The Term Structure of Option-Implied Volatility and Future Realized Volatility

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ABSTRACT: We extract the short-, medium-, and long-term factors from the term structure of the option-implied volatility (OIV) of the S&P 500, the FTSE 100, and the Chinese 50 Exchange-Traded Funds (ETF), using an extension of the Nelson-Siegel (N-S) model and use estimated factors to predict future realized volatility (FRV) in the US, UK, and Chinese markets. Several interesting findings emerged from our study. First, we confirmed that the VIX is more informative than historical realized volatility (HRV) in predicting FRV. Second, we find that the volatility term structure contains some additional information compared with the VIX and HRV. Third, we verify that the three factors extracted from the N-S model are strongly cointegrated, related to volatilities. Moreover, based on the normalized error term of the cointegrated pairs, we construct straddles and delta-hedging option trading strategies. Without taking transaction costs into account, the straddle call trading strategy achieves a mean return of 37.59% monthly, and, at the same time, the exponential cumulative returns for the straddle call strategies are 4.2411 at a threshold of 1.1 in the S&P 500. As the threshold increases, the volume of transactions declines, leading to a fall in cumulative mean returns.

KEYWORDS: delta hedging strategy, future realized volatility, Nelson-Siegel model, option implied volatility, straddle trading strategy

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Introduction

Empirical studies on the term structure of option-implied volatility (OIV) demonstrate its usefulness in forecasting future realized volatility (FRV) (Stein, 1989; Diz & Finucane, 1993; Poteshman, 2001; Busch et al., 2011; Qian et al., 2014). Theoretically, every Treasury bond has a corresponding yield upon maturity while every equity option has a corresponding implied volatility. Hence, Treasury bonds can construct a term structure of interest rates while stock options can build a term structure of implied volatility. Moreover, the shape of the term structure of interest rates reflects the market's expectation of future average interest rates in different periods while that of an implied volatility term structure represents the future average implied volatility across different maturities from a market perspective.

Motivated by the similarities between the term structure of interest rates and the OIV, we attempt to bridge the modeling of interest rates and the OIV via the Nelson-Siegel (N-S) model. By reversing the Black-Scholes model with the market option price, the OIV that corresponds to the different expiration date and strike price can be calculated. On this basis, the term structure of implied volatility can be obtained by interpolation. We select the at-the-money (ATM) volatility term structure and calibrate some known points on the volatility term structure using the Nelson-Siegel model. So, we obtain the estimated value of short-, medium-, and long-term OIV. The N-S model extracts the information on future volatility implied by options according to the time length, so the three factors are a special form of OIV.

The forecasting of future realized volatility (FRV) has been widely studied in the literature (Byoun, Kwok, & Park, 2003; Chalamandaris et al., 2001; Kemna et al., 1994). The OIV can be regarded as a benchmark for forecasting FRV. Although the predictability of implied volatility to FRV is different at different stages, they still have a high correlation.

Nelson and Siegel (1987) first proposed a parametrically parsimonious model to represent the shapes of yield curves. They empirically show that 96 percent of the variation in US Treasury bond yields can be explained by their model. Diebold and Li (2006) extend Nelson and Siegel's (1987) framework to distill the entire yield curve into three factors: a short-term factor that represents trends in constantly changing financial market conditions or the degree of concern among investors; a medium-term factor that can be regarded as financial market default risk; and a long-term factor that captures macroeconomic variables. This finding further enhances the validity of the N-S model connected with economic explanations. Moreover, Guo, Han, and Zhao (2014) indicate the N-S model has better performance in forecasting OIV than a deterministic OIV function and restricted two-factor model. Their results indicate that short-and long-term factors are both highly related to the VIX. Based on extant findings, they explore whether the information extracted from implied volatility can forecast the volatility of stock index futures. Scharfstein and Stein (1993) use S&P 100 index options to test a model in order to capture the relationship between implied volatilities with different maturities. They find that the implied volatility followed a mean-reversion process in the long term. Moreover, their results show that long-term volatilities, rather than short-term volatilities, tended to overreact. In addition, Poteshman (2001) and Mixon (2007) argue that the option market has three tendencies. First, option market investors had an obvious lag in tracking and judging the information from daily trading. Second, as stated by Diz and Finucane (1989), investors tended to underestimate the impact of market information on option prices, leading to inefficient execution. Third, when investors misunderstand prior market information, more aggressive or more conservative remedies are adopted, resulting in higher volatility in the options market. The evidence presented above suggests that a single-factor model fails to capture the variation in volatility term structure. Guo, Han, and Zhao (2014) decompose the implied volatility into three components using the N-S model: short-, medium-, and long-term volatilities. They empirically show that these three components of volatility are highly correlated with the level, slope, and curvature of the implied volatility term structure. Their results also indicate that long-term volatility is affected by macroeconomic financial policy; the VIX, which reflects investor sentiment, has significant explanatory power for short-term volatility; medium-term volatility is highly correlated with default risk in financial markets. In sum, related studies in the literature show that the N-S model can be used not only for modeling the term structure of interest rates but also for modeling the term structure of implied volatility. Unlike previous studies, this paper comprehensively investigates the predictive power of the factors extracted from the N-S model in forecasting FRV.

We evaluate our N-S model against spline methods in several ways. First, the spot rate curve has different shapes at different times, including monotonically increasing, monotonically decreasing, humped, and S shapes. Second, this model is more accurate and more flexible in describing the entire yield curve. Third, the model has a better fit for short- and long-term structures because the short-term structure depends on changes in the medium and long term. Moreover, the yield curve of spot rates fitted by the N-S model is a continuously smooth curve, which can accurately depict interest rate curve changes in a variety of shapes, and 96 percent of the variations in a yield curve can be captured by the N-S model.

Furthermore, we investigate the relationship between OIV and FRV and then examine the predictive power of OIV to FRV. For comparison, Christensen and Prabhala (1998) use lower-frequency data (one month) than previous research and extend the data acquisition time. The conclusion shows that the predictability of implied volatility is not independent of other factors, consistent with historical realized volatility (HRV), in predicting FRV. Furthermore, they evaluate the performance of implied volatility forecasts both before and after the financial crisis (1987 stock market crisis). They find that, although the stock market crisis led to higher OIV, the high OIV did not result in high FRV in the stock market. According to their research, after the stock market crisis, the accuracy of predictability of implied volatility improved. Moreover, the effectiveness of the implied volatility of the Black-Scholes model was proved by empirical tests by the author, rather than by quoting option prices (Merton, 1974).

The advantages of our method with the traditional option pricing models are we have two steps in data dealing, the first step is using the traditional option pricing model such as Black – Scholes model to obtain the implied volatility at each strike price and each date, but the problem is the grid density of data acquisition points and those points can't be ascribed to one or two parameters. So, we apply the second step, we employ N–S model to extract the less figures (points) to represent short-, medium-, and long-term factors to capture option implied volatility to avoid the calibration risks (parametric instability) of traditional models such as Merton model and Stochastic volatility model etc. Not only that, the traditional option pricing models are just method to measure the parameters, which does not provide the predictive power to future realized volatility.

This paper empirically investigates four questions. First, we explore whether estimated parameters extracted from the N-S model on an implied volatility term structure can provide valuable information for predicting FRV in the S&P 500 and the FTSE 100. Second, we examine whether these extracted parameters contain more information than the VIX in predicting FRV and stock index performance. Third, we construct straddle and delta-hedged trading strategies to calculate monthly returns. Fourth, we explore whether the N-S model can be applied to China's 50 Exchange-Traded Funds (ETF) and forecast FRV.

The paper is organized as follows: Section 2 provides a brief review of previous studies on modeling the term structure of OIV and forecasting FRV. Section 3 introduces the Nelson-Siegel model as well as its extension. Section 4 introduces the data and fits the extension of the N-S model. Based on the extension of N-S model and the term structure of OIV, section 5 investigates the statistical and economic significance of three estimated factors in predicting FRV. Section 6 summarizes our main findings.

Methodology

The mechanism of the original N-S model (1987) and its extension (2006) simulates entire surface of the curve, while the spline function simulates the curve at a different time interval. The N-S model plays an irreplaceable role in modeling the term structure of interest rates.

$$r(m) = \beta_0 + \beta_1 \cdot e^{-\frac{m}{\tau}} + \beta_2 \cdot [(\frac{m}{\tau}) \cdot e^{\frac{-m}{\tau}}] \text{ (1)}$$

where $r(m)$ is the forward interest rate, m is time to maturity, β_0, β_1, and β_2 are parameters associated with the initial situation, and τ is the time constant. Furthermore, the average forward interest rate, denoted $R(m)$, can be calculated by integrating $r(x)$ from 0 to m and dividing by m.

$$R(m) = \frac{1}{m}\int_0^m r(x)dx \text{ (2)}$$

Then $R(m)$ can be represented as:

$$R(m) = \beta_0 + (\beta_1 + \beta_2) \cdot \frac{1-e^{-\frac{m}{\tau}}}{\frac{m}{\tau}} - \beta_2 \cdot e^{-\frac{m}{\tau}} \text{ (3)}$$

Diebold and Li (2006), based on the N-S model, attempt to use β_0, β_1, and β_3 to predict the yield curve and then propose a more parsimonious model:

$$f(\tau) = \beta_1 + \beta_2 \cdot e^{-\lambda\tau} + \beta_3\lambda \cdot e^{-\lambda\tau} \text{ (4)}$$

where $f(\tau)$ is a function of the forward interest rate, λ is the attenuation rate of the function, and τ is a different time point. Then the average interest rate function can be calculated by integrating $f(\tau)$ and dividing by τ.

$$y = \beta_1 + \beta_2\frac{1-e^{-\lambda\tau}}{\lambda\tau} + \beta_3(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}) \text{ (5)}$$

β_0, β_1, and β_2 represent different economic implications, and both the yield curves of forward interest rates and spot interest rates are affected by these three parameters. The first load of β_0 be a long-term factor, and it would not decay to zero in the limit. The second load associated with β_1, $(1 - e^{-\lambda\tau})/\lambda\tau$, is a function driven by λ and τ, with monotonically decreasing and continuous decays from 1 to 0. Due to the short duration of the effect on β_1, which tends to decrease, β_1 can be used as a short-term factor. The third load associated with β_2 is a function also driven by λ and τ. It starts at 0, then the value of the function increases gradually, before gradually decreasing to 0. In addition, the effect of β_2 is between that of β_0 and β_1 and similar to curvature. It has a weak impact on both the long- and short-term interest rate yield curves while increasing the medium-term interest rate yield curve. β_0, β_1, and β_2 are long-, short-, and medium-term factors, respectively.

As in the analogy stated above, modeling the term structure of OIV is a natural extension of the N-S model. β_0, β_1, and β_2 can be understood as playing the same role of level, slope, and curvature in the term structure of interest rates.

