Hyperedge bundling: A practical solution to spurious interactions in MEG/EEG source connectivity analyses

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ABSTRACT

Inter-areal functional connectivity (FC), neuronal synchronization in particular, is thought to constitute a key systems-level mechanism for coordination of neuronal processing and communication between brain regions. Evidence to support this hypothesis has been gained largely using invasive electrophysiological approaches. In humans, neuronal activity can be non-invasively recorded only with magneto- and electroencephalography (MEG/EEG), which have been used to assess FC networks with high temporal resolution and whole-scalp coverage. However, even in source-reconstructed MEG/EEG data, signal mixing, or “source leakage”, is a significant confounder for FC analyses and network localization.

Signal mixing leads to two distinct kinds of false-positive observations: artificial interactions (AI) caused directly by mixing and spurious interactions (SI) arising indirectly from the spread of signals from true interacting sources to nearby false loci. To date, several interaction metrics have been developed to solve the AI problem, but the SI problem has remained largely intractable in MEG/EEG all-to-all source connectivity studies. Here, we advance a novel approach for correcting SIs in FC analyses using source-reconstructed MEG/EEG data.

Our approach is to bundle observed FC connections into hyperedges by their adjacency in signal mixing. Using realistic simulations, we show here that bundling yields hyperedges with good separability of true positives and little loss in the true positive rate. Hyperedge bundling thus significantly decreases graph noise by minimizing the false-positive to true-positive ratio. Finally, we demonstrate the advantage of edge bundling in the visualization of large-scale cortical networks with real MEG data. We propose that hypergraphs yielded by bundling represent well the set of true cortical interactions that are detectable and dissociable in MEG/EEG connectivity analysis.

Introduction

Large-scale neuronal networks, e.g., manifested by functional, directed, and effective connectivity (Karl, 2011), are thought to be critical for healthy brain functions while their abnormalities are thought to underlie many brain diseases (Brookes et al., 2016; Bullmore and Sporns, 2009, 2012; Fornito et al., 2015; Papo et al., 2014; Petersen and Sporns, 2015; Rubinov 2015; Sporns, 2014; Uhlhaas and Singer 2006, 2010). Currently, magneto- and electro-encephalography (MEG/EEG) are the only non-invasive electrophysiological tools for studying connectivity networks with millisecond-range temporal resolution and good coverage of the cortical surface (Kujala et al., 2008; Palva and Palva, 2012; S. Baillet et al., 2001; Salmelin and Baillet, 2009). Accurately identifying interaction dynamics from MEG/EEG data is of crucial importance for understanding their role in human cognition and its deficits.

To date, numerous interaction metrics have been developed and utilized to assess functional connectivity (FC) in terms of amplitude-, phase-, and phase-amplitude correlations within or across frequency bands for pairs of electrophysiological signals (Bastos and Schoffelen, 2016; Kreuz, 2011; O’Neill et al., 2015). These pairwise metrics are typically applied to estimate FC among all brain regions, i.e., to obtain “all-to-all” FC connectomes (Sporns et al., 2005). Networks of inter-areal FC are often represented as graphs where brain areas constitute the nodes...
(or vertices) and observed inter-areal connections the edges (Bullmore and Sporns, 2009; Rubinov and Sporns, 2010).

FC graphs estimated from MEG/EEG sensor space data are neuro-anatomically uninformative and severely confounded by signal mixing. Signal mixing has two facets: first, any focal neuronal signal is picked up by several sensors. Conversely, one sensor detects a mixture of signals from several distinct sources. Source reconstruction can be used to reduce signal mixing and, importantly, elucidate the likely neuroanatomical sources of the MEG/EEG signals (Buzsáki et al., 2012; Gross et al., 2013; Hamalainen et al., 1993; Palva and Palva, 2012; Schoffelen and Gross, 2009). Yet, because of ill-posed nature of the inverse problem, no source reconstruction approach can yield an unambiguous estimate of the source topography. Residual signal mixing in source space, signal leakage, is quantitatively dependent on the source-reconstruction method of choice but qualitatively characteristic to all such methods.

Because of signal leakage, FC measures exhibit two distinct types of false positive observations: artificial interactions (AI) and spurious interactions (SI), see Box 2 in Palva and Palva (2012). Als arise directly from the signal mixing by one true signal being smeared to multiple sensors or sources, regardless of whether true interactions are present. SIs are “ghost” interactions caused by the leakage of the signals from two true connected nodes to their surrounding nodes that in turn become falsely connected like the truly connected nodes (Colclough et al., 2015; Farahibozorg et al., 2017; Korhonen et al., 2014; Palva et al., 2012). Als can be suppressed by a number of bivariate metrics that typically aim to remove linear coupling terms, and therefore removing artificial and true interactions with zero- and anti-phase-lag coupling (for a review see (Palva et al., 2017)). However, the problem of SIs is much less acknowledged and more difficult to solve because SIs stem from multivariate mixing effects. With typical distributed source modeling approaches, signal leakage causes a large number of SIs that render both the network localization and graph property estimates inaccurate (Drakesmith et al., 2015). To date, one solution has been proposed for correcting SIs in oscillation amplitude correlation estimates, which simultaneously orthogonalizes all source time series through the Löwdin procedure (Colclough et al., 2015, 2016). Despite this promising advance, no solutions have yet been proposed to suppress SIs for other interaction metrics.

Here we advance a novel approach, hyperedge bundling, to alleviate the SIs problem in connectivity analyses performed with any interaction metric. Instead of correcting the mixing effects in source signals per se, the approach is based on quantifying the extent of mixing between all sources, evaluation of mixing similarity among all edges, and then clustering the raw interaction metric edges into hyperedge bundles. This procedure aims to yield a hypergraph where each hyperedge represents a true interaction and its spurious reflections.

