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Two-way capital flows: A risk-sharing approach

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Abstract

The two-way capital flows has been a persistent pattern existing in international capital market, i.e. net bond asset flows from developed countries to developing countries as a whole while net equity asset goes the other way around at the same time. In this paper, I construct a model of two-country open economy within which each country is subject to New-Keynesian frictions. Using new techniques of computing portfolio choices in macroeconomic models, I solve for the country holdings of equity and bond assets in such a general framework. Based on the recent work which estimate New-Keynesian macroeconomic model of US and Chinese economy, I introduce empirically relevant cross-country asymmetries with regard to different economic structure, country openness, monetary policy stance and severity of frictions, etc. in the model and show that the pattern of the two-way capital flows emerges as a result of agents seeking to attain high level of risk-sharing across countries through optimal portfolio allocation.

Keywords: International portfolio choices, Two-way capital flows, Emerging markets.

JEL Codes: F32, F41

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1 Introduction

In international macroeconomics, the so-called two-way capital flows between developing and developed countries is an interesting phenomenon, i.e. net bond asset flows towards developing countries while net equity asset flows towards developed countries as a whole. (See Lane and Milesi-Ferretti, 2001, 2007a, 2007b, Ju and Wei, 2010.) What is the reason for this phenomenon? Why do equity and bond assets flow in such ways rather than the other way around? Are there any casual links between the stage of development of a country and their preference over different types of international assets? If there are, what are they and how do they work? This paper seeks to answer these questions.

There has been an increasingly large literature studying net capital flows between developing and developed worlds since 1990 when Lucas (1990) proposed the famous question of why does capital not flow from developed countries to developing countries. Or even though it does why is this flow not stronger than observed. Based on standard neoclassical models, capital tends to flow to where it is able to yield a higher return. And the most basic reason for a differing return in such models is the degree of capital scarcity. Since developing countries are usually capital scarce in comparison to developed countries, the model predicts that net capital should flow from the latter to the former. The puzzle might not have gained so much attention if it was just a problem of size in a world of balanced international payments. In a world featuring global imbalances, as has emerged since 1990, this becomes even more puzzling because net capital actually flows the opposite way to that predicted by the neoclassical model.

Various theories have been proposed to explain this puzzling fact. Explanations include policy misalignments (Obstfeld and Rogoff, 2007, Summers, 2004, Blanchard et al., 2005, etc.), difference in productivity growth (Hunt et al., 2005, Engle and Rogers, 2006), demographic dynamics (Henriksen, 2005, Attanasio et al., 2006), volatility of the business cycle (Fogli and Perri, 2006) and a global savings glut (Bernanke, 2005) etc. In particular, one strand of the literature emphasizes the importance of financial underdevelopment of developing countries in reconciling the facts. According to these studies, various financial frictions, for instance lack of enforceability of financial contracts (Mendoza et al., 2009), incapability in supplying a sufficient asset stock (Caballero et al., 2009) or/and in insuring away idiosyncratic risk (Angeletos and Panousi, 2011) etc., can distort the decisions of saving and investment in emerging markets, which in turn results in both a lower interest...
rate and a lower capital stock in autarky. While saving cannot be effectively channelled to investment domestically due to these financial frictions, under financial integration, excess saving must find its way to developed countries in the form of a net capital flow.

There is also an expanding literature on ‘two-way capital flows’ (i.e. where bonds and equity flows in opposite directions). Most of this literature also focuses exploring the effects of financial distortions on the choices of different types of asset. Ju and Wei (2010) attribute the major reason to financial market imperfections and related institutions such as property rights protection. The mechanism of financial capital flowing out while investment arriving in the form of FDI can serve as a nice vehicle bypassing the adverse effect of an inefficient financial system within developing countries. Hagen and Zhang (2011) model financial development as an endowment fixed in the short run. With the comparative advantage of providing financial service, developed countries will find it optimal to import financial capital and export FDI while the developing countries follow the opposite pattern. Wang et al. (2015) show that the common presence of underdevelopment factors in the credit market of developing countries can lower the rate of return of financial capital while raise that of fixed capital at the same time. So under capital liberalization, financial capital flows out while the fixed capital flows in.

Rather than mainly focusing on the return and mobility aspects of assets in the above literature, another strand of literature such as Devereux and Sutherland (2010, 2011) and Tille and Wincoop (2010) pay attention to different risk characteristics of international assets and the role they play in determining capital flows. The asset holdings of a country are determined because all assets have different risk characteristics and thus satisfy specific demands of households in different countries for risk-hedging devices. This approach allows for the analysis of many other potential factors in addition to financial frictions that behind net capital flows. The current paper falls in the category of this literature in explaining two-way capital flows. However, Devereux and Sutherland (2010, 2011) focus on methodological usefulness while Tille and Wincoop (2010) on (both net and gross) portfolio dynamics in a world of two symmetric countries. In terms of two-way capital flows, asymmetries must be involved.

The analytical framework in this paper is a model of a two-country world. Two types of assets, equity and bond, are assumed to be present. In separation, each country can be described by a medium-scale full-fledged model of the New Keynesian approach. So
as a whole, the model of the two economies also represents an extension of the literature such as Woodford (2003), Gali (2008), Christiano et al. (2010), etc. to the context of international economy with endogenous portfolio choices. Specifically, the environment in each country is very close to that of Smets and Wouters (2007). For our purpose of distinguishing developing and developed country, we assume different values of structural parameters for them. These parameters capture various aspects in which the two economies may differ, including those of economic structure, policy stance, severity of various (real and nominal) frictions and properties of economic shocks, etc.. The studies in the literature employing econometric techniques to estimate the DSGE models of developed (for instance Smets and Wouters, 2003, 2007, etc.) and developing countries (for instance Sun and Sen, 2012, Dai, 2012 and Miao and Peng 2012, etc.) provide us with these parameter values of empirical relevance. Given the presence of the country asymmetries, optimal portfolio choices are computed and then assessed from the perspective of conforming to or contradicting the pattern of two-way capital flows. Through this process, we uncover which asymmetries matter and to what extent they matter.

To summerize our findings, firstly, we find that the asymmetries associated with country’s industrial structure, severity of nominal rigidities, trade openness, consumption home bias, investment adjustment frictions, monetary policy stance, market competitiveness and pricing strategy of international trade, etc. can cause the two-way capital flows between developing and developed countries. Secondly, among these factors, those from the real side of economy are more important than those from the nominal side. Lastly, we simulate the model with fully asymmetric parameter values and find that it yields a portfolio allocation that are broadly consistent with the pattern of two-way capital flows. Besides, if we take into account of the situation where international bonds can only be issued by the developed country (as it is often the case in reality), this result still holds.

This work is closely related to Devereux and Sutherland (2009). The latter considers asymmetry in asset market structure and finds that under the pattern of two-way capital flows the economies achieve a relatively high level of international risk-sharing, which supplies evidence in support of the emergence of the pattern. We follow a similar idea in this paper, however, with substantial extension of the model and analysis. This, on the other hand, explains why we need such a general framework of New Keynesian approach (with each economy being modelled with rich features) in this paper. Non-trivial mon-
etary policy is present so that bond assets’ return can be defined while many frictions, price/wage rigidities and costly investment adjustments for instance, are assumed here so that a long list of asymmetries associated with these features can be examined in the analysis. The work is also linked to Devereux et al. (2014) when it comes to decomposing the hedging properties of assets into correlation and variability effects which sheds light on the machinery of each asymmetry. With the presence of the central role of differing hedging properties of different types of asset in the model, it also connects to the literature on (symmetric) asset home bias in international macroeconomics. Coeurdacier and Rey (2012) give a survey of the literature on this topic.

The rest of the paper is structured as follows. Section 2 presents the model. Section 3 discusses the determination, representation and interpretation of optimal country portfolios in the general model. Section 4 simulates the model symmetrically. Section 5 simulates the model asymmetrically and assesses country asymmetries’ impact on the pattern of two-way capital flows. Section 6 concludes.

2 Model

The model assumes a world consisting of two countries, Home and Foreign. For the reader’s convenience, a figure, Figure 1, is employed to summarize the economic structure of the two countries. At the top of the figure is a diagram of resource flows while on the lower half are some key points of information. The two countries are the same in terms of economic structure, which is reflected by the fact that the flows in the foreign country are drawn to be a mirror image of those in the home country. As shown in the diagram, each economy consists of five sectors. From left to right, they are the sector of households, labour union, intermediate goods sector, final goods sector and government. The lines linking sectors represent resource flows with the arrows showing the direction of flow. In each economy, households consume final goods from both home and foreign countries. They supply, domestically, their labour to labour unions for wages and capital to intermediate goods firms for capital rental. The labour unions distribute the labour supplies. And the intermediate goods firms combine the labour and capital collected to produce intermediate goods whose usefulness is only to be sold to the final goods sector. The firms in the final goods sector produce the final goods which are then ready for use
for consumption and investment.

Following the literature, the intermediate and final goods firms are further divided into two parallel sectors of traded and non-traded goods production in both countries. In the diagram, this is reflected by the fact that the traded goods sectors are circled in a shadowed area. The traded and non-traded goods sectors are different such that the final goods produced by non-traded sectors can only be sold to domestic households while the final goods produced by the traded sector can be sold to both domestic and foreign households. There is one public sector, government, in the economy. They tax and consume on the one hand and implement fiscal and monetary policies according to rules on the other hand.

On the lower half of the figure, the first row lists the frictions embedded in the private sector and the policy rules adopted by the governments while the second row lists the shocks that are present. Being put forward without explanation, they are gathered here to give a better general description of the whole model and will be explained in more detail below. In what follows, the complete behaviours of each sector will be specified. However, because the two economies have the same structure, we will focus on the case of the home country. As a convention, when it is necessary to mention foreign country variables, an asterisk is used.
2.1 Households

Assume a continuum of household \( z \in (0, 1) \). The representative household \( z \) is an intertemporal optimizer whose objective is to maximize the following utility function:

\[
E_t \sum_{i=0}^{\infty} \beta^i \left\{ \frac{C_{Xt+i}^1(z)}{1 - \rho} - \chi_{t+i} \frac{L_{t+i}^\mu}{\mu} \right\} \tag{2.1}
\]

The function is an expected summation of an infinite series of single period utility. The latter equals the utility from consumption of a composite good \( C, \frac{C_{Xt+i}^1(z)}{1 - \rho} \), less the disutility from hours worked, \( \chi_{t+i} \frac{L_{t+i}^\mu}{\mu} \). \( \beta, \rho \) and \( \mu \) are respectively the discount factor, the risk aversion parameter (or inverse of the elasticity of intertemporal substitution) and the elasticity of labour supply. \( \chi \) represents a weight between consumption and working hours. It is assumed to be a labour supply shock following the process \( \tilde{\chi}_t = \delta \tilde{\chi}_{t-1} + \varepsilon_{xt} \) where a hat over a variable indicates a log-deviation from the steady state. Here if \( \varepsilon_{xt} \) is realized to be positive, there is negative shock to the labour supply.

The household \( z \) faces two restrictions when maximizing the above utility function. First, there is an (external) habit formation process

\[
C_{Xt+i}(z) = C_{t+i}(z) - hC_{t+i-1} \tag{2.2}
\]

where \( h \) is the degree of habit persistence.

Second, the household should meet the intertemporal budget constraint as follows:

\[
F_t = \sum_{i=1}^{4} r_{it} \alpha_{it-1} + \frac{w_t}{P_t} L_t - C_t(z) + \Pi_t + \Theta_t - T_t \tag{2.3}
\]

where \( F_t \) is the net wealth of households at the end of time \( t \). In the model of representative agents, it also denotes per capita net foreign asset (\( NFA \)) of the country. We assume that both the home and foreign countries issue equities and bonds. So there are \( 2 \times 2 = 4 \) assets in total in the model. To understand the budget constraint, note that we denote the households’ holding of asset \( i \) at the end of time \( t \) as \( \alpha_{it} \), so \( F_t = \sum_{i=1}^{4} \alpha_{it} \). We further denote the gross rate of return for asset \( i \) during period \( t \) as \( r_{it} \), so the total return by holding the time-(\( t-1 \)) portfolio to the end of time \( t \) is given by \( \sum_{i=1}^{4} r_{it} \alpha_{it-1} \) which explains the first term on the right hand side of Eq.(2.3). For the rest of the terms on the right hand side, \( w_t \) is the nominal wage received by households. \( P_t \) is home country \( CPI \), i.e. price index of composite good \( C \). \( L_t \) is labour supply so \( \frac{w_t}{P_t} L_t \) is labour income.
We assume that households own firms and the labour unions. $\Pi_t$ and $\Theta_t$ in the equation denote the profits of firms and labour unions that are received by households. $C_t$ and $T_t$ are households’ spending on consumptions and taxation. So the budget constraint states that the amount of net total wealth each period is given by the sum of the gross return by holding existing portfolio and the newly earned saving.

The households’ choice variables include the levels of consumption $C$, labour supply $L$ and portfolio holdings $\alpha_i$s. The first-order conditions associated with optimal $C$, $L$ and $\alpha_i$s are respectively:

$$\Omega_{t+i} = \beta^i C^\rho_{Xt+i} \quad (2.4)$$
$$w_t = \chi_t L^{\rho-1}_t P_t \frac{1}{\Omega_t} = \chi_t L^{\rho-1}_t P_t C^\rho_{Xt} \quad (2.5)$$
$$C^\rho_{Xt} = \beta E_t \left[ C^\rho_{Xt+1} \right] \quad (2.6)$$

where $\Omega_{t+i}$ are multipliers for budget constraints at time $t + i$. Eqs.(2.4) and (2.5) are familiar intertemporal and intratemporal optimal conditions which define optimal $C$ and $L$. Eq.(2.6) determines the optimal portfolio choices $\alpha_i$. To understand it, it asserts that at the optimum, the marginal loss of utility by forgoing consumption (and investing in an asset) today should be equal to the marginal gain of utility by reaping the asset return tomorrow after discounting.

Once $C$ is determined, following the literature, we assume the composite good is made up of non-traded and traded goods by the Dixit-Stiglitz aggregation relation as follows:

$$C = \left[ \kappa^{\frac{\phi-1}{\sigma}} C_N^{\frac{\phi}{\sigma}} + (1 - \kappa)^{\frac{\phi-1}{\sigma}} C_T^{\frac{\phi}{\sigma}} \right]^{\frac{\sigma}{\phi-1}} \quad (2.7)$$

where $C_N$ and $C_T$ are consumptions of non-traded and traded goods. Their weights in the basket are respectively $\kappa$ and $(1 - \kappa)$. $\phi$ is the elasticity of substitution between the two types of good.