Based on a single-volatility model, Park (2011) and Stein (1989) construct a two-volatility-factor model to obtain better capacity and better fit prediction accuracy. Assuming instantaneous volatility, σ_t follows a continuous-time mean-reversion process,

$$\mathrm{d}\sigma_t = -\alpha(\sigma_t - \bar{\sigma})\,\mathrm{d}t + \beta\sigma_t\varepsilon\sqrt{\mathrm{d}t} \quad (6)$$

where $\varepsilon \sim \emptyset(0, 1)$, α and β are coefficients of the continuous-time mean-reversion process, $-\alpha(\sigma_t - \bar{\sigma})$ is the drift rate, and $\beta\sigma_t$ is the variance rate. At time $t + j$, the expectation of σ_{t+j} can be expressed as

$$E_t(\sigma_{t+j}) = \bar{\sigma}_t + \rho^j(\sigma_t - \bar{\sigma}) \quad (7)$$

where ρ^j is the correlation between σ_t and $\bar{\sigma}$. The average OIV is denoted $i_t(T)$, which means that the observation time of an option is t, and T determines the time to maturity. Then the OIV can be expressed as:

$$i_t(T) = \frac{1}{T}\int_{j=0}^{T}[\bar{\sigma} + \rho^j(\sigma_t - \bar{\sigma})] \quad (8)$$
$$= \bar{\sigma}_t + \frac{\rho^T - 1}{T\ln\rho}[\sigma_t - \bar{\sigma}]$$

According to this equation, when $\rho^j = \mathrm{e}^{-\alpha}$, the functional form is similar to that of the N-S model, which is

$$i_t(T) = \bar{\sigma}_t + \frac{1 - e^{-\alpha T}}{\alpha T}(\sigma_t - \bar{\sigma}) \quad (9)$$

where α is the speed of the mean-reversion process. The correlation between OIV and $(\sigma_t - \bar{\sigma})$ is negative on the first derivative and positive on the second derivative.

Although Park (2011) and Stein (1989) prove the feasibility of a two-volatility-factor model for term structure, they still have problems in simulating the shape, for example, of humps. To overcome this limitation, Christoffersen et al. (2008) introduce a useful extension in modeling OIV, which divides the mean-reversion process of σ_t into two sub-mean-reversion processes

$$\mathrm{d}\sigma_t = -\alpha(\sigma_t - \bar{\sigma})\,dt + \beta\sigma_t\varepsilon\sqrt{dt} \quad (10)$$
$$\mathrm{d}\bar{\sigma}_t = -\kappa(\sigma_t - \bar{\bar{\sigma}})\,dt + \xi\bar{\sigma}_t\varepsilon\sqrt{dt} \quad (11)$$

Accordingly, the sub-mean-reversion process of $\mathrm{d}\sigma_t$ reverts to the first stage of volatility $\bar{\sigma}_t$. At time $t + j$, the expectation of σ_{t+j} can be expressed as:

$$E_t(\sigma_{t+j}) = \mathrm{E}(\bar{\sigma}_{t+j}) + \rho^j[\sigma_t + E(\bar{\sigma}_{t+j})] \quad (12)$$

$$E_t(\bar{\sigma}_{t+j}) = \bar{\bar{\sigma}}_t + \tau^j(\sigma_t - \bar{\bar{\sigma}}_t) \ (13)$$

where ρ^j is the correlation between σ_t and $\bar{\sigma}$, τ^j is the correlation between $\bar{\sigma}_t$ and $\bar{\sigma}$, and both of them are less than 1. The functional form is:

$$i_t(T) = \frac{1}{T}\int_{j=0}^{T}[\bar{\bar{\sigma}}_t + \rho^j(\sigma_t - \bar{\bar{\sigma}}_t) + \rho^j\sigma_t - \tau^j(\sigma_t - \bar{\bar{\sigma}}_t) - \bar{\bar{\sigma}}_t]dj \ (14)$$

$$= \bar{\bar{\sigma}}_t + \frac{\rho^T - 1}{T\ln\rho}[\sigma_t - \bar{\bar{\sigma}}_t] - \rho^T(\sigma_t - \bar{\bar{\sigma}}_t)$$

When $\rho = \mathrm{e}^{-\alpha}$, the OIV can be expressed as

$$i_t(T) = \bar{\bar{\sigma}}_t + \frac{1-e^{-\alpha T}}{\alpha T}(\sigma_t - \bar{\sigma}) - \mathrm{e}^{-\alpha \mathrm{T}}(\sigma_t - \bar{\bar{\sigma}}_t) \ (15)$$

Finally, β_0, β_1, and β_2 can be used to replace $\bar{\bar{\sigma}}_t$, $(\sigma_t - \bar{\sigma})$, and $(\sigma_t - \bar{\bar{\sigma}}_t)$, and the final functional form is

$$i_t(T) = \beta_{0t} + \beta_{1t}\frac{1-e^{-\alpha_t\tau}}{\alpha_t\tau} + \beta_{2t}(\frac{1-e^{-\alpha_t\tau}}{\alpha T} - e^{-\alpha T}) \ (16)$$

$$= \beta_{0t} + \beta_{1t}\frac{1-e^{-\lambda\tau}}{\lambda\tau} + \beta_{2t}(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau})$$

In $i_t(T)$, the observation time is *t,* and *T* is the remaining time to maturity. λ determines the exponential decay rate and the decay rate of β_1 and β_2. If the value of λ is small, the decay rates of β_1 and β_2 tend to be slow. At the same time, the decay rate of the short- and medium-term factors accelerates, which leads to a more accurate fit on the long-term yield curve.

(Insert Table 1 here)

(Insert Figure 1 here)

As described above regarding the N-S model, this process not only proves the feasibility of the N-S model for estimation of the OIV but also provides the basis for estimation of the model parameter values. Moreover, the sign of the coefficients plays a decisive role in the shape of interest rate yield curves.

Data

We collect the index options data on the S&P 500 and the FTSE 100 from the Option Metrics Ivy database by Wharton Research Data Services (WRDS). The Chinese 50 ETF data are from the Wind database. We use the daily options data for the S&P 500 and the FTSE 100, from February 2, 2001, to April 29, 2016, and January 2, 2002, to January 23, 2018, respectively. Chinese 50 ETF data are from November 11, 2014, to October 20, 2017.

To obtain reliable and less mismatched data from the market for data calibration, we apply four screening criteria. First, the exercise style is European, which eliminates the biases in the Black-Scholes model implied volatility inferred from American options. Second, the expiration date of the option is set at 30, 60, 91, 122, 152, 182, 273, 365, 547, and 730 days respectively. Third, delta (δ) with various maturity dates should equal 50 to ensure they are at the money (ATM). Fourth, the mean of the closing bid and ask prices is calculated as a daily options price.

To match the daily trading data in these three markets, we extracted the corresponding volatility indices from these three countries respectively. Specifically, the VIX measures the expected stock implied volatility of the S&P 500, so we obtained VIX data from the WRDS with daily frequency from January 2, 1990, to August 2, 2017, consisting of 6,951 data points to match the S&P 500. In the UK and Chinese markets, we use the VFTSE (FTSE 100 volatility index) and the IVIX (50 ETF VIX) to measure investors' fear in the FTSE 100 and the 50 ETF markets respectively. The time span of those two indices is also matched with the daily market transaction data.

The realized volatilities of HRV and FRV are calculated based on the log returns of the daily closing price on the three indices. Normally, there are 252 trading days per year and 22 trading days per month. For instance, assuming that the observation time is January 3, 2001, FRV is the standard deviation of the yield between January 3, 2001, and February 2, 2001 (after 21 trading days). Similarly, if the observation time is September 1, 1987, the calculation of HRV is the standard deviation of the yield between September 1, 1987, and August 3, 1987 (21 trading days earlier).

(Insert Table 2a & Table 2b here)

Panel A in Tables 2a and 2b indicates the mean, maximum, minimum, standard deviation, and autocorrelation of implied volatility in the S&P 500 and the FTSE 100. On average, an increase in maturity leads to an increase in implied volatility. In Table 2a, the mean of implied volatility with a 30-day maturity is 0.1836, while that with a 730-day maturity is 0.2000. The Vega risk is higher in options with longer maturity. More specifically, the maximum value and standard deviation of implied volatility tend to decrease with increasing maturity in the S&P 500 and the FTSE 100. The last four columns show the autocorrelation of different maturities: 10, 30, 60, and 180 days. It shows a gradual decline with an increase in the length of maturity.

$$slope = y(365) - y(30) \quad (17)$$

$$curvature = y(122) - 0.5 * [y(30) + y(365)] \quad (18)$$

We define the level as the 365-day implied volatility. The slope can be defined as the difference in implied volatility between 365 and 30 days, and the curvature can be calculated as 122-day implied volatility minus 0.5 times the sum of 30- and 365-day implied volatility. Additionally, the value of the level suggests relative stability compared to the other factors. With increasing maturity, the curve of implied volatility slopes upward. The negative curvature shows a hump in the term structure.

The implied volatility curve can be fitted by the N-S model,

$$I(\lambda, \tau) = \beta_{0t} + \beta_{1t}\frac{1-e^{-\lambda\tau}}{\lambda\tau} + \beta_{2t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) \quad (19)$$

(Insert Figure 2a & Figure 2b here)

(Insert Table 3a & Table 3b here)

Figures 2a and 2b show the three estimated parameters of the N-S model in the S&P 500 and the FTSE 100 over four different time intervals. The performance of β_0 is relatively stable in the S&P 500 index, even during the economic crisis, as it represents a long-term factor. β_1 is a short-term factor, which is an indicator of market risk and the fear index. It is more volatile than the other two factors, especially during the economic crisis during 2007-2009. In Tables 3a and 3b, λ is a variable that changes over time, and τ is the remaining time to maturity. In Table 3a, the mean of β_0 (0.2047) is similar to the mean in the data from February 2, 2001, to April 29, 2016. It is clear that the autocorrelations of β_0 with different time displacements are larger than those for β_1 and β_2. The correlations of different pairs (β_0 and β_1, β_0 and β_2, β_0 and λ, β_1 and β_2, β_1 and λ, β_2 and λ) are also reported in Panel B.

Panel C in Tables 3a and 3b shows that the null hypothesis is that a unit root exists, which means the regression equation is unstable over time. After the first-order difference, β_1 in the S&P 500 is stationary, while in the FTSE 100, all three-parameter series are stationary. The correlation coefficient of lagged items gradually decreases.

Figure 3a reports their correlations. β_1 and β_0 can be used for predicting short- and long-term factors in the S&P 500 index. The figure shows larger volatility around 2000 in the x-axis, indicating the 2008 economic crisis, which greatly affected short-, medium-, and long-term factors.

(Insert Figure 3a & Figure 3b here)

(Insert Table 4a & Table 4b here)

Tables 4a and 4b report the correlation coefficients between parameters and corresponding significance levels. Specifically, for the US market, the relationship between β_0 and level is strongly positive (0.9371), the correlation between β_2 and curvature (0.3793) is higher than slope (-0.2355) and level (0.3126), while β_1 and slope shows a strong negative relationship (-0.9789). All the correlations are significant at 1% level. Regarding the UK market, results are consistent with the correlations in the US market. For instance, the correlations between β_0 and level is 0.9371, β_2 and curvature is -0.3144 which is also higher than other parameters, while β_1 and slope shows a strong negative relationship (-0.8859) slightly lower than US market (-0.9789). Since the difference of the attenuation rate λ and the different time point τ lead to short-term factor represented by slope corresponding to β_1 in the N-S model, long-term factor represented by level corresponding to β_0 in the N-S model and medium-term factor represented by curvature corresponding to β_2 in the N-S model. All the correlations are significant at 1% level, except the one between β_2 and level.

