STRESS AND STRAIN VARIABLES FOR FINE-GRAINED GASSY SOIL

Stress variables

As discussed in the previous section, fine-grained gassy soil is essentially a

composite material. A composite approach is thus used to model the stress-strain relationship of fine-grained gassy soil. According to a simple rule of mixture for composite materials (Tsai & Hahn, 1980; Pietruszczak & Pande, 1996; Diambra *et al.*, 2010; Shi *et al.*, 2019), the following equations for the stress quantities are assumed for gassy clay

$$p = (1 - f)p_m + fu_g \quad (1)$$

$$p_m = p'_m + u_w \quad (2)$$

$$q = (1 - f)q_m \quad (3)$$

where f is the volume fraction of the gas cavities (Wheeler, 1986), p_m and p'_m are the total and mean effective stress of the saturated clay matrix, q_m is the deviator stress of the saturated soil matrix, u_g is the gas pressure, u_w is the pore water pressure. q is not affected by the stress of gas because the gas bubbles cannot sustain shear stress. A three-phase diagram of the composite material is shown in Fig. 2. The volume fraction of cavities f is expressed as

$$f = \frac{V_c}{V} \quad (4)$$

where V_c is the cavity volume and V is the total soil volume. When the gas bubbles are not flooded, the volume of the gas is the same as that of the cavity with

$$V_c = V_g = (1 - S_r)e \quad (5)$$

$$f = \frac{V_c}{V} = \frac{(1 - S_r)e}{1 + e} \quad (6)$$

where $S_r (= \frac{V_w}{V_v})$ is the degree of saturation and e is the global void ratio. Once bubble flooding occurs (Fig. 3), the phase diagram shown in Fig. 2 must be used to calculate f using Eq. (3), because there will be both water and gas in the cavity (i.e., $V_c > V_g$).

In most cases, f is very small (less than 0.05). Therefore, contribution of the gas pressure to the total stress can be negligible. In this paper, the following relationship is assumed for the stress state for fine-grained gassy soil by neglecting the effect of f and u_g :

$$p = p_m \quad (7)$$

$$p' = p - u_w = p'_m \quad (8)$$

$$q = q_m \quad (9)$$

Fig. 2 Phase diagram for fine-grained gassy soil (water in both the saturated matrix and cavity).

Fig. 3 Cavity and compressed gas bubble in fine-grained gassy soil after bubble flooding ($V_c > V_g$)

Indeed, the assumption in Eqs. (7)-(9) is supported by experimental test results on fine-grained gassy soil (Thomas, 1987; Wheeler, 1988b; Sills *et al.*, 1991; Hong *et al.*, 2017). First, it is found that there is a unique relationship between the matrix void ratio (or water void ratio) $e_m (= \frac{V_m}{V_s})$ and p' in isotropic consolidation of gassy clay (Wheeler, 1986; Hong *et al.*, 2017), where V_m is the volume of the saturated soil matrix and V_s is the volume of soil particles. As the Terzaghi's effective stress principle only works in the saturated soil matrix, it is expected that the $e_m - p'_m$ relationship must be unique, which means that $p' = p'_m$ and $p = p_m$. Secondly, it is found that the critical state η observed in undrained triaxial compression is the same ($= M$) for both saturated and gassy clay (Wheeler, 1986; Sham, 1989; Hong *et al.*, 2017). Similarly, the Terzaghi's effective stress is only valid in the saturated soil matrix, the critical state $\eta_m (= q_m/p'_m)$ for gassy clay should be M . Since $p' = p'_m$, q_m should be the same as q .

It should be noted that for simplicity, Eqs. (1) to (3) are based on Terzaghi's effective stress principle (which only works in the saturated soil matrix) and rule of mixture for composite materials, without considering the principles of thermodynamics. This could be improved by using effective stress expressions, which work for the entire gassy soil while obeying principles of thermodynamics. Such expression was first proposed by Borja (2004) and Borja (2006), who rigorously derived an effective stress tensor for three-phase (solid-water-air) partially saturated porous continua emerging from principles of thermodynamics. In light of the unique internal structure and soil-bubble interaction (e.g., bubble flooding) of gassy clay, one is anticipated to re-derive the effective stress for this specific internal structure following the theoretical framework

of Borja (2004, 2006). This will bring a theoretically sound alternative to the existing stress variables for better modelling gassy clay, and remains to be the authors' future purist.

Strain variables

The total volumetric strain of a composite material can be decomposed as below using the rule of mixture (Tsai & Hahn, 1980; Pietruszczak & Pande, 1996; Diambra *et al.*, 2010):

$$\varepsilon_v = (1 - f)\varepsilon_v^m + f\varepsilon_v^c \quad (10)$$

where ε_v^m is the volumetric strain of the saturated soil matrix and ε_v^c is the volumetric strain of the cavity. Though f is small, the term $f\varepsilon_v^c$ in Eq. (10) cannot be neglected, because $f\varepsilon_v^c$ can be much bigger than $(1 - f)\varepsilon_v^m$ in some cases. For instance, in a globally undrained test on fine-grained gassy soil without bubble flooding, $\varepsilon_v > 0$ due to the compressibility of gas bubbles but $\varepsilon_v^m = 0$. In this case, $\varepsilon_v = f\varepsilon_v^c > 0$. If $f\varepsilon_v^c$ is neglected, one will always get $\varepsilon_v = 0$, which is physically unreasonable.