Investment goods are assumed to be aggregated in the same way, so

$$I = \left[ \kappa^{\frac{\phi-1}{\sigma}} I_N^{\frac{\phi}{\sigma}} + (1 - \kappa)^{\frac{\phi-1}{\sigma}} I_T^{\frac{\phi}{\sigma}} \right]^{\frac{\sigma}{\phi-1}} \quad (2.8)$$

Given the aggregation relations of spending above, the demands for non-traded and traded goods in the home country are given by

$$D_N = \kappa (C + I) \left[ \frac{P_N}{P} \right]^{-\phi} \quad (2.9)$$
\[ DT = (1 - \kappa) (C + I) \left( \frac{P_T}{P} \right)^{-\phi} \]  

(2.10)

where \( P_T \) and \( P_N \) denote price indices for traded and non-traded goods. Moreover, the price index of the composite good at home \( P \) is

\[ P = \left[ \kappa P_N^{1-\phi} + (1 - \kappa) P_T^{1-\phi} \right]^{\frac{1}{1-\phi}} \]  

(2.11)

Further assume that the demand for traded goods is made up of home and foreign traded goods (with subscript of \( H \) and \( F \) respectively) by the same technology with the weight and elasticity of substitution being now \( \gamma \) and \( \theta \):

\[ C_T = \left[ \gamma^\frac{1}{2} C_H^{\theta-1} + (1 - \gamma)^{\frac{1}{2}} C_F^{\theta-1} \right]^{\frac{\theta}{\theta-1}} \]  

(2.12)

\[ I_T = \left[ \gamma^\frac{1}{2} I_H^{\theta-1} + (1 - \gamma)^{\frac{1}{2}} I_F^{\theta-1} \right]^{\frac{\theta}{\theta-1}} \]  

(2.13)

Combining with their foreign counterparts, it follows that the home demands of home and foreign traded goods are respectively:

\[ D_H = \gamma D_T \left[ \frac{P_D}{P_T} \right]^{-\theta} \]  

(2.14)

\[ D_F = (1 - \gamma) D_T \left[ \frac{S^* P_X^*}{P_T} \right]^{-\theta} \]  

(2.15)

and the foreign demands of home and foreign traded goods are respectively:

\[ D_H^* = (1 - \gamma) D_T^* \left[ \frac{S^{-\eta} P_X}{P_T^*} \right]^{-\theta} \]  

(2.16)

\[ D_F^* = \gamma D_T^* \left[ \frac{P_D^*}{P_T^*} \right]^{-\theta} \]  

(2.17)

where \( P_D \) and \( P_X \) are prices of home traded goods for home and foreign buyers. \( P_D^* \) and \( P_X^* \) are prices of foreign traded goods for foreign and home buyers. Note that in Eqs.(2.15) and (2.16), prices of exports \( P_X^* \) and \( P_X \) are converted to local terms if they are not set through local currency pricing (LCP) but rather the producer currency pricing (PCP). The nominal exchange rate \( S \), defined as the price of foreign currency in terms of home currency, is thus involved in the above equations. Note we use a switch parameter of
different pricing strategies $\eta$ here. It takes the value of 1 in the PCP case or 0 in the LCP case.

The price index of the home traded goods is thus

$$P_T = \left[ \gamma P^{1-\theta}_D + (1 - \gamma) \left( S^{\nu} P^*_X \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}$$ (2.18)

The price index of the foreign traded goods $P^*_T$ has a similar expression.

### 2.2 Labour unions

The representative labour union $z$ buys labour from households and sells it to intermediate goods producers. Their problem is to maximize the following profit function

$$E_t \sum_{i=0}^{\infty} \Omega_{t+i} \Theta_{t+i}$$ (2.19)

with subject to

$$\Omega_{t+i} = \beta^i C_{X_{t+i}}^{-\rho}$$ (2.20)

$$L_t(z) = L_t \left( \frac{w_t(z)}{W_t} \right)^{-\xi}$$ (2.21)

$$\Theta_t = L_t(z) \frac{w_t(z)}{P_t} - L_t(z) \frac{w_t}{P_t}$$ (2.22)

We assume that they use the same discount factor as the one used by households, which leads to Eq.(2.20). $w(z)$ and $W$ denote respectively the optimal (nominal) wage which is set by $z$ and the aggregate wage index of labour sold to intermediate goods sector. With a constant elasticity of substitution between different types of labour supply $\xi$, the labour amount sold by the labour union is given by $L_t(z)$ by Eq.(2.21). Using $w_t$ to represent the nominal wage paid by the labour union to households, we obtain the labour union’s period profit function, i.e. Eq.(2.22).

This defines the problem of how $w_t(z)$ is chosen optimally. Moreover, we assume that the process of wage setting suffers from a rigidity friction. Wages adjust infrequently through a Calvo-type contract. Each time only a fraction of all wages $(1 - \zeta)$ can be reset and the rest of wages $\zeta$ are indexed to past inflation automatically with an indexation degree of $\varpi$. 

9
To solve the labour union’s problem, note that the related Lagrangian equation is:

\[ E_t \sum_{i=0}^{\infty} \Omega_{t+i}^i \left\{ \frac{L_{t+i}}{W_{t+i}} \left( \frac{W_{t+i-1}}{W_{t-1}} \right) \right\}^{-\xi} \frac{W_{t+i}}{P_{t+i}} \left( \frac{w_{t+i-1}}{w_{t-1}} \right)^{1-\xi} \]

Equation (2.23)

By the associated first-order condition, the optimal wage rate set at time \( t \) can be obtained as:

\[ w_t(z) = \frac{\xi}{\xi - 1} \frac{E_t \sum_{i=0}^{\infty} \Omega_{t+i}^i L_{t+i} \frac{W_{t+i}^i}{P_{t+i}^i} \left[ \left( \frac{W_{t+i-1}}{W_{t-1}} \right)^{\omega} \right]^{-\xi} W_{t+i}}{E_t \sum_{i=0}^{\infty} \Omega_{t+i}^i L_{t+i} \frac{W_{t+i}^i}{P_{t+i}^i} \left[ \left( \frac{W_{t+i-1}}{W_{t-1}} \right)^{\omega} \right]^{1-\xi}} \]

Equation (2.24)

from which it is clear that the optimal wage is a mark-up over a weighted average of future marginal cost of labour \( w_{t+i} \). The weight is affected by the degree of wage rigidity \( \zeta \) and other variables. The stronger the degree of wage rigidity, i.e. a high \( \zeta \), the less is the importance of the current marginal cost comparing to the future marginal cost. The mark-up factor \( \frac{\xi}{\xi - 1} \) is a function of the elasticity of labour substitution \( \xi \). The lower is the substitution rate \( \xi \), the lower is the market competitiveness and the higher is the mark-up. We introduce a mark-up shock \( V = \frac{\xi}{\xi - 1} \) here and we assume that it follows the process \( \dot{V}_t = \delta V \dot{V}_{t-1} + \varepsilon_V \). When there is a positive realization of \( \varepsilon_V \), there is a negative shock to market power in labour market.

Given the optimal wage \( X_w t = w_t(z) \), by aggregation, the aggregate wage index \( W_t \) is given by:

\[ W_t = \left\{ \zeta \left[ \frac{W_{t-1}}{W_{t-2}} \right]^{\omega} \right\}^{1-\xi} + \left( 1 - \zeta \right) X_w t \]

Equation (2.25)

2.3 Intermediate goods firms

As mentioned before, there are two parallel intermediate goods sectors within each country. In either sector, the firms only supply intermediate goods to final goods firms of the same sector. Except for this difference, the structure of the two intermediate goods sectors is the same. So in this subsection, unless it is necessary, we only discuss the behaviour of the traded sector. The related equations for non-traded sector are similar.

The intermediate firms buy labour and capital and combine them to produce the
intermediate goods. For a representative firm \( z \), its problem is to maximize its profit:

\[
E_t \sum_{i=0}^{\infty} \Omega_{t+i} \Pi_{Mt+i}
\]

with subject to

\[
\Pi_{Mt+i} = \frac{q_t Y_t}{P_t} - \frac{W_t}{P_t} L_t - I_t - \frac{\psi (\varepsilon_t I_t - \bar{T})^2}{2T}
\]

\[
K_{t+1} = I_t + (1 - \delta) K_t
\]

\[
Y_t = A_t K_t^{1-a} L_t^a
\]

The production function is assumed to be of the Cobb-Douglas form, Eq. (2.29). The share of labour \( L \) and capital \( K \) in the output are respectively \( a \) and \( 1 - a \). The factors of technology or efficiency enter the function through variable \( A \). Following the literature (for instance Corsetti et al., 2008 and Devereux et al. 2014), the exogenous state vector of technology \( \hat{A} \equiv [\hat{A}_T \hat{A}_N] \) are assumed to evolve according to

\[
\hat{A}_T = \delta_{TT1} \hat{A}_{T_{t-1}} + \delta_{TT2} \hat{A}_{T_{t-1}}^* + \delta_{TN1} \hat{A}_{N_{t-1}}^* + \delta_{TN2} \hat{A}_{N_{t-1}}^* + \varepsilon_T
\]

\[
\hat{A}_N = \delta_{NT1} \hat{A}_{T_{t-1}}^* + \delta_{NT2} \hat{A}_{T_{t-1}}^* + \delta_{NN1} \hat{A}_{N_{t-1}} - \delta_{NN2} \hat{A}_{N_{t-1}}^* + \varepsilon_N
\]

where \([\varepsilon_T \varepsilon_N]\) are disturbances to technology.

Eq. (2.28) is the standard capital accumulation equation. Capital at the end of time \( t \), \( K_{t+1} \), equals the sum of the investment this period, \( I_t \), and the depreciation-adjusted capital stock, \( (1 - \delta) K_t \). The capital depreciation rate is \( \delta \).

Eq. (2.27) gives the profit function for the intermediate goods firm. \( q \) is the price of intermediate goods. The first term of the equation represents the income by selling the goods. The second and third terms on the right hand side of the equation represent the cost of the labour and capital inputs respectively. We assume a cost of investment adjustment, i.e. \( \frac{\psi (\varepsilon_t I_t - \bar{T})^2}{2T} \). The cost function is set to be a quadratic form mainly out of tractability. Moreover, it also implies that both accumulation and decumulation of capital will incur adjustment cost and the cost is marginally increasing. The parameter \( \psi \) is used to govern the degree of the friction. We assume there is a shock variable \( \varepsilon_t \) that affects investment-adjustment cost which follows the process of \( \hat{\varepsilon}_t = \delta_{\varepsilon} \hat{\varepsilon}_{t-1} + \varepsilon_{\varepsilon} \).
The choice variables for intermediate goods firm are labour demand $L$, investment $I$ and capital stock $K$. The associated first-order conditions are:

$$MPL_t = \frac{W_t}{q_t} \quad (2.32)$$

$$\Psi_t = \left[ 1 + \frac{\psi (\varepsilon_t I_t - I^*)}{I^*} \right] \quad (2.33)$$

$$\Omega_t \Psi_t = E_t \Omega_{t+1} \left[ \frac{q_{t+1}}{P_{t+1}} MPK_{t+1} + (1 - \delta) \Psi_{t+1} \right] = E_t \Omega_{t+1} R_{Kt+1} \quad (2.34)$$

The optimal $L$ is determined by Eq.(2.32). The condition states that at the optimum the marginal product of labour should be equal to the real wage, which should be familiar. Eq.(2.33) is a type of Tobin’s $Q$ equation where the price of the investment goods is set to be the same as the price of the final goods which is normalized to 1. The $\Psi$ on the left hand side of the equation is the multiplier associated with the constraint of Eq.(2.28). It also stands for the marginal product of investment. In equilibrium, it should be equal to the marginal cost of investment on the right hand side. This equation ties down the optimal investment $I_t$. Eq.(2.34) determines the optimal capital stock $K_t$. It balances the intertemporal use of capital. Existing capital can either be used today or be invested as capital tomorrow. At optimum, there should be no difference between the marginal benefits of the two different uses.

### 2.4 Final goods firms

The final goods sector is also divided into traded and non-traded sectors. As before, in this subsection, we only consider the traded sector. The equations for the non-traded sector are similar. In addition, because the firms in the traded sector have to set the price for exports, this again involves different pricing strategies, i.e. whether $PCP$ or $LCP$ is adopted. In what follows, as before, this is represented by the cases of $\eta = 1$ for $PCP$ and $\eta = 0$ for $LCP$.

The structure of the problem of the final goods sector is similar to that of the labour unions. The firms buy intermediate goods from the intermediate goods sector, transform them into final goods and sell the goods to domestic and foreign buyers. The goods have some degree of heterogeneity so firms have power to set prices. However, the prices cannot change every period. The change is subject to a Calvo-type price rigidity.
A representative firm $z$ chooses $p_{Dt}(z)$ and $p_{Xt}(z)$ to maximize the profit function:

$$
E_t \sum_{i=0}^{\infty} \Omega_{t+i} \Pi_{F_{t+i}}
$$

subject to

$$
\Pi_{F_t} = y_{1t}(z) \frac{p_{Dt}(z)}{P_{Dt}} \frac{P_{Dt}}{P_t} + y_{2t}(z) \frac{p_{Xt}(z)}{P_{Xt}} \frac{S_{t+i}^{1-q}P_{Xt}}{P_t} - y_{1t}(z) \frac{q_{Dt}}{P_t} - y_{2t}(z) \frac{q_{Dt}}{P_t}
$$

$$
y_{1t}(z) = D_t \left[ \frac{p_{Dt}(z)}{P_{Dt}} \right]^{-\varphi}
$$

$$
y_{2t}(z) = D_t^* \left[ \frac{p_{Xt}(z)}{P_{Xt}} \right]^{-\varphi}
$$

$p_{Dt}(z)$ and $p_{Xt}(z)$ are the prices of home traded goods for home and foreign buyers respectively. With the assumptions of a constant elasticity of substitution $\varphi$, the demand for $z$’s goods from home and foreign countries $y_{1t}(z)$ and $y_{2t}(z)$ are given by Eqs. (2.37) and (2.38). So the first two terms on the right hand side of Eq. (2.36) are the related income by selling final goods while the last two terms are the costs of buying intermediate goods. By taking the difference of the two, Eq. (2.36) gives the profit of firm $z$ at period $t$.

