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Non-Linear Transformation Guidance for Aero-Assisted Lunar Return Trajectories

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Abstract

The use of non-linear transformations is investigated in the context of an aero-assisted orbital transfer from lunar return to a space station. Analytic prediction of the minimum altitude for a skip entry into the atmosphere is used to provide the reference for the constant altitude guidance. Feedback linearisation is used to maintain this altitude while a matched asymptotic solution to the exit trajectory problem is used to target the required apogee and so determine the "pull-up" point. During the exit phase the predictions made in targeting the apogee are used to generate an analytic reference trajectory in flight. Feedback linearisation is then used to guide the vehicle along the predicted trajectory, artificially improving the accuracy of the predictions and closely matching the actual apogee to the desired apogee. Autonomous return to the station has advantages over a return to ground in terms of both time and manpower.

1. Introduction

The planned construction of International Space Station Alpha (ISSA) may make feasible a larger scale return to the moon than the single mission programs such as Clementine which have been proposed or undertaken in recent years. The cost of mounting a multi-mission program from the Earth is likely to prove prohibitive, the construction of ISSA may provide a platform for the launch of lunar missions and an orbiting laboratory for sample study if the vehicles could return to the station rather than Earth[1].

One current ESA proposal is for a rover equipped lander mission to the south lunar pole, intended to assess the suitability of the moon as an off-world observatory. It is also proposed that the vehicle perform some soil sample analysis looking for oxygen and helium-3 for respiratory and fusion fuel usage respectively. More in-depth analysis of the lunar geology will be necessary if a manned base is to be established at some point in the future.

A single mission of the type proposed by ESA is limited in both the area it can cover and the experiments it may perform. A series of smaller sample return vehicles could achieve greater coverage in less time, and, with the possible use of ISSA as an orbiting laboratory, perform more detailed analysis for a lower financial outlay. With autonomous on-board guidance the problem of communication loss with the return vehicle becomes less of a concern provided the guidance algorithms employed are sufficiently robust. The use of smaller vehicles would also minimise the loss, both financial and scientific of any single vehicle should a failure occur.

Returning to Earth would require an effective ΔV of the order of 11km/s if the return is to ground. This compares with a ΔV of around 4km/s to achieve space station altitude from the return trajectory, making return to the station the more attractive option in terms of the required ΔV and consequently the total heat load experienced by the vehicle. In addition, although a ground return could be carried out using aerodynamic forces to provide a significant part of the required ΔV, the accuracy with which the landing site can be determined is limited. Historically this has led to ocean 'landings' and such an approach requires a large amount of hardware and personnel to be on hand to retrieve the vehicle. If the vehicle were returned to ISSA then once its orbit had been circularised, rendezvous with the station could be achieved with a limited number of personnel, no more hardware than would be on hand to track the vehicle anyway, and greater flexibility in the time scale.

The 4km/s ΔV requirement for return to the station would still demand a significant fuel load for a purely propulsive return. Use of aero-assisted trajectories provides an alternative means of achieving the required ΔV for space station rendezvous, without incurring a large fuel penalty.

Previous work in the field of orbital transfer[2] has examined the use of analytic modelling techniques to predict the trajectories of aero-assisted orbital transfer vehicles and as the basis for non-linear feedback guidance. The approach presented here uses a non-linear transformation technique to ensure stability of a trajectory about a reference condition by compensating for the non-linear terms in the motion of the vehicle, and so artificially linearising the system.

Any suitable reference condition may be used in developing the control expressions. There are two reference conditions used here, a constant altitude and an analytically produced trajectory model. These are discussed further below.

The method of matched asymptotic expansions has proved a useful tool in the analysis of transatmospheric vehicle motion, and has been proposed as the basis for a number of guidance schemes[2,3,4]. Solution by matched asymptotic expansions makes use of the approximation, first made by Allen and Eggers[5], that gravity may be ignored for high speed atmospheric motion in comparison with the aerodynamic forces experienced. With this assumption, the motion of the vehicle is considered in two parts, Keplerian and atmospheric. Individual solutions obtained for each of these regions are combined by asymptotic matching to produce a uniformly valid composite solution. Such solutions have been shown to yield accurate predictions of transatmospheric trajectory data and these predictions have enabled the development of some robust, low-complexity, inexpensive guidance schemes.

In this study it is proposed to marry the use of feedback linearisation guidance with the analytic predictions obtained via the method of matched asymptotic expansions.