The increment of ε_v^c is related to the change of the cavity size

$$d\varepsilon_v^c = \frac{dV_c}{V_c} \quad (11)$$

where dV_c denotes the volume change of the cavity. Before bubble flooding, the volume change of the cavity has to be the same as that of the gas bubble. Once bubble flooding occurs, part of the cavity is flooded by pore water and volume change for the cavity and gas bubble is not identical. More discussion on this will be presented in the following section on the constitutive model.

The volumetric strain increment of the saturated clay matrix $d\varepsilon_v^m$ is dependent on

the volume change due to water flow out/into the matrix at the outer boundary of the saturated matrix (dV_b) and bubble flooding (dV_f)

$$d\varepsilon_v^m = \frac{dV_b + dV_f}{V_m} = d\varepsilon_v^b + d\varepsilon_v^f \quad (12)$$

where $d\varepsilon_v^b$ and $d\varepsilon_v^f$ denote the volumetric strain increments due to water flow at the boundary and bubble flooding. In a globally undrained test, $dV_b = d\varepsilon_v^b = 0$ while $d\varepsilon_v^f > 0$ if bubble flooding occurs. Consideration of $d\varepsilon_v^f$ is crucial for modelling the beneficial effect of gas cavities on the undrained shear strength of fine-grained gassy soil (Wheeler, 1986; Sham, 1989; Sills *et al.*, 1991).

The effect of gas bubbles on the total shear strain ε_q is neglected for simplicity, such that the total shear strain of the composite material is assumed as:

$$\varepsilon_q = \varepsilon_q^m \quad (13)$$

where ε_q^m is the shear strain of the saturated soil matrix.

A CONSTITUTIVE MODEL FOR FINE-GRAINED GASSY SOIL

A constitutive model for fine-grained gassy soil is presented in this section based on the framework above. The constitutive relationship of the saturated soil matrix is developed within the framework of critical state soil mechanics (Roscoe & Burland, 1968; Wood, 1990). A new hardening law is formulated to describe the damaging effect of gas bubbles on the stiffness and shear strength of gassy clay and bubble flooding is considered to capture the beneficial effect. Volume change of gas bubbles is assumed to follow the Boyle's law, which states that the product of pressure and volume is a constant for a given mass of confined gas at a constant temperature.

Constitutive relationship for the saturated clay matrix

The yield function F of modified Cam-Clay model is adopted, as follows:

$$F = q^2 - M^2 p'(p'_0 - p') = 0 \quad (14)$$

where p'_0 denotes the size of the current yield surface and M is the critical state stress ratio in triaxial compression. The associated plastic flow rule is employed, as follows:

$$d\varepsilon_v^{mp} = \langle L \rangle \frac{\partial F}{\partial p'} \quad (15)$$

$$d\varepsilon_q^{mp} = \langle L \rangle \frac{\partial F}{\partial q} \quad (16)$$

where $d\varepsilon_v^{mp}$ is the plastic volumetric strain increment, $d\varepsilon_q^{mp}$ is the plastic shear strain increment, L is the loading index and $\langle \cdot \rangle$ are the McCauley brackets [$\langle L \rangle = L$ for $L > 0$ and $\langle L \rangle = 0$ otherwise].

The elastic bulk modulus K_m and shear modulus G_m for the saturated matrix are

$$K_m = \frac{1 + e_m}{\kappa} p' \quad (17)$$

$$G_m = K_m \frac{3(1 - 2\nu)}{2(1 + \nu)} \quad (18)$$

where κ is the swelling index in the $e - \ln p'$ space and ν is the Poisson's ratio which is routinely assumed to be a constant in the model. Other forms of K_m and G_m which account for the effect of f can be used, because gas bubbles cause stress concentration in the soil around them, which has detrimental effect on the elastic stiffness of the saturated matrix (Wheeler, 1986). Such formulations can improve the model prediction of the soil response at relatively small strain level, but more parameters may have to be introduced.

The following hardening law is employed for this model

$$dp'_0 = \langle L \rangle (r_1 - r_2) = \langle L \rangle \frac{(1+e_0)p'_0}{\lambda-\kappa} \frac{\partial F}{\partial p'} \left[1 - \sqrt{f} \frac{q}{Mp'} \left(\frac{u_g+p_a}{p'_0} \right)^\xi \right] \quad (19)$$