In this study, we performed a large set of connectivity simulations and realistic all-to-all MEG source space analyses, in which we estimated phase synchrony as a measure of FC with an AI-insensitive metric. We show that in simulated graphs, hyperedge bundling greatly decreases the phase synchrony as a measure of FC with an AI-insensitive metric. We realistic all-to-all MEG source space analyses, in which we estimated phase synchrony as a measure of FC with an AI-insensitive metric. We show that in simulated graphs, hyperedge bundling greatly decreases the phase synchrony as a measure of FC with an AI-insensitive metric.

**Theory**

Signal mixing results in false positive artificial (AI) and spurious interactions (SI)

Let us consider a scenario where a true phase correlation is present between two distant (unmixed) sources $V_1$ and $V_2$ (Fig. 1A top). The signals from $V_1$ and $V_2$ are mixed with signals of their nearby and mutually uncorrelated neighbors $V_3$ and $V_4$. Estimating phase FC among all four nodes with the phase-locking value (PLV) will reveal both the true edge $E(V_1, V_2)$ and false positive “short-range” Als between the nearby nodes $E(V_1, V_3)$ and $E(V_2, V_4)$, because PLV is inflated by mixing (thick gray edges, Fig. 1A bottom). However, due to leakage of the signal from $V_1$ and $V_2$ to their neighbors $V_3$ and $V_4$, false positive “long-range” SIs $E(V_1, V_3), E(V_2, V_3)$, and $E(V_1, V_4)$ will also be observed (thin dashed edges). These SIs are thus only indirectly caused by mixing and, unlike the zero-phase-lag Als (see 2.2), SIs inherit the phase-lag of the true interaction (Colclough et al., 2015; Farahibozorg et al., 2017). Mixing-insensitive bivariate metrics such as the imaginary part of PLV (iPLV) can remove Als but do not eliminate SIs if the true coupling has non-zero phase lag.

Quantifying the mixing between reconstructed sources

Signal mixing/leakage between two sources is instantaneous and therefore always leads to inflated zero-phase-lag correlations between the sources. Mixing does not vary over time or across frequency bands (Brookes et al., 2012, 2014; Drakesmith et al., 2013; Nolte et al., 2004; Palva and Palva, 2012).

Source-reconstruction

Suppose we have a data matrix $X = \{x_1, x_2, ..., x_n\} \in \mathbb{R}^{n \times t}$ representing narrow-band time series of $t$ samples from $n$ neuronal populations. Simulating a MEG/EEG recording, $X$ can be linearly projected to source-space (Hamalainen and Ilmoniemi, 1994):

$$ Y = \Gamma X + \epsilon $$

where $Y \in \mathbb{R}^{s \times t}$ represents the forward-modeled time series from $s$ sensors ($n > s$). Here, $\Gamma \in \mathbb{R}^{s \times n}$ is the forward operator (or the lead field) and $\epsilon \in \mathbb{R}^{s \times t}$ is the model prediction error derived from measurement noise. Next, $Y$ can be projected back into the source-space, e.g., by minimum-norm estimation (MNE) based inverse modeling:

$$ \hat{X} = W Y = R I^T (\Gamma R I^T + \lambda^2 \chi^{-2})^{-1} Y $$

where $W \in \mathbb{R}^{s \times s}$ is the inverse operator (sources \times sensors), the regularization parameter $\lambda^2 = 0.1, R$ is the source covariance matrix, and $\chi$ is the noise covariance matrix. After inverse modeling, the $5000$–$10000$ source time series are collapsed into parcel time series for a cortical parcellation with $50$–$400$ parcels. In the present study, we used reconstruction-accuracy optimized collapsing (Korhonen et al., 2014) and a resolution of $400$ parcels covering the whole cortex.

Cross-talk function and resolution matrix

In MEG/EEG source connectivity studies, a resolution matrix $P = W R (P \in \mathbb{R}^{s \times s})$ is often used to describe the relationship between true signals and modeled signals from $n$ sources in the absence of noise (Farahibozorg et al., 2017; Hauk and Stenroos, 2014; Hauk et al., 2011; Liu et al., 2002). In $P$, each diagonal element quantifies the sensitivity for estimating signals from that source. Each row of $P$ is the “cross-talk” function (CTF) that describes the amount of mixing between one source and all other sources. Each column of $P$ is a “point-spread” function (PSF), which describes how the modeled signal from any one source is spread across all other sources.

The mixing function

For the reconstruction accuracy (fidelity) optimized cortical parcellation (Korhonen et al., 2014), we approximated the resolution matrix $P$...
numerically with a mixing matrix $A_{\text{mix}}$ of dimension $n \times n$ parcels. Each element of $A_{\text{mix}}$ is a mixing function ($f_{\text{mix}}$) that characterizes the signal mixing between two parcels. For uncorrelated true source signals, the magnitude of zero-lag correlations between reconstructed signals measures the forward- and inverse-transform caused mixing between the sources. Thus, $f_{\text{mix}}$ can be quantified by the zero-lag correlation between parcel time series estimated using simulated MEG/EEG measurements of uncorrelated source noise.