Moreover, compared to the VIX volatility in the two markets, β_0 tends to be stable and smooth. **Its mean is 0.2047 (Table 3a, Panel A).** Additionally, the correlation between the VIX and β_{0t} is 0.7546 (Table 4a); thus, β_0 captures most of the VIX trend and volatility. The correlation between β_2 and the VIX is 0.3207. The highest correlation is between β_1 and the VIX, 0.8431, which shows that the two variables are highly positively related. In summary, a significant correlation is found between β_1 and the VIX

(Insert Figure 4a & Figure 4b & Figure 4c here)

Compared with the US market, the Chinese 50 ETF does not have ideal data for the N-S model in Figure 4c. Specifically, the correlations are: 0.1356 between β_1 and the VIX, 0.1351 between β_2 and the VIX, and 0.7828 between β_3 and the VIX. Because of data unavailability of Chinese 50 ETF, suitable data for the N-S model are not available.

Empirical Results

This section discusses the results of our three empirical tests. First, we test the relationship

among FRV, the VIX, and HRV. Second, three parameters (*β_0, β_1, β_2*) estimated from the N-S model on the OIV term structure are applied in predicting FRV. Finally, we test the pair-wise cointegration between the three parameters, the VIX, HRV, and the slope/curvature, all the parameters have been adjusted by the Newey-West test to eliminating heteroscedasticity and autocorrelation.

Forecast Stock FRV with the VIX and HRV

We explore the relationship among FRV, the VIX, and HRV, using three regressions, in which FRV is regarded as a dependent variable while the VIX and HRV are independent variables, as follows:

$$FRV_t = \alpha + \beta_t * VIX_t + \varepsilon_t \; (20)$$

$$FRV_t = \alpha + \beta_t * HRV_t + \varepsilon_t \; (21)$$

$$FRV_t = \alpha + \beta_1 * VIX_t + \beta_2 * HRV_t + \varepsilon_t \; (22)$$

The regression results are shown in Tables 5a and 6a, in which the t-statistic of the VIX in Panel A is 6.3110 while that of the constant is -7.0710. The adjusted R^2 of the VIX and HRV are 60.17% and 50.43% respectively. The VIX is more effective in predicting FRV in the FTSE 100. In Table 6a, Panel A, the adjusted R^2 of the VIX and HRV are 76.87% and 56.17% respectively Thus, we conclude that the VIX has more information on variations in FRV than HRV. The adjusted R^2, combining the VIX and HRV, increases by 0.71 and 0.025 percent compared with that of using only the VIX in the S&P 500 and the FTSE 100 respectively.

Table 5b shows the results of AIC and BIC analysis, and the regression of FRV with the VIX and HRV has a minimum value according to both the AIC and the BIC. The results indicate that the combination of the VIX and HRV can provide a more accurate prediction of variations in FRV.

(Insert Table 5a & Table 6a here)

Compared with a regression using two single factors, combining the two factors can explain FRV more accurately, as shown by the increase in the adjusted R^2 to 60.88% and 79.39%. In summary, the majority of information on future volatility is provided by the VIX, and HRV provides additional information for estimating FRV.

Forecast Stock FRV with LTF_t (β_0), STF_t (β_1), and MTF_t (β_2)

Regression equations of FRV on LTF_t (β_0), STF_t (β_1), and MTF_t (β_2) are fitted to examine their relationship and to test the potential predictive power of these three factors on FRV. In the following equations, LTF_t (β_0) is a long-term factor, STF_t (β_1) is a short-term factor, and MTF_t (β_2) is a medium-term factor, the coefficients have been adjusted by the Newey-West test. Tables 7a and 8a report the regression results.

$$FRV_t = \alpha + \beta_t * LTF_t + \varepsilon_t \ (23)$$

$$FRV_t = \alpha + \beta_t * STF_t + \varepsilon_t \ (24)$$

$$FRV_t = \alpha + \beta_t * MTF_t + \varepsilon_t \ (25)$$

$$FRV_t = \alpha + \beta_1 * LTF_t + \beta_2 * STF_t + \beta_3 * MTF_t + \varepsilon_t \ (26)$$

(Insert Table 7a & Table 8a here)

According to Table 7a, the coefficient of β_0 in Panel A is 1.3725 while that of β_1 is 1.1377. The p-values of β_0 and β_1 are both positively significant at the 1 percent level. Moreover, β_1 explains more variation in FHV than β_0, and the adjusted R^2s are 50.98% and 29.47%, respectively. The adjusted R^2 of β_1 is nearly twenty percentage points higher than that of β_0. Panel C shows *MTF* (β_2) has an adjusted R^2 of 4.68%, thus it is significant but can explain little of the variation in FHV. In Panel D, the combination of β_0, β_1, and β_2 provides a better explanation of the variation in FHV and is positively significant at the 1 percent level, with an adjusted R^2 of 61.96%. In Table 8a, the result is consistent with Table 7a, so it is noteworthy that in Panel D, the adjusted R^2 is 77.78%, much higher than the S&P 500. Thus, a comparison of the four regressions shows that combining the three factors can provide more valuable information for predicting future volatility.

Comparing the results of Tables 7b and 8b, the results in the US and the UK markets are consistent. The short-term factor has better performance than the other factors in predicting FRV, because the increments of AIC and BIC are smaller than those of the Tables 5b and 6b, the number of explanatory variables was increased to improve the optimal fit. AIC encouraged the optimal data fit but avoided overfitting as much as possible. When all explanatory variables were regressed with FRV, these two regression variables had the minimum AIC and BIC values. The regression results on FRV using the VIX and HRV are generally better than those using three single estimated factors. The results imply that the predictability of FRV using the VIX and HRV is greater than that from using the three single estimated factors. However, the combination of the three estimated factors provides more valuable information on FRV than the combination of the VIX and HRV in the S&P 500 and the FTSE 100.

Forecasting Stock FRV with the VIX, HRV, and Estimated Factors Based on Extension of the Nelson-Siegel Model

$$FRV_t = \alpha + \beta_1 * VIX_t + \beta_2 * HRV_t + \beta_3 * STF_t + \varepsilon_t \quad (27)$$
$$FRV_t = \alpha + \beta_1 * VIX_t + \beta_2 * HRV_t + \beta_3 * LTF_t + \varepsilon_t \quad (28)$$
$$FRV_t = \alpha + \beta_1 * VIX_t + \beta_2 * HRV_t + \beta_3 * MTF_t + \varepsilon_t \quad (29)$$
$$FRV_t = \alpha + \beta_1 * VIX_t + \beta_2 * HRV_t + \beta_3 * STF_t + \beta_4 * MTF_t + \beta_5 * LTF_t + \varepsilon_t \quad (30)$$

(Insert Table 9a & Table 10a here)

As shown in Table 9a, Panel A indicates that the coefficients of the VIX, HRV, and the short-term factor are all significant at the 1 percent level, the coefficient 0.0698 of MTF(β2) should be significant at the 5% significance level. Panels B and C are similar to Panel A. Panel B adds the long-term factor to the regression with the VIX and HRV. Panel C presents the results of the three-factor regression for the medium-term factor, the VIX, and HRV. Additionally, its adjusted R^2 is 61.18%, which is a little lower than in Panel A. In Panel C, the adjusted R^2 is 61.02%. Panel D shows the regression results of stock FRV on the VIX, HRV, and the three estimated factors. The VIX does not predict FRV because the three estimated factors include the information in the VIX, resulting in its insignificance. We conclude that when the VIX is regressed separately from other factors, it provides sufficient explanatory power for predicting FRV. But when we regress the three factors extracted from N-S model with the VIX, the information provided by the VIX is replaced by the information in the three factors. However, the information provided by the three factors is limited.

The results in Table 10a are consistent with those in Table 9a as well. The results are generally better in the US market than in the FTSE 100: VIX provides enough information to predict future volatility. The difference between the two results is when the regress the three estimated factors with VIX, it is still significant in the UK market.

Combining the VIX, HRV, and the short-term factor can provide more information on FRV than without the short-term factor. The results indicate that the short-term factor can provide some useful information on forecasting FRV that is different from the VIX and HRV. In addition, combining the VIX, HRV, the long- and medium-term factor also provides additional information on FRV. Thus, we conclude that the information extracted from the three single estimated factors contributes less to predicting FRV than information from the VIX. The

VIX contains the most information on FRV.

Although the three factors extracted from the extension of the N-S model provide less information than the VIX in predicting FRV, it is undeniable that they contain different information from the VIX and HRV and still play a positive role in enhancing predictions of FRV.

Cointegration between Different Levels of Volatility

Factors extracted from the extension of the N-S model help improve the predictability of FRV. The mean-reversion relationship between different levels of volatility is tested in this paper. Moreover, the mean-reversion relationship can be applied as a mean-reversion trading strategy. A mean-reversion strategy is a trading activity that assumes that the price of an asset changes over time and eventually returns to its average level. In a trading strategy on cross-market information, the prices of the two assets are assumed to return eventually to their average level. Before final convergence, speculation can be profitable, if the trader buys low and sells high. Investors can apply this strategy to capture mispricing opportunities.

Cointegrated relationships between the three parameters and slope/level/curvature, as well as VIX and HRV, are tested. According to Balvers, Wu, and Gilliland (2000), the residual of each pair can be standardized by $\frac{\varepsilon_{i,t}-\mu_i}{\sigma_i}$, where $i = [1,2,3,4 \ldots \ldots N]$, and μ_i and σ_i are the mean and standard deviation of $\varepsilon_{i,t}$.

The augmented Dickey-Fuller (ADF) tests are conducted on all the factors in the previous section to predict FRV. The two parameters are cointegrated if ε_t is stationary. Specifically, if the spread between the VIX and HRV is cointegrated, then the spread between the VIX and HRV should be stationary over time. So, the ADF test is used to determine whether ε_t (the spread between two parameters) in a regression is stationary.

$$VIX_{i,t} = \beta_i * HRV_{i,t} + \varepsilon_{i,t}, i = [1, 2, 3, 4 \ldots \ldots N] \quad (31)$$

$$\varepsilon_{i,t} = \beta_i * HRV_{i,t} - VIX_{i,t}, i = [1, 2, 3, 4 \ldots \ldots N] \quad (32)$$

(Insert Table 11 here)

Residual standardization and the ADF test are carried out based on all the attributes. All pairs of factors are significantly cointegrated, which means all the factors complement one another. The trends in volatility and parameters extracted from the N-S model on volatility term structure are similar.

The Straddles and Delta-Hedging Trading Strategy

Straddles and delta-hedging option portfolios are constructed in the following three steps. First, the first trading day in each month is detected (Monday dominates). Second, two criteria are applied in selecting call options: the delta should be the nearest 0.5; the maturity date is the next month. We use delta at 0.5 because the modeling on the implied volatility term structure in this paper uses it for OIV. Then, the put option with the same maturity and strike price is selected to construct straddles. This reduces the total delta of the straddle to nearly 0.