where r_1 and r_2 are self-evident, e_0 is the initial value of e_m , p_a is the atmospheric pressure (101 kPa), λ is the compression index, κ is the swelling index and ξ is a model parameter. The term r_2 is used to model the detrimental effect of gas bubbles on the soil behavior. It is proposed based on the experimental observation that, due to strain and pore water pressure localization caused by the gas bubbles, the tangent shear modulus of fine-grained gassy soil at strain levels beyond the elastic region decreases as f and $\frac{u_g+p_a}{(s_u)_{sat}}$ increase (Wheeler, 1986; Sham, 1989; Hong *et al.*, 2017), where $(s_u)_{sat}$ is the undrained shear strength of the saturated clay matrix. In this model, $\frac{u_g+p_a}{p'_0}$ is used instead of $\frac{u_g+p_a}{(s_u)_{sat}}$ [which is suggested by Wheeler (1988a)] to simplify the model formulations, because p'_0 is proportional to $(s_u)_{sat}$ for a normally consolidated clay according to the Cam-Clay model. The term $\frac{q}{Mp'}$ is introduced to make the damaging effect on hardening vanish in isotropic compression, as it was found that the effective pre-consolidation pressure p'_0 of isotopically consolidated gassy specimens is not altered by addition of gas (Hong *et al.*, 2017). Meanwhile, it is also evident in this study that consideration of gas bubbles as compressible inclusion is sufficient for modelling the response of fine-grained gassy soil in compression (see Fig. 6 in the model validation section). When there are no cavities inside the soil, i.e., $f = 0$ and $r_2 = 0$, the hardening law becomes identical to that of the modified Cam-Clay model.

It is important to point out that this hardening law does not always make the undrained shear strength of a gassy clay sample lower than that of a saturated one with the same initial p'_0 and overconsolidation ratio. When the bubble flooding $d\varepsilon_v^f$ (its expression will be given in the subsequent sections) is 0 or very small compared to $d\varepsilon_v^b$,

the detrimental effect dominates for gassy clay and lower undrained shear strength will be predicted, because dp'_0 and the critical state p'_0 are smaller for it (higher critical state p'_0 indicates higher undrained shear strength). On the other hand, when $d\varepsilon_v^f$ is much bigger than $d\varepsilon_v^b$ (e.g., in an undrained triaxial compression test with $d\varepsilon_v^b=0$), there will be extra positive plastic volumetric strain increment associated with $d\varepsilon_v^f$, which causes extra expansion of the yield surface (or more increases in dp'_0) for the gassy clay. In this case, a gassy clay can have higher undrained shear strength compared than a saturated one. This is further illustrated in the section on model validations.

The plastic modulus for the saturated matrix is:

$$K_p = -\frac{\partial F}{\partial p'_0}(r_1 + r_2) \quad (20)$$

The loading index L can be obtained based on the condition of consistency for the yield function as below:

$$L = \frac{\frac{\partial F}{\partial p'} K_m d\varepsilon_v^m + \frac{\partial F}{\partial q} 3G_m d\varepsilon_q^m}{K_p + \frac{\partial F}{\partial p'} K_m \frac{\partial F}{\partial p'} + \frac{\partial F}{\partial q} 3G_m \frac{\partial F}{\partial q}} = \Lambda_p d\varepsilon_v^m + \Lambda_q d\varepsilon_q^m \quad (21)$$

where $d\varepsilon_v^m$ and $d\varepsilon_q^m$ denote the volumetric and shear strain increment for the saturated matrix, respectively. Since $d\varepsilon_q = d\varepsilon_q^m$ according to the assumption made in Eq. (13), $d\varepsilon_q^m$ in the equation above can be replaced by $d\varepsilon_q$.

Assuming that the total strain increment is the summation of the elastic and plastic parts ($d\varepsilon_v^m = d\varepsilon_v^{me} + d\varepsilon_v^{mp}$; $d\varepsilon_q^m = d\varepsilon_q^{me} + d\varepsilon_q^{mp}$), where $d\varepsilon_v^{me}$ and $d\varepsilon_q^{me}$ respectively denote the elastic volumetric and shear strain increment for the saturated matrix, the constitutive equation for the saturated clay matrix can then obtained based on the condition of consistency for the yield function and Eqs. (15)-(21) as below:

$$\begin{aligned}
\begin{bmatrix} dp' \\ dq \end{bmatrix} &= \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} d\varepsilon_v^m \\ d\varepsilon_q \end{bmatrix} \\
&= \begin{bmatrix} K_m - h(L)K_m\Lambda_p \frac{\partial F}{\partial p'} & -h(L)K_m\Lambda_q \frac{\partial F}{\partial p'} \\ -h(L)3G_m\Lambda_p \frac{\partial F}{\partial q} & 3G_m - h(L)3G_m\Lambda_q \frac{\partial F}{\partial q} \end{bmatrix} \begin{bmatrix} d\varepsilon_v^m \\ d\varepsilon_q \end{bmatrix} \quad (22)
\end{aligned}$$

where $h(L)$ is the Heaviside function [$h(L) = 1$ when $L > 0$ and $h(L) = 0$ when $L \leq 0$].