We assume that the degree of price rigidity and price indexation are given by $\lambda$ and $\omega$ respectively, the related Lagrangian equation of the final goods firm’s problem can be set up following the same logic as in Eq. (2.23). The associated first-order conditions lead to the optimal $p_{Dt}(z)$

$$
p_{Dt}(z) = \frac{\varphi}{1-\varphi} \frac{E_t \sum_{i=0}^{\infty} \Omega_{t+i} \lambda^i D_{t+i}^{Dt+i} P_{Dt}^\varphi }{E_t \sum_{i=0}^{\infty} \Omega_{t+i} \lambda^i D_{t+i}^{Dt+i} P_{Dt}^\varphi } \left[ \left( \frac{P_{Dt+i-1}}{P_{Dt-1}} \right)^\omega \right]^{-\varphi} q_{Dt+i}
$$

and the optimal $p_{Xt}(z)$

$$
p_{Xt}(z) = \frac{\varphi}{1-\varphi} \frac{E_t \sum_{i=0}^{\infty} \Omega_{t+i} \lambda^i D_{t+i}^{Xt+i} P_{Xt}^\varphi }{E_t \sum_{i=0}^{\infty} \Omega_{t+i} \lambda^i D_{t+i}^{Xt+i} P_{Xt}^\varphi } \left[ \left( \frac{P_{Xt+i-1}}{P_{Xt-1}} \right)^\omega \right]^{-\varphi} q_{Xt+i} S_{t+i}^{1-q}
$$

As before, the optimal prices under the nominal rigidity are markups over weighted average of the current and future marginal costs $q_{Dt+i}$ and $q_{Xt+i}$. The weight over time is
affected by how serious is the price rigidity, i.e. \( \lambda \). And the markup is mainly controlled by the degree of the market competitiveness i.e. \( \varphi \). As in the case of the labour union, we assume \( V = \frac{\varphi}{1 - \varphi} \) is a price markup shock and assume that it follows the process of \( \dot{V}_t = \delta_V \dot{V}_{t-1} + \varepsilon_V \).

### 2.5 Government

The government implements both fiscal and monetary policies. The fiscal policy is assumed to be aimed at a balanced budget. So we have the following rule

\[
P_{Gt} G_t = P_t T_t \quad (2.41)
\]

As for the scale of government, we assume that the total expenditure of government in the steady state amounts to a fixed proportion of the total output in steady state. Parameter \( g \) governs the ratio:

\[
G = gY \quad (2.42)
\]

where for \( G \) and \( Y \) the time subscript \( t \) is dropped to indicate a steady state value of them.

Government spending is assumed to be subject to a fiscal policy shock:

\[
\dot{G}_t = \delta_G \dot{G}_{t-1} + \varepsilon_{Gt} \quad (2.43)
\]

We further assume that the government buys both traded and non-traded goods. And the shares are consistent with that of private spending, i.e. a constant proportion of the total expenditure \( \kappa \) goes to non-traded goods and the remaining proportion \( 1 - \kappa \) goes to traded goods. We assume that the government only buys domestic traded goods. So we have:

\[
G_{Nt} = \kappa G_t \quad (2.44)
\]
\[
G_{Ht} = (1 - \kappa) G_t \quad (2.45)
\]

Monetary policy follows a standard Taylor-type rule. By assumption, the deviation of the chosen interest rate from its steady state can be broken down into terms of interest rate smoothing, inflation feedback, output gap feedback and monetary shock respectively. In particular, the rule takes the form:

\[
\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\delta_R} \left[ \left( \frac{\pi_t}{\pi} \right)^{\delta_\pi} \left( \frac{Y_t}{Y} \right)^{\delta_Y} \right]^{1 - \delta_R} \left( \frac{rr_t}{rr} \right) \quad (2.46)
\]
where \( R_t \) denotes the deviation of the interest rate from its steady state. \( \delta_R \) is the degree of interest rate smoothing. \( \delta_x \) and \( \delta_y \) are respectively feedback parameters of inflation and output gap. And \( rr \) stands for a monetary shock which follows the process of \( \hat{r}r_t = \delta_{rr}\hat{r}r_{t-1} + \epsilon_{rr} \).

### 2.6 Financial markets

In this subsection, let us define the rate of return for the assets available in the international financial market. As mentioned, both countries can issue equities and nominal bond. For home and foreign equities, we assume that they represent claims on the profit made by the firms in the issuer country. The gross (real) rate of return for home and foreign equities are thus given by:

\[
\begin{align*}
    r_{1t} &= \frac{\Pi_t + Z_{1t}}{Z_{1t-1}} \quad (2.47) \\
    r_{2t} &= \frac{\Pi_t^* \cdot Q_t + Z_{2t}}{Z_{2t-1}} \quad (2.48)
\end{align*}
\]

where \( \Pi_t = \Pi_{Mt} + \Pi_{Ft} + \Theta_t \) and \( \Pi_t^* = \Pi_{Mt}^* + \Pi_{Ft}^* + \Theta_t^* \) are the total profits of firms, i.e. the profits belonging to intermediate and final goods firms of both traded and non-traded goods sectors plus labour unions, in the two countries. \( Z_{1t} \) and \( Z_{2t} \) are the real prices of home and foreign equities. \( Q_t = (S_t \cdot P_t^*) / P_t \) in Eq.(2.48) is the real exchange rate representing the price of foreign consumption basket in terms of home consumption basket. The rate of return of the foreign equity \( r_{2t} \) is defined in terms of home basket and is comparable to \( r_{1t} \).

For the home and foreign bonds, we assume that they represent claims on one unit of currency per period in the issuer country. The gross (real) rates of return for them are thus given by:

\[
\begin{align*}
    r_{3t} &= \frac{1/P_t + Z_{3t}}{Z_{1t-1}} \quad (2.49) \\
    r_{4t} &= \frac{(1/P_t^*) \cdot Q_t + Z_{4t}}{Z_{4t-1}} \quad (2.50)
\end{align*}
\]

where \( 1/P_t \) and \( (1/P_t^*) \cdot Q \) denote real payoffs of one unit of home and foreign bonds. Again, \( Q \) is used to convert the foreign payoff into terms of the home consumption basket.
2.7 Market clearing

In equilibrium, all markets should clear. These include market clearing in the goods market, the labour market and asset markets.

In the goods market, for the non-traded sector, we should have

$$ Y_{Nt} = D_{Nt} + \kappa G_t $$

(2.50)

where $D_N$ is the private demand for home non-tradables and $\kappa G$ is the public spending on them. Note that as explained there is no demand coming from the foreign country for home non-tradables.

For the traded sector, we have

$$ Y_{Tt} = D_{Ht} + D_{Ht}^* + (1 - \kappa) G_t $$

(2.51)

where $D_{H1}$ and $D_{H2}$ are the private demands for home tradables from the home and foreign countries, whose formulae are given by Eqs.(2.14) and (2.16), and $(1 - \kappa) G$ is the public spending on home tradables.

Aggregating the goods demands across sectors leads to the total demand for goods

$$ Y_t = Y_{Nt} + Y_{Tt} $$

(2.52)

In the labour market, the total labour supply $L$ is made up of that of traded sectors $L_T$ and that of non-traded sectors $L_N$

$$ L_t = L_{Tt} + L_{Nt} $$

(2.53)

In the foreign country, these conditions are similar.

In asset markets, all assets are in net supply of zero, so

$$ \alpha_{it} + \alpha_{it}^* = 0 $$

(2.54)

for $i = 1, 2, 3, 4$. Note that $i$ is an index of assets and the $\alpha$s with asterisk are foreign holdings. By the market clearing conditions of assets, once (steady-state) asset holdings of home country are obtained, those of foreign country are simply $\alpha_i^* = -\alpha_i$. So in what follows, we only focus on the solutions of home portfolio choices, i.e. the $\alpha_i$s.
3 Optimal portfolios in the general model

After specifying the details of the model, in this section, we are ready to discuss the determination of the optimal portfolios, i.e. the $\alpha_i$s. We first derive the optimality condition that can be used to tie down the $\alpha_i$s from the Euler equations. It turns out that the $\alpha_i$s are determined by first-order behaviour of the cross-country consumption differential and asset excess returns. We approximate the budget constraints of the two countries and apply them to the optimality condition to yield $\alpha_i$s as variance-covariance ratios. The ‘correlation’ and ‘variability’ effects are defined and derived following the literature, which provide useful hints about the way of how the optimal portfolios are structured.

3.1 Optimality condition

As noted in the previous section, the optimal portfolio choices are determined by equation set (2.6) and its foreign counterpart. In the home country, Eq.(2.6) gives us the following three restrictions that need to be satisfied:

$$E[C_{Xt+1}^\rho r_{it+1}] = E[C_{Xt+1}^\rho r_{it+1}^4]$$

for $i = 1, 2, 3$. Following Devereux and Sutherland (2011) (and also Tille and Wincoop, 2010), to obtain the zero-order $\alpha_i$s, at least second-order approximations of the portfolio conditions are required. So we approximate the above conditions in a standard way up to second-order accuracy. Combined with the foreign approximated conditions, we can arrive at the following covariance condition

$$E\left[(\hat{C}_{Xt+1} - \hat{C}_{Xt+1}^\ast - \hat{Q}_{t+1}/\rho)\hat{r}_{ixt+1}\right] = 0 + O(\epsilon^3)$$

(3.2)

where, except for $\hat{r}_{ixt+1}$ which is defined as $(\hat{r}_{it+1} - \hat{r}_{4t+1})$, all other variables with hats represent log deviations from their steady states. For example, $\hat{C}_{Xt+1} = \log[(C_{Xt+1} - C_X)/C_X]$ where $C_X$ is steady-state $C_{Xt}$. $\hat{C}_{Xt+1}^\ast$ and $\hat{Q}_{t+1}$ are defined similarly.

Eq.(3.2) can serve as the condition to tie down the $\alpha_i$s for $i = 1, 2, 3$. Note by this equation, the $\alpha_i$s are determined by two first-order behaviours. There are $\hat{C}_{Xt+1}^D = \left(\hat{C}_{Xt+1} - \hat{C}_{Xt+1}^\ast - \hat{Q}_{t+1}/\rho\right)$, which is referred to as the cross-country consumption differential (with habit formation), and $\hat{r}_{xt+1}$, which is referred to as the excess returns of
asset \( i \) over asset 4 which is the numeraire asset in the model. At the optimum, the \( \alpha_i \)s are chosen so that the covariance between the two is zero, or the two are orthogonal, which indicates the optimal portfolios as hedging vehicles smoothing relative consumption fluctuations through generating relative asset returns.

Once \( \alpha_1 \) to \( \alpha_3 \) are derived from Eq.(3.2), \( \alpha_4 \) can be obtained by the fact of \( \alpha_4 = F - (\alpha_1 + \alpha_2 + \alpha_3) \) where \( F \) is steady-state NFA in the home country. Because in this paper, we assume that the steady state autarky interest rates are equalized across countries \( r = r^* = \frac{1}{\beta} \). There is no reason for capital flows to particular country in net terms. Steady state net foreign assets in equilibrium is thus zero, i.e. \( F = 0 \).

### 3.2 Approximating budget constraints

Obviously, \( \hat{C}^D_{X_t} \) is endogenous and it depends on the optimal portfolio \( \alpha_i \)s in the model. Most basically, consumptions link to portfolios through budget constraints. By writing out the links between them, we can establish expressions of portfolios explicitly instead of implicitly as in Eq.(3.2). This procedure is usually very useful in providing us with intuitions on which kind of motive drives the emergence of the observed portfolios, i.e. the motive to hedge away certain income risks. In this subsection, we obtain the links by approximating the budget constraints of countries. In the next subsection, we derive the portfolios as a variance-covariance ratio representing them explicitly.

Let us start with the home budget constraint, Eq.(2.3), which can be rewritten as

\[
F_t = \alpha'_{t-1} r_{xt} + r_4 F_{t-1} + Y_{ct} - C_t
\]

(3.3)

where we define portfolio vector \( \alpha'_{t-1} = [\alpha_{1t-1} \alpha_{2t-1} \alpha_{3t-1}] \), excess vector \( r_{xt} = [r_{1xt} r_{2xt} r_{3xt}]' \) and disposable income \( Y_{ct} = \frac{w}{P_t} L_t + \Pi_t + \Theta_t - T_t \).

First-order approximating the equation around the steady-state yields

\[
\hat{C}_t = \hat{Y}_{ct} + \frac{1}{c} \hat{\alpha'} \hat{r}_{xt} + \frac{1}{c} \frac{1}{\beta} \hat{F}_{t-1} - \frac{1}{c} \hat{F}_t
\]

(3.4)

where \( \hat{Y}_{ct} = \log[(Y_{ct} - Y_c) / Y_c] \) and \( \hat{C}_t = \log[(C_t - C) / C] \). Because in steady state, \( F = 0 \), \( \hat{F}_t \) is defined here as deviation of \( F_t \) from its steady state (of zero) as a percentage of equilibrium income \( Y \) instead of \( F \), i.e. \( \hat{F}_t = \log[F_t / Y] \). Besides, we define \( \hat{\alpha'} = \frac{1}{\beta Y} \alpha' \) and \( \hat{r}_{xt} = [r_{1xt} r_{2xt} r_{3xt}]' \). \( c \) is the steady-state ratio of consumption to income \( c = C / Y \).
The budget constraint in the foreign country is

\[
\frac{F_t^*}{Q_t} = \frac{1}{Q_t} \left( \alpha_{t-1}^* r_{xt} + r_4 F_{t-1}^* \right) + Y_{ct}^* - C_t^* \tag{3.5}
\]

Note that exchange rate appears in the constraint because all asset returns are in terms of the home consumption basket while foreign consumption and disposable income are in terms of foreign consumption basket.

Similarly, approximating this constraint yields

\[
\hat{C}_t^* = \hat{Y}_{ct}^* + \frac{1}{c^t Q} \hat{\alpha}^* \hat{r}_{xt} + \frac{1}{cQ \beta} \hat{F}_{t-1}^* - \frac{1}{cQ} \hat{F}_t^* \tag{3.6}
\]

where variables are defined analogously.