Based on the threshold, if the standard Z-score (error term of the co-integrated difference between β_1 and the VIX) is higher than the threshold, then we will short the straddle or delta-hedging option portfolio; if the standard Z-score is lower than the negative value of the threshold, then we will long the straddle or delta-hedging option portfolio; if Z-score is between the negative value of the threshold and the threshold, we do not trade.

$$\varepsilon_{i,t} = \beta_{i,t} - \alpha_i * VIX_{i,t}, i = [1,2,3,4 \dots \dots N] \ (33)$$

Third, on the first trading day of the next month, we track the OptionID (from the Option Metrics Ivy database Wharton Research Data Services) for options if we traded the previous month to calculate the Straddle and Delta-hedging strategies returns and rebalance the portfolio according to step 2. Our trading sample comprises 180 months of options contracts. Specifically, we trade every month from January 2001 to December 2016.

(Insert Table 12a & Table 12b here)

Trading strategies for straddle and delta-hedged calls are compared, and these trading strategies have two characteristics. First, both have increases in mean returns as the thresholds increase. Without taking transaction costs into account, the straddle call trading strategy achieves a mean return of 37.59% monthly, and at the same time, the exponential cumulative returns for the straddle calls strategies are 4.2411 at a threshold of 1.1 in the S&P 500. Second, as the threshold increases, the volume of transactions declines, leading to a decline in cumulative mean returns. In addition, mean returns are higher for straddle calls than delta-hedged calls. The mean returns for straddle calls dramatically increase from 2.53 percent to

37.59 percent, then remain steady around 14 percent, while the mean returns of delta-hedged calls are around 0.4 percent monthly.

This is consistent with Guo et al. (2016), who find that information on OIV surfaces can help predict implied volatility. They apply an adapted N-S model, combining it with an AR (Autoregressive Model) model to construct delta-hedging option portfolios to trade volatility. Guo's model is predictive while the model in this paper is based on mean reversion.

Guo (2016) uses the cointegrated difference of CIV (CDS inferred volatility) and OIV to estimate FRV, whereas this paper applies the cointegrated difference of the short-term factor and the VIX to estimate FRV. A regression combining CIV, OIV, and HRV is more efficient in predicting FRV than a regression that uses the OIV and HRV of investment groups, junk investors, and a combination of all firms. The combination of the three estimated factors extracted from the extension of the N-S model can predict FRV with an adjusted R^2 of 61.96 percent. Moreover, after adding the VIX and HRV to the regression, the adjusted R^2 is 62.52 percent, which is clearly higher than the results from adding CIV, OIV, and HRV. In summary, the predictive ability of the three estimated factors is much greater from the N-S model than from the combination of CIV, OIV, and HRV. Another difference is that Guo (2016) uses firm-level data with different ratings to predict FRV, while this paper uses market-level data.

Our findings are consistent with the results in Guo et al. (2014). They use the fixed decay rate to fit the extension of the N-S model, whereas our model allows for a dynamic decay rate, which is similar to Barrett, Gosnell, and Heuson (1995) and Diebold and Li (2006).

Overall, the parameters extracted from the N-S model on implied volatility contribute to the predictions of future volatility. Even though the VIX and historical volatility can explain a majority of the variation in future volatility, the three parameters can explain 1.7 percent additional variation in future volatility. Thirty-day future volatility is driven mainly by short-term effects, following by long-term effects, with little impact on future volatility from medium-term effects.

Concluding Remarks

This paper predicts future volatility with more information extracted from the term structure of implied volatility. Our findings indicate that the three factors estimated from the term structure of OIV and using an extension of the Nelson-Siegel model can provide additional information for forecasting FRV of stock indexes.

We extract information from the implied volatility term structure to predict future volatility in stock indexes. Moreover, these extracted factors were shown to influence future fluctuations in stock indexes. The results also show that the VIX provides the majority of information for predicting FRV in these three extracted parameters.

We conduct empirical tests that yield six main findings. First, the VIX contains more information than historical volatility for predicting FRV in both the S&P 500 and the FTSE 100. Second, the short-term effect is highly related to FRV over 30 days, following by long- and medium-term effects. Third, the three parameters estimated from extension of the N-S model contain some valuable information that is different from the VIX and historical volatility in predicting FRV. Thus, combining the three parameters with the VIX and historical volatility will improve the predictive power of FRV more than using only the VIX and historical volatility. Fourth, a cointegrated relationship exists between volatility and the three parameters extracted from the N-S model. This means similar trends exist in volatility and the three parameters. Fifth, because of the cointegrated relationship between volatility pairs and the extracted parameters, we use logistic regression to examine predictions of one-month S&P 500 ups and downs using volatility information and find that more than half the volatility pairs are significantly related to market ups and downs, but this relationship is not linear because of the low adjusted R^2. Finally, based on mean reversion between β_2 and the VIX, the difference is the proxy for trading signals compared with a fixed threshold. Monthly rebalanced straddle and delta-hedging options portfolios are constructed using the S&P 500 and FTSE 100 options. When the threshold is increased, both options portfolios perform better. Specifically, straddle calls achieve their highest mean return (37.59%), while exponential cumulative returns for the straddle calls strategies are 4.2411 at a threshold of 1.1 in the S&P 500. Both straddle call and delta-hedging call option portfolios perform better with an increase in the threshold.

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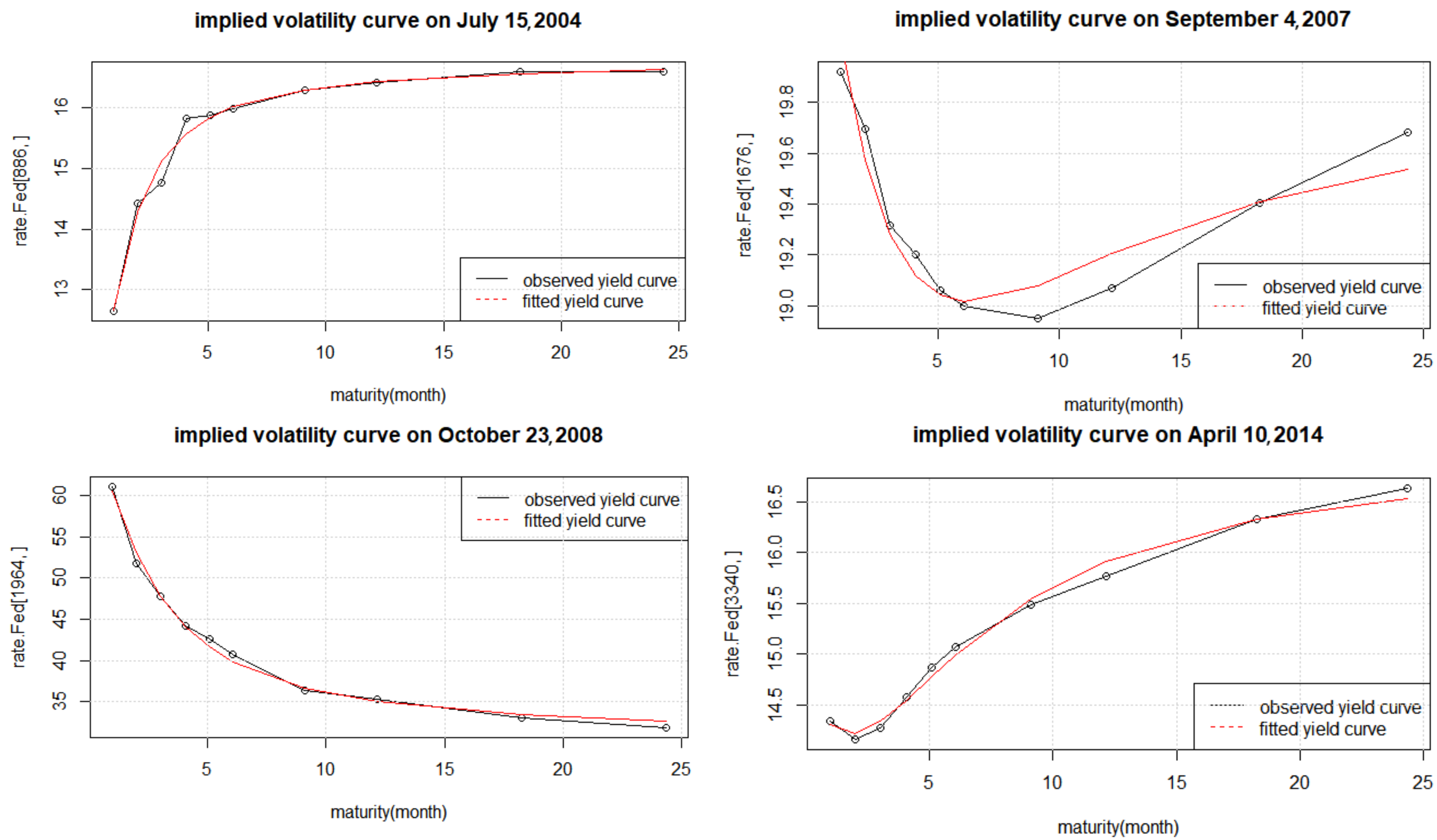
Tables and Figures

FIGURE 1. Implied Volatility Curves at Different Observation Times in the S&P 500 Using N-S Model



Notes: The x-axis is maturity in months, and the y-axis is implied volatility in percentage. The N-S model can simulate diverse shapes in the implied volatility term structure, such as monotonically increasing (July 15, 2014), monotonically decreasing (October 23, 2008), humped (September 4, 2007), and S shaped (April 10, 2014)

