Statistical Emulation of Cardiac Mechanics: 
An Important Step towards a Clinical Decision Support System

Dirk Husmeier1, Alan Lazarus1, Umberto Noè2, Vinny Davies3, Agnieszka Borowska1, 
Benn Macdonald1, Hao Gao1, Colin Berry4 and Xiaoyu Luo1

1School of Mathematics and Statistics, University of Glasgow 
University Place, Glasgow G12 8SQ, United Kingdom 
dirk.husmeier@glasgow.ac.uk, a.lazarus.1@research.gla.ac.uk, agnieszka.borowska@glasgow.ac.uk, 
benn.macdonal@glasgow.ac.uk, hao.gao@glasgow.ac.uk, xiaoyu.luo@glasgow.ac.uk 
2German Centre for Neurodegenerative Diseases (DZNE), 
Sigmund-Freud-Straße 27, 53127 Bonn, Germany. 
umberto.noe@dzne.de 
3School of Computing Science, University of Glasgow 
18 Lilybank Gardens, Glasgow G12 8RZ, United Kingdom 
vinn.y.davies@glasgow.ac.uk 
4BHF Glasgow Cardiovascular Research Centre, University of Glasgow 
University Place, Glasgow G12 8TA, United Kingdom 
colin.berry@glasgow.ac.uk

Extended Abstract

In recent years we have witnessed substantial advances in the mathematical modelling of the biomechanical processes 
underlying the dynamics of the cardiac soft-tissue. In particular, in our recent work [1] we have demonstrated that the 
parameters underlying the biomechanical model have diagnostic value for prognosticating the risk of myocardial infarction 
(heart attack). However, determining the unknown parameters is computationally challenging, with the optimization process 
taking days or weeks to converge, even with a modern multi-core workstation. The primary reason for this is the high 
computational expense of simulating from the biomechanical model, which requires a numerical integration of the underlying 
partial differential equations with finite element discretization. This procedure has to be repeated hundreds or thousands of 
times during the iterative optimization of the material parameters. As a result of these high computational costs of simulating 
the biomechanical model, estimating myocardial properties using a process which uses this model as a simulator is not 
suitable for real-time clinical diagnosis.

A potential approach to overcome this problem is statistical emulation [2], which has recently been explored in the 
closely related contexts of cardiovascular fluid dynamics [3] and ventricular mechanics [4]. Emulation methods are far more 
computationally efficient as most of the computation can be done in advance, making the in-clinic diagnosis faster. With 
emulation approaches, we simulate a large number of samples at different parameter specifications in advance and use these 
simulations combined with an interpolation method to replace the computationally expensive simulator in the optimization 
procedure.

The work presented here is designed as a proof of concept study to assess the accuracy of alternative emulation strategies 
with Gaussian processes (GPs) [5] for learning the material properties of the left ventricle of the heart based on only non-
invasive, in vivo magnetic resonance image (MRI) data. We compare different emulation strategies, loss functions and 
methods for dealing with high output dimensions. Our results can be summarized as follows.

Emulation strategies: We have compared output emulation, where the outputs of the mathematical model are emulated 
directly, with loss emulation, where we emulate the loss function that quantifies the agreement between the mathematical 
model and the data. Our simulations show that output emulation consistently outperforms loss emulation.

Gaussian process paradigm: For large data sets, it is not computationally feasible to train a GP, as the computational 
complexity is of the order of the third power of the data set size. Two paradigms for dealing with this issue have widely been 
used: sparse GPs [6] and local GPs [7]. Our results suggest that local GPs consistently outperform sparse GPs.
Dealing with high-dimensional outputs: We show that modelling high-dimensional outputs with a multivariate-output GP gives better results than using several univariate output GPs, and we quantify the performance improvement achieved in this way.

Computational cost reduction: Most importantly, we have achieved a dramatic reduction of the computational complexity. While conventional parameter estimation based on numerical simulations from the cardiac mechanical model, following the approach described in [1], leads to computational costs in the order of weeks, the proposed emulation method reduces the computational complexity to the order of the quarter of an hour, while effectively maintaining the same level of accuracy.

This is an important step towards a clinical decision support system that can assist a clinical practitioner in real time.

Acknowledgements
This work was funded by the UK Engineering and Physical Sciences Research Council (EPSRC), grant number EP/N014642/1. Dirk Husmeier is supported by a grant from the Royal Society of Edinburgh, award number 62335.

References