Notice that in a two-country world we have \( F_t^* = -F_t \) so

\[
\hat{F}_t^* = -\frac{Y}{Y^*} \hat{F}_t
\]

By the conditions of asset market clearing, we have

\[
\hat{\alpha}_t^* = \frac{\alpha^*}{\beta Y^*} = \frac{-\alpha Y}{\beta Y^*} = -\frac{Y}{Y^*} \hat{\alpha}
\]

Making use of these facts, we can rewrite Eq.(3.6) as

\[
\hat{C}_t^* = \hat{Y}_{ct}^* - \frac{Y}{Y^* c^t Q} \hat{\alpha}^* \hat{r}_{xt} - \frac{Y}{Y^* c^t Q \beta} \hat{F}_{t-1} + \frac{Y}{Y^* c^t Q} \hat{F}_t \tag{3.7}
\]

### 3.3 Variance-covariance representation of portfolios

In this subsection, we represent \( \hat{\alpha} \) as a variance-covariance ratio. For convenience, approximated home and foreign budget constraints that were obtained above are put together as follows

\[
\hat{C}_t = \hat{Y}_{ct} + \frac{1}{c} \hat{\alpha} \hat{r}_{xt} + \frac{1}{c \beta} \hat{F}_{t-1} - \frac{1}{c} \hat{F}_t
\]

\[
\hat{C}_t^* = \hat{Y}_{ct}^* - \frac{Y}{Y^* c^t Q} \hat{\alpha}^* \hat{r}_{xt} - \frac{Y}{Y^* c^t Q \beta} \hat{F}_{t-1} + \frac{Y}{Y^* c^t Q} \hat{F}_t
\]

According to Eq.(2.2), i.e. \( C_{Xt+1} = C_{t+1} - hC_t \), we have

\[
(1 - h) \hat{C}_{Xt+1} = \hat{C}_{t+1} - h \hat{C}_t
\]
which can be used to rewrite $\hat{C}^D_{t+1}$ as

$$\hat{C}^D_{t+1} = \frac{1}{1 - h} (\hat{C}_{t+1} - h\hat{C}_t) - \frac{1}{1 - h^*} (\hat{C}^*_t - h\hat{C}^*_t) - \frac{1}{\rho} \hat{Q}_{t+1}$$

With the expressions of consumption behaviours above, it follows that

$$\sum_{i=0}^{\infty} \beta^i \hat{C}^D_{t+1+i} = \sum_{i=0}^{\infty} \beta^i \left[ \frac{1}{1 - h} (\hat{C}_{t+1+i} - h\hat{C}_{t+i}) - \frac{1}{1 - h^*} (\hat{C}^*_{t+1+i} - h\hat{C}^*_{t+i}) \right] - \frac{1}{\rho} \hat{Q}_{t+1+i}$$

$$= \sum_{i=0}^{\infty} \beta^i \left[ \frac{1}{1 - h} (\hat{Y}_{t+1+i} - h\hat{Y}_{t+i}) - \frac{1}{1 - h^*} (\hat{Y}^*_{t+1+i} - h\hat{Y}^*_{t+i}) \right] + \sum_{i=0}^{\infty} \beta^i \left[ \tau_1 \cdot 2\hat{r}_{t+1+i} - \tau_2 \cdot 2\hat{r}_{t+i} - \frac{1}{\rho} \hat{Q}_{t+i} \right] + t.i.$$ 

where

$$\tau_1 = \left[ \frac{1}{1 - h} \frac{1}{2c} + \frac{1}{1 - h^*} \frac{Y}{2c^*Q} \right]$$

$$\tau_2 = \left[ \frac{h}{1 - h} \frac{1}{2c} + \frac{h^*}{1 - h^*} \frac{Y}{2c^*Q} \right]$$

and $t.i.$ denotes terms of irrelevance (whose covariance with $\hat{r}_{t+1}$ is 0). The summation is equivalent to

$$\sum_{i=0}^{\infty} \beta^i \hat{C}^D_{t+1+i} = \frac{1}{1 - \beta} \hat{C}^D_{t+1}$$

$$= \sum_{i=0}^{\infty} \beta^i \left[ \frac{1 - \beta h}{1 - h} \hat{Y}_{t+1+i} - \frac{1 - \beta h^*}{1 - h^*} \hat{Y}^*_{t+1+i} \right] + \sum_{i=0}^{\infty} \beta^i \left[ \tau_1 \cdot 2\hat{r}_{t+1+i} - \tau_2 \cdot 2\hat{r}_{t+i} - \frac{1}{\rho} \hat{Q}_{t+i} \right] + t.i.$$ 

or

$$\hat{C}^D_{t+1} = (1 - \beta) (\Gamma_{yt+1} + \tau \cdot 2\hat{r}_{t+1} + t.i.)$$

(3.8)

where

$$\Gamma_{yt+1} = \sum_{i=0}^{\infty} \beta^i \left[ \frac{1 - \beta h}{1 - h} \hat{Y}_{t+1+i} - \frac{1 - \beta h^*}{1 - h^*} \hat{Y}^*_{t+1+i} - \frac{1}{\rho} \hat{Q}_{t+i} \right]$$

(3.9)

denotes the sum of discounted expected fluctuations in relative disposable incomes and

$$\tau = \tau_1 - \beta \tau_2$$

(3.10)

denotes a wedge whose value depends on the severity of the habit friction and the degree of country differences in the general model.
Putting Eq. (3.8) back into Eq. (3.2) leads to
\[ E_t \{ \hat{r}_{xt+1} (\Gamma_{yt+1} + \tau \cdot 2\tilde{\alpha}\hat{r}_{xt+1}) \} = 0 \]
or
\[ \tilde{\alpha}_i = -\frac{1}{2\tau} \frac{cov (\zeta_{yt+1}, \hat{r}_{xt+1})}{var (\hat{r}_{xt+1})} \]  
(3.11)
where \( \tilde{\alpha}_i \) for \( i = 1, 2, 3 \) is element of \( \tilde{\alpha} \). \( \zeta_{yt+1} = \Gamma_{yt+1} - E_t \Gamma_{yt+1} \) is the sum of discounted expected innovations in relative disposable incomes while \( \hat{r}_{xt+1} = \hat{r}_{xt+1} - E_t \hat{r}_{xt+1} \) is the innovations in excess return of assets\(^1\). Eq. (3.11) states that the optimal portfolios \( \tilde{\alpha} \) depends on how the innovations in discounted expected relative disposable incomes co-vary with that in excess return of assets. The equation coincides with Eq. (24) of Devereux et al. (2014) if we ignore the presence of \( \tau \).\(^2\) While in Devereux et al. (2014), \( \tau \) collapses (into \( 1/C \)) because the two countries are entirely symmetric and they do not consider the situation where households form habits, in the current model we are interested in the portfolio choices in an asymmetric world. And to consider possible asymmetry in habit persistence between developing and developed countries and its effects on portfolio choices, habit formation is taken into account. So \( \tau \) emerges as one measure of how \( \tilde{\alpha} \) differs in the asymmetric model from that in a symmetric model. While \( \tau \) has a multiplicative effect on the size of portfolio holdings, the fundamental force underlying the determination of \( \tilde{\alpha} \) is essentially the same as that in the symmetric model, i.e. households’ motive to hedge against those risks that disturb their desired smooth schedule of relative consumption. Eq. (3.11) makes sense given that relative consumption is always supported by relative disposable income.

### 3.4 Correlation and variability effects

We now define and derive the ‘correlation’ and ‘variability’ effects. These effects provide a useful decomposition of the portfolio expressions which will be used in the analysis

\(^1\)Note that \( E_t \hat{r}_{xt+1} = 0 \) is derived from the first-order approximation of Eq. (3.1). Both Devereux and Sutherland (2011) and Tille and Wincoop (2010) also share this property. Later on in Eq. (3.27) of next chapter, we show in more detail how this can be the case in a similar context.

\(^2\)Except that \( \zeta_{yt+1} \) is also defined in a slightly different way. Specifically, in their paper, \( \zeta_{yt+1} \) is multiplied by steady-state consumption \( C \) which is equalized across countries in their model. The degree of asymmetry in the model of this chapter is instead reflected in \( \tau \) here.
reported below. By Eq. (3.11), \( \tau \) is the same across \( \hat{\alpha}_i \)'s, i.e. elements in \( \hat{\alpha} \). If there are differences among the \( \hat{\alpha}_i \)'s they must come from the differences among the variance-covariance ratios. The correlation and variability effects will provide some clues about the causes of these differences across assets.

Note that the \( \hat{\alpha}_i \)'s in Eq. (3.11) can be re-written as

\[
\hat{\alpha}_1 = -\frac{1}{2\tau} \text{corr} \left( \zeta_{yt+1}, \hat{r}_{1xt+1}, \hat{r}_{2xt+1} \right) \frac{\text{STD} \left( \zeta_{yt+1} \mid \hat{r}_{2xt+1}, \hat{r}_{3xt+1} \right)}{\text{STD} \left( \hat{r}_{1xt+1} \mid \hat{r}_{2xt+1}, \hat{r}_{3xt+1} \right)} \tag{3.12}
\]

\[
\hat{\alpha}_2 = -\frac{1}{2\tau} \text{corr} \left( \zeta_{yt+1}, \hat{r}_{2xt+1}, \hat{r}_{1xt+1} \right) \frac{\text{STD} \left( \zeta_{yt+1} \mid \hat{r}_{1xt+1}, \hat{r}_{3xt+1} \right)}{\text{STD} \left( \hat{r}_{2xt+1} \mid \hat{r}_{1xt+1}, \hat{r}_{3xt+1} \right)} \tag{3.13}
\]

\[
\hat{\alpha}_3 = -\frac{1}{2\tau} \text{corr} \left( \zeta_{yt+1}, \hat{r}_{3xt+1}, \hat{r}_{1xt+1} \right) \frac{\text{STD} \left( \zeta_{yt+1} \mid \hat{r}_{1xt+1}, \hat{r}_{2xt+1} \right)}{\text{STD} \left( \hat{r}_{3xt+1} \mid \hat{r}_{1xt+1}, \hat{r}_{2xt+1} \right)} \tag{3.14}
\]

According to above formulae, the signs of asset holdings are determined by the correlation between relative disposable income and the excess return of the asset conditional on the excess returns of other assets, i.e. \( \text{corr} \left( \zeta_{yt+1}, \hat{r}_{ixxt+1} \mid \hat{r}_{-ixxt+1} \right) \) where to ease notation we define \( \hat{r}_{-ixxt+1} \) as a vector consisting of all elements of \( \hat{r}_{ixxt+1} \) except for \( \hat{r}_{ixxt+1} \). In other words, the short or long positions of asset holdings depend on the (conditional) hedging properties of related assets. Suppose for asset \( i \), given the presence of the other assets, its excess return co-moves negatively with the relative disposable income, so \( \text{corr} \left( \zeta_{yt+1}, \hat{r}_{ixxt+1} \mid \hat{r}_{-ixxt+1} \right) < 0 \). This means after a shock, households’ relative income moves in one direction while the asset yields returns that move in the offsetting direction. The asset is able to stabilize households’ relative consumption. In this sense the asset is deemed as a good hedge and will be held in long position. Otherwise, if its excess return co-move positively with the relative incomes \( \text{corr} \left( \zeta_{yt+1}, \hat{r}_{ixxt+1} \mid \hat{r}_{-ixxt+1} \right) > 0 \), holding the asset would exaggerate the effects of the risks. This means that in order to provide a good hedge the asset will be held in a short position by households.

Coming back to our model, \( \hat{\alpha}_1 \) and \( \hat{\alpha}_3 \) are gross holdings of home assets which are supplied by the home country by default, so they are expected to be negative. That is to say, the two associated correlations are expected to be positive. \( \hat{\alpha}_2 \) and \( \hat{\alpha}_4 \) are gross (and also net) holdings of foreign assets, so they are expected to be positive. That is to say, the two associated correlations are expected to be negative. (\( \hat{\alpha}_4 \)'s expression can be obtained if another asset, say asset 2, is chosen as the numeraire asset. The representation
is analogous. Note that the choice of numeraire asset does not matter in the sense that they all yield the same portfolio solutions $\tilde{\alpha}$.)

The size of asset holdings are determined by both $corr(\zeta_{y,t+1}, \tilde{r}_{ix,t+1}| \tilde{r}_{-ix,t+1})$ and the ratio of $\frac{StD(\zeta_{y,t+1}| \tilde{r}_{-ix,t+1})}{StD(\tilde{r}_{ix,t+1}| \tilde{r}_{-ix,t+1})}$. Following the literature, from now on, we refer them respectively as the ‘correlation’ and ‘variability’ effects. The two effects have very intuitive interpretations when it comes to affecting the size of $\tilde{\alpha}_i$s.

The size of the $\tilde{\alpha}_i$s positively depend on the correlation effect. This is because the higher is the conditional correlation (in absolute value), the closer is the co-movement between the relative disposable income and excess return, the more significant is the role of asset in serving as a good hedge against risks. So the households desire to hold a more substantial amount of it, positively or negatively. The effect can be thought of as a quality effect, i.e. the assets which are more efficient in hedging (or exaggerating) risks will be bought (or sold) more. The correlation effect measures how relevant are the assets. The more relevant they are in risk-hedging, the more important they are in portfolios.

The size of the $\tilde{\alpha}_i$s also depends positively on the variability effect as well. Note that the latter is the ratio of the conditional standard deviation of relative disposable income to that of excess return. It tells us how much the volatility of the relative disposable income is relative to that of the excess return. While the former volatility provides us with a measure of total amount of risks to be hedged against, the latter provides a measure of the amount of hedging that is made available by holding one unit of certain asset. A higher value of the ratio implies that more units of the asset is required. So the effect can be thought of as a quantity effect, i.e. more income volatility requires more units of hedging.

In the case of two-way capital flows, the developing country imports equities while exports bonds in net terms. If we define the net holding of equities and bonds as, respectively, $\tilde{\alpha}_E = \tilde{\alpha}_1 + \tilde{\alpha}_2$ and $\tilde{\alpha}_B = \tilde{\alpha}_3 + \tilde{\alpha}_4$, then two-way capital flows implies $\tilde{\alpha}_E < 0$ and $\tilde{\alpha}_B > 0$. Because $\tilde{\alpha}_1$ and $\tilde{\alpha}_3$ have negative signs, so they are equivalent to the pattern of $|\tilde{\alpha}_1| > |\tilde{\alpha}_2|$ and $|\tilde{\alpha}_3| < |\tilde{\alpha}_4|$ in optimal portfolios, i.e. the size of $\tilde{\alpha}_1$ is larger than that of $\tilde{\alpha}_2$ while the size of $\tilde{\alpha}_3$ is less than that of $\tilde{\alpha}_4$. Applying the above analysis, we know that this pattern can be the result of a certain combination of correlation and variability effects. As a central analysis of this paper, in Section 5 we will assess the effect of various asymmetries between countries in generating the two-way capital flows. The correlation
and variability effects we define here will provide useful devices in order to understand the findings there.

To end this section, we have to remind that neither Eq (3.11) nor Eqs. (3.12 – 14) are full reduced forms because as the determinants of $\hat{\alpha}$ in Eq. (3.2), the second moments in these formulae are also in themselves depending on $\hat{\alpha}$. In other words, both Eq. (3.2) and Eqs. (3.11 – 14) indicates $\hat{\alpha}$ as a fixed point except that the former defines it implicitly while the latter explicitly and thus provide intuitions for the results. To sum up, we apply Devereux and Sutherland (2011)’s method to Eq. (3.2) to obtain $\hat{\alpha}$ and make use of this $\hat{\alpha}$ and Eqs. (3.12) to decompose portfolios into correlation and variability effects. In the sections below, we analyse the model numerically.