FIGURE 2a. Time Series of Three Parameters in the S&P 500

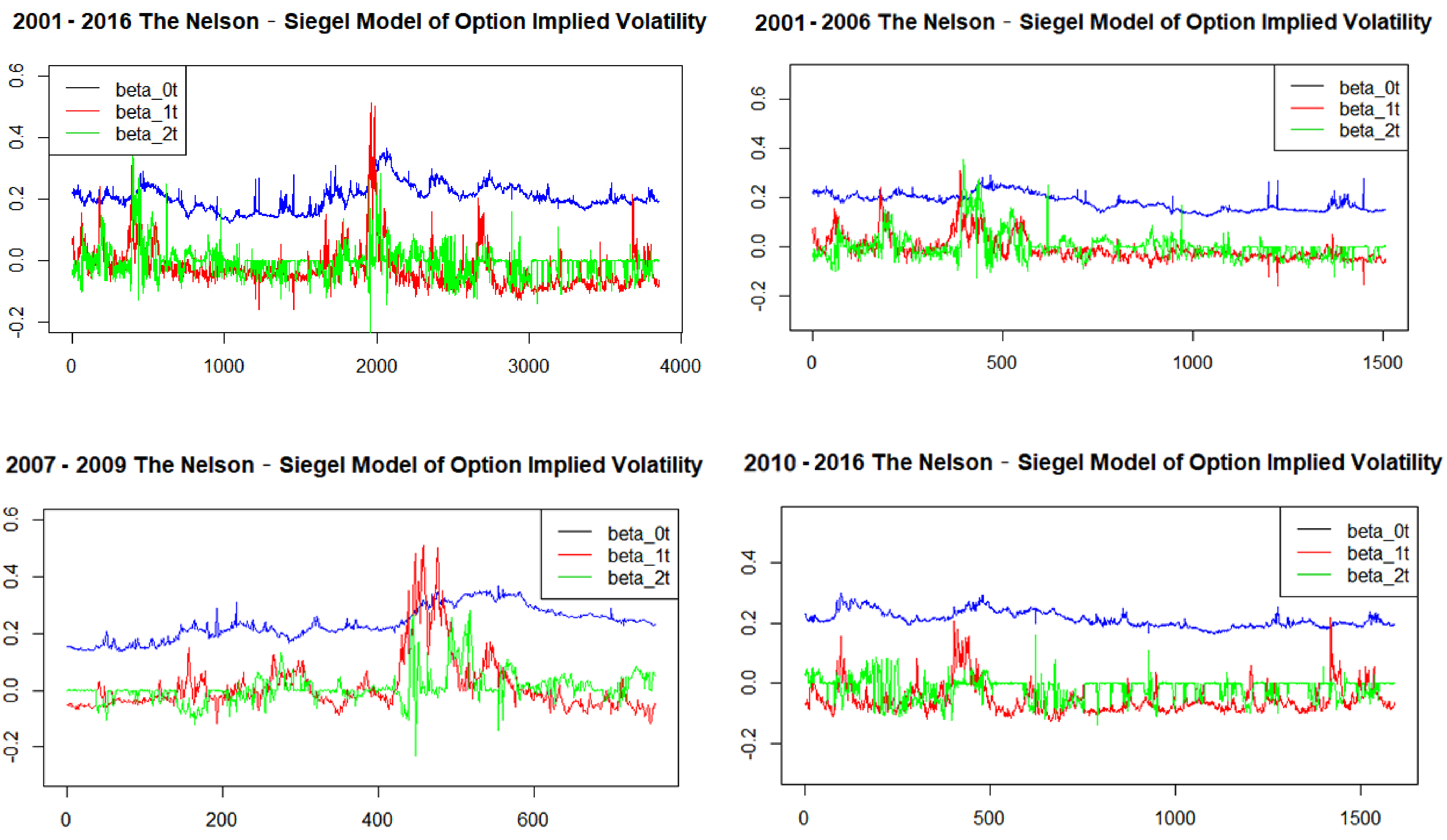


FIGURE 2b. Time Series of Three Parameters in the FTSE 100

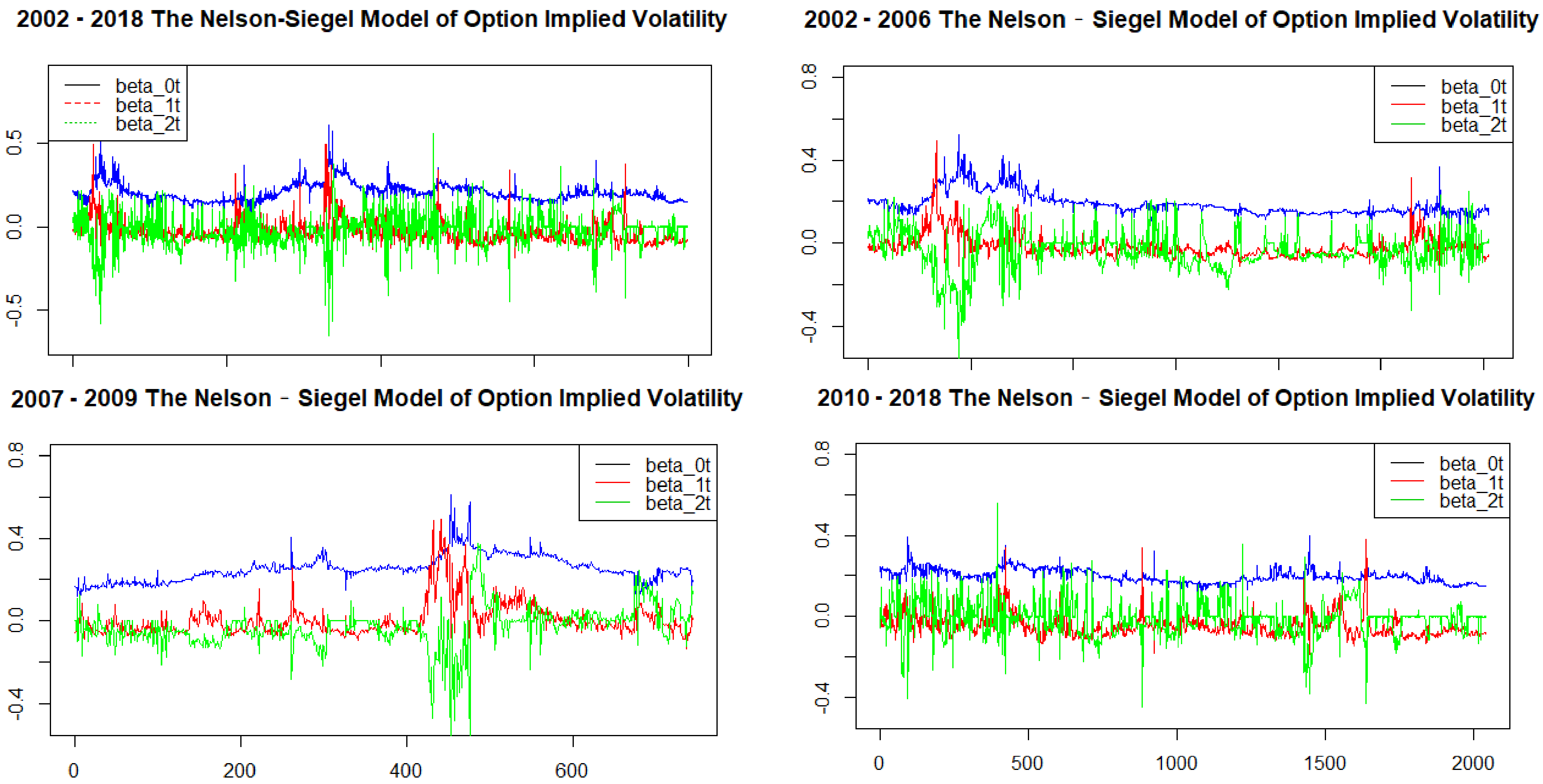


FIGURE 3a. Comparison between the VIX and β_0, β_1, and β_2 in the S&P 500 Index

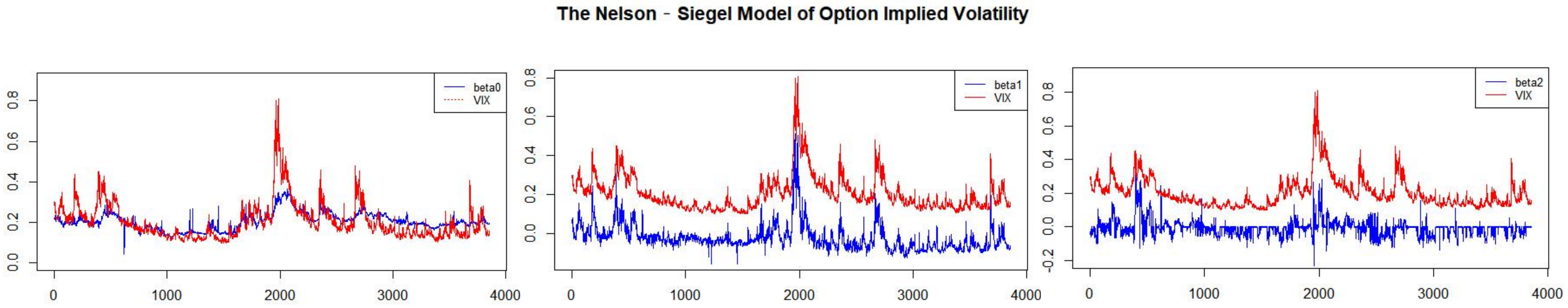


FIGURE 3b. Comparison between the VIX and β_0, β_1, and β_2 in the FTSE 100 Index

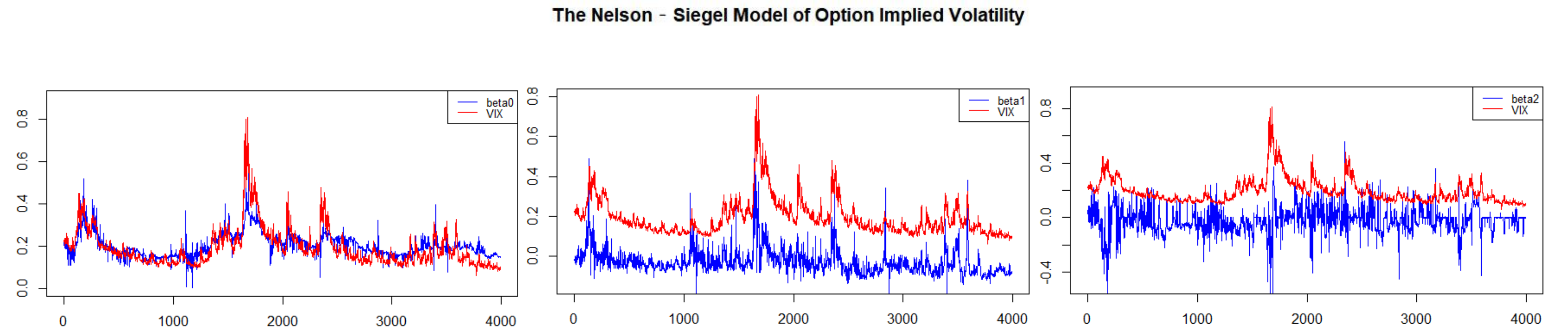
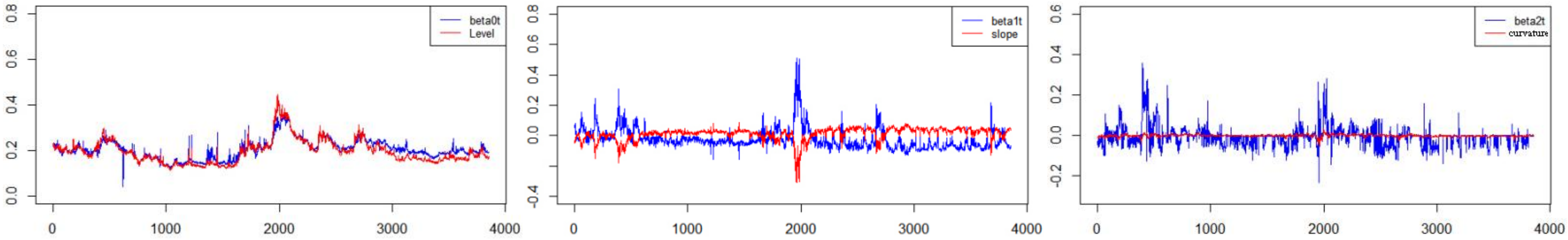
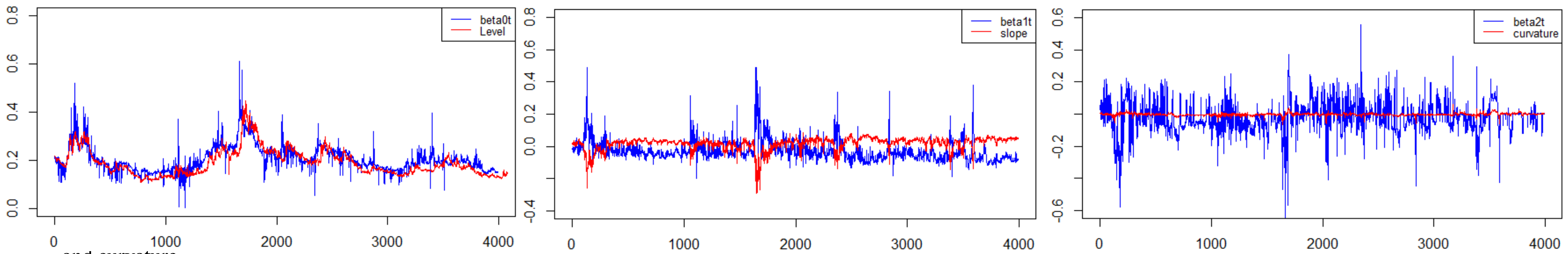