As the vehicle enters the atmosphere at a moderate bank angle ($\cos\sigma \approx 0.6$; σ measured from the vertical) analytic prediction of the skip trajectory is used to estimate the minimum altitude that the vehicle will reach. Feedback linearisation is used to track this altitude while matched asymptotic expansions are used to predict the apogee which will result should the in-plane lift component be suddenly increased by rolling the vehicle to a predetermined pull-up bank angle ($\cos\sigma \approx 0.8$). When the predicted apogee lies within acceptable tolerances of the desired apogee the vehicle "pulls-up". The pull-up bank angle is chosen such that the in-plane lift force is not saturated, leaving some degree of control authority over the exit phase.

The apogee that would be achieved by this manoeuvre would be close to the desired apogee and moderate correction needed as the analytic relations used to predict the apogee are an approximation to the motion. The required correction could be achieved propulsively, however the scheme proposed here uses aerodynamic control during the exit phase. The predictions used to determine the pull-up point are used to provide an analytic reference trajectory. Non-linear transformations are then used to guide the vehicle along the reference trajectory, artificially improving the analytic predictions.

As the reference trajectory data is produced analytically it may be generated in flight removing the need for numerical integration or storage of trajectory data and so freeing valuable computer power for other functions. In addition, the trajectory data obtained is altitude dependent and consequently there will be zero altitude error for any given point along the path towards apogee. Given this, convergence of the climb-rate to the reference condition will guarantee attainment of the desired apogee.

The concept of a reference trajectory is somewhat misleading in this application as the 'trajectory' used actually comprises the velocity and flight path angle prediction data, and the vehicle is guided along the altitude profiles obtained. The term trajectory will be used in this study for convenience. Since the reference data is obtained from the same trajectory predictions used in determining the point of pull-up from level flight, the actual trajectory is guaranteed to be near the reference trajectory. How near that is obviously depends on the accuracy of the analytic model, but by guiding the vehicle along the predicted path it is found that the error in the predictions may be attenuated, matching the actual apogee as closely as possible to the targeted apogee.

The solution of the exit trajectory problem using asymptotic matching produces uniformly valid expressions for the velocity and flight path angle altitude profiles. Implementation of a linear feedback guidance scheme based on these then requires no more than the solution of simple algebraic expressions with no derivative terms involved.

In summary, vehicle control is implemented via bank angle modification of the in-plane lift component and it is assumed that the desired plane change is achieved using periodic roll-reversals. The control strategy falls into three sections:-

i) entry trajectory - vehicle enters at a moderate bank angle ($\cos\sigma \approx 0.6$) and the minimum altitude is predicted analytically.

Prediction of the apogee resulting from a sudden increase in upward lift component ($\cos\sigma \approx 0.8$) is constantly made to allow for unfavourable atmospheric conditions. This pull-up bank angle is only achieved if the predicted apogee lies within an acceptable tolerance of the desired apogee. At this stage the apogee prediction is really only a monitor, intended to check that the vehicle is not experiencing extreme atmospheric conditions.

ii) constant altitude guidance - when the flight path angle approaches zero the vehicle tracks the predicted minimum altitude until the desired apogee is predicted. The vehicle is then rolled to the pull-up bank angle and commences atmospheric exit.

iii) exit trajectory - the analytic trajectory solutions used to predict the apogee now provide a reference trajectory along which feedback linearisation is used to guide the vehicle to apogee.

The pull-up bank angle is chosen such that the vertical lift component is not saturated, thus leaving some control authority over the exit trajectory.

2. System Dynamics

In this study a non-linear transformation guidance method is presented which is based on matched asymptotic predictions of the vehicle's trajectory. To assist solution of the problem in this manner it is assumed that only in-plane motion is considered about a spherical, wind-free, non-rotating Earth. The equations of motion for this system are therefore

$$\frac{dV}{dt} = -\frac{\rho V^2 S C_D}{2m} - \frac{\mu}{r^2}\sin\gamma \tag{1}$$

$$V\frac{d\gamma}{dt} = -\left\{\frac{V^2}{r} + \frac{\mu}{r^2}\right\}\cos\gamma - \frac{\rho V^2 S C_L}{2m} \tag{2}$$

$$\frac{dr}{dt} = v\sin\gamma \tag{3}$$

where the assumed density model has the standard exponential form

$$\rho = \rho_0 \exp\left(\frac{-h}{H}\right) \tag{4}$$

It is further assumed that both the lift and drag coefficients remain constant over the atmospheric passage.