Bubble flooding and cavity compression

Eq. (12) shows that $d\varepsilon_v^m$ consists of two parts, i.e., $d\varepsilon_v^b$ and $d\varepsilon_v^f$. $d\varepsilon_v^b$ can be obtained based on the boundary condition but $d\varepsilon_v^f$ is dependent on the state of fine-grained gassy soil, especially the difference between u_g and u_w . u_g is typically greater than u_w due to the surface tension for the air-water interface (Wheeler, 1986). There is a lower limit (u_l) which is the water entry value and upper limit (u_u) for $u_g - u_w$, which is affected by the gas bubble size and critical radius of the water menisci (Wheeler *et al.*, 1990). Bubble flooding will occur when $u_g - u_w = u_l$ (Wheeler *et al.*, 1990). Since u_l is different for each bubble in the sample (u_l is bigger for smaller bubbles), an approximate method is used in modelling the bubble flooding. It is assumed that the rate of bubble flooding always occurs but increases as $u_g - u_w$ decreases when the entire sample is considered. Bubble flooding can occur when u_g decreases or u_w increases. Since u_g changes passively as u_w changes in most cases (Thomas, 1987; Sham, 1989), it can be assumed that bubble flooding occurs only when u_w increases. Based on such observation, the following expression for $d\varepsilon_v^f$ is used

$$d\varepsilon_v^f = A\langle du_w \rangle = (1 - S_r) \left(\frac{p'_0}{u_g + p_a} \right)^\xi \exp \left(1 - \frac{u_g - u_w}{xp'_0} \right) \frac{\langle du_w \rangle}{u_g + p_a} \quad (23)$$

where x is a positive model parameter. It is evident that $d\varepsilon_v^f = 0$ when the bubble is completely flooded with $S_r = 1$. Bubble flooding is also more significant ($d\varepsilon_v^f$ is higher) when u_g is lower, which is supported by experimental observations (Wheeler, 1988b; Sham, 1989). The term $\left(\frac{p'_0}{u_g + p_a}\right)^\xi$ is used to make the rate of bubble flooding increase as p'_0 increases based on the test results (Wheeler, 1986; Sham, 1989). The term $\exp\left(1 - \frac{u_g - u_w}{xp_a}\right)$ is used to render that the rate of bubble flooding decreases as $u_g - u_w$ increases. The McCauley brackets $\langle \rangle$ are used to guarantee that bubble flooding occurs only when u_w increases. It must be mentioned that Eq. (23) is a homogenization method for modelling bubble flooding, which is suitable for an entire sample but not each individual bubble.

In this model, dissolving of gas into the pore water is neglected during the loading process, due to the extremely low solubility of the bio-gases. The volume change of the gas cavity before bubble flooding can be described using the Boyle's law for ideal gas. Once the bubble is flooded, however, compressibility of the cavity becomes more difficult to characterize because it will be dependent on the cavity volume, gas volume (smaller than the cavity volume) and gas pressure. A simplified method for modelling the cavity volume change is used based on the Boyle's law for ideal gas

$$d\varepsilon_v^c = Bdp = B(dp' + du_w) \quad (24)$$

$$B = \begin{cases} 0 & (\text{for } S_r = 1) \\ \frac{1}{u_g + p_a} & (\text{for } S_r < 1) \end{cases} \quad (25)$$

It is evident that $B = 0$ when $S_r = 1$, which indicates that the cavity size remains unchanged when the sample is completely saturated (all the bubbles are completely

flooded). Note that du_g is more appropriate for calculating $d\varepsilon_v^c$. However, it is generally very difficult to measure u_g and dp is used instead. Indeed, Thomas (1987) showed that Eqs. (24) and (25) are sufficient for modelling the bubble compression without bubble flooding.

Initial gas pressure and evolution of gas pressure and volume

The initial gas pressure before a test is difficult to measure or estimate, though there are upper and lower bounds for it (Sham, 1989). In this paper, the initial gas pressure u_{gi} is assumed to vary between the initial pore water pressure u_{wi} and initial total stress p_i as below (Sham, 1989)

$$u_{gi} = u_{wi} + \delta (p_i - u_{wi}) = u_{wi} + \frac{u_{wi} + p_a}{p_i + p_a} (p_i - u_{wi}) \quad (26)$$

A constant δ is used in Sham (1989) while a variable one is used here. $u_{gi} = u_{wi}$ when $\delta = 0$ and $u_{gi} = p_i$ when $\delta = 1$. δ in Eq. (26) varies between 0 and 1 as $u_{wi} + p_a \geq 0$ and $u_{wi} \leq p_i$. δ increases as u_{wi} increases, making u_{gi} increase and get closer to p_i . The variable δ used in Eq. (26) is based on the consideration that as u_{wi} increases at the same p_i , the mean effective stress p' and shear strength of the saturated matrix becomes smaller, indicating that the soil is closer to the cavity expansion state. As shown by Sham (1989), u_{gi} becomes higher when the soil is closer to cavity expansion.