We first generate uncorrelated signals $X_i \in \mathbb{R}^{n \times t}$, $t$ samples for $n$ parcels, and forward transform them to obtain sensor signals $Y_i$ (eq. (1)). We next inverse transform $Y_i$ to obtain reconstructed signals $\hat{X}_i$ (eq. (2)). In this process, the reconstructed signals $\hat{X}_i(v_i)$, $\hat{X}_j(v_j)$ of any two nearby sources $v_i$ and $v_j$ become correlated to a certain degree due to mixing. Thus, the mixing from the simulated “true” signal $x_0(v_i)$ to the reconstructed signal $\hat{x}_0(v_j)$ can be quantified as:

$$f_{\text{mix}}(v_i, v_j) = |\text{re}(c\text{PLV}(x_0(v_i), \hat{x}_0(v_j)))|$$

where $\text{re}()$ denotes the real part of a complex number and $c\text{PLV}$ is the complex-valued phase locking value (Lachaux et al., 1999):

$$c\text{PLV}(A, B) = \frac{1}{T} \sum_{t=1}^{T} \left[ e^{i \theta_A(t)} - \bar{e}^{i \theta_B(t)} \right] \frac{1}{\sum_{t=1}^{T} |S_A(t)|^2 |S_B(t)|^2}$$

where $T$ denotes the number of samples, $\theta_A$ and $\theta_B$ are the instantaneous phases of signal $A$ and $B$; $S_A$ and $S_B$ are complex-valued narrow-band signals from $A$ and $B$, and $\bar{z}$ is the complex conjugate of $z$. Because mixing is instantaneous, $\text{re}(c\text{PLV}(A, B))$ captures all correlations caused by mixing. For parcel pairs that do not become correlated by signal mixing, $f_{\text{mix}}$ is near zero. For parcel pairs influenced by signal mixing, $f_{\text{mix}}$ reaches 1 for complete mixing. It is important to note that although we here measured mixing using phase correlation, practically identical quantification would be achieved by, for example, the correlation coefficient as well as by deriving the mixing values analytically from CTFs and PSFs. $A_{\text{mix}}$ is thus interaction-metric independent.

**Signal mixing smears a true interaction into multiple spurious interactions**

For a simplified illustration of how signal mixing/source leakage produces SIs, we used toy model with a $13 \times 13$ grid of point sources. The
mixing matrix $A_{mix}$ of dimension $169 \times 169$, was defined so that mixing between any two sources was a 2D Gaussian distribution decreasing with distance between the two sources (inset, Fig. 1B, methods see (Wang et al., 2018)).

We simulated one true edge by setting two sources $V_1$ and $V_2$ to have phase coupling of 0.9 with non-zero phase lag and keeping the remaining 167 sources uncorrelated. Next, we introduced mixing between reconstructed sources and mapped all-to-all phase PC with an AI-free metric, the imaginary part of the phase-locking-value (IPLV) (Palva and Palva, 2012)

$$iPLV = \left|\text{Im}(cPLV)\right|$$ (5)

The IPLV, like the imaginary coherency (Nolte et al., 2004), removes zero-lag couplings by excluding the real part of cPLV. Therefore, IPLV yields only the true phase-lagged interactions and their false positive ghosts (SIs). In this simulation, visualization of the strongest 0.1% of IPLV edges revealed the true edge and several SIs, all of which connected zero-lag couplings by excluding the real part of distance between the two sources (inset, Fig. 1B, methods see (Wang et al., 2018)).

For visualizing hyperedges, we utilized a method that both indicates the adjacency of the constituent nodes from $A_{mix}$ and is necessary for computing the similarity matrix $S_E$. The resolution of hyperedge bundling is defined by the cutoff limit (CL) that dictates the “resolution” of a hypergraph. CL is defined as the ratio of desired number of clusters to the number of available raw edges to be clustered. We now introduce a control parameter, the cutoff limit (CL) that dictates the “resolution” of a hypergraph. CL is defined as the ratio of desired number of clusters to the number of available raw edges to be clustered. For example, for a graph of 1000 edges, a CL of 0.1 causes the clustering effect a shared true underlying interaction.

**Hyperedge bundling for multiple true interactions**

To demonstrate that bundling could be extended to separate multiple true interactions, we expanded the simulation and modeled interactions with three degrees of adjacency: “kin”, “nearby”, and “far”. The estimated raw graph yielded the true-positive (TP) edges surrounded by numerous false positive (FP) SIs (Fig. 2D). Estimating and partitioning the edge similarity matrix $S_E$ revealed that: 1) two “kin” edges were inseparable and together with their SIs they merged into the largest hyperedge $HE_1$ (Fig. 2E); 2) the “far” pair was clustered into two clearly separable hyperedges $HE_2$ and $HE_3$; 3) the “nearby” pair and their SIs were also clustered into two distinct hyperedges $HE_4$ and $HE_5$ with greater inter-hyperedge similarity as measured by mean-linkage (green box) than the “far” pair (magenta box); 4) a few scattered random false positive edges were also clustered into hyperedges (gray box), but they were much smaller in size than any of the hyperedges containing a true edge.

If a hyperedge containing at least one true raw edge is considered as a TP observation, bundling greatly decreased graph noise in terms of the FP/TP ratio. FP/TP in raw graph was 239/6 and 4/5 in the hypergraph, which marks a reduction in the fraction of FPs by a factor of 50. Visualizing these bundles showed that the hypergraph had less visual clutter and facilitated identification of the true interactions compared to the raw graph (Fig. 2F).

**Estimation of the edge similarity matrix $S_E$**

Hyperedge bundling is based on the raw connectivity graph $A_E$ (a sparse matrix containing only significant edges), and the mixing matrix $A_{mix}$ (Fig. 2A, C). We first parsed the edges in $A_E$ into a list of node pairs (Fig. 2B). We next find the mixing function $f_{mix}$ between all involved nodes from $A_{mix}$ (Fig. 2C, and illustrated geometrically in Fig. 2D) to compute the edge-to-edge adjacency in signal mixing.

**The edge adjacency matrix ($A_E$)**

For a raw graph with $m$ edges, the edge-to-edge adjacency matrix $A_E \in \mathbb{R}^{m \times m}$ represents the pairwise mixing adjacency among all raw edges and is necessary for computing the similarity matrix $S_E$. The adjacency between two edges $E_i(V_1,V_2)$ and $E_j(V_3,V_4)$ was defined as follows (Fig. 2D): if $V_1 - V_4$ are distinct nodes

$$A_E(i,j) = \max \{ f_{mix}(V_1,V_3), f_{mix}(V_2,V_4), f_{mix}(V_1,V_4), f_{mix}(V_2,V_3) \}$$

elseif $V_1 = = V_2$: $A_E(i,j) = f_{mix}(V_2, V_4)^2$

elseif $V_2 = = V_4$: $A_E(i,j) = f_{mix}(V_1,V_3)^2$

elseif $V_1 = = V_4$: $A_E(i,j) = f_{mix}(V_2, V_3)^2$

elseif $V_2 = = V_3$: $A_E(i,j) = f_{mix}(V_1,V_4)^2$

elseif $i = = j$: $A_E(i,j) = 0$ diagonal of $A_E$ (6)

here “$=$” is assertion, “$=$” is assignment. This algorithm is applied for all pairs of edges in the raw graph to populate the $A_E$ matrix (Fig. 2E).