4 Model simulation: Symmetric case

We will compute the numerical solution of equilibrium portfolios by simulating the model. As a benchmark, the two countries are firstly calibrated symmetrically in this section. We choose parameter values at their standard levels of calibration in the literature which are basically descriptions of advanced economies or/and from the estimates that are based on U.S. data. So we will see what the portfolios will look like without country asymmetry. In the next section, we will take into account the existence of a developing country by considering asymmetric simulations.

4.1 Parameterization

The frequency is assumed to be quarterly which is consistent with the literature on business cycles. The discount factor $\beta$ is set at 0.99 which implies an annual interest rate of 4 percent. The elasticity of substitution between home and foreign traded goods is set at $\theta = 1.5$ which conforms to that of Backus et al. (1994). As for the values of the share of home traded goods in traded consumption basket $\gamma$, the share of nontraded goods in the total consumption basket $\kappa$ and the elasticity of substitution between traded and nontraded goods $\phi$, we choose them based on an average of values used in Benigno and Thoenissen (2008), Corsetti et al. (2008) and Stockman and Tesar (1995). The elasticity of substitution among individual final goods is set at 10 which implies an approximate 10 percent price mark-up over marginal cost.
For the production technology, the labour share of income $a$ is calibrated to approximately $2/3$ which is common in the literature and consistent with U.S. data. Based on the same grounds, the share of government spending in total expenditure $g$ is assumed to be 0.18. The depreciation rate of capital $\delta$ is set at 0.025 implying an annual depreciation rate of capital of 10%. The coefficient of investment adjustment cost $\psi$ is chosen as 0.25 so that the variance of total investment is approximately 3 times the variance of GDP which is consistent with U.S. data.

The values for the remaining parameters come from the median estimates by Smets and Wouters (2007) based on the data of the U.S. economy. These parameters include those related to preference (such as risk aversion $\rho$, labour supply elasticity $\mu$ and habit persistence $h$), Calvo price-setting, the monetary policy rule and structural shocks. Note by the parameter values, the U.S. households feature a persistent habit formation with $h = 0.7$. The price and wage adjust infrequently and the average duration of a price is about 3 quarters, $\lambda = 0.66$ and $\varsigma = 0.7$. In addition, the price and wage index to previous levels to some degree and the degree of wage indexation is higher than that of price, $\omega = 0.24$ while $\bar{\omega} = 0.58$. The interest rate is highly persistent with a persistence of $\delta_R = 0.81$. The related feedback coefficients of monetary policy with regard to inflation and output gap are respectively 2 and 0.1. The table 2.1 lists all values of parameters used in the benchmark calibration.

### 4.2 Symmetric case: Benchmark

Table 2.2 reports the result for equilibrium portfolios (divided by $\beta Y$) under the benchmark calibration. The home households’ holdings of home and foreign equity are $-2.2985$ and $2.2985$ (times of steady-state income) while their holdings of home and foreign bonds are $-0.7756$ and $0.7756$ (times of steady-state income). The home demands of home assets $\tilde{a}_1$ and $\tilde{a}_3$ are negative reflecting the fact that the home country is net supplier of home assets. The home demands of foreign assets $\tilde{a}_2$ and $\tilde{a}_4$ are positive reflecting the fact that the home country is net demander of foreign assets. The home net holdings of equities and bonds are both equal to zero $\tilde{a}_E = \tilde{a}_B = 0$ because the two countries are the same. In the light of portfolio decomposition, the symmetry of the countries implies
<table>
<thead>
<tr>
<th>Description</th>
<th>Variable values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calvo price rigidity parameter</td>
<td>$\lambda = 0.66$</td>
</tr>
<tr>
<td>Calvo wage rigidity parameter</td>
<td>$\zeta = 0.70$</td>
</tr>
<tr>
<td>Price indexation</td>
<td>$\omega = 0.24$</td>
</tr>
<tr>
<td>Wage indexation</td>
<td>$\varpi = 0.58$</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta = 0.99$</td>
</tr>
<tr>
<td>Habit persistence</td>
<td>$h = 0.70$</td>
</tr>
<tr>
<td>Risk aversion coefficient</td>
<td>$\rho = 1.38$</td>
</tr>
<tr>
<td>Labour supply elasticity</td>
<td>$\mu = 2.83$</td>
</tr>
<tr>
<td>Share of home traded goods in traded basket</td>
<td>$\gamma = 0.58$</td>
</tr>
<tr>
<td>Share of nontraded goods in consumption</td>
<td>$\kappa = 0.40$</td>
</tr>
<tr>
<td>Substitutability between traded goods</td>
<td>$\theta = 1.50$</td>
</tr>
<tr>
<td>Substitutability between traded and nontraded goods</td>
<td>$\phi = 0.45$</td>
</tr>
<tr>
<td>Substitutability among individual goods</td>
<td>$\varphi = 10$</td>
</tr>
<tr>
<td>Labour share of income in traded goods sector</td>
<td>$a_T = 0.67$</td>
</tr>
<tr>
<td>Labour share of income in nontraded goods sector</td>
<td>$a_N = 0.67$</td>
</tr>
<tr>
<td>Capital depreciation rate</td>
<td>$\delta = 0.025$</td>
</tr>
<tr>
<td>Investment adjustment cost</td>
<td>$\psi = 0.25$</td>
</tr>
<tr>
<td>Share of government spending</td>
<td>$g = 0.18$</td>
</tr>
<tr>
<td>Interest rate smoothing factor in Taylor rule</td>
<td>$\delta_R = 0.81$</td>
</tr>
<tr>
<td>Inflation feedback in Taylor rule</td>
<td>$\delta_{\pi} = 2$</td>
</tr>
<tr>
<td>Output feedback in Taylor rule</td>
<td>$\delta_Y = 0.1$</td>
</tr>
<tr>
<td>Pricing strategy</td>
<td>$\eta = 0$</td>
</tr>
<tr>
<td>Persistence of technology shock in traded sector</td>
<td>$\delta_{TT1} = 0.95$, $\delta_{TT2} = 0$</td>
</tr>
<tr>
<td>Variance of technology shock in traded sector</td>
<td>$\sigma_T = 0.0045$</td>
</tr>
<tr>
<td>Persistence of technology shock in non-traded sector</td>
<td>$\delta_{NN1} = 0.95$, $\delta_{NN2} = 0$</td>
</tr>
<tr>
<td>Variance of technology shock in non-traded sector</td>
<td>$\sigma_N = 0.0045$</td>
</tr>
<tr>
<td>Cross terms of technology shocks</td>
<td>$\delta_{TN1} = \delta_{TN2} = 0.60$</td>
</tr>
<tr>
<td></td>
<td>$\delta_{NT1} = \delta_{NT2} = 0$</td>
</tr>
<tr>
<td>Monetary policy shock</td>
<td>$\delta_{rr} = 0.15$, $\sigma_{rr} = 0.0024$</td>
</tr>
<tr>
<td>Government spending shock</td>
<td>$\delta_G = 0.97$, $\sigma_G = 0.0053$</td>
</tr>
<tr>
<td>Mark-up shock</td>
<td>$\delta_V = 0.89$, $\sigma_V = 0.002$</td>
</tr>
<tr>
<td>Labour supply shock</td>
<td>$\delta_{\chi} = 0.90$, $\sigma_{\chi} = 0.025$</td>
</tr>
<tr>
<td>Investment adjustment cost shock</td>
<td>$\delta_{\psi} = 0.71$, $\sigma_{\psi} = 0.0045$</td>
</tr>
</tbody>
</table>

Table 1: Parameter values: Symmetric case
that the correlation and variability effects of the same type of assets across countries are also equal to each other in absolute value. (The correlation effects have opposite signs because of different country identity.) As references, the value of $\tau$ here is 1.6818. The correlation and variability effects associated with $\hat{\alpha}_1$ are respectively 0.1763 and 43.8628 while those associated with $\hat{\alpha}_2$ are $-0.1763$ and 43.8628. For bond assets, the two effects associated with $\hat{\alpha}_3$ are respectively 0.4156 and 6.2776 while those associated with $\hat{\alpha}_4$ are $-0.4156$ and 6.2776. One can verify that these values are consistent with the optimal portfolios via Eqs. (3.12 – 14). It also follows by inspection of the effects that the (conditional) correlation between the innovation in the equity excess return and that of relative disposable income is relatively low while the correlation between the innovation in the bond excess return and that of relative disposable income is relatively high. The bond assets’ return moves more closely with relative disposable income in the model. According to the analysis in the last section, more sizable bond positions should be held in optimal portfolios due to the relative correlation effect. In contrast, the (conditional) variability effect belonging to equity assets is relatively high while that belonging to bond assets is relatively low. Due to this relative variability effect, however, more sizable equity positions should be held in optimal portfolios. It turns out that the relative variability effect dominates the correlation effect, so in the end we observe that the size of equity positions outweighs that of bond positions.

The key information conveyed by the benchmark calibration is that the pattern of two-way capital flows cannot arise in a symmetric model. There must be some asymmetries between the two countries which make this happen. By design, our model is general enough to allow for assessments of various asymmetries’ impact on the capital flows. The next section is thus dedicated to such assessments in which course the result of the symmetric simulation in this section is always used as a comparison.

<table>
<thead>
<tr>
<th>Assets menu</th>
<th>Optimal portfolio choices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home equity</td>
<td>$\hat{\alpha}_1 = -2.2985$</td>
</tr>
<tr>
<td>Foreign equity</td>
<td>$\hat{\alpha}_2 = 2.2985$</td>
</tr>
<tr>
<td>Home bond</td>
<td>$\hat{\alpha}_3 = -0.7756$</td>
</tr>
<tr>
<td>Foreign bond</td>
<td>$\hat{\alpha}_4 = 0.7756$</td>
</tr>
</tbody>
</table>

Table 2: Optimal portfolio choices: Symmetric case
5 The two-way capital flows: developing vs developed countries

Now we turn to consider asymmetric situations in this section. The integration of developing country into the world economy is considered. To distinguish, in what follows, the home country is viewed as developing country while the foreign country is viewed as developed country. Because it is very likely the case that between the two types of countries various asymmetries coexist at the same time, we take two steps to investigate their impacts. First of all, we consider the individual effect of each asymmetry on net portfolio positions and two-way capital flows. Through the exercise, we will know whether the asymmetry considered matters for the emergence of the pattern of two-way capital flows. Moreover, if we find that an asymmetry does generate a two-way capital flow we also examine the question of in which direction the asymmetry plays its role (i.e. does it cause equity capital to flow to or from the developing country). The correlation and variability effects will also be traced during the course of the analysis in order to uncover the main channels in operation. After checking these individual effects, we put all asymmetries together into the same picture. By picking different sets of parameter values for the two countries, we simulate a fully asymmetric model mimicking a world of developing and developed countries that differ along multiple dimensions. We will thus check the composite effect of all asymmetries on portfolio choices.

5.1 Asymmetric cases: Single factors

To separate the effects of the asymmetries from each other, in this subsection, we examine them one by one. The process is as follows. We treat the foreign country as a control group and fix all foreign country parameter values at the benchmark levels. For each asymmetry, in the home country, we change the value of the associated parameter over a range around the benchmark value. Our target is to see how the net foreign equity and bond positions, \( \hat{\alpha}_E \) and \( \hat{\alpha}_B \), respond to such changes.
Figure 2: Labour intensity of technology $a_T$ and $a_N$
5.1.1 Labour intensity

As the first experiment, we look at labour intensity of technology. The parameter characterizing this aspect is \( a \). In the experiment, the foreign labour share \( a^* \) is fixed at the standard value of 0.67 while the home share \( a \) ranges from 0.55 to 0.79. The results are depicted as Figure 2. In this figure, panels (a) and (b) demonstrate the variations in \( \hat{\alpha}_E \) and \( \hat{\alpha}_B \) respectively. At the horizontal middle, \( \hat{\alpha}_E \) and \( \hat{\alpha}_B \) are both equal to zero which corresponds to the benchmark case of \( a = a^* = 0.67 \). To the right hand side of the point, \( a > a^* \). We observe \( \hat{\alpha}_E < 0 \) and \( \hat{\alpha}_B > 0 \). That is to say, when the labour share is higher in the home country than in the foreign country, the home country holds a negative net equity position and a positive net bond position, i.e. there are two-way capital flows in the form observed for developing countries. Moreover, as the magnitude of the asymmetry grows, i.e. when \( a \) is much higher than \( a^* \), the pattern in capital flows become more significant, \( \hat{\alpha}_E \) and \( \hat{\alpha}_B \) both increase in absolute size.

To explore why this is the case, we decompose the portfolios into associated correlation and variability effects, whose results are documented in the remaining panels of the figure. Since we will present the results of other asymmetries in the same way, some explanation on how to read these figures will be useful. Panels (c) and (e) report the correlation and variability effects for equities (in absolute value), i.e. \( \hat{\alpha}_1 \) and \( \hat{\alpha}_2 \). Panels (d) and (e) do the same for bonds, i.e. \( \hat{\alpha}_3 \) and \( \hat{\alpha}_4 \). Because the variability effect is a ratio between two volatilities, the latter are also displayed as bottom panels, i.e. in panels (g) and (i) are the conditional volatility of relative disposable income and that of the excess return belonging to the two equities while in panels (h) and (j) are conditional volatility of relative disposable income and that of the excess return belonging to the two bonds. In all these panels, solid lines are used for home assets while dashed lines for foreign assets.

According to (c) and (d), \( \hat{\alpha}_E \) decreases because both the correlation and variability effects associated with \( \hat{\alpha}_1 \) are higher than those of \( \hat{\alpha}_2 \). As is shown, as \( a \) increases, the correlation effects of both equities increase, which implies an enhancement of equities’ role as a good hedge against income risks. However, the increase in the correlation effect for home equity is more significant. On the other hand, the variability effects of both equities decrease, which implies lower gross positions are required to hedge against risks. (This is in turn due to a decrease in the volatility of relative incomes while there is an increase in the volatility of asset returns based on the facts in panels (g) and (i)). However, the
decrease in the variability effect of the home equity is less significant. Both facts point to a relative rise in the size of $\tilde{\alpha}_1$ which favours presence of a negative $\tilde{\alpha}_E$.