During the loading process, evolution of u_g is affected by cavity volume change and bubble flooding. Assuming that all the volume change of cavities is related to compression/expansion of the gas bubbles, the increment of the volumetric strain of gas bubbles $d\varepsilon_v^g$ is:

$$d\varepsilon_v^g = \frac{V_c}{V_g} d\varepsilon_v^c + \frac{V_m}{V_g} d\varepsilon_v^f \quad (27)$$

The terms $\frac{V_c}{V_g}$ and $\frac{V_m}{V_g}$ are used because $d\varepsilon_v^c$ and $d\varepsilon_v^f$ are measured with reference to the volume of the cavity and saturated clay matrix, respectively. Based on the Boyle's law, the expression for du_g can be obtained as below:

$$du_g = d\varepsilon_v^g (u_g + p_a) \quad (28)$$

Elasto-plastic relationship for the composite material

In the present model, fine-grained gassy soil is considered as a three-phase material. u_g , V_g and V_c are used as internal variables, which should be updated based on the evolution of p' , q and u_w . Since u_w is needed in the constitutive equation, an extra term $d\varepsilon_v^w$ defined as below is used

$$d\varepsilon_v^w = \frac{dV_w}{V_w} \quad (29)$$

where dV_w is the volume change of water in the soil. For this soil, the increment of u_w is expressed as

$$du_w = K_w d\varepsilon_v^w \quad (30)$$

where K_w is the bulk modulus of water which is assumed infinite in this study.

Combining Eqs. (12), (22), (23) and (34) one can get

$$\begin{bmatrix} dp' \\ dq \\ du_w \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & 0 \\ 0 & 0 & K_w \end{bmatrix} \begin{bmatrix} d\varepsilon_v^b + AK_w \langle d\varepsilon_v^w \rangle \\ d\varepsilon_q \\ d\varepsilon_v^w \end{bmatrix} \quad (31)$$

Eq. (31) is needed in finite element modelling, which will be discussed in the following subsection. For simple single element tests, alternative ways without calculating $d\varepsilon_v^w$ can be used in using the model: (a) In a globally undrained triaxial test, $d\varepsilon_v^b = 0$. For each increment of $d\varepsilon_q$, one can get the expression for dp' and dq in terms of $d\varepsilon_q$

(known) and du_w by replacing $AK_w\langle d\varepsilon_v^w \rangle$ with $A\langle du_w \rangle$ in Eq. (31). du_w can then be calculated based on the total stress path [e.g., $dq = 3(dp' + du_w)$]. Once du_w is obtained, dp' and dq can be calculated using Eq. (31). $d\varepsilon_v^c$, $d\varepsilon_v^g$ and du_g can then be calculated using Eqs. (24), (25), (27) and (28); (b) In an isotropic consolidation test or drained triaxial test, $du_w = 0$ and $d\varepsilon_v^w = 0$ which means that there is no ‘bubble flooding’. Eq. (31) can thus be used to calculate dp' and dq based on this condition. The remaining calculations are the same as (a). The volume change of cavity, gas, saturated matrix and entire soil should be calculated using the following equations

$$dV_c = V_c d\varepsilon_v^c \quad (32)$$

$$dV_g = dV_c + V_m d\varepsilon_v^f \quad (33)$$

$$dV_m = V_m d\varepsilon_v^m \quad (34)$$

$$dV_v = dV = V_m d\varepsilon_v^m + V_c d\varepsilon_v^c \quad (35)$$

Numerical implementation of the proposed constitutive model

Application of the new model in simulating simple laboratory tests is discussed in the section above. The model must be implemented in a finite element (FE) code for solving practical boundary value problems. As this model is developed following the conventional plasticity theory, it can be easily implemented using either the explicit or implicit numerical schemes (e.g., Borja et al., 1998; Borja, 2004; Andrade and Borja, 2006; Gao and Zhao, 2013). One key issue for the implementation is that the main FE code must be able to consider the drainage of water from the saturated clay matrix into gas cavities (i.e., bubble flooding) and the associated volume change (Eqs. 10 and 12). This means that one may not be able to implement this model in some commercial FE

software packages which cannot properly simulate the process of bubble flooding. It is therefore more convenient to implement the model in open-source FE software packages, where users can make necessary changes to the main code to account for bubble flooding. Research is being done in this regard.

DETERMINATION OF MODEL PARAMETERS AND PARAMETRIC STUDIES

The new model consists of 7 parameters, 5 of which are identical to those in the MCC model ($M, \lambda, \kappa, \nu, N$), where N is value of $1 + e_m$ at $p'_m = 1$ kPa for the normal compression line in the $(1 + e_m) - \ln p'_m$ space. These parameters can be determined following standard procedures based on the conventional triaxial and oedometer/isotropic consolidation test results of saturated soils. The effect of ξ and x on the model simulation will be studied using the undrained triaxial tests on gassy Malaysian kaolin (Hong *et al.* 2017). Determination of them will then be presented based on these parametric studies.

Parametric studies for ξ and x

The parameter ξ is used in both the plastic hardening law and bubble flooding. An increasing value of ξ renders more reduction in the plastic modulus and less bubble flooding at the same $\frac{u_w + p_a}{p'_0}$, leading to a more detrimental effect of bubbles. Fig. 4 shows the effect of ξ on model simulation of the undrained triaxial compression tests on gassy clay. A very small x is used to render no bubble flooding. The remaining parameters are the same as those for gassy Malaysian kaolin shown in Table 1. The model gives very little difference between a gassy clay and saturated one when $\xi = 0$,

indicating very small (but not 0) damaging effect of gas bubbles on the plastic hardening and shear strength. Note that the damaging effect is small but does not vanish when $\xi = 0$ (Eq. 19). There is no damaging effect only when $f = 0$ (Eq. 19). More damaging effect (lower undrained shear strength for gassy clay) is seen at higher ξ . Fig. 4c shows the evolution of total soil volume V and volume of the saturated soil matrix V_m . There is small reduction in V for gassy clay due to compression of the gas cavities. No change in V_m is predicted for the gassy samples, because bubble flooding is made 0 for these simulations.