**Evaluation of edge similarity ($S_E$) with correlation of edge mixing profiles in $A_E$**

We denote rows of the $A_E$ matrix as the signal mixing profiles so that $A_E(i)$ and $A_E(j)$ are the mixing profiles of edges $E_i$ and $E_j$, respectively, and thus indicate their mixing adjacency to all the other raw edges in the graph. If $E_i$ and $E_j$ are similar to each other, i.e., a high correlation between $A_E(i)$ and $A_E(j)$, edge $E_j$ will be similar to all the edges in the raw graph that $E_j$ is similar to, and vice versa (Fig. 2F, G). Such pattern can be already observed in the simplified models (Fig. 1) where SIs of any given true edge are all close to each other and adjacent to the true interaction. Conversely, if two edges are far apart in mixing, their mixing profiles exhibit little to no correlation. Using correlation estimates of mixing profiles, it is thus possible to assess the significant similarity of all pairs of edges in $A_E$ and populate the similarity matrix $S_E \in \mathbb{R}^{m \times m}$ (Fig. 2H).

Hyperedge bundling is based on the notion that a $S_E$ can be partitioned into clusters of raw edges that are similar to each other in mixing within each cluster and therefore to collectively reflect a shared true underlying interaction.

**The resolution of hyperedge bundling is defined by the cutoff limit**

We partition the edge similarity matrix $S_E$ into clusters of “hyperedges” so that within any one hyperedge, the raw edges are mixing-wise close (large $S_E$ values) to each other and distant (small $S_E$ values) from raw edges of other hyperedges. We now introduce a control parameter, the cutoff limit (CL) that dictates the “resolution” of a hypergraph. CL is defined as the ratio of desired number of clusters to the number of available raw edges to be clustered. For example, for a graph of 1000 edges, a CL of 0.1 causes the clustering method to partition the $S_E$ matrix into 100 hyperedges. We chose to control clustering using the CL for better comparability of clustering methods or graphs of different sizes. The similarity matrix $S_E \in \mathbb{R}^{m \times m}$ can
be partitioned into any number of clusters from 1 to \( m - 1 \), i.e., \( C_L \) ranging from \( 1/m \) to \((m - 1)/m\) (Fig. 2I, for technical details on how \( C_L \) is related to the depth at which dendrogram was cut into clusters, see (Wang et al., 2018)).

**Validate the stability of hyperedge clustering**

To ensure that the hyperedges are not random outcomes of partitioning the similarity matrix, the “stability” of partitioning solutions must be evaluated. We ask, at any resolution \( \{C_L\} \), if the differences between the partitioning solutions of \( n \) randomly perturbed versions of a similarity matrix \( S_E \) is statistically smaller than their surrogate counterparts, the partitioning solution can be considered as stable (Fig 2 (Wang et al., 2018)). The distance between two partitioning solutions can be estimated with the variation of information (Meilă, 2007). The independent perturbations to a similarity matrix can be acquired by randomly deleting a small subset, e.g., 10 or 20%, of the elements in the similarity matrix (Ben-Hur et al., 2002; Williams et al., 2015). The surrogates can be obtained by randomly permuting the perturbed similarity matrix.

**Methods**

The goal of this study was to assess the performance and applicability of hyperedge bundling in suppressing spurious interactions (SI) in MEG/EEG source connectivity graphs. To this end, we obtained large numbers of functional connectivity (FC) graph estimates from simulated data with realistic sources and inverse modeling. We next evaluated the efficacy of hyperedge bundling in capturing true positive (TP) interactions and rejecting false positive (FP) SIs. Finally, we demonstrated the bundling of FC graphs estimated from MEG data recorded in a visual working memory (VWM) experiment.

This section includes the procedural outlines of the simulations and evaluation of bundling efficacy. The preprocessing pipeline, technical details of the simulations and preprocessing of the VWM experiment are described in Wang et al. (2018). The Python 2.7 and National Instruments™ LabVIEW version of the hyperedge bundling program can be
Simulating "truth" time series of varying coupling strengths

In real electrophysiological data, mixing across source loci and subjects is inhomogeneous (Brookes et al., 2014) and coupling strengths of neuronal interactions also exhibit great spatiotemporal and inter-subject variability (Preti et al., 2016; Zalesky et al., 2014). To account for such variability, we created 1000 distinct truth graphs containing 200 randomly generated true interactions between 400 cortical parcels in a standard cortical source space (Destrieux et al., 2010). Each node thus connected only to a single other node, which allows an unbiased survey of the whole cortical surface in every graph realization. We did not simulate structured networks therefore excluding the impact of higher order SI. These higher order SI can arise from common drive, third-party sources, and cascade effects, although identifying them is of equal importance (Mannino and Bressler, 2015; Wollstadt et al., 2015).

For every truth graph, we simulated ten sets of coupled time series, representing two different modes of coupling, i.e., gamma distribution ($C_\gamma$) with maximum coupling of 0.9 and order parameter $r$ ranging from 1 to 20 or uniform distribution ($C_u$) at five different levels of coupling strength each (Fig 3 (Wang et al., 2018)). A set of uncorrelated null hypothesis time series was also simulated for each truth graph. These null hypothesis time series were used for estimating the parcel mixing properties (3.2) and as the baseline condition against coupled conditions in group analysis.