For $\tilde{\alpha}_B$, we look at panels (d) and (f). As $a$ increases, the correlation effect of $\tilde{\alpha}_3$ decreases while that of $\tilde{\alpha}_4$ increases, which favours presence of a positive $\tilde{\alpha}_B$. On the other hand, the variability effect of $\tilde{\alpha}_3$ increases while that of $\tilde{\alpha}_4$ decreases. (Based on the facts in panels (h) and (j), the rise in $\tilde{\alpha}_3$ is because the associated volatility of relative income decreases less than that of the asset return while the decline in $\tilde{\alpha}_4$ is because the associated volatility of relative income increases less than that of the asset return.) So the change in the variability effect favours the presence of a negative $\tilde{\alpha}_B$ instead. It turns out that in the race between the two effects the former one wins out and $\tilde{\alpha}_B$ becomes positive.

5.1.2 Nominal rigidity

We consider both price and wage rigidities in this subsection.

First, for the degree of price stickiness $\lambda$, we set the foreign value at the standard value of 0.66 while we vary the home value from 0.54 to 0.78. As is shown in Figure 3, on either side of the middle point of $\lambda = \lambda^* = 0.66$, the pattern of two-way capital flows emerges with $\tilde{\alpha}_E < 0$ while $\tilde{\alpha}_B > 0$, so the home country has a negative net position in equities and a positive net position in bonds in the way observed in the data for developing countries.

It is rather surprising that, in the case illustrated in Figure 3, the direction of the asymmetry in price rigidity appears to be unimportant in generating an outcome with $\tilde{\alpha}_E < 0$ while $\tilde{\alpha}_B > 0$. To test the sensitivity of this result, we conduct further experiments in which $\lambda^*$ (i.e. the foreign degree of price rigidity) is different from 0.66. These experiments are illustrated in Figures 4 and 5. These figures show the effects of varying $\lambda$ on $\tilde{\alpha}_E$ and $\tilde{\alpha}_B$ for a high value of $\lambda^*$ (Figure 4) and a low value of $\lambda^*$ (Figure 5). By Smets and Wouters' estimation, the value of $\lambda$ lies in a confidence interval of 0.56 and 0.74 so we use these two extremes as values for $\lambda^*$. These two figures show that in general the effects of $\lambda$ and $\lambda^*$ on $\tilde{\alpha}_E$ and $\tilde{\alpha}_B$ are quite complicated. Both figures show that the plots for $\tilde{\alpha}_E$ and $\tilde{\alpha}_B$ cross at two values of $\lambda$. For either high values of $\lambda$ or low values of $\lambda$ the pattern of two-way capital flows is observed with $\tilde{\alpha}_E < 0$ and $\tilde{\alpha}_B > 0$. But for intermediate values of $\lambda$ the opposite result emerges.

From the last paragraph, the impact of asymmetry in $\lambda$ on two-way capital flows is
Figure 3: Nominal (price) rigidity $\lambda$
Figure 4: Price rigidity: High $\lambda^*$

Figure 5: Price rigidity: low $\lambda^*$
in general complicated in terms of signs. However, in terms of magnitude, it turns out that the asymmetry in \( \lambda \) is always a factor of little importance. The sizes of \( \tilde{\alpha}_E \) and \( \tilde{\alpha}_B \) under asymmetric cases are generally below 0.01. This is consistent with the results of the decomposition into correlation and variability effects. The other panels in Figure 3 show that the conditional second moments that are associated with home and foreign assets are generally very similar regardless of the value of \( \lambda \).

Turning now to the degree of wage stickiness \( \varsigma \), we set \( \varsigma^* \) at the standard value of 0.7 while we vary \( \varsigma \) from 0.58 to 0.82. The result is shown in Figure 6. It is obvious that when \( \varsigma > \varsigma^* \), \( \tilde{\alpha}_E < 0 \) and \( \tilde{\alpha}_B > 0 \). The more severe is the problem of wage stickiness in the home country, the more significant is the pattern of two-way capital flows in the model. For different foreign values, the result is robust.

When \( \varsigma > \varsigma^* \), a rise in \( \varsigma \) increases the correlation effect of equities to approximately the same degree (see panel (c)). It also increases the variability effect, however, with that belonging to \( \tilde{\alpha}_3 \) more significantly according to panel (e). (By panel (i), this is in turn because the excess return of the home equity becomes relatively less volatile.) This leads to \( \tilde{\alpha}_E < 0 \).

A rise in \( \varsigma \) moves the correlation and variability effects of bonds as well. While the correlation effect associated with \( \tilde{\alpha}_3 \) is higher than that of \( \tilde{\alpha}_4 \), its variability effect is lower than that of \( \tilde{\alpha}_4 \). It turns out that the correlation effect dominates the variability effect so \( \tilde{\alpha}_B > 0 \).

As was seen with the asymmetry in price stickiness, the pattern of two-way capital flows is insensitive to the asymmetry in wage stickiness, with the sizes of \( \tilde{\alpha}_E \) and \( \tilde{\alpha}_B \) under asymmetric calibrations being generally below 0.01 (in panels (a) and (b)) so we can conclude that asymmetries in the degree of both wage and price stickiness are of little importance in generating large two-way capital flows.

5.1.3 Home good bias

The parameter that determines the steady state share of home traded goods in the traded consumption basket, \( \gamma \), governs the severity of home good bias. The higher is the value of \( \gamma \), the more severe is home good bias. We set \( \gamma^* \) at the standard value of 0.58 and vary \( \gamma \) from 0.46 to 0.70. Figure 7 reports the results for this experiment. From panels (a) and (b), when \( \gamma < \gamma^* \), we obtain \( \tilde{\alpha}_E < 0 \) and \( \tilde{\alpha}_B > 0 \). So a less severe home good bias
Figure 6: Nominal (wage) rigidity $\varsigma$
Figure 7: Home good bias $\gamma$
in the home country will lead to two-way capital flows, with the home country holding a net negative position in equities and a net positive position in bonds (as observed in the data for developing countries).

Panel (c) tells us that when $\gamma < \gamma^*$, the relative return on home equity is more closely correlated with relative income than that of the foreign equity, which implies a relatively large absolute position of $\tilde{\alpha}_1$. This is the reason for a negative $\tilde{\alpha}_E$. By panel (e), the relative variability effect actually works in the other direction. When $\gamma < \gamma^*$, the relative returns conditional on $r_{-1x}$ and $r_{-2x}$ have the same volatility (panel (g)), but because the excess return of home equity has a relatively high volatility compared to that of the foreign equity (panel (i)), the variability effect is lower (panel (e)), which entails a relatively small position of $\tilde{\alpha}_1$. This partially offsets the relative correlation effect.

For bond positions, when $\gamma < \gamma^*$, the relative variability effect between home and foreign assets are similar to that of equity assets. The variability effect associated with home bond is relatively low (panel (f)), which entails a relatively small position of $\tilde{\alpha}_3$ (and a relatively large position of $\tilde{\alpha}_4$ correspondingly). This is the reason for a positive $\tilde{\alpha}_B$. The relative correlation effects between $\tilde{\alpha}_3$ and $\tilde{\alpha}_4$ are approximately zero, i.e. the lines representing the two effects overlap each other (panel (d)).

5.1.4 Trade openness

Trade openness can be represented by the share of nontraded goods in the total consumption basket, which is determined by the parameter $\kappa$. The higher is the value of $\kappa$, the less open is trade in the country. We set $\kappa^*$ at the standard value of 0.4 and vary $\kappa$ from 0.28 to 0.52. As is shown in the Figure 8, the result is that as the home country has a smaller share of nontraded goods in the consumption basket, the more pronounced are two-way capital flows (panels (a) and (b)) i.e. where $\tilde{\alpha}_E < 0$ and $\tilde{\alpha}_B > 0$. The pattern therefore resembles that of home bias shown above.

In terms of decomposition into correlation and variability effects, we observe that, when $\kappa < \kappa^*$, the correlation effect associated with $\tilde{\alpha}_1$ is always higher than that of $\tilde{\alpha}_2$ (panel (c)). This gives rise to a large position of $\tilde{\alpha}_1$ and a negative $\tilde{\alpha}_E$. In addition, both the conditional volatility of relative income and that of the excess return associated with home equity are higher than those associated with foreign equity (panels (g) and (i)). But the volatility of the excess return rises more than that of relative income. This generates
Figure 8: Trade openness $\kappa$
Figure 9: Substitutability between home and foreign tradables $\theta$

a lower variability effect of $\tilde{\alpha}_1$ compared to that of $\tilde{\alpha}_2$ (panel (e)), which partially offsets the relative correlation effect.

For bond positions, when $\kappa < \kappa^*$, the correlation effect associated with $\tilde{\alpha}_3$ is always greater than that associated with $\tilde{\alpha}_4$ (panel (d)), which implies a relatively large position of $\tilde{\alpha}_3$ and a negative $\tilde{\alpha}_B$. However, the variability effect associated with $\tilde{\alpha}_3$ is always below that associated with $\tilde{\alpha}_4$ (panel (f)), which, in contrast, implies a relatively small position of $\tilde{\alpha}_3$ and a positive $\tilde{\alpha}_B$. The importance of the relative variability effect quantitatively outweighs that of the relative correlation effect. This justifies the presence of a positive $\tilde{\alpha}_B$.

5.1.5 Household preferences

In this subsection, we consider asymmetries associated with two parameters of households’ preferences. These are the elasticity of substitution between home and foreign tradables, $\theta$, and the elasticity of substitution between tradables and non-tradables, $\phi$.

For the former, $\theta^*$ is set at 1.5 while $\theta$ ranges from 1 to 2. The result is shown in Figure 9. It is obvious that there is no effect of this asymmetry on $\tilde{\alpha}_E$ and $\tilde{\alpha}_B$. The asymmetry associated with $\theta$ seems to be an irrelevant factor when it comes to two-way
Figure 10: Substitutability between tradables and non-tradables $\phi$
capital flows.

For the latter parameter, \( \phi^* \) is set at 0.45 while \( \phi \) ranges from 0.33 to 0.57. It is clear from Figure 10 that as \( \phi \) increases, the portfolio pattern displays two-way capital flows where the home country has a negative net position in equities and a positive net position in bonds in line with observed data on developing countries. The higher is \( \phi \) relative to \( \phi^* \), the more significant are the two-way capital flows.

When \( \phi > \phi^* \), the correlation effect associated with \( \tilde{\alpha}_1 \) is greater than that associated with \( \tilde{\alpha}_2 \) (panel (c)), which implies that home equity as a hedge against income risks is relatively superior to foreign equity. This tends to generate a negative \( \tilde{\alpha}_E \). However, the variability effect associated with \( \tilde{\alpha}_1 \) is less than that associated with \( \tilde{\alpha}_2 \) (panel (e)), which implies, given the presence the other assets, it requires a relatively smaller \( \tilde{\alpha}_1 \) to hedge against the related income risks. This tends to generate a positive \( \tilde{\alpha}_E \). It turns out that the relative correlation effect is more important, so \( \tilde{\alpha}_E < 0 \) is observed.

For bond positions, when \( \phi > \phi^* \), the correlation effect associated with \( \tilde{\alpha}_3 \) is below that associated with \( \tilde{\alpha}_4 \) (panel (d)) which tends to generate a positive \( \tilde{\alpha}_B \). Moreover, the variability effect associated with \( \tilde{\alpha}_3 \) is below that associated with \( \tilde{\alpha}_4 \) (panel (f)), which tends to reinforce the relative correlation effect in generating a positive \( \tilde{\alpha}_B \).

### 5.1.6 Capital adjustment costs

It is possible that marginal costs of capital adjustment in developing and developed countries are not equal. What is the consequence of this asymmetry on country portfolios? In our model, this can be determined by manipulating the parameter \( \psi \). We set \( \psi^* \) at its benchmark level of 0.25 and vary \( \psi \) from 0.13 to 0.37. The results are displayed in Figure 11. By panels (a) and (b), when \( \psi < \psi^* \), we have \( \tilde{\alpha}_E < 0 \) and \( \tilde{\alpha}_B > 0 \). Thus the lower is \( \psi \) relative to \( \psi^* \), the more significant is the pattern of two-way capital flows (with the home country holdings a negative net position in equities and a positive net position in bonds).

By panel (c), \( \tilde{\alpha}_E \) is negative mainly because the correlation effect associated with \( \tilde{\alpha}_1 \) is above that associated with \( \tilde{\alpha}_2 \). The variability effect associated with \( \tilde{\alpha}_1 \) is, by panel (e) however, below that associated with \( \tilde{\alpha}_2 \). This is in turn because even though the conditional relative income and excess return of home equity both are more volatile than that of foreign equity, the volatility in excess return dominates.
Figure 11: Capital adjustment costs $\psi$
By panel (d) and (f), the correlation and variability effect associated with $\tilde{\alpha}_3$ are both below that associated with $\tilde{\alpha}_4$. They combine to lower the size of $\tilde{\alpha}_3$ comparing to that of $\tilde{\alpha}_4$, which explains why $\tilde{\alpha}_B$ is positive. Besides, by panels (h) and (j), the relatively low variability effect of the home bond is due to the relative low volatility of disposable income and relative high volatility of excess return when $\psi < \psi^*$. 

5.1.7 Monetary policy

Monetary policies in developing and developed countries may be conducted in different ways. In this subsection, we explore the possibility that they put different weights on inflation and output gap stabilization. This is captured by asymmetries associated with the two feedback coefficients of Taylor rule in the model, i.e. inflation feedback coefficient $\delta_\pi$ and output gap feedback coefficient $\delta_y$ respectively.

For the former, we set $\delta_\pi$ at the benchmark level of 2 and vary $\delta_\pi$ from 1.1 to 2.8. We plot the results in Figure 12. By panels (a) and (b), it is obvious that when $\delta_\pi < \delta_\pi^*$, we have $\tilde{\alpha}_E < 0$ and $\tilde{\alpha}_B > 0$, i.e. if the home country puts relatively less weight on inflation stabilization when conducting monetary policy, there tends to be a two-way capital flows with the home country holding a negative net position in equities and a positive net position in bonds.

When $\delta_\pi < \delta_\pi^*$, by panel (c), the correlation effect associated with $\tilde{\alpha}_1$ is above that associated with $\tilde{\alpha}_2$, which involves a relatively large negative position in home equity and thus a negative net equity position. However, there is a minor conflicting effect from the variability effect. By panel (e), the variability effect associated with $\tilde{\alpha}_1$ is below that associated with $\tilde{\alpha}_2$, partially offsetting the correlation effect.

By panel (d), when $\delta_\pi < \delta_\pi^*$, both the correlation and variability effects associated with the home bond are lower than those associated with the foreign bond. This means the position in the home bond should be smaller than that of foreign bond, which explains a positive net bond position.