Fig. 4 Effect of parameter ξ on the model simulation for undrained triaxial tests on gassy clay (Malaysian kaolin)

The parameter x affects the rate of bubble flooding. A larger value of x indicates a

higher rate of bubble flooding, enabling more beneficial effect of bubbles on the strength and stiffness of fine-grained gassy soil. Fig. 5 shows the effect of x on the model simulation for gassy clay in undrained triaxial compression. $\xi = 0$ is used to make the damaging effect small.

Fig. 5 Effect of parameter x on the model simulation for undrained triaxial tests on gassy clay (Malaysian kaolin)

It is evident that the model gives higher undrained shear strength for gassy clay at $x = 2.5$, indicating more beneficial effect. The evolution of V and V_m is shown in Fig. 5c. In undrained tests, the change in V_m is only caused by bubble flooding. There is more reduction in V_m for the case with $x = 2.5$ due to more bubble flooding. More

reduction in V_m means that more pore water drains out of the saturated soil matrix, which makes the undrained shear strength of gassy clay higher. There is reduction in V for both unsaturated cases. This is due to both bubble flooding and compression of the cavities (Eq. 35).

Determination of ξ and x for gassy Malaysian kaolin silt

Generally, bubble flooding is more significant when f is larger and u_w is smaller (Wheeler, 1986; Sham, 1989; Hong *et al.*, 2017). It is recommended that x and ξ should be calibrated against the stress-strain relationship of two typical gassy specimens, i.e., one with high f and low u_w (in which beneficial effect of bubbles dominates) while the other with low f and high u_w (where detrimental effect of bubbles is more obvious). For gassy Malaysian kaolin, ξ and x are determined based on the undrained triaxial tests with $u_{wi} = 0$ kPa, where the beneficial effect on the undrained shear strength dominates, and $u_{wi} = 1000$ kPa, in which the detrimental effect of gas bubbles on the undrained shear strength dominates. Since ξ has effect on both plastic hardening and bubble flooding, it is not possible to determine the two parameters separately. Therefore, they have to be fine-tuned simultaneously using a trial-and-error procedure to best fit the test data. The model is then used to predict the soil response under other loading conditions. Similar procedure has been used for the other two gassy clays, Combwich mud (Wheeler, 1986) and Speciwhite kaolin clay (Sham, 1989). All the model parameters are listed in Table 1.

MODEL VALIDATION

In this section, the predictive capability of the new model proposed in this study is validated against the experimental results of isotropic compression and undrained triaxial compression of three different types of fine-grained gassy soils, namely gassy Malaysian kaolin silt (Hong *et al.*, 2017), gassy Combwich mud (Wheeler, 1986) and Specwhite kaolin clay (Sham, 1989). Comparisons between the measured and predicted results are presented and discussed in the following sections.

Table 1 **Model parameters**

Materials	M	λ	κ	N	ν	ξ	x
Malaysian kaolin with nitrogen (Hong <i>et al.</i> , 2017)	1.00	0.25	0.06	3.81	0.3	2.2	2.5
Combwich mud with methane (Wheeler, 1986)	1.33	0.174	0.0297	3.06	0.2	4	0.2
Kaolin with Helium (Sham, 1989)	0.87	0.23	0.014	3.35	0.2	2.2	0.01

Tests on gassy Malaysian kaolin

Both isotropic compression and undrained triaxial compression tests have been reported in Hong *et al.* (2017) on Malaysian kaolin. Details of the sample preparation method, initial conditions and loading conditions can be found in Hong *et al.* (2017). Fig. 6 shows the comparison between the measured and the predicted isotropic compression behavior of gassy Malaysian kaolin (Hong *et al.*, 2017). There is a unique relationship between e_m and p' , which is the same as the $e - p'$ relationship for the saturated sample. The model gives good prediction for the test data, with slightly higher e for the tests with $u_w = 120$ kPa and $u_w = 600$ kPa at higher p' . The model gives a

unique $e_m - p'$ relationship as the effective stress principle works in the saturated soil matrix.

Fig. 6 Comparison between the measured and predicted isotropic compression behavior of gassy Malaysian kaolin (Data from Hong *et al.*, 2017)

Fig. 7 shows the comparison between the measured and the predicted stress-strain relationships of fine-grained gassy Malaysian kaolin in undrained triaxial compression tests. Two of the gassy clay tests ($u_{wi} = 0$ kPa and $u_{wi} = 1000$ kPa) are used for parameter determination and the rest three are used for model validation. Prior to the undrained shearing, each gassy specimen was isotopically consolidated to a given mean effective stress ($p'_c = 200$ kPa) but at different initial pore water pressures (i.e., with u_{wi} ranging from 0 to 1000 kPa). The result of a benchmark test in saturated specimen ($p'_c = 200$ kPa) is also included in the two figures for comparison. More details of the sample preparation, test procedure and initial degree of saturation S_{ri} can be found in Hong *et al.* (2017).