Estimation of mixing properties using the H$0$ time series

Mixing in source reconstructed MEG/EEG data is essentially captured in the forward and inverse operators used in source reconstruction. These operators are determined by the data acquisition system and specific of the individual source model (Wens, 2015). In addition to the mixing function $f_{mix}$ (see 2.2.3), we characterized the source model used here with a set of additional mixing metrics obtained from the 12 subjects from the VWM experiment (Figs. 5 to 7 (Wang et al., 2018)):

1) Parcel fidelity quantifies the reconstruction accuracy and is defined as the phase correlation between the simulated null hypothesis time series $x_0$ and the reconstructed null hypothesis time series $\hat{x}_0$ of parcel $v_i$.

$$f_s(v_i) = |\text{re}(cPLV}(x_0(v_i), \hat{x}_0(v_i)))|,$$

(7)

2) Edge fidelity, $f_{e}(v_i, v_j) = f_{mix}(v_i, v_j)$, that quantifies the reconstruction accuracy of raw edges connecting two parcels $v_i$ and $v_j$.

3) Residual spread function is the correlation between two parcels reconstructed null hypothesis time series.

$$PLV_0(v_i, v_j) = |\text{re}(cPLV}(\hat{x}_0(v_i), \hat{x}_0(v_j)))|,$$

(8)

The definition of $PLV_0$ appears similar to that of $f_{mix}$ but they are conceptually different. The $f_{mix}$ measures how much of each source's true signals are picked up in other sources' reconstructed signals. $PLV_0$, on the other hand, is the correlation between any two sources' modeled time series that both are contaminated by mixing with numerous other sources. Because the $iPLV$ estimates can be biased by mixing (Palva et al., 2017), we used $PLV_0$ to exclude edges connecting sources with large mixing.

Elimination of poorly measurable edges with the intractable-edge-mask (IEM)

We applied an intractable-edge-mask (IEM) to exclude edges that connect sources with poor reconstruction accuracy. True interactions between these sources may exist, but cannot be reliably detected because estimations of connectivity between them are unreliable due to the limitations of the source model. We utilized the mixing properties and construct a group-level IEM in two steps:

1) With average edge-fidelity $<f_s>$ and the residual spread $<PLV_0>$, we create two Boolean masks:

i. The edge-fidelity mask ($M_{pf}$) to exclude edges with low fidelity, thereby removing edges connecting poorly reconstructed sources.  
ii. The residual spread mask ($M_{PLV0}$) to exclude edges with large $PLV_0$, thereby removing edges whose FC estimates likely are much distorted by mixing between these loci (Palva et al., 2017).

2) The IEM is the union of these two masks.

In this study, we set 0.1 as the threshold for $M_{pf}$, which removed the 40% most poorly reconstructed edges from all 79,800 ($N(N-1)/2$, $N = 400$) possible edges in raw graphs (for anatomical details of the mixing properties and the IEM, see Figs. 8C and 9 (Wang et al., 2018)). The $M_{PLV0}$ was acquired by deleting edges whose $PLV_0$ was greater than the 95th percentile of the $PLV_0$ matrix.

Estimation of group-level FC of simulated graphs

The group-level significant $iPLV$ estimates thresholded with the IEM were used as raw graphs for hyperedge bundling. The group-level analysis for the simulated graphs and for real MEG/EEG data in the VWM experiment were carried out in the same manner. For simulated graphs, we forward- and inverse-modeled the coupled truth time series into 12 subjects’ individual source space, thereby introducing mixing into reconstructed signals (Schoffelen and Gross, 2009). We next estimated $iPLV$ connectivity for these subjects. We then tested across subjects, for each edge in every estimated FC graph, whether there was a significant difference (one-tailed t-test) in the $iPLV$ estimate between the coupled and the $H_0$ condition. Those edges that showed a significant difference were identified as raw edges (corrected for multiple comparisons within each FC graph). We acquired FC graphs with three significance levels $p < 0.05, 0.01$, and 0.001 for each of the ten coupled time series.

Hyperedge bundling with two clustering methods

After applying the IEM to all group-level FC matrices, we followed the procedures described in Theory to obtain the similarity matrix $S_p$ for each FC. We next partitioned each $S_p$ into clusters of hyperedges with two clustering methods. The unweighted pair group method with arithmetic mean (UPGMA) is an agglomerative hierarchical clustering method that builds a rooted hierarchical tree to represent the distance in signal mixing between all raw edges (Jain et al., 1999). The Louvain method for community detection extracts communities by optimizing the modularity of clusters through a gradient descent procedure (Blondel et al., 2008).

Comparing hypergraphs with raw graphs

We denoted the TPs as the edges from truth graphs that were identified as significant edges in the group-level FC matrix, and FPs as significant edges in the group-level FC matrix but absent in the truth graph. Thus, the true positive rate (TPR, sensitivity) is given by $TPR = TP/N_{true}$, where $N_{true}^*$ is the number of "detectable true edges" referring to the number of simulated true edge that passed the intractable-edge-mask. We further defined the noise as the FP to TP ratio. An ideal group-level FC should capture as many of the true interactions as possible while rejecting other edges, i.e., high TPR and low FP/TP.

We used TPR and FP/TP as the main criteria to characterize raw graphs instead of the commonly used receiver operating characteristic curve (ROC) for two reasons. First, the ROC is derived from the TPR and false positive rate (FPR) which are not directly comparable between raw graphs and hypergraphs, as these are different constructs; second, because the number of FP is disproportionally larger than that of TP (as shown later with an example), the shape of the ROC is misleadingly optimal when limiting the number of raw edges with varying edge weight.
We defined a TP hyperedge (TPHE) as a hyperedge capturing at least one TP raw edge, whereas a FP hyperedge (FPHE) contained only FP raw edges. Hyperedges may also contain multiple TP raw edges. To quantify this, we defined separability as the fraction of true positive hyperedges that contain only one TP raw edge out of all true positive hyperedges. An ideal hypergraph should balance high TPR and separability against low noise (FP/TP).