FIGURE 4a. Comparison of β_0 and Level, β_1 and Slope, and β_2 and Curvature in the S&P 500



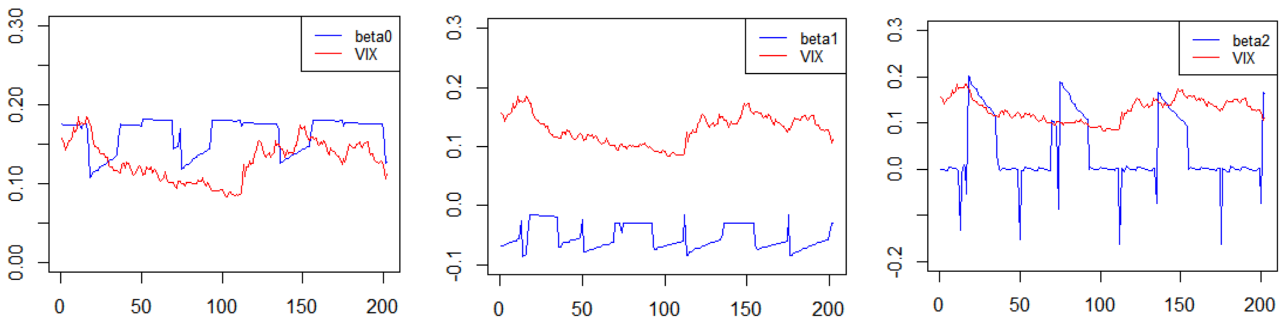
Notes: The x-axis is the daily observation from 2001 to 2016 in the S&P 500 index, and the y-axis is the magnitude of the three parameters inferred from the N-S model, level, slope, and curvature.

FIGURE 4b. Comparison of β_0 and Level, β_1 and Slope, and β_2 and Curvature in the FTSE 100



and curvature.

FIGURE 4c. Comparison of β0 and Level, β1and Slope, and β2 and Curvature in the Chinese 50 ETF



Notes: The x-axis is daily observations from December 5, 2016, to October 27, 2017, and the y-axis is the magnitude of the three parameters inferred from the N-S model and the VIX.

TABLE 1. Relationship between the Sign of the Coefficients and the Shape of the Curve

Sign of the coefficients	$\beta_1 > 0, \beta_2 > 0$	$\beta_1 > 0, \beta_2 < 0$	$\beta_1 < 0, \beta_2 > 0$	$\beta_1 < 0, \beta_2 < 0$
Curved shape	Negative slope	Negative slope & U shape	Positive slope & inverted U shape	Positive slope

Notes: The sign of β_1 and β_2 determines the shape of the curve.

TABLE 2a. Summary Statistics of Implied Volatility of the S&P 500

Maturity	*Mean*	*Max*	*Min*	*S.D.*	$\hat{\rho}(10)$	$\hat{\rho}(30)$	$\hat{\rho}(60)$	$\hat{\rho}(180)$
Panel A: Implied volatility								
30	0.1836	0.7483	0.0814	0.0815	0.9029	0.7645	0.6232	0.3393
60	0.1859	0.6722	0.0908	0.0739	0.9258	0.8135	0.6756	0.3745
91	0.1875	0.6045	0.0970	0.0690	0.9352	0.8350	0.7068	0.4009
122	0.1889	0.5744	0.1023	0.0647	0.9427	0.8523	0.7258	0.4175
152	0.1903	0.5384	0.1045	0.0612	0.9481	0.8631	0.7455	0.4351
182	0.1915	0.5384	0.1060	0.0586	0.9519	0.8726	0.7621	0.4503
273	0.1936	0.4648	0.1096	0.0541	0.9585	0.8893	0.7856	0.4739
365	0.1950	0.4448	0.1125	0.0516	0.9615	0.8961	0.7971	0.4853
547	0.1978	0.4019	0.1161	0.0474	0.9651	0.9062	0.8167	0.5056
730	0.2000	0.3841	0.1174	0.0456	0.9659	0.9089	0.8210	0.5098
Factor	*Mean*	*Max*	*Min*	*S.D.*	$\hat{\rho}(10)$	$\hat{\rho}(30)$	$\hat{\rho}(60)$	$\hat{\rho}(180)$
Panel B: Implied volatility curve level, slope, and curvature								
Level	0.1950	0.4448	0.1125	0.0516	0.9615	0.8961	0.7971	0.4853
Slope	0.0114	0.0841	-0.3070	0.0407	0.8102	0.5593	0.3634	0.2213
Curvature	-0.000	0.0287	-0.0545	0.0056	0.4964	0.0453	0.0772	0.0911

Notes: The period of the sample is January 2, 2001, to April 29, 2016. The last four columns show the autocorrelation of different times to maturity with 10, 30, 60, and 180 days' displacement.

TABLE 2b. Summary Statistics of Implied Volatility of the FTSE 100

Maturity	*Mean*	*Max*	*Min*	*S.D.*	$\hat{\rho}$ (10)	$\hat{\rho}$ (30)	$\hat{\rho}$ (60)	$\hat{\rho}$ (180)
Panel A: Implied volatility								
30	0.1745	0.7119	0.0687	0.0852	0.8985	0.9651	0.9651	0.9651
60	0.1759	0.6207	0.0794	0.0772	0.9243	0.9651	0.9651	0.9651
91	0.1780	0.5927	0.0842	0.0724	0.9340	0.9651	0.9651	0.9651
122	0.1798	0.5817	0.0850	0.0694	0.9380	0.9651	0.9651	0.9651
152	0.1818	0.5512	0.0886	0.0658	0.9427	0.9651	0.9651	0.9651
182	0.1834	0.5176	0.0935	0.0628	0.9470	0.9651	0.9651	0.9651
273	0.1870	0.4707	0.1000	0.0574	0.9580	0.9651	0.9651	0.9651
365	0.1900	0.4458	0.1073	0.0544	0.9636	0.9651	0.9651	0.9651
547	0.1949	0.4270	0.1082	0.0509	0.9663	0.9651	0.9651	0.9651
730	0.1974	0.4233	0.0727	0.0495	0.9651	0.9651	0.9651	0.9651
Factor	*Mean*	*Max*	*Min*	*S.D.*	$\hat{\rho}$ (10)	$\hat{\rho}$ (30)	$\hat{\rho}$ (60)	$\hat{\rho}$ (180)
Panel B: Implied volatility curve level, slope, and curvature								
Level	0.1905	0.4458	0.1074	0.0542	0.9629	0.8965	0.8015	0.5001
Slope	0.0159	0.0835	-0.2926	0.0401	0.7589	0.4988	0.3398	0.1392
Curvature	-0.0025	0.5990	-0.067	0.0081	0.4361	0.1241	0.0075	0.0755

Notes: The period of the sample is January 2, 2002, to August 22, 2018. The last four columns show the autocorrelation of different times to maturity with 10, 30, 60, and 180 days' displacement.

TABLE 3a. Descriptive Statistics of the Estimated Factors of the S&P 500

	Mean	*Max*	*Min*	*S.D.*	$\hat{\rho}(10)$	$\hat{\rho}(30)$	$\hat{\rho}(60)$	$\hat{\rho}(180)$
Panel A: Estimated Factors								
$ß_0$	0.2047	0.3651	0.0434	0.0421	0.9390	0.8765	0.7936	0.5048
$ß_1$	-0.0231	0.5116	-0.1591	0.0668	0.7818	0.5290	0.3399	0.2241
$ß_2$	-0.0050	0.3557	-0.2330	0.0500	0.4471	0.2228	0.0976	0.0482
λ	04847	0.9999	0.0747	0.3048	0.4095	0.2565	0.1514	0.1697
	$ß_0$	$ß_1$	$ß_2$	λ				
Panel B: Correlations between β_0, β_1, β_2, and λ								
$ß_0$	1							
$ß_1$	0.3263	1						
$ß_2$	0.1847	0.1742	1					
λ	0.0857	0.2404	0.1133	1				
				T-Statistic	*P-Value*			
Panel C: Augmented Dickey-Fuller Test (ADF Test)								
$ß_0$				-2.602	0.3234			
First difference of β_0				-4.9525	0.01			
$ß_1$				-5.729	0.00			
$ß_2$				-8.3499	0.00			
λ				-8.3752	0.00			

Notes: β_0, β_1, and β_2 are long-, short-, and medium-term factors, respectively. λ determines the decay rate of the exponential. The last four columns of Panel A show the autocorrelation of different times to maturity with 10, 30, 60, and 180 days' displacement, respectively.

TABLE 3b. Descriptive Statistics of the Estimated Factors of the FTSE 100

	Mean	*Max*	*Min*	*SD*	$\hat{\rho}$ (10)	$\hat{\rho}$ (30)	$\hat{\rho}$ (60)	$\hat{\rho}$ (180)
Panel A: Estimated Factors								
ß$_0$	0.2043	0.6102	0.0044	0.0531	0.8296	0.7397	0.6452	0.3391
ß$_1$	-0.2839	0.4904	-0.1999	0.0639	0.6001	0.3501	0.2430	0.1299
ß$_2$	-0.0230	0.5587	-0.6521	0.0923	0.3337	0.1224	0.0102	0.0246
λ	0.3997	0.9999	0.0747	0.2821	0.2458	0.0157	0.0470	0.0078
	ß$_0$	*ß$_1$*	*ß$_2$*	λ				
Panel B: Correlations between β_0, β_1, β_2, and λ								
ß$_0$	1							
ß$_1$	0.3263	1						
ß$_2$	0.1847	0.1742	1					
λ	0.0298	0.1829	-0.1825	1				
				T-Statistic	*P-Value*			
Panel C: Augmented Dickey-Fuller Test (ADF Test)								
ß$_0$				-9.7413	0.00			
ß$_1$				-3.4482	0.00			
ß$_2$				-2.4412	0.00			
λ				-13.3116	0.00			

Notes: β_0, β_1, and β_2 are long-, short-, and medium-term factors, respectively. λ determines the decay rate of the exponential. The last four columns of Panel A show the autocorrelation of different times to maturity with 10, 30, 60, and 180 days' displacement, respectively.