For the asymmetry in $\delta_y$, we set $\delta_y^*$ at the benchmark level of 0.1 and change the home value from 0.01 to 0.19. As shown in Figure 13, it turns out that when $\delta_y > \delta_y^*$, $\tilde{\alpha}_E < 0$ and $\tilde{\alpha}_B > 0$, i.e. if the monetary policy in home country reacts more to the output gap than in foreign country, there tends to be a two-way capital flow between the two countries, with the home country holding a net negative position in equities and a
Figure 12: Monetary policy: Inflation feedback $\delta_\pi$
Figure 13: Monetary policy: Output gap feedback $\delta_y$
net positive position in bonds.

When \( \delta_y > \delta_y^* \), the correlation effect associated with the home equity is well above that associated with the foreign equity (panel (c)) while the variability effect associated with the home equity is slightly below that associated with the foreign equity (panel (e)), so in total, the position of the home equity will exceeds that of the foreign equity which leads to a negative \( \tilde{\alpha}_E \).

For bond assets however, when \( \delta_y > \delta_y^* \), both the correlation and variability effects associated with the home bond are below those associated with the foreign bond (panels (d) and (f)). The emphasis on output stabilization in the developing country at the same time (relatively) undermines the relevance of the home bond in risk hedging and the risk amount to be hedged against by it, which implies a smaller position in the home bond compared to that of the foreign bond and thus a positive \( \tilde{\alpha}_B \).

5.1.8 Price/Wage indexation

We turn to asymmetries in price and wage indexation across countries in this subsection. For price indexation, we set \( \omega^* \) at the standard value of 0.24 and change the home value from 0.12 to 0.36 while for wage indexation, we set \( \varpi^* \) at the standard value of 0.58 and change the home value from 0.46 to 0.70. As is shown in Figure 14 and 2.15, when \( \omega > \omega^* \) or/and \( \varpi > \varpi^* \), then \( \tilde{\alpha}_E < 0 \) and \( \tilde{\alpha}_B > 0 \), so there is a two-way capital flow with the home country holding a negative net position in equities and a positive net position in bonds. The results tend to suggest that a high degree of price and wage indexation in developing countries is consistent with the emergence of two-way capital flows between the two groups of countries. However, as in the case of asymmetries in the degree of price and wage rigidity (\( \lambda \) and \( \zeta \)), the asymmetries in \( \omega \) and \( \varpi \) have a very small effect on net equity and bond positions.

5.1.9 Habit formation

The degree of habit formation is governed by the parameter \( h \). To assess the effect of asymmetry in \( h \) on two-way capital flows, we set \( h^* \) at the benchmark value of 0.7 and vary the value of \( h \) from 0.58 to 0.82. Figure 16 plots the result. It is clear from panels (a) and (b) that when \( h < h^* \), we have \( \tilde{\alpha}_E < 0 \) and \( \tilde{\alpha}_B > 0 \), i.e. if the home households have a lower degree of habit formation than foreign households, this will result in two-way capital
Figure 14: Price indexation $\omega$
Figure 15: Wage indexation \( \varpi \)
flows with the home country holding a negative net position in equities and a positive net position in bonds.

When \( h < h^* \), we have \( \tilde{\alpha}_E < 0 \) because both the correlation and variability effect associated with the home equity are above those associated with the foreign equity (panels (c) and (e)). For bond positions, when \( h < h^* \), we have \( \tilde{\alpha}_B > 0 \) because on the one hand, the correlation effect associated with the home bond is relatively lower than that of the foreign bond, on the other hand, the variability effect associated with it is relatively higher but the correlation effect dominates.

5.1.10 Market competitiveness

We can use the parameter of \( \varphi \) to represent the degree of competitiveness in an economy. The lower is \( \varphi \), the lower is the substitutability between varieties and so the more power firms have when setting prices. Also note that the optimal price of final good is a mark-up over associated marginal cost of production, \( \varphi \frac{\varphi}{\varphi - 1} \), so the lower is \( \varphi \) the higher is the mark-up. In other words, the lower is \( \varphi \), the lower is the degree of market competitiveness.

To check the effect of the asymmetry associated with \( \varphi \) on two-way capital flows, we set the value of \( \varphi^* \) at 10 as in the benchmark calibration and vary the value of \( \varphi \) from 7 to 13 which corresponds to a price mark-up from about 8.3\% to 16.7\% in economy. As is shown in Figure 17, two-way capital flows arise if \( \varphi \) is less than \( \varphi^* \), i.e. the home market is less competitive than the foreign market.

When \( \varphi < \varphi^* \), the correlation effect associated with the home equity is below that associated with the foreign equity while the variability effect associated with the home equity is above that associated with the foreign equity (panels (c) and (e)). The difference in variability effect is quantitatively more important, so the gross position in the home equity is relatively large (in absolute value) and \( \tilde{\alpha}_E < 0 \). For bond assets, the correlation effect associated with the home bond is also below that associated with the foreign bond while the variability effect associated with the home bond is above that associated with the foreign bond (panels (d) and (f)). However, the difference in correlation effect is quantitatively more important, so the gross position in the home bond is relatively small (in absolute value) and \( \tilde{\alpha}_B > 0 \).
Figure 16: Habit formation $h$
Figure 17: Market competitiveness $\varphi$
5.1.11 Pricing strategy

Different pricing strategies, PCP or LCP, have different implications for behaviour of import prices. So it is worthwhile to check the cases in which the developing and developed countries price products according to different strategies. Because, without loss of generality, the home country is viewed as developing country and the currencies used in international transactions are usually those of developed country, it is natural to believe that the firms in the home country use LCP while the firms in the foreign country use PCP. Based on this belief, in what follows we consider two experiments. First, suppose the firms in the home country all set prices of tradables according to LCP and the foreign country’s pricing strategy stands at different position between perfect LCP and PCP, one can interpret this as such that some foreign firms adopt LCP while others adopt PCP. Second, suppose conversely that the firms in foreign country all set prices of tradables according to PCP and the home country’s pricing strategy stands at different positions between perfect LCP and PCP, again one can interpret this as such that some home firms adopt LCP while others adopt PCP.

For the former experiment, we set $\eta = 0$ and vary value of $\eta^*$ from 0 to 1. The result is displayed in Figure 18. Note that the symmetric benchmark corresponds to the allocation at the left-hand side in all panels in the figure. It is obvious from the figure, as the foreign country’s choice of pricing strategy approaches PCP, the portfolio allocations exhibit two-way capital flows, i.e. when $0 = \eta < \eta^*$, we have $\tilde{\alpha}_E < 0$ while $\tilde{\alpha}_B > 0$ (panels (a) and (b)), so the home country has a net negative holding of equities and a net positive holding of bonds. Further investigation shows that when $0 = \eta < \eta^*$, the correlation effects associated with home assets are roughly the same as those associated with foreign assets, however, the variability effect associated with the home equity is above that associated with the foreign equity (panel (e)) while the variability effect associated with the home bond is below that associated with the foreign bond (panel (f)), so the net equity position is negative while the net bond position is positive.

For the second experiment, we set $\eta^*$ at 1 and vary the value of $\eta$ from 0 to 1. The result is displayed in Figure 19. Note that the allocation at the right-hand side in all panels in the figure corresponds to a symmetric case. By panels (a) and (b), as the home country’s choice of pricing strategy approaches LCP, the portfolio allocations always exhibit two-way capital flows, i.e. when $\eta < \eta^* = 1$, we have $\tilde{\alpha}_E < 0$ while $\tilde{\alpha}_B > 0$, so the home
Figure 18: Pricing strategy: Foreign country moving to PCP $\eta^*$
Figure 19: Price strategy: Home country moving to LCP $\eta$

- (a) Net foreign equity
- (b) Net foreign bond
- (c) Corr($\zeta y, r_{1x}$) & Corr($\zeta y, r_{2x}$)
- (d) Corr($\zeta y, r_{3x}$) & Corr($\zeta y, r_{4x}$)
- (e) StD($\zeta y, r_{1x}$)/StD($r_{1x}$) & StD($\zeta y, r_{2x}$)/StD($r_{2x}$)
- (f) StD($\zeta y, r_{3x}$)/StD($r_{3x}$) & StD($\zeta y, r_{4x}$)/StD($r_{4x}$)
- (g) StD($\zeta y, r_{1x}$) & StD($\zeta y, r_{2x}$)
- (h) StD($\zeta y, r_{3x}$) & StD($\zeta y, r_{4x}$)
- (i) StD($r_{1x}$) & StD($r_{2x}$)
- (j) StD($r_{3x}$) & StD($r_{4x}$)
country has a negative net holding of equities and a positive net holding of bonds. Further investigation shows that when \( \eta < \eta^* = 1 \), as in the previous experiment, the correlation effects of home and foreign assets are roughly the same, however, the variability effect associated with the home equity is above that associated with the foreign equity (panel \((e)\)) while the variability effect associated with the home bond is below that associated with the foreign bond (panel \((f)\)), so the net equity position is negative while the net bond position is positive.

So to sum up, if the home country has a lower \( \eta \) compared to the foreign country, i.e. the developing country’s pricing strategy is relatively close to \( LCP \) while the developed country’s strategy is relatively close to \( PCP \), two-way capital flows arise. However, again we have to notice that the magnitude of the effect that the asymmetry has on net positions is very small. It turns out that it is always below 0.001 so we also view the asymmetry as a minor factor in affecting the pattern of two-way capital flows.

5.1.12 Short summary

In this section, we have examined the various asymmetries’ role in generating two-way capital flows between the two types of countries. We have obtained at least two sets of result. The first set of result concerns the question of which direction the asymmetries impact the pattern of capital flows. And we have found that the following facts are candidates in favour of the emergence of two-way capital flows (which are consistent with observed data for developing countries). Compared to a developed country, in a developing country, if firms use more labour intensive technology \( a > a^* \); it is less costly for them to adjust investment \( \psi < \psi^* \); when setting prices for products and labour (given that both countries feature high nominal rigidity) they are confronted with more frictions \( \lambda > \lambda^* \) and/or \( \zeta > \zeta^* \); in the traded sector, firms set the prices through \( LCP \) more often, \( \eta < \eta^* \); households consume more traded goods \( \kappa < \kappa^* \) and imports \( \gamma < \gamma^* \); traded and non-traded goods are more substitutable \( \phi > \phi^* \); there is less persistent habit formation \( h < h^* \); the monetary authority responds more intensely to the output gap while less so to inflation, \( \delta_y > \delta_y^* \) and/or \( \delta_n < \delta_n^* \); the market in developing country is less competitive \( \varphi < \varphi^* \); and the degree of price/wage indexation is higher \( \omega > \omega^* \) and/or \( \pi > \pi^* \). The second set of results concerns the magnitude of the effects of asymmetries on two-way capital flows. While some asymmetries that we mentioned do affect the pattern of capital
flows, they are not that important because the magnitude of their effects is relatively low. These include the asymmetries associated with nominal rigidities, the degree of price/wage indexation and pricing strategy. So we see that the pattern of two-way capital flows is more likely driven by asymmetries in real factors instead of asymmetries in nominal factors. In addition, we found that not all asymmetries in the model are relevant for the question of two-way capital flows. For instance, asymmetry in the substitutability between home and foreign traded goods has no effect on the pattern of two-way capital flows.

5.2 A fully asymmetric simulation

After investigating the effect of each asymmetry on the pattern of country portfolios, we will now undertake another exercise, i.e. taking into account all asymmetries at the same time. In this section, we simulate the model in a fully asymmetric way. This will yield steady-state portfolios allowing us to assess the composite effect of coexistence of multiple asymmetries.

Following our convention, the home and foreign countries are labelled as developing and developed country respectively. Our strategy is to choose parameter values for the home country from the estimates based on the data of developing countries, especially China, (if they are available) while choosing parameter values for the foreign country from the estimates based on the data of developed countries, especially U.S.. The task of choosing parameter values for the foreign country is already done in the symmetric simulation. Now we describe how we choose parameter values for the home country.

In the model, the value of many parameters in the foreign country is obtained from Smets and Wouters (2007). That paper estimates a New Keynesian model of the U.S. economy. Recently, there are many studies applying the framework to emerging markets, in particular China, and these provide us with estimates of parameters in the context of developing countries. The main contributions to this empirical literature include Mehrotra et al. (2011), Sun and Sen (2012), Dai (2012) and Miao and Peng (2012) among others. In the following exercise, we mainly rely on Sun and Sen’s (2012) estimation in choosing parameter values. These parameters include the degrees of price/wage stickiness \( \lambda \) and \( \varsigma \), the degrees of price/wage indexation \( \omega \) and \( \varpi \), habit persistence \( h \), the feedback
<table>
<thead>
<tr>
<th>Description</th>
<th>Variable values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calvo price rigidity parameter</td>
<td>$\lambda = 0.95$, $\lambda^* = 0.66$</td>
</tr>
<tr>
<td>Calvo wage rigidity parameter</td>
<td>$\zeta = 0.79$, $\zeta^* = 0.70$</td>
</tr>
<tr>
<td>Price indexation</td>
<td>$\omega = 0.97$, $\omega^* = 0.24$</td>
</tr>
<tr>
<td>Wage indexation</td>
<td>$\varpi = 0.61$, $\varpi^* = 0.58$</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta = \beta^* = 0.99$</td>
</tr>
<tr>
<td>Habit persistence</td>
<td>$h = 0.81$, $h^* = 0.70$</td>
</tr>
<tr>
<td>Risk aversion coefficient</td>
<td>$\rho = \rho^* = 1.38$</td>
</tr>
<tr>
<td>Labour supply elasticity</td>
<td>$\mu = \mu^* = 2.83$</td>
</tr>
<tr>
<td>Share of home traded goods in traded basket</td>
<td>$\gamma = \gamma^* = 0.58$</td>
</tr>
<tr>
<td>Share of nontraded goods in consumption</td>
<td>$\kappa = \kappa^* = 0.40$</td>
</tr>
<tr>
<td>Substitutability between Home and Foreign tradables</td>
<td>$\theta = \theta^* = 1.50$</td>
</tr>
<tr>
<td>Substitutability between nontraded and traded goods</td>
<td>$\phi = \phi^* = 0.45$</td>
</tr>
<tr>
<td>Substitutability between individual goods</td>
<td>$\varphi = \varphi^* = 10$</td>
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<tr>
<td>Labour share of income in traded goods sector</td>
<td>$a_T = 0.5$, $a_T^* = 0.67$</td>
</tr>
<tr>
<td>Labour share of income in nontraded goods sector</td>
<td>$a_N = 0.5$, $a_N^* = 0.67$</td>
</tr>
<tr>
<td>Capital depreciation rate</td>
<td>$\delta = \delta^* = 0.025$</td>
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<tr>
<td>Investment adjustment cost</td>
<td>$g = g^* = 0.18$</td>
</tr>
<tr>
<td>Share of government spending</td>
<td>$\psi = \psi^* = 0.25$</td>
</tr>
<tr>
<td>Interest rate smoothing factor in Taylor rule</td>
<td>$\delta_R = 0.98$, $\delta_R^* = 0.81$</td>
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<tr>
<td>Inflation feedback in Taylor rule</td>
<td>$\delta_{\pi} = 1.67$, $\delta_{\pi}^* = 2$</td>
</tr>
<tr>
<td>Output feedback in Taylor rule</td>
<td>$\delta_Y = 0.15$, $\delta_Y^* = 0.1$</td>
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<tr>
<td>Pricing strategies</td>
<td>$\eta = 0$, $\eta^* = 1$</td>
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<tr>
<td>Persistence of technology shock in traded sector</td>
<td>$\delta_{TT1} = 0.93$, $\delta_{TT2} = 0$</td>
</tr>
<tr>
<td>Variance of technology shock in traded sector</td>
<td>$\sigma_T = 0.0277$, $\sigma_T^* = 0.0045$</td>
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<td>Technology shock in non-traded sector</td>
<td>$\delta_{NN1} = 0.93$, $\delta_{NN2} = 0$</td>
</tr>
<tr>
<td>Variance of technology shock in non-traded sector</td>
<td>$\sigma_N = 0.0277$, $\sigma_N^* = 0.0045$</td>
</tr>
<tr>
<td>Cross terms of technology shocks</td>
<td>$\delta_{TN1} = \delta_{TN2} = 0.60$, $\delta_{NT1} = \delta_{NT2} = 0$</td>
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<tr>
<td>Monetary policy shock</td>
<td>$\delta_{rr} = 0.15$, $\sigma_{rr} = 0.0015$</td>
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<tr>
<td>Government spending shock</td>
<td>$\delta_G = 0.90$, $\sigma_G = 0.0877$</td>
</tr>
<tr>
<td>Mark-up shock</td>
<td>$\delta_V = 0.89$, $\sigma_V = 0.05$</td>
</tr>
</tbody>
</table>

*Table 3: Parameter values: Asymmetric case*
coefficients in monetary policy $\delta_R$, $\delta_y$ and $\delta_y$ and the persistence and volatility of various shocks.