Finally, the range angle is not considered here as it does not influence the other state variables and has no effect on the guidance strategy.

In order to prepare these expressions for solution by matched asymptotic expansions the equations are non-dimensionalised and re-written in terms of non-dimensionalised altitude, h, as the independent variable. The following substitutions are used,

$$\overline{V} = V\sqrt{\frac{R}{\mu}} \tag{5}$$

$$h = \frac{r-R}{R} \tag{6}$$

$$\overline{\rho} = \frac{\rho H}{m/A} \tag{7}$$

$$\varepsilon = \frac{H}{R} \tag{8}$$

the reduced equation set in terms of h, is now,

$$\frac{d\overline{v}^2}{dh} = -\frac{\overline{\rho}\overline{v}^2 SC_D}{\varepsilon \sin\gamma} - \frac{2}{(1+h)^2} \tag{9}$$

$$\frac{d\cos\gamma}{dh} = -\frac{\overline{\rho}C_L}{2\varepsilon} - \cos\gamma\left\{\frac{1}{1+h} - \frac{1}{(1+h)^2\overline{V}^2}\right\} \tag{10}$$

This system is now solved by the method of matched asymptotic expansions.

3. Solution by Matched Asymptotic Expansions

The skip-trajectory uses atmospheric drag to slow the vehicle, reducing the energy of the orbit such that the resultant apogee is as close as possible to the desired apogee.

Initially the motion of the vehicle is classical Keplerian, then, as the vehicle enters the atmosphere, aerodynamic forces take over and the contribution of gravity to the motion may be neglected. Finally, as the vehicle exits the atmosphere, aerodynamic effects disappear and the vehicle's motion is once again under the sole influence of gravity.

The clear dominance of gravitational force outwith the atmosphere and of aerodynamic force within allow the analysis of the motion to be split into two sections; the outer, or Keplerian region, and the inner, or aerodynamic region. This approximation allows the closed-form solution of a simple skip trajectory by matched asymptotic expansions. In this approach expressions for the motion in each region are obtained separately and then combined by asymptotic matching to produce a uniformly valid composite solution.

The following variable substitutions are made for clarification,

$$u = V^2 \text{ and } \omega = \cos\gamma \tag{11}$$

The equations of motion for the system are now given by

$$\frac{du}{dh} = -\frac{\overline{\rho}_0 u C_D \exp(-h/\varepsilon)}{\varepsilon\sqrt{1-\omega^2}} - \frac{2}{(1+h)^2} \tag{12}$$

$$\frac{d\omega}{dh} = -\frac{\overline{\rho}_0 C_L}{2\varepsilon}\exp(-h/\varepsilon) - \omega\left\{\frac{1}{1+h} - \frac{1}{(1+h)^2 u}\right\} \tag{13}$$

Solutions are now obtained for the inner and outer regions.

3.1 Inner region

The method of matched asymptotic expansions can be applied to systems of differential equations in which a small parameter multiplies the highest derivative. This derivative may then be ignored except in regions of rapid change, or boundary layers. In the case being considered this boundary layer is the sensible atmosphere close to the surface of the planet, the inner region. A stretched independent variable is used in the inner region to help obtain a solution. The variable employed, $\tilde{h} = h/\varepsilon$, acts as a 'mathematical microscope', artificially expanding the region of interest. With this substitution the system is now written as

$$\frac{d\tilde{u}}{d\tilde{h}} = -\frac{\overline{\rho}_0 \tilde{u} C_D}{\sqrt{1-\tilde{\omega}^2}}\exp\left(-\tilde{h}\right) - \frac{2\varepsilon}{\left(1+\varepsilon\tilde{h}\right)^2} \tag{14}$$

$$\frac{d\tilde{\omega}}{d\tilde{h}} = -\frac{\overline{\rho}_0 C_L}{2}\exp\left(-\tilde{h}\right) - \varepsilon\tilde{\omega}\left\{\frac{1}{1+\varepsilon\tilde{h}} - \frac{1}{\left(1+\varepsilon\tilde{h}\right)^2 \tilde{u}}\right\} \tag{15}$$