TABLE 4a. Correlation Matrix for the Regression Variables of the S&P 500

	ß₀	*ß₁*	*ß₂*	*VIX*	*Level*	*Slope*	*Curvature*
β_0	1						
β_1	0.3263***	1					
β_2	0.1847***	0.1742***	1				
VIX	0.7546***	0.8431***	0.3207***	1			
Level	0.9371***	0.5685***	0.3126***	0.9031***	1		
Slope	-0.3370***	-0.9789***	-0.2355***	0.8439***	-0.5536***	1	
Curvature	0.0139***	-0.4314***	0.3793***	-0.1945***	0.0211***	0.4175***	1

TABLE 4b. Correlation Matrix for the Regression Variables of the FTSE 100

	β_0	β_1	β_2	*VIX*	*Level*	*Slope*	*Curvature*
β_0	1						
β_1	0.3305***	1					
β_2	-0.3144***	-0.1948***	1				
VIX	0.8120***	0.7142***	-0.1235***	1			
Level	0.8812***	0.5573***	-0.0068	0.9236***	1		
Slope	-0.5687***	-0.8859***	0.3703***	-0.0915***	-0.8059***	1	
Curvature	0.1232***	-0.4226***	0.4621***	0.0231***	0.1396***	0.2556***	1

Notes: Table 4a&b illustrate the correlation matrix for the regression variables of S&P 500 and FTSE 100 respectively, *** denotes at 0.01 significance level.

TABLE 5a. Predicting the S&P 500 FRV Using VIX and HRV

	Estimate	*t-Statistics*	*P-Value*
Panel A: FRV with the VIX			
Constant	-0.0190***	-7.0710	0.0000
VIX	0.9176***	6.3110	0.0000
Adj. R^2	0.6017		
Panel B: FRV with HRV			
Constant	0.0439***	2.1500	0.0000
HRV	0.7354***	7.6700	0.0000
Adj. R^2	0.5043		
Panel C: FRV with the VIX and HRV			
Constant	-0.0093**	-3.2070	0.0014
VIX	0.7069***	5.4780	0.0000
HRV	0.1968***	8.4080	0.0000
Adj. R^2	0.6088		

Notes: The coefficients have been adjusted using Newey-West. ***, **, and * are significance levels of 1%, 5%, and 10% respectively.

TABLE 5b. Results of AIC and BIC Analysis for Three Regressions in the S&P 500

Model	Int_i	*VIX*	*HRV*	$Log(L)_i$	AIC_i	Δ_i *(AIC)*	BIC_i	Δ_i *(BIC)*
Panel A	-0.0190	0.9176		3164.7	-9873.1	68.11	-9854.32	61.86
Panel B	0.0439		0.7354	4674.4	-9342.8	598.37	-9324.07	592.11
Panel C	-0.0093	0.7069	0.1968	4974.6	-9941.2	0.00	-9916.18	0.00

Notes: AIC_i is the Akaike information criterion, Δ_i (AIC) = AIC_i – min(AIC), while BIC_i is the Bayesian information criterion, Δ_i (BIC) = BIC_i – min(AIC).

TABLE 6a. Predicting the FTSE 100 FRV Using VIX and HRV

	Estimate	*t-Statistics*	*P-Value*
Panel A: FRV with the VIX			
Constant	-0.0091***	-11.96	0.0000
VIX	0.4110***	14.91	0.0000
Adj. R^2	0.7687		
Panel B: FRV with HRV			
Constant	0.0188***	2.138	0.0000
HRV	0.7346***	8.634	0.0000
Adj. R^2	0.5417		
Panel C: FRV with the VIX and HRV			
Constant	-0.0096***	-3.4070	0.0000
VIX	0.3453***	7.111	0.0000
HRV	0.1888***	6.656	0.0000
Adj. R^2	0.7839		

Notes: The coefficients have been adjusted using Newey-West. ***, **, and * are significance levels of 1%, 5%, and 10% respectively.

TABLE 6b. Results of AIC and BIC Analysis for Three Regressions in the FTSE 100

Model	*Int_i*	*VIX*	*HRV*	*$Log(L)_i$*	*AIC_i*	*Δ_i (AIC)*	*BIC_i*	*Δ_i (BIC)*
Panel A	-0.0092	0.4113		9906.0	-19523.3	280.67	-19504.49	274.39
Panel B	0.0188		0.7346	8419.6	-16833.2	2970.76	-16814.40	2964.48
Panel C	-0.0097	0.3453	0.1888	9764.7	-19804.0	0.00	-19778.88	0.00

Notes: AIC_i is the Akaike information criterion, Δ_i (AIC) = AIC_i – min(AIC), while BIC_i is the Bayesian information criterion, Δ_i (BIC) = BIC_i – min(AIC).

TABLE 7a. Predicting the S&P 500 FRV Using STF, MTF, and LTF

	Estimate	*t-Statistics*	*P-Value*
Panel A: FRV with LTF			
Constant	-0.1121***	-5.7000	0.0000
LTF (β_0)	1.3725***	4.1400	0.0000
Adj. R^2	0.2947		
Panel B: FRV with STF			
Constant	0.1951***	5.5100	0.0000
STF (β_1)	1.1377***	4.3100	0.0000
Adj. R^2	0.5098		
Panel C: FRV with MTF			
Constant	0.1711***	5.6700	0.0000
MTF (β_2)	0.4625***	4.7900	0.0000
Adj. R^2	0.0468		
Panel D: FRV with LTF, STF, and MTF			
Constant	0.0155**	2.6960	0.0070
LTF (β_0)	0.8587***	4.0210	0.0000
STF (β_1)	0.9470***	5.128	0.0000
MTF (β_2)	0.1080***	4.9720	0.0000
Adj. R^2	0.6196		

Notes: The coefficients have been adjusted using Newey-West. ***, **, and * are significance level of 1%, 5%, and 10% separately.

TABLE 7b. Results of AIC and BIC Analysis for Three Regressions in the S&P 500

Model	*Int*$_i$	*LTF*	*STF*	*MTF*	*Log(L)*$_i$	*AIC*$_i$	Δ_i *(AIC)*	*BIC*$_i$	Δ_i *(BIC)*
Panel A	-0.0993	1.3160			3838.2	-7670.3	2378.21	-7651.55	2365.7
Panel B	0.1951		1.1380		4539.2	-9072.4	976.09	-9053.67	963.58
Panel C	0.1711			0.4625	3257.6	-6509.2	3539.37	-6490.37	3526.88
Panel D	0.1546	0.8587	0.9470	0.1080	5029.3	-10048.5	0.00	-10017.25	0.00

Notes: AIC$_i$ is the Akaike information criterion, Δ_i (AIC) = AIC$_i$ – min(AIC), while BIC$_i$ is the Bayesian information criterion, Δ_i (BIC) = BIC$_i$ – min(AIC).

TABLE 8a. Predicting the FTSE 100 Using FRV STF, MTF, and LTF

	Estimate	*t-Statistics*	*P-Value*
Panel A: FRV with LTF			
Constant	-0.0548***	-3.2020	0.0000
LTF (β0)	0.6106***	7.5333	0.0000
Adj. R^2	0.5882		
Panel B: FRV with STF			
Constant	0.0823***	4.8161	0.0000
STF (β_1)	0.4344***	5.4840	0.0000
Adj. R^2	0.4308		
Panel C: FRV with MTF			
Constant	0.0678***	9.9690	0.0000
MTF (β_2)	-0.0926***	-2.198	0.0000
Adj. R^2	0.0404		
Panel D: FRV with LTF, STF, and MTF			
Constant	-0.0248 ***	-7.1750	0.0000
LTF (β_0)	0.5118***	7.8120	0.0000
STF (β_1)	0.3056***	5.7960	0.0000
MTF (β_2)	0.0416***	11.440	0.0000
Adj. R^2	0.7778		

Notes: The coefficients have been adjusted using Newey-West. ***, **, and * are significance level of 1%, 5%, and 10% separately.

TABLE 8b. Results of AIC and BIC Analysis for Three Regressions in the FTSE 100

Model	*Int_i*	*LTF*	*STF*	*MTF*	*$Log(L)_i$*	*AIC_i*	*Δ_i (AIC)*	*BIC_i*	*Δ_i (BIC)*
Panel A	-0.0547	0.6103			8623.8	-17241.7	2440.88	-17222.87	2428.33
Panel B	0.0824		0.4336		7985.7	-15965.3	3717.28	-15946.47	3704.73
Panel C	0.0680			-0.0922	6959.1	-13912.2	5770.43	-13893.32	5757.88
Panel D	0.0252	0.5126	0.3060	0.0417	9846.3	-19682.6	0.00	-19651.20	0.00

Notes: AIC_i is the Akaike information criterion, Δ_i (AIC) = AIC_i – min(AIC), while BIC_i is the Bayesian information criterion, Δ_i (BIC) = BIC_i – min(AIC).

TABLE 9a. Predicting the S&P 500 FRV Using Three N-S Model Inferred Parameters, VIX, and HRV

	Estimate	*t-Statistics*	*P-Value*
Panel A: FRV with the VIX, HRV, and STF(β_1)			
Constant	-0.0127***	-4.1890	0.0000
VIX	0.7166***	5.7690	0.0000
HRV	0.2023***	8.6430	0.0000
STF(β_1)	-0.0877***	-3.8660	0.0001
Adj. R^2	0.6194		
Panel B: FRV with the VIX, HRV, and LTF(β_0)			
Constant	0.0185**	3.1840	0.0015
VIX	0.7983***	4.7870	0.0000
HRV	0.1823***	7.7730	0.0000
LTF(β_0)	-0.2151***	-5.5290	0.0000
Adj. R^2	0.6118		
Panel C: FRV with the VIX, HRV, and MTF(β_2)			
Constant	-0.0128***	-4.1890	0.0000
VIX	0.7166***	8.7690	0.0000
HRV	0.2023***	8.6430	0.0000
MTF(β_2)	-0.0877***	-3.8660	0.0000
Adj. R^2	0.6102		
Panel D: FRV with the VIX, HRV, STF_t (β_{1t}), MTF_t (β_{2t}), and LTF_t (β_{0t})			
Constant	0.0197***	3.4450	0.0006
VIX	-0.0465	-0.5470	0.5841
HRV	0.1743***	7.5410	0.0000
LTF(β_0)	0.7239***	7.6070	0.0000
STF(β_1)	0.8139***	4.9510	0.0000
MTF(β_2)	**0.0698****	2.5710	0.0102
Adj. R^2	0.6252		

Notes: The coefficients have been adjusted by the Newey-West. ***, **, and * are significance levels of 1%, 5%, and 10% respectively.

TABLE 9b. Results of AIC and BIC Analysis for Three Regressions in the S&P 500

Model	*Int$_i$*	*VIX*	*HRV*	*LTF*	*STF*	*MTF*	*Log(L)$_i$*	*AIC$_i$*	*Δ_i (AIC)*	*BIC$_i$*	*Δ_i (BIC)*
Panel A	0.0359	0.5412	0.1716		0.3078		5027.9	-10045.9	58.85	-10014.67	44.64
Panel B	0.0185	0.7983	0.1824	-0.2151			4989.8	-9969.7	135.11	-9938.40	120.91
Panel C	-0.0127	0.7166	0.2023			-0.0877	498201	-9954.1	150.66	-9922.86	136.45
Panel D	0.0197	-0.0465	0.1743	0.0698	0.7239	0.8139	5058.6	-10103.1	0.00	-10059.31	0.00

Notes: AIC$_i$ is the Akaike information criterion, Δ_i (AIC) = AIC$_i$ – min(AIC), while BIC$_i$ is the Bayesian information criterion, Δ_i (BIC) = BIC$_i$ – min(AIC).