Results

This section includes three parts: 1) Demographics of group-level FC of the simulated graphs; 2) Efficacy of hyperedge bundling; 3) Application of hyperedge bundling to real MEG data.

Group-level FC as raw graphs

In individual subjects, mixing introduced by the virtual MEG experiment distorted PLV, iPLV and the phase-lag of all measured graphs of varying coupling strength including the H0 time series (Fig 10 (Wang et al., 2018)). To find group-level significant edges, we tested for each edge whether there was a difference in iPLV value between the coupled condition and the H0 condition (Fig. 3A, see 3.4). Edges that showed a significant difference were reported as raw edges. Thus, we obtained FC graphs for each of the ten sets of coupled graphs at three significance levels of p < 0.05, 0.01 and 0.001.
Raw graphs of iPLV edges are noisy

Overall, the number of significant iPLV edges increased as coupling strengths increased (Fig. 3B). The group-level graphs at all 3 significance levels captured over 75% of all detectable TP edges, except in the case of weak uniform coupling, $C_{\theta}(0.1)$ (Fig. 3C). We simulated 200 random edges in each ground truth graph and computed the true positive rate (TPR) for each measured group graph as the number of significant edges divided by the number of all simulated true edges that passed through the intractable-edge-mask (IEM). Despite the high TPR, there was a large variability in graph noise (FP/TP) among these graphs (Fig. 3D).

Is strict statistical thresholding a good solution for pruning FPs?

We chose the graphs of gamma coupling ($C_{\theta}$) with order parameter $r$ of 15 and uniform coupling ($C_\theta$) with coupling of 0.5 to test statistical thresholding (below) and hyperedge bundling because they had comparable TPR (Fig. 3C) and equivalent true edge strengths (see distribution of Fig 3 (Wang et al., 2018)). Moreover, both contained only $\approx 750$ edges, which mitigated computational overhead in later clustering analyses.

One sensible way to identify key structures in FC graphs is to apply a statistical threshold to iPLV values. We found that by increasing the significance iPLV threshold, the number of FP edges decreased at a faster rate than the number of TP edges in both graphs (Fig. 3E). Around 120 of the 640 strongest edges were TP, giving a TPR $>90\%$ for 125 detectable true edges, but a FP/TP ratio of 4. When retaining the 20 strongest edges reduced the FP/TP to 0.1 (Fig. 3F) but at the cost of reduced TPR, (TPR $=0.15$). Overall we found that the mean iPLV of TP edges was larger than that of FP edges’ (Fig. 3G), which suggests that strict thresholding is an applicable solution for reducing FP/TP but comes at a price of an elevated false negative rate, although the shape of ROC curve appeared to be optimal (inset Fig. 3E).

Hypergraphs yields better FP/TP than raw graphs with reasonable TPR cost

The stability of clusters

Evaluating the stability of clustering was a necessary step prior to further analysis of the properties of hyperedge clusters. The resolution of clustering and thereby of the hypergraphs was controlled by the cutoff limit (CL, see 2.6). We used bootstrapping to identify the CL range that yielded stable partitioning of the raw graphs (see Methods and Fig 2 of (Wang et al., 2018)). We found that at CL $<0.4$, both UPGMA and Louvain clustering yielded significantly more stable partitions for simulated graphs than their randomly rewired counterparts (Fig. 4A). For the 640 raw edge graphs, this CL upper bound corresponded to $\approx 250$ hyperedges. In the following analysis, we thus tested bundling with CL ranging from 0.05 to 0.45.

Cluster-size distribution

We next quantified the distributions of hyperedge sizes (numbers of raw edges per hyperedge, Fig. 4B) by pooling hyperedges from 500 clustered graphs with CL ranging from 0.05 to 0.45. As expected, we found a systematic shift towards smaller hyperedges with increasing resolution/CL. The Louvain method consistently yielded more small hyperedges than UPGMA.

Hyperedge-bundling performance: trade-offs between separability, TPR and graph noise

Hyperedge bundling aims to detect and separate as many TP interactions as possible while rejecting as many FP as possible. We tried to find an optimal balance among these competing outcomes by taking into account two aspects of hyperedge bundling: separability and noise. We defined separability as the ratio between singleton TP hyperedges (containing only one TP raw edge) and all TP hyperedges, and noise as the FP/TP ratio of the hyperedges. An ideal hyperedge partitioning would thus have separability $\approx 1$, FP/TP $\approx 0$, and a TPR equal to the TPR of raw edges.

We observed that by increasing the hyperedge resolution (CL from 0.05 to 0.45), the separability increased but noise also increased with both clustering methods (Fig. 4C, D). Thus at coarse resolutions (low CL), multiple TP raw edges were partitioned into one hyperedge but there were very few FP hyperedges, likely because there were less small-sized hyperedges. Conversely, at fine resolutions (high CL), separability was improved but at the cost of having greater numbers of FPs.

Knowing that small hyperedges are more likely to be FPs than large hyperedges (Fig. 1E), we further tested whether excluding hyperedges by size would decrease noise. At each resolution level, excluding small hyperedges lead to a decrease in noise (FP/TP decreased with increasing $\theta_{\text{threshold}}$ Fig. 4C, D). Nevertheless, this was accompanied by reduced separability (y axis, Fig. 4C, D) and a reduced TPR (Fig. 4E, F) caused by the removal of small-sized TP hyperedges together with FP hyperedges.

