LOW-DIMENSIONAL CHARACTERIZATION
AND CONTROL OF BLUFF-BODY WAKES

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Low-dimensional characterization and control of bluff body wakes.
Final report for EPSRC contract number GR/L59030

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March 2000
1 Abstract

This report is a summary of research conducted for the EPSRC on contract GR/L/59030—“Low-dimensional characterization and control of bluff-body wakes”. The motivation for the work was to investigate active control schemes for stabilizing the low-Reynolds number bluff-body wake, which is an archetypal unstable flow exhibiting self-sustained flow oscillations as a result of global flow instability. Control of bluff-body wake oscillations is of use in drag reduction, noise suppression and prevention of flow induced structural oscillations. Moreover, suppression of closely related unstable flows, such as growth of a dynamic stall vortex on a pitching helicopter blade, may be possible using a similar strategy.

To this end, a numerical model of an unstable bluff-body flow was developed and validated by comparison with published literature. Various control strategies involving low-dimensional models of the flow and combinations of distributed sensors and actuators in the near and far wake were investigated. The control results of this study are unique, in that successful control of the flow has been demonstrated further away from criticality than by any other scheme. These results provide a base for continued research in this area and in other related flows (for instance control of helicopter dynamic stall) and has contributed to a publication [1] and several others in preparation.

2 Objectives

The work was inspired by an earlier study [1] which showed the promise of distributed sensing and low-dimensional models of the flow for design of a control strategy. A major objective of the current study was to demonstrate feedback control, with distributed sensing, of a numerical wake model containing all of the salient features of a 2D bluff-body wake (rather than the simplified prototype flow used in the earlier study). It was also appropriate to provide a control strategy that could be more readily implemented in a real world flow.

3 Description of work programme and sample results

3.1 Wake model

A numerical model of the bluff-body wake with control feedback was required to investigate various control strategies.

There are two qualitatively different types of local fluid flow instability: convective instability and absolute instability. Convectively unstable flows are noise amplifiers: examples are boundary layer flows, which transit from laminar to turbulent profile by amplification of small disturbances (Tollmein-Schlichting waves) or flows over open cavities, which may exhibit global flow oscillations (cavity or edge tones) as a result of amplification of pressure waves fed back from the downstream edge of the cavity. Unless a convectively unstable flow incorporates some sort of pressure feedback (as in the cavity case) the flow is globally stable. Active control of these flows is possible by removing the external noise source (an then the flow disturbances are swept downstream) or by removing the source of pressure feedback (e.g by out of phase control actuation).

Absolutely unstable flows, an archetype of which is the bluff-body wake, are intrinsically unstable and demonstrate self-excited oscillations even when all sources of noise are removed. Global instability, such as the vortex shedding oscillations from a bluff-body, results when the flow contains a large enough region of absolute instability, such as that found in the near wake of a bluff-body [2]. These absolutely unstable flows present difficulties for control schemes as they are relatively insensitive to external forcing and often interact with the control in a non-linear fashion [8].

Numerical and experimental studies have shown that suppression of one globally unstable mode of the wake by single sensor feedback merely results in destabilization of another mode and continued wake oscillations [3]. An appropriate flow model is the 1D, complex Ginzburg-Landau equation [4][5], which is deriveable from the Navier-Stokes equations and contains all of the stability features of the 2D bluff body wake pertinent to control [4] [6] [7]. This wake model has been used frequently in the literature for wake control studies and has been shown to allow semi-qualitative predictions of the wake with feedback [6]. Significantly, the Ginzburg-Landau model demonstrates the ineffectiveness of single sensor feedback in controlling the wake past a critical Reynolds number [7]: like the bluff-body wake it has many unstable global modes. The Ginzburg-Landau model is, however, relatively straightforward to integrate numerically and allows rapid prototyping of control strategies.
The wake model chosen for the study was of the following form [7]:

\[ \frac{\partial A}{\partial t} + U \frac{\partial A}{\partial x} = \mu(x)A + (1 + j c_d) \frac{\partial^2 A}{\partial x^2} - (1 + j c_n)|A|^2 A \]

where \( A(x, t) \) is the complex amplitude and \( U, c_d, c_n \) and \( \mu(x) \) are real. The spatial inhomogeneity is confined to the growth rate parameter

\[ \mu(x) = \mu_0 + \mu' x \]

where \( \mu_0 \) is similar to a wake Reynolds number. For \( \mu' < 0 \) the stability features of this ‘prototype’ wake are similar to the stability features of a 2D bluff body wake, as shown in figure 1. Like a low Reynolds number cylinder wake, the Ginzburg-Landau ‘wake’ is absolutely unstable near the origin, convectively unstable further downstream and ultimately stable far downstream.