Fig. 7 Comparison between the measured and predicted stress-strain behaviour of gassy Malaysian kaolin in undrained triaxial compression (Data from Hong *et al.*, 2017)

Both u_{wi} and S_{ri} are different for these tests (Hong *et al.*, 2017). Generally, the model can capture both the beneficial and detrimental effect of gas bubbles on the undrained shear strength (Figs. 7), while existing models can only capture the beneficial effect (Pietruszczak & Pande, 1996; Grozic *et al.*, 2005). It gives reasonable prediction for the undrained shear strength for all the cases, but overestimates the shear modulus, particularly at relatively small strains (when axial strain $< 2\%$). The expression for the elastic stiffness could be modified to account for the effect of gas bubbles, which would in turn introduce more parameters for such improvement.

Figs. 8(a) and 8(b) compares the measured and predicted effective stress paths of fine-grained gassy Malaysian kaolin silt in undrained triaxial compression tests. The model can capture the trends of effective stress paths exhibited by the gassy specimens with varying u_{wi} . Quantitatively, however, the measured and predicted results show some discrepancies at relatively high initial pore water pressure (i.e., $u_{wi} > 250$ kPa) and $u_{wi} = 0$ kPa. This implies that the actual shape of the yield surface may deviate from the assumed elliptical shape when $u_{wi} > 250$ kPa (Fig. 8b). A flatter shape of

yield surface on the “wet” side may give better predictions for gassy specimens at relatively high initial pore water pressures. The model gives reasonable prediction for the excess pore water pressure Δu_w for most of the tests, except the one with $u_{wi} = 0$ kPa (Fig. 9). The predicted Δu_w is higher than the measured value, which makes the model give lower undrained shear strength for this test (Fig. 7). The model prediction for this test can be improved by using a better formulation for the bubble flooding, which will be pursued in the future.

Fig. 8 Comparison between the measured and predicted effective stress path of gassy Malaysian kaolin in undrained triaxial compression (Data from Hong *et al.*, 2017)

Fig. 9 Comparison between the measured and predicted excess pore water pressure of gassy Malaysian kaolin in undrained triaxial compression (Data from Hong *et al.*, 2017)

Tests on gassy Combwich mud

Wheeler (1986) have carried out extensive laboratory tests on gassy Combwich mud, which include both isotropic compression and undrained triaxial compression tests on normally consolidated soils. Details of the test procedure can be found in Wheeler (1986). But only one set of stress-strain relationship is available (see Fig. 10(a)). The test results in Fig. 10 (beneficial effect dominates) and Fig. 11(c) (detrimental effect is obvious) have been used to determine the parameters for modelling the effect of gas bubbles (ξ and x). Figs. 10(a), 10(b) and 10(c) shows the comparisons between the measured and the predicted stress-strain relationship, effective stress path and excess pore pressure of Combwich mud with methane (Wheeler, 1986) subjected to undrained triaxial compression, respectively. As illustrated, the model gives reasonable predictions for the measured stress-strain relationship and effective stress path. It overestimates the excess pore water pressure for axial strain between 3 and 10%, which is due to the discrepancy between the measured and predicted effective stress paths (Fig. 10(b)).

The undrained shear strength (s_u) of fine-grained gassy soil is of great importance for various offshore geotechnical engineering problems. Although Wheeler (1986) has derived the upper and lower bounds for s_u of fine-grained gassy soil, these bounds cannot be used to give a specific value for each case. Fig. 11 shows the model prediction of the undrained shear strength s_u of Combwich mud with methane. Generally, the model captures both the detrimental and beneficial effect of gas bubbles on the undrained shear strength. The model satisfactorily predicts the tests with $\sigma_{3i} =$

400 kPa and $\sigma_{3i} = 100$ kPa, but gives overestimation for tests with $\sigma_{3i} - u_{wi} =$
200 kPa. Overestimation for the test data with $\sigma_{3i} - u_{wi} = 200$ kPa could be
attributed to the factor that the initial gas pressure u_{gi} calculated using Eq. (26) is too
low. Higher u_{gi} would render less bubble flooding and therefore a lower s_u .

Fig. 10 Comparison between the measured and predicted gassy Combwich mud in
undrained triaxial compression with $\sigma_{3i} = 400$ kPa and $u_{wi} = 0$ kPa: (a) stress-strain
relationship; (b) effective stress path; (c) excess pore water pressure (Data from
Wheeler, 1986)

Fig. 11 Comparison between the measured and predicted undrained shear strength of gassy Combwich mud in undrained triaxial compression: (a) $\sigma_{3i}-u_{wi}=400 \text{ kPa}$; (b) $\sigma_{3i}-u_{wi}=200 \text{ kPa}$; (c) $\sigma_{3i}-u_{wi}=100 \text{ kPa}$ (Data from Wheeler, 1986)

Tests on gassy Speciwhite kaolin clay

Sham (1989) have carried out experiments on Speciwhite kaolin clay using the same test methods in Wheeler (1986). Similar to the Combwich mud, only one group of the stress-strain relationship of gassy Speciwhite kaolin is available (Fig. 12). The results in Fig. 12 (undrained shear strength is higher for gassy samples) and Fig. 13(c) (undrained shear strength is lower for gassy samples) have thus been used to determine the parameters for gassy soil. For this soil, it is found that the detrimental effect of gas on the soil strength dominates. A smaller α (lower rate of bubble flooding) is thus used, as compared to that for gassy Malaysian silt and gassy Combwich mud (see Table 1).