To summarize, at all graph resolutions, hypergraphs were less noisy than raw edge graphs. In the least noisy hypergraph (e.g., Louvain, CL $= 0.05$ and $\theta_{\text{threshold}} >8$), 87% of the 125 TP raw edges were retained while achieving a $10^2$-fold decrease in noise compared to the underlying raw graphs, i.e., FP/TP decreased from (640-125)/125 $\approx 4.1$ (C raw graphs in Fig 3E) to $3.8 \times 10^{-3}$ (leftmost filled box on the cyan curve, Fig. 4F). Nevertheless, this improvement came at the cost of poor separability, meaning many hyperedges in CL $= 0.05$ graphs contained several true edges. To balance an optimal trade-off, we decided to use CL $\approx 0.15$ and $\theta_{\text{threshold}} > 2$, expecting to achieve a reduction of FP/TP to 0.1 from (4.1 in raw edges) with negligible reduction in TPR and adequate separability (0.5).

Louvain clustering yields less noisy hypergraphs but lower separability than UPGMA clustering

The Louvain method produced more small hyperedges than the UPGMA method (Fig. 4B). Although the Louvain hypergraphs had higher level of noise when retaining singleton hyperedges ($\theta_{\text{threshold}} = 0$), this relation was inverted when singleton hyperedges were screened (Fig. 5A). This indicates that the majority of the singleton hyperedges yielded by Louvain were FPs. Moreover, the Louvain hypergraphs had greater TPR when CL values were between 0.15 and 0.25 (Fig. 5B). These advantages, however, came at the cost of separability, which was better with UPGMA throughout the tested range (Fig. 5C).

Visual working memory networks: real MEG data

To assess the feasibility of using hyperedge bundling with real MEG/EEG data, we applied bundling to raw graphs that reflected significant strengthening of inter-areal phase synchronization during memory retention compared to pre-stimulus baseline during a visual working memory task (see (Wang et al., 2018) and Honkanen et al., 2015).

We found that the iPLV estimates in alpha- and gamma-frequency bands were greater during memory retention than in pre-stimulus baseline. Here, we picked the 1000 strongest iPLV edges and drew them as lines linking the synchronized parcels on a flattened cortical surface (Fig. 6A, B). We also illustrated a randomly picked graph from our simulations as a comparison (Fig. 6C). We applied hyperedge bundling (UPGMA with CL $= 0.15$, $\theta_{\text{threshold}} > 6$) to these raw graphs. The resulting hypergraphs, the real MEG and simulated FC graphs alike, offered better visualization of large-scale FC than raw graphs, emphasizing the long-range synchronizations between brain regions (Fig. 6D–F).

Discussion

MEG and EEG have great potential for yielding insight into the spatio-temporal structure of brain connectivity. Nonetheless, due to the ill-posed nature of the inverse problem, linear mixing and inaccurate source localization complicate MEG/EEG connectivity analyses both by distorting phase and amplitude estimates and by leading to false positive observations of artificial (AIs) and spurious interactions (SIs). We advance here a novel methodological framework, hyperedge bundling, to suppress SIs in brain connectivity graphs. We found that hyperedge bundling can be used to reduce the false positive rate with moderate to little decrease on the true positive rate.
Fig. 4. Hyperedge bundling outperformed raw edges. A) The hypergraphs created with both clustering methods were stable below CL of 0.4. B) The cumulative distribution function (cdf) of hyperedge size at different levels of CL, computed with hyperedges pooled from 500 graphs with 100 iterations within each graph. For both clustering methods C) and D), increasingly strict hyperedge size threshold ($\theta_{\text{HEsize}}$ varying from 0 to 8) caused separability and noise level (FP/TP) to decrease. E,F) The retained true positive raw edges also decreased as hyperedge size threshold increased.
Hyperedge bundling has several features that are advantageous and facilitate its application. First, since it is done only after interaction analyses, it does not require sophisticated preprocessing to suppress mixing effects in the original source time series. Hyperedge bundling only requires the forward and inverse operators and a mixing function estimated analytically or from simulations. Accordingly, it inherently takes the source-model heterogeneity appropriately into account. Hyperedge bundling is also independent of the interaction metric and can be applied to connectomes estimated with any bivariate interaction metric. Finally, the nodal groups in the hypergraph obtained from hyperedge bundling constitute data-driven coarsening of originally high-resolution source parcellations. We suggest that these nodal groups are more
representative of the true co-active local areas than a priori constructed low-resolution parcellations. This can be a subject for future work.

In summary, hyperedge bundling can be used to suppress SIs and identify putative true edges in brain connectivity data and thereby to improve the localization of true interacting neuronal networks.

Hyperedge bundling vs. edge thresholding: reducing FP/TP while maintaining acceptable true positive rate

Some connectivity studies have reduced the amount of edges by applying strict criteria on edge selection. However, biases and instability of graph properties can be introduced when using arbitrary threshold criteria on raw edges (Drakesmith et al., 2015; van Wijk et al., 2010) and weak connections may also play an important role in cognitive functions (Santaronechi et al., 2014). Nevertheless, imposing strict criteria for thresholding is an attractive option for increasing the fraction of true positives among all observations, i.e., decreasing the FP/TP ratio (see Fig. 3E, F) and for focusing the outcome on most robust effects. However, this approach, while effective in excluding FPs (SIs), also excludes a large fraction of true positives. For example, we found that in raw graphs when we applied a threshold strict to decrease noise (FP/TP dropped from 4 to 0.1), but the TPR was reduced to 0.15. In contrast, with hyperedge bundling we could obtain the same noise level (FP/TP of 0.1) while preserving a TPR of up to 0.88 (see brown line, Fig. 4F). Hyperedge bundling is thus superior to strict thresholding in attenuating FP/TP with little decrease in TPR.

Importantly, our simulations show that the raw edges with the largest estimates of interaction strengths might not correspond to the strongest or most important neurophysiological couplings, because the interaction estimates are biased by reconstruction accuracy (Fig 9A (Wang et al., 2018)). Considering that the reconstruction accuracy is highly heterogeneous across the cortical surface (Korhonen et al., 2014; Wang et al., 2018), this bias is another reason for including weak observations in FC graphs. In particular, thresholding FC graphs by correlation strengths will predispose the resulting network to be composed of nodes with the greatest reconstruction accuracy.