The Ginzburg-Landau model was solved numerically on a domain with \( 0 < x < 120 \) using 1000 grid points with boundary conditions \( A(0, t) = 0 \) (which simulates the bluff-body) and \( A(120, t) = 0 \). Fixed parameters were \( U = 5, \mu' = -0.0434, c_d = 1, c_n = 0 \), to allow comparison with earlier wake control studies [7]. The advective part of the model was solved using a weighted average flux method, which resulted in minimum numerical dissipation. Diffusive and source terms were given timesteps approximately 10 per cent of the advective solution, which was chosen to have a Courant number of 0.9. Explicit integration was used for all terms except the diffusive part, for which an implicit scheme was used. Numerical integration of the model compared well to similar results in the literature. The wake model exhibits self excited wake oscillations above \( \mu_0 = \mu_{crit} = 3.43 \).

### 3.2 Actuator modelling

In order to provide control forcing of the wake an actuation function was placed in the near wake (within the absolutely unstable region). Simple delta functions have been used in previous work and were employed in this study. The actuator provided a step perturbation to the complex amplitude over the spatial range \( 0 < x < 2.0 \), which is well within the absolutely unstable near wake. Time dependent forcing of \( A(x, t) \) below \( \mu_{crit} \) resulted in a wake similar to that during self-excited oscillations above \( \mu_{crit} \). The ability to promote sub-critical wake oscillations is a desired feature in any proposed wake control actuator [8]. This type of actuation could simulate actuation at the bluff body (e.g. by cylinder rotations or alternate suction and blowing close to the separation points), or alternatively a vibrating control wire or flapping foil immediately behind the bluff-body. Cylinder rotations are a preferred actuation for a real cylinder wake as the actuation is constant along the cylinder span and locks the shedding into a 2D pattern [1].

### 3.3 Wake sensors

It was an objective of the current study to investigate active feedback control of the wake— in which sensors are placed in the flow and provide information to drive the control actuator. Previous studies have shown that multiple, distributed feedback sensor control holds more promise for alleviating vortex shedding wake oscillations than single-sensor feedback control [1] [4]. The original inspiration for the current study was [1], in which flow-field pictures were used to generate modes of the flow using Proper Orthogonal Decomposition. These modes were used as control feedback signals in a simple prototype wake model.

In order to design a control strategy that might, in future work, be implemented in an experiment, distributed point wake measurements were selected as feedback signals rather than the flow-field pictures employed in the previous study [1]. In an experimental flow these could be readily implemented by hotwires. Control strategies that minimized the number of intrusive wake sensors were of specific interest in the current study.

Experimentation with sensor locations, together with experimental evidence [3] showed that successful control could only be achieved (even at low Reynolds numbers) when the feedback sensors were placed within the absolutely unstable region of the near wake.

### 3.4 Single sensor proportional feedback

Control of the wake by using a single sensor and proportional feedback gain was investigated and the results compared to similar studies of the Ginzburg-Landau wake and of the low Reynolds number circular cylinder wake. At very low Reynolds numbers the cylinder wake is controllable by single sensor proportional feedback
Orthogonal Decomposition (POD) [1] could be performed on the data from these sensors and the POD mode $x_0$. To simplify the information from these spatially distributed measurements it was suggested that Proper Orthogonal Decomposition (POD) could be performed on the data from these sensors and the POD mode $x_0$. It was desired to incorporate a large number of sensors in the near wake to control the flow at even higher $\mu_\alpha$. To simplify the information from these spatially distributed measurements it was suggested that Proper Orthogonal Decomposition (POD) [1] could be performed on the data from these sensors and the POD mode $x_0$.

Complex, the search space grows with powers of two which meant that searching for suitable gains for three distributed sensors might be able to extend this controllable range. It was conjectured that an increased number of (and higher numbers of) sensors was time consuming and impractical.

Theoretical studies [10], and the prototype study of [1] have suggested that spatially distributed sensors are required for complete control of the wake and all of its global modes. It was also noted from the single sensor feedback experiments of §3.4 that the single sensor feedback was insensitive to small random perturbation of the wake. It was conjectured that the original single sensor control could be augmented by superimposing a small feedback signal from another sensor downstream of the first. This dual sensor feedback control improved the controllable range of the wake.

The arrangement is shown in figure 4a for a control run at $\mu_\alpha = 3.85$, or 12.5% above criticality (where single sensor control would fail), and sample results shown in figure 4b. The control actuation is provided by the summation of a feedback signal, gain $g_1 = 0.05$, from a sensor at $x = 4.8$ and another signal, gain $g_2 = -0.00006 - 0.0005j$, from a sensor at $x = 9.36$. Complete control of the supercritical wake above $\mu_1$ was achieved with this modified control strategy. This represents a significant improvement over previous wake control strategies which fail at $\mu_1$. Spatially distributed sensing increases the controllable range for the wake.

Figure 5a shows the position of the wavepacket (the oscillations in the wake) during runs with no control, single sensor control and dual sensor control at 12.5% above criticality. With no control the centroid of the wavepacket is within the convectively unstable region of the wake but, because the near wake absolutely unstable region acts as a 'wavemaker' for the flow the wavepacket remains at a fixed location indicating a self-excited (globally unstable) wake. With single sensor control the wavepacket convects all the way downstream and oscillations eventually subside indicating that the wake is now convectively unstable and globally stable.