TABLE 10a. Predicting the FTSE 100 FRV Using Three N-S Model Inferred Parameters, the VIX, and HRV

	Estimate	*t-Statistics*	*P-Value*		*Estimate*	*t-Statistics*	*P-Value*
Panel A: FRV with the VIX, HRV, and STF (β_1)				*Panel C: FRV with the VIX, HRV, and MTF (β_2)*			
Constant	-0.0080***	-6.832	0.0000	Constant	-0.0096***	-13.060	0.0000
VIX	0.3405***	5.8342	0.0000	*VIX*	0.3486***	6.781	0.0000
HRV	0.1839***	16.177	0.0000	*HRV*	0.1676***	15.052	0.0000
STF (β_1)	-0.0129	1.795	0.0727	*MTF (β_2)*	-0.0327***	-9.477	0.0000
Adj. R^2	0.7841			Adj. R^2	0.7887		
Panel B: FRV with the VIX, HRV, and LTF (β_0)				*Panel D: FRV with the VIX, HRV, STF_t (β_{1t}), MTF_t (β_{2t}), and LTF_t (β_{0t})*			
Constant	-0.0228***	-17.09	0.0000	Constant	-0.0202***	-14.650	0.0000
VIX	0.2926***	3.411	0.0000	*VIX*	0.1986***	7.042	0.0000
HRV	0.1798***	6.154	0.0000	*HRV*	0.1275***	11.154	0.0000
LTF (β_0)	0.1168***	11.76	0.0000	*LTF (β_0)*	0.2271***	4.509	0.0000
Adj. R^2	0.7912			*STF (β_1)*	0.1184***	11.338	0.0000
				MTF (β_2)	0.0020***	0.495	0.6210
				Adj. R^2	0.7993		

Notes: The coefficients have been adjusted by the Newey-West. ***, **, and * are significance levels of 1%, 5%, and 10% respectively.

TABLE 10b. Results of AIC and BIC Analysis for Three Regressions in the FTSE 100

Model	*Int$_i$*	*VIX*	*HRV*	*LTF*	*STF*	*MTF*	*Log(L)$_i$*	*AIC$_i$*	*Δ_i (AIC)*	*BIC$_i$*	*Δ_i (BIC)*
Panel A	0.1839	0.3405	0.1839		0.0129		9907.6	-19805.2	289.43	-19773.83	275.11
Panel B	-0.0228	0.2923	0.1798	0.1168			9974.1	-19938.1	156.55	-19906.70	142.24
Panel C	-0.0096	0.3486	0.1676			-0.0327	9950.5	-19890.9	203.75	-19859.50	189.44
Panel D	-0.0202	0.1986	0.1275	0.2271	0.1184	0.0020	10053.5	-20092.9	0.00	-20048.94	0.00

Notes: AIC$_i$ is the Akaike information criterion, Δ_i (AIC) = AIC$_i$ – min(AIC), while BIC$_i$ is the Bayesian information criterion, Δ_i (BIC) = BIC$_i$ – min(AIC).

TABLE 11. Cointegrated Relationship

	VIX	*HRV*	*ß$_{0t}$*	*ß$_{1t}$*	*ß$_{2t}$*	*slope*	*curvature*
VIX	1						
HRV	0.0100	1					
ß$_{0t}$	0.0100	0.0100	1				
ß$_{1t}$	0.0100	0.0100	0.0100	1			
ß$_{2t}$	0.0100	0.0100	0.0100	0.0100	1		
slope	0.0100	0.0100	0.0100	0.0100	0.0100	1	
curvature	0.0100	0.0100	0.0100	0.0100	0.0100	0.0100	1

Notes: After standardizing the residuals of each pair in Table 11, the ADF test reports that the residuals of each pair are stationary, furthermore, there are cointegrated.

Table 12a. Summary Statistics for Straddle and Delta-Hedged Call in the S&P 500

Panel A：Straddle Returns (monthly)								
Threshold	*Mean*	*S.D.*	*Max*	*Min*	*Cumulative*	*N*	*Long (ε<0)*	*Short (ε>0)*
0	0.047088	0.568724	4.577011	-0.838388	-12.3018	180	103	77
0.1	0.025342	0.571772	4.577011	-0.838388	-15.2701	168	102	66
0.2	0.038256	0.588736	4.577011	-0.838388	-12.6717	153	91	62
0.3	0.019958	0.602006	4.577011	-0.838388	-14.3197	137	86	51
0.4	0.004078	0.611491	4.577011	-0.838388	-14.531	119	74	45
0.5	0.037623	0.636488	4.577011	-0.838388	-9.40647	104	60	44
0.6	0.054723	0.667301	4.577011	-0.838388	-7.61587	92	53	39
0.7	0.065742	0.718819	4.577011	-0.798729	-5.39693	72	42	30
0.8	0.093930	0.748227	4.577011	-0.798729	-3.38092	63	35	28
0.9	0.125506	0.758889	4.577011	-0.519553	0.091694	47	24	23
1	0.227465	0.857641	4.577011	-0.442667	2.68706	34	15	19
1.1	0.375927	0.991145	4.577011	-0.427009	4.241112	23	9	14
1.2	0.146596	0.368351	0.838384	-0.427009	1.536353	18	8	10
1.3	0.152900	0.379844	0.838384	-0.427009	1.411262	16	7	9
1.4	0.130501	0.382081	0.838384	-0.427009	1.013232	15	7	8
1.5	0.147087	0.390860	0.838384	-0.427009	1.120491	14	6	8

Panel B: Delta-Hedged Call Returns (monthly)								
Threshold	*Mean*	*S.D.*	*Max*	*Min*	*Cumulative*	*N*	*Long (ε<0)*	*Short (ε>0)*
0	0.000246	0.019769	0.062373	-0.062239	0.009526	180	103	77
0.1	-0.000665	0.019451	0.062373	-0.062239	-0.1433	168	102	66
0.2	0.000092	0.019396	0.062373	-0.062239	-0.01443	153	91	62
0.3	-0.000515	0.019453	0.062373	-0.062239	-0.09618	137	86	51
0.4	-0.001204	0.019326	0.062373	-0.062239	-0.16532	119	74	45
0.5	-0.000322	0.020061	0.062373	-0.062239	-0.05411	104	60	44
0.6	0.000240	0.020803	0.062373	-0.062239	0.002487	92	53	39
0.7	0.000173	0.022254	0.062373	-0.062239	-0.00501	72	42	30
0.8	0.001119	0.023003	0.062373	-0.062239	0.054153	63	35	28
0.9	0.001160	0.023729	0.050710	-0.062239	0.041539	47	24	23
1	0.003115	0.026915	0.050710	-0.062239	0.093805	34	15	19
1.1	0.005528	0.031201	0.050710	-0.062239	0.11607	23	9	14
1.2	0.002049	0.031832	0.044087	-0.062239	0.028177	18	8	10
1.3	0.001847	0.033045	0.044087	-0.062239	0.021268	16	7	9
1.4	0.000316	0.033613	0.044087	-0.062239	-0.00323	15	7	8
1.5	0.001040	0.034760	0.044087	-0.062239	0.006628	14	6	8

Notes: Cumulative is cumulative returns over 15 years of trading, from January 2001 to December 2015. *N* is trading time over 15 years (180 months). Long (ε<0) is the number of long strategies when ε is smaller than the negative value of the threshold; short (ε>0) is the number of short strategies when ε is larger than the threshold.

Table 12b. Summary Statistics for Straddle and Delta-Hedged Call in the FTSE 100

Panel A：Straddle Returns (monthly)								
Threshold	*Mean*	*S.D.*	*Max*	*Min*	*Cumulative*	*N*	*Long (ε<0)*	*Short (ε>0)*
0	0.015944	0.520967	4.362651	-0.796106	0.000002	180	109	71
0.1	0.029556	0.506377	4.362651	-0.796106	0.000003	168	105	63
0.2	0.023717	0.699021	4.362651	-0.796106	0.000001	153	94	59
0.3	0.019958	0.601159	4.362651	-0.796106	0.000002	137	83	54
0.4	0.026608	0.601949	4.362651	-0.796106	0.000000	119	71	48
0.5	0.022941	0.696713	4.362651	-0.796106	0.000081	104	47	47
0.6	0.019127	0.697620	4.362651	-0.796106	0.000451	92	50	42
0.7	0.062385	0.702000	4.362651	-0.718629	0.005641	72	39	33
0.8	0.098524	0.792943	4.362651	-0.718629	0.035076	63	32	31
0.9	0.115926	0.850848	4.362651	-0.519113	1.087131	47	21	26
1	0.235498	0.896433	4.362651	-0.442667	13.652424	34	12	22
1.1	0.369562	0.910896	4.362651	-0.4258491	57.864187	23	6	17
1.2	0.135264	0.344764	0.754161	-0.4258491	3.946109	18	5	13
1.3	0.148292	0.352096	0.754161	-0.4258491	4.635486	16	4	12
1.4	0.126351	0.369053	0.754161	-0.4258491	2.751952	15	4	11
1.5	0.139584	0.360794	0.754161	-0.4258491	2.284157	14	3	11
Panel B: Delta-Hedged Call Returns (monthly)								
Threshold	*Mean*	*S.D.*	*Max*	*Min*	*Cumulative*	*N*	*Long (ε<0)*	*Short (ε>0)*
0	0.010329	0.021211	0.061129	-0.063135	1.007572	180	99	81
0.1	-0.000665	0.014943	0.061129	-0.063135	0.516491	168	98	70
0.2	0.000012	0.016214	0.061129	-0.063135	0.975167	153	92	61
0.3	-0.000724	0.022330	0.061129	-0.063135	0.908756	137	84	53
0.4	0.002684	0.014447	0.061129	-0.063135	0.842193	119	75	44
0.5	-0.000216	0.021392	0.061129	-0.063135	0.935679	104	63	41
0.6	0.000149	0.026762	0.061129	-0.063135	1.003050	92	55	37
0.7	0.000142	0.022254	0.061129	-0.063135	0.985997	72	43	29
0.8	0.001670	0.024723	0.061129	-0.063135	1.046568	63	33	30
0.9	0.001215	0.026896	0.051097	-0.063135	1.050213	47	27	20
1	0.001224	0.026915	0.051097	-0.063135	1.088562	34	17	21
1.1	0.000201	0.031201	0.051097	-0.063135	1.136151	23	11	12
1.2	0.001026	0.031842	0.046213	-0.063135	1.035184	18	10	8
1.3	0.001356	0.034657	0.046213	-0.063135	1.029181	16	6	10
1.4	0.000421	0.034321	0.046213	-0.063135	0.985621	15	6	9
1.5	0.001641	0.035714	0.046213	-0.063135	1.015628	14	5	9

Notes: Cumulative is cumulative returns over 15 years of trading, from January 2002 to March 2017. N is trading time over 15 years (180 months). Long (ε<0) is the number of long strategies when ε is smaller than the negative value of the threshold; short (ε>0) is the number of short strategies when ε is larger than the threshold.