Control parameters of hyperedge determine resolution and the balance among FP/TP, TPR, separability

In the current implementation, hyperedge bundling is controlled by the cutoff limit (CL) and the hyperedge size threshold ($\theta_{HEsize}$). CL determines the resolution of the hypergraph and the balance between noise (FP/TP) and separability of true hyperedges. Low CL values lead to low noise in hypergraphs but poor separability of true raw edges into distinct hyperedges. $\theta_{HEsize}$ can be used to prune the smallest hyperedges to further reduce noise, albeit at a cost of pruning TP hyperedges.

We compared two clustering methods, UPGMA and Louvain. While the results showed clearly that by and large both clustering methods yielded similar performance, each method had interesting advantages. Louvain yielded better TPR than UPGMA for CL values between 0.05 and 0.25 (see Fig. 5B), and lower noise when singleton hyperedges were excluded (see Fig. 5A). UPGMA, on the other hand, yielded better separability of TP hyperedges throughout the control parameter ranges. Overall, using either clustering method with $CL = 0.15–0.25$ and $\theta_{HEsize} = 1–2$ will yield a large reduction in FP/TP (from 4 to 0.1–0.2) with good separability and negligible reduction in TPR.

In applications to real data where the truth graph is unknown, choosing parameters, i.e., to control the trade-off between suppressing noise and maintaining high TPR and separability, can be based on both our simulation results and objectives of the research. If the objective of the hypothesis requires good separability (e.g., establishing connectivity between specific visual areas to inferior parietal region), one should create high resolution hypergraphs, but this will be accompanied by suboptimal noise reduction. Conversely, if the objective is to establish connectivity between the visual and parietal regions, a low resolution hypergraph (with low noise) is pertinent.

Comparison of hyperedge bundling and symmetric orthogonalization

Symmetric orthogonalization is a pioneering solution to the overall problem of SIs in the context of amplitude correlation estimation (Colclough et al., 2015). Its predecessor, pairwise orthogonalization (Brookes et al., 2012; Hipp et al., 2012) excluded instantaneous mixing and evaluated amplitude correlations for each time-series pair at a time. The cross correlation between orthogonalized is a bivariate AI-free metric and therefore not immune to SIs (Palva et al., 2017).

Symmetric orthogonalization overcomes the problem of SIs by simultaneously removing zero phase-lag components from all source time series through a gradient descent procedure known as the Lówdin orthogonalization (Esvendorf, 1999; Lówdin, 1950). Next, all-to-all amplitude correlations are estimated with partial correlation of amplitude envelopes to keep direct and remove indirect interactions (Marrelec et al., 2006). Because the partial correlation matrix is expected to be sparse, a graphical lasso regularization of the inverse covariance matrix is applied to penalize near-zero elements (Banerjee et al., 2008; Friedman et al., 2008), which reduces noise in the partial correlation graph.

Symmetric orthogonalization effectively attenuates SIs caused both by signal leakage and by indirect true couplings (i.e., $A \leftrightarrow B \rightarrow C$). Two limitations of this method are: (i) it is applicable only to the estimation of amplitude correlations, (ii) it is limited by the rank of the data due to its dependence on singular value decomposition. For MEG/EEG data that are preprocessed with signal space separation (SSS) and temporal SSS methods, the rank of the data (degrees of freedom) is often limited to 60–70 (Hauamnn et al., 2016). Thus, symmetric orthogonalization should be applied to cortical networks with less than 60–70 independent sources, such as the 19 regions per hemisphere used in (Colclough et al., 2015). For studying FC with greater parcellation resolutions (70) or with interaction metrics other than amplitude correlations, hyperedge bundling thus provides an alternative method for SI suppression. The similarities and differences between symmetric orthogonalization and hyperedge bundling are summarized in Table 1.

The choice of source modeling and interaction metric

The choice of the MEG/EEG source reconstruction method influences source-level connectivity analysis through their differences in sensitivity to point-like, distributed, or time-varying source topographies (Hincapié et al., 2017). Although we used linear L2 minimum-norm based inverse operators (Hamalainen and Sarvas, 1989; Hamalainen and Ilmoniemi, 1994; Lin et al., 2006), hyperedge bundling can also be used with other source reconstruction methods as long as the amount of mixing among the source/parcels can be quantified. Likewise, any parcellation-generation approach, such as the reconstruction-accuracy-optimized (Korhonen et al., 2014) or adaptive (Farahibozorg et al., 2017) parcellations, will function with hyperedge bundling.

Finally, although we here evaluated the mixing function with the phase-locking value and used the imaginary part of the complex phase-locking value for estimating connectivity, both the approach and the results are generalizable to other interaction metrics (Palva et al., 2017). Essentially identical mixing functions may be obtained by other metrics that quantify a linear relationship, such as the correlation coefficient, or analytically, as suggested by Farahibozorg et al. (2017), and hence there is no need to adapt the mixing function estimation to the choice of the interaction metric applied in the data analyses.

High resolution parcellations are beneficial for hyperedge bundling

Parcel numbers in recent MEG/EEG source connectivity studies range from tens of parcels, e.g., 38 in (Colclough et al., 2015) and around 70 in
Hyperedge bundling is likely to be suboptimal for low-resolution source spaces, where the mixing similarity between observed edges is not limited by the rank of the source time series. Not limited by the rank of the source time series, which allows a high spatial resolution source estimates
Hyperedge bundling can be used with any directed or undirected interaction metric

- Limitations
  - Blind to true zero-phase-lag interactions
  - Likely insensitive to low SNR sources
  - Limited by the rank of the time series
  - Risk of reporting false positives if the time series are non-Gaussian

References