The discount factors are assumed to be equalized to be consistent with our assumptions of equalized autarky interest rates across countries and $F = 0$ in steady state. For the same reason, the values of the parameters appearing in households’ preference are assumed to be in line with the benchmark. These include $\rho$ and $\mu$. Mehrotra et al (2011) estimates the elasticity of investment with respect to the current price of installed capital in China, $1/\psi$, and finds it is very close to that found in the U.S. by Christiano et al. (2005), which make us to choose $\psi = \psi^*$. Based on Miao and Peng’s (2012) estimation, the values of $g$ and $\delta$ are also the same as their foreign counterparts. For the choice of labour share of production $a$, there is a wide spectrum. According to Chinese data (that reported in china statistical yearbook), the labour income share is at around 0.5 which is much lower than that in the U.S.. However, the current literature suggests that the real share in China should be higher than this and view the reported level as puzzling. Based on the literature, the reason for a reported low $a$ are possibly due to measurement problems (Golin 2002) or/and misallocation frictions (Hsieh and Klelow 2009). Na (2015) estimates an average labour share for emerging countries of 0.7. For our simulation, because $a^*$ is chosen based on reported share we also use the reported level of $a = 0.5$. Note that according to our analysis in the last section, a higher $a$ tends to strengthen the pattern of two-way capital flows.

It is another challenge to obtain the estimates of the parameters that associated with open economy for the developing country. These include the share of traded goods in all tradables, $\gamma$, the share of non-traded goods in the consumption basket, $\kappa$, and the elasticity of substitution between home and foreign traded goods, $\theta$, and that between traded and non-traded goods, $\phi$. Schmitt-Grohe and Uribe (2015), using data from 38 poor and emerging countries, calibrate $\kappa$ and $\phi$ at 0.44 and 0.5 which are still within the range of the parameter estimation for developed countries. Some literature, such as Laxton et al. (2010) and Prasad and Zhang (2015), use the same value of these parameters for the different types of country. Due to the lack of accurate estimate for these parameters for developing countries and the fact that (to our knowledge) no evidence shows a significant difference between these estimates in developing and developed countries, we follow the approach of Laxton et al. (2010) and Prasad and Zhang (2015) to be on the safe side.
in our simulation. (We also assume that the elasticity of substitution among individual goods $\varphi$ are the same across countries.) For simplicity, we assume that the firms in the home country use $LCP$ to price their exports while those in the foreign country use $PCP$, so $\eta = 0$ and $\eta^* = 1$.

Given that the differences between the two countries are specified by the parameter values as in Table 2.3, the result of fully asymmetric simulation of the model is documented in Table 2.4. According to the results, the home country sells the home equity to the amount of 0.42 (multiplied by $\beta Y$) while it buys the foreign equity to the amount of 0.13, which results in a negative net position of equity, $\tilde{\alpha}_E = -0.28 < 0$. On the other hand, the home country also sells the home bond to the amount of 0.07 while it buys the foreign bond to the amount of 0.36, which results in a positive net position in bonds, $\tilde{\alpha}_B = 0.28 > 0$. By the condition of asset market clearing, the short position of an asset at home is a long position of the asset in the foreign country, $\tilde{\alpha}_i = -\tilde{\alpha}^*_i$. This leads to the fact that in foreign country we must have $\tilde{\alpha}^*_E > 0$ and $\tilde{\alpha}^*_B < 0$. Putting these facts together, we observe that the optimal portfolio allocations between the two asymmetric countries can be just described by the pattern of two-way capital flows, i.e. the home (developing) country ends up with a negative net position in equities and a positive net position in bonds while the foreign (developed) country ends up with a positive net position in equities and a negative net position in bonds.

<table>
<thead>
<tr>
<th>Assets menu</th>
<th>Optimal portfolio choices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home equity</td>
<td>$\tilde{\alpha}_1 = -0.4177$</td>
</tr>
<tr>
<td>Foreign equity</td>
<td>$\tilde{\alpha}_2 = 0.1341$</td>
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<tr>
<td>Net equity asset</td>
<td>$\tilde{\alpha}_E = -0.2836$</td>
</tr>
<tr>
<td>Home bond</td>
<td>$\tilde{\alpha}_3 = -0.0739$</td>
</tr>
<tr>
<td>Foreign bond</td>
<td>$\tilde{\alpha}_4 = 0.3575$</td>
</tr>
<tr>
<td>Net bond asset</td>
<td>$\tilde{\alpha}_B = 0.2836$</td>
</tr>
</tbody>
</table>

Table 4: Optimal portfolio choices: Fully asymmetric case
Table 5: Optimal portfolio choices: Asymmetry in asset menu

<table>
<thead>
<tr>
<th>Assets menu</th>
<th>Optimal portfolio choices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home equity</td>
<td>$\tilde{\alpha}_1 = -0.4442$</td>
</tr>
<tr>
<td>Foreign equity</td>
<td>$\tilde{\alpha}_2 = 0.1081$</td>
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<tr>
<td>Net equity asset</td>
<td>$\tilde{\alpha}_E = -0.3361$</td>
</tr>
<tr>
<td>Foreign bond</td>
<td>$\tilde{\alpha}_4 = 0.3361$</td>
</tr>
<tr>
<td>Net bond asset</td>
<td>$\tilde{\alpha}_B = 0.3361$</td>
</tr>
</tbody>
</table>

5.3 Asymmetry in asset menu

In our model, we assume that both countries can issue equities and bonds. However, due to financial underdevelopment and high risk of default in emerging markets, international bonds that are frequently transacted are those issued by advanced economies. In this subsection, let us consider the situation where the asset menu offered by the two countries in international financial market is asymmetric. Specifically, suppose that the home (developing) country can only issue home equity while the developed country can issue both foreign equity and a bond. The specification of other aspects of the model is the same as before. Under the current fully asymmetric parameterization, the optimal country portfolios is computed and displayed in Table 2.5. By this result, the home country sells the home equity to the amount of 0.44 (multiplied by $\beta Y$) and buys the foreign equity to the amount of 0.11, which, again, implies a negative net position of equity $\tilde{\alpha}_E = -0.34 < 0$. At the same time, the home country buys the foreign bond to the amount of 0.34. The home bond being absent, this also implies the net position in bonds of the same volume $\tilde{\alpha}_B = 0.34 > 0$. By the same argument, the reverse pattern of net asset positions will be seen in the foreign country. As in the last subsection, with the asymmetric asset menu, a two-way capital flow between the two countries persists. Moreover, because net positions of equities and bonds are both higher (in absolute value) than before, the asymmetry in fact strengthens the pattern of capital flows.
6 Conclusion

There is a noticeable heterogeneity in country’s asset composition of the gross flows and positions. In the literature, this is documented as the pattern of “short equity, long bond” in (many) developing countries and “long equity, short bond” in developed countries. We present an international macroeconomic model of both equity and bond portfolios in this paper. It shows that the presence of a selection of empirically relevant asymmetries between two countries can generate such a pattern of capital flows.

We find that these asymmetries include those related to industrial structure, severity of nominal rigidities, trade openness, consumption home bias, investment adjustment frictions, monetary policy stance, market competitiveness and pricing strategy of international trade, etc. In particular, for the two-way capital flows to happen, it is found that this can be the case if the developing country relies on more labour intensive technology to produce, or/and is more dependent on international trade, or/and features less local goods preference, or/and faces a relative low cost of investment adjustment, or/and is less focused on inflation stabilization while more focused on stabilization of the output gap when conduct monetary policy, or/and has a less competitive goods market. We also find that the factors from the real side of economy are more important than those from the nominal side. With the help of other empirical studies’ results of parameter estimation, the fully simulated model yields optimal portfolio holdings that are broadly consistent with the pattern of two-way capital flows. Moreover, if we assume that international bonds can only be issued by the developed country (as it is often the case in reality) the result is strengthened.

The paper highlights the role of correlation and variability effects in understanding the size of gross positions of certain types of asset which have particular importance in driving two-way capital flows. The correlation effect reflects how relevant the asset is in hedging risks while the variability effect reflects how much the amount of risk exposure is for the asset to hedge against. It turns out that the size of portfolio holdings are increasing in both of the effects.

The contribution of this work is at least threefold. Firstly, we use an open economy model with full-fledged New Keynesian features and endogenous portfolio choices. This framework is very general and obviously convenient to be modified for the purposes of understanding many other international macroeconomic issues where the presence of dis-
tinct country portfolios is required. Secondly, we identify a selection of factors that matter in accounting for heterogeneous asset composition. This is not only useful for explaining the two-way capital flows between developing and developed countries. As an example, the patterns of international capital flows within the group of developed countries or that of developing countries can be explored. Lastly, we make use of the recent estimation of structural parameter values that are based on the data of the U.S. and China when simulating our model. The results contribute to the related literature on emerging markets, especially China.
Appendix

A Price setting in the final goods sector (LCP)

In this section, we show how optimal prices are chosen in the final goods sector. The case of the traded sector is considered while the case of non-traded sector can be obtained similarly. Besides, the case of LCP is considered while the case of PCP can be obtained by removing $S$ from the profit function and then following similar derivations.

The firms’ problem has been described by Eqs.(2.35 – 38) in the main text. The related Lagrangian function for the problem is

$$\Lambda_t = E_t \sum_{i=0}^{\infty} \Omega_{t+i} \lambda^i \left\{ \frac{D_{t+i}}{P_{t+i}} \left[ \frac{p_{Di}(z)}{P_{Di+i}} \left( \frac{P_{Di+i-1}}{P_{Di-1}} \right)^\omega \right] 1 - \varphi \frac{P_{Di+i}}{P_{Di-1}} \right\}$$

$$+ E_t \sum_{i=0}^{\infty} \Omega_{t+i} \lambda^i \left\{ \frac{D^*_t}{P_{t+i}} \left[ \frac{p_{Xi}(z)}{P_{Xi+i}} \left( \frac{P_{Xi+i-1}}{P_{Xi-1}} \right)^\omega \right] 1 - \varphi \frac{S_{t+i}}{P_{Xi+i}} \right\}$$

First-order condition with respect to $p_{Di} (z)$ is

$$\frac{\partial \Lambda_t}{\partial p_{Di} (z)} = E_t \sum_{i=0}^{\infty} \Omega_{t+i} \lambda^i \left( 1 - \varphi \right) \frac{D_{t+i}}{P_{t+i}} \left[ \frac{p_{Di}(z)}{P_{Di+i}} \left( \frac{P_{Di+i-1}}{P_{Di-1}} \right)^\omega \right] ^{-\varphi} \frac{P_{Di+i-1}}{P_{Di-1}}$$

$$+ E_t \sum_{i=0}^{\infty} \Omega_{t+i} \lambda^i \varphi \frac{D_{t+i}}{P_{t+i}} \left[ \frac{p_{Di}(z)}{P_{Di+i}} \left( \frac{P_{Di+i-1}}{P_{Di-1}} \right)^\omega \right] ^{-\varphi} \frac{q_{t+i}}{P_{Di+i}} \left( \frac{P_{Di+i-1}}{P_{Di-1}} \right)^\omega$$

$$= 0$$

so

$$E_t \sum_{i=0}^{\infty} \Omega_{t+i} \lambda^i \frac{D_{t+i}}{P_{t+i}} \left( \frac{P_{Di+i-1}}{P_{Di-1}} \right)^\omega \left\{ (\varphi - 1) \left[ \frac{p_{Di}(z)}{P_{Di+i}} \left( \frac{P_{Di+i-1}}{P_{Di-1}} \right)^\omega \right] ^{-\varphi} - \frac{q_{t+i}}{P_{Di+i}} \left( \frac{P_{Di+i-1}}{P_{Di-1}} \right)^\omega \right\} = 0$$

so

$$E_t \sum_{i=0}^{\infty} \Omega_{t+i} \lambda^i \frac{D_{t+i}}{P_{t+i}} \left( \frac{P_{Di+i-1}}{P_{Di-1}} \right)^\omega \left\{ (\varphi - 1) \left[ p_{Di}(z) - \frac{q_{t+i}}{P_{Di+i}} \left( \frac{P_{Di+i-1}}{P_{Di-1}} \right)^\omega \right] ^{-\varphi} - \frac{q_{t+i}}{P_{Di+i}} \left( \frac{P_{Di+i-1}}{P_{Di-1}} \right)^\omega \right\} = 0$$

Rearranging the equation, one obtains Eq.(2.39).

Similarly, first-order condition with respect to $p_{Xi} (z)$, $\frac{\partial \Lambda_t}{\partial p_{Di} (z)} = 0$, leads to Eq.(2.40).

We omit the derivations here.
References