This could be due that the gas bubbles formed in soil with finer particles tend to be larger than those in soil with coarser particles, given the stress level of the two are identical (Hong *et al.*, 2019a). Since Speciwhite kaolin clay is finer than the other two soils reported herein, it contains bubbles of larger size, which is less susceptible to bubble flooding (being beneficial) but could have caused more yielding around the large cavities (Wheeler *et al.*, 1990). Figs. 12(a) and 12(b) illustrate the comparisons between the measured and the predicted stress-strain relationship and effective stress path of gassy Speciwhite kaolin clay in undrained triaxial compression (Sham, 1989), respectively. The model gives good predictions for both the stress-strain relationship and effective stress path. Fig. 13 shows the model prediction for the undrained shear strength of gassy Speciwhite kaolin clay. Despite some discrepancies between the test data and the predictions, the model does capture both the detrimental and beneficial effect of gas bubbles on the undrained shear strength with a unified set of parameters.

Fig. 12 Comparison between the measured and predicted response of gassy Speciwhite Kaolin in undrained triaxial compression: (a) stress-strain relationship; (b) effective stress path (Data from Sham, 1986)

Fig. 13 Comparison between the measured and predicted undrained shear strength of gassy Speciwhite Kaolin with various initial stress conditions in undrained triaxial compression (Data from Sham, 1986)

614 CONCLUSIONS

615 A critical-state-based constitutive model for describing the distinct mechanical
616 behavior of fine-grained soil containing large gas bubbles is developed. The model has
617 the following features:

618 (a) Gassy clay is considered as a three-phase composite material consisting of saturated
619 soil matrix and gas cavities, which are compressible and allows for water entry from
620 the saturated matrix (“bubble flooding”). The stress and strain state of fine-grained
621 gassy soil is described using the rule of mixtures for composite materials. Since the
622 volume fraction of bubbles is typically very small, contribution of gas pressure to
623 the total stress is neglected. The effective stress principle is used for describing the
624 constitutive relationship of the saturated matrix.

625 (b) In conjunction with the bubble flooding equation, the hardening law can describe
626 both the detrimental and beneficial effects of gas bubbles on mechanical behavior
627 of fine-grained gassy soils. When the volumetric strain increment of the saturated
628 matrix associated with bubble flooding is very small, the detrimental effect
629 dominates (lower shear strength for a gassy sample). When there is significant
630 amount of water drainage from the saturated matrix to gas bubbles, extra yield
631 surface expansion occurs, making a gassy clay sample have higher undrained shear
632 strength.

633 Compared to the MCC model, only 2 additional parameters are introduced to

634 characterize the effect of gas bubbles on soil response. The proposed model is shown
635 to adequately capture the experimental data of three types of fine-grained gassy soils
636 under various initial conditions using a unified set of parameters.

637 The model may be further improved in the following aspects: (a) The model needs
638 to be improved to capture the response of gassy clay under unloading. There gas bubble
639 can expand when the gassy soil is subjected to unloading (e.g., decrease of total mean
640 stress during sampling of the gassy soil sample from the sea), which causes damage to
641 the soil structure and affects the soil response during subsequent loading (Sultan *et al.*,
642 2012; Sultan & Garziglia, 2014). But the model cannot describe such soil response,
643 because it gives purely elastic response in unloading. Future work will be done to
644 improve the model in this regard based on the work by Sultan & Garziglia (2014) and
645 the bounding surface concept (Dafalias, 1986); (b) The gas solubility is ignored in this
646 model. When the u_w changes during loading, the undissolved gas could get dissolved
647 in the pore water. This can cause extra volume contraction of the saturated matrix,
648 which is similar to the effect of bubble flooding. For the three gassy clays used in this
649 paper, such effect is small due to the extremely low solubility of typical types of biogas
650 (Lin *et al.*, 2004); (c) The effect of gas bubbles on yield surface shape (e.g., Yu, 1998;
651 Yin *et al.*, 2013; Gao *et al.*, 2017; Chen and Yang, 2017; Hong *et al.*, 2020) and dilatancy
652 relationship (Hong *et al.*, 2019b) should be considered to get better prediction of the
653 effective stress paths in undrained triaxial compression.

654 ACKNOWLEDGEMENTS

655 The authors would like to acknowledge the fine contribution by Professor Simon
656 Wheeler at University of Glasgow through various discussions. The second and the
657 third authors are grateful for the financial supports from the National Key Research and
658 Development Program (2018YFE0109500), National Natural Science Foundation of
659 China (51779221 and 51939010) and Zhejiang Provincial Key Research and
660 Development program (2018C03031). The assistance provided by Mr. Xuanyu Chen
661 of Zhejiang University in preparing the figures of this manuscript is also gratefully
662 acknowledged.

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