The second sensor control gains which achieve this stabilization are explored in figure 5b. With a fixed upstream sensor gain of 0.05, the figure shows which combinations of gain result in wake stabilization (corresponding to the zero asymptotic oscillation amplitude). It is interesting to note that this 'window' of gains is quite small and shrinks with increasing $\mu_\alpha$. On either side of the 'gain window' there are plateaus: the left corresponding to oscillations in the original mode; the right to oscillations in another mode, destabilized by the control.

The dual sensor control was found to be successful up to $\mu_\alpha = 3.876$, or 13% above criticality (compared to single sensor control which fails at 5% above critical). It was conjectured that an increased number of distributed sensors might be able to extend this controllable range.

As there is no a priori method of selecting control gains for a non-linear spatio-temporal system like the Ginzburg-Landau wake, the gains for the dual sensor case were selected by searching the gain parameter space and observing the resultant asymptotic wake magnitude at each gain combination. As each gain is complex, the search space grows with powers of two which meant that searching for suitable gains for three (and higher numbers of) sensors was time consuming and impractical.

It was desired to incorporate a large number of sensors in the near wake to control the flow at even higher $\mu_\alpha$'s. To simplify the information from these spatially distributed measurements it was suggested that Proper Orthogonal Decomposition (POD) [1] could be performed on the data from these sensors and the POD mode.
amplitudes used as states for the controller. Simplification of the control algorithm could be achieved by neglecting POD modes with little energy.

This process was attempted for the Ginzburg-Landau wake using 30 sensors in the near wake region at $\mu_0 = 3.9$. The POD process was performed on the wake during a combination of random and periodic frequency sweep actuation. Eight modes of the wake had significant energy and the rest were neglected. As $\mu_0$ was increased the spatial extent of the absolutely unstable region grew—the feedback sensors became more numerous and spread out in the wake. Significant time differences became apparent between the upstream and downstream sensors.

The delays inherent in wake flows (i.e. upstream actuation takes a significant time to convect downstream and the flow response is therefore sensed by the control sensors at different times) cause significant problems in controlling the wake—even when using a small number of POD modes as control states. Traditional control theory is often only valid for systems where there is an instantaneous response to actuation (or one with a very small delay). It was observed that some of the POD modes changed amplitudes soon after actuation of the wake, whereas others responded hardly at all and much later. Traditional control methods drove the flow unstable because of the inherent wake delays (a factor which has been ignored in most of the wake control literature). The delays in the wake are quite evident in figure 4b during the dual sensor control case.

Predictive controllers, using auto-regressive models with exogeneous input (ARX models), have been developed for various systems with large delays [11]. It was attempted to design an ARX model of the forced Ginzburg-Landau wake and use the predictive control of [11] to design the control gains. This process, however, was unsuccessful, leading to very large (unstable) gains. The linear ARX models were unable to fully capture the non-linear features of the wake. It is suggested, however, that predictive controllers based on an ARX model of the wake may hold the most promise for extending the wake's controllable range. This is an important area of future research.

4 Variations from original proposal

The work was successful in achieving the aims of the contract. Multiple, spatially distributed sensors were employed in wake control and found to extend the wake's controllable range. This is a significant improvement over previous control strategies.

The dual sensor control adopted could also be readily employed in an experimental wake flow with hot-wires as sensors and cylinder rotations as actuation.

The work also investigated using a low-dimensional model of the flow based on POD modes. The POD modes were able to capture, in a small number of mode amplitudes, the salient features of the wake discerned from a large set of spatially distributed sensors. Experimentation with various linear control strategies showed that the delayed response of some of the POD modes to actuation requires a predictive control strategy.

5 Conclusions

In conclusion, distributed sensing of the near wake of the globally unstable Ginzburg-Landau equation increases the controllable range of the wake. This was demonstrated by control of the wake further from criticality (13%) than possible before. This is a significant improvement in wake control, over single sensor feedback schemes. It may be inferred that multiple, spatially distributed control schemes will increase the Reynolds number at which feedback control can stabilize the vortex shedding oscillations of the low Reynolds number bluff-body wake. The most promising control strategy for suppressing the bluff-body wake at higher Reynolds numbers is a predictive control strategy because of the inherent delays in feedback response from the spatially distributed sensors.

References


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a) Schematic of cylinder wake

![Schematic of cylinder wake](image1)

b) Ginzburg-Landau 'wake'

![Ginzburg-Landau 'wake'](image2)

Figure 1: Qualitative stability regions in the bluff-body and Ginzburg-Landau wakes

a) Wake signal at AU/CU boundary

![Wake signal at AU/CU boundary](image3)

b) Effect of sensor location

![Effect of sensor location](image4)

Figure 2: Single sensor feedback control at 2% above critical
a) Wake signal

Figure 3: Single sensor feedback control at 5% above critical — destabilization of second mode

a) Control arrangement

b) Signals in near wake (top), AU/CU boundary (bottom)

Figure 4: Two sensor feedback control at 12.5% above critical

a) Convection of wavepacket for 0,1,2 sensors

b) Wake magnitude for varying sensor gains

Figure 5: Wavepacket convection and wake magnitude at AU/CU boundary (12.5% above critical)