In the limit $\varepsilon \to 0$, $\tilde{h} \to \infty$ and the atmosphere is effectively expanded to an infinite distance. Applying this limit, with all non-dimensionalised variables held constant, and assuming the following expansions for the velocity and flight path angle terms,

$$\tilde{u} = \sum_{i=0}^{n} \varepsilon^i \tilde{u}_i\left(\tilde{h}\right) + O\left(\varepsilon^{n+1}\right) \tag{16}$$

$$\tilde{\omega} = \sum_{i=0}^{n} \varepsilon^i \tilde{\omega}_i\left(\tilde{h}\right) + O\left(\varepsilon^{n+1}\right) \tag{17}$$

the relations become, to lowest order,

$$\frac{d\tilde{u}_0}{d\tilde{h}} = -\frac{\overline{\rho}_0 \tilde{u}_0 C_D}{\sqrt{1-\tilde{\omega}_0^2}}\exp\left(-\tilde{h}\right) \tag{18}$$

$$\frac{d\tilde{\omega}_0}{d\tilde{h}} = -\frac{\overline{\rho}_0 \tilde{\omega}_0 C_L}{2}\exp\left(-\tilde{h}\right) \tag{19}$$

These expressions are then integrated to give

$$\tilde{u}_0 = \tilde{u}_* \exp(-2\tilde{\gamma}/\lambda) \tag{20}$$

$$\tilde{\omega}_0 = \tilde{\omega}_* + \frac{\overline{\rho}_0 \lambda C_D}{2}\exp\left(-\tilde{h}\right) \tag{21}$$

where $\lambda = C_L/C_D$ and $\tilde{u}_*$ and $\tilde{\omega}_*$ are constants of integration.

3.2. Outer Region

The solution to the outer region is obtained in a similar fashion, although here no substitution is made for the independent variable and the limit $\varepsilon \to 0$ effectively shrinks the atmosphere to the Earth's surface, so that only exoatmospheric motion is considered. The equations of motion for this region are then

$$\frac{du}{dh} = -\frac{2}{(1+h)^2} \tag{22}$$

$$\frac{d\omega}{dh} = -\omega\left\{\frac{1}{1+h} - \frac{1}{(1+h)^2 u}\right\} \tag{23}$$

Series expansions are again assumed for u and ω of the form

$$u = \sum_{i=0}^{n} \varepsilon^i u_i(h) + O\left(\varepsilon^{n+1}\right) \tag{24}$$

$$\omega = \sum_{i=0}^{n} \varepsilon^i \omega_i(h) + O\left(\varepsilon^{n+1}\right) \tag{25}$$

Integrating the resultant lowest order expressions the outer solutions are obtained as

$$u_0 = u_* + \frac{2}{1+h} \tag{26}$$

$$\omega_0 = \frac{\omega_*}{\sqrt{u_*(1+h)^2 + 2(1+h)}} \tag{27}$$

where u_* and ω_* are the outer constants of integration.

3.3. Composite Solution

The composite solution is obtained by combining the outer and inner solutions and relating the integration constants by asymptotic matching. This process involves expanding the inner solutions for $\tilde{h} \to \infty$ and the outer for $h \to 0$. By equating the equivalent expressions from each region the integration constants are found to follow the relations

$$u_* = \tilde{u}_* \exp\left(-2\tilde{\gamma}_*/\lambda\right) - 2 \tag{28}$$

$$\omega_* = \tilde{\omega}_* \sqrt{u_* + 2} \tag{29}$$

Finally, there exists a constant solution which is common to both the inner and the outer regions and this must be subtracted from the sum of the solutions for the two regions so that it is not included twice in the composite expressions. The common solution is obtained by expressing the outer solution in the inner variable and taking the limit as $\varepsilon \to 0$, giving

$$u = u_* + 2 \tag{30}$$

$$\omega = \frac{\omega_*}{\sqrt{u_* + 2}} \tag{31}$$

The composite solutions for non-dimensionalised velocity and flight path angle in terms of altitude are now given as

$$u = (u_* + 2)\exp\left(-2(\tilde{\gamma} - \tilde{\gamma}_*)/\lambda\right) - \frac{2h}{1+h} \tag{32}$$

$$\omega = \frac{\overline{\rho}_0 \lambda C_D}{2}\exp(-h/\varepsilon) + \frac{\omega_*}{\sqrt{u_*(1+h)^2 + 2(1+h)}} \tag{33}$$

with

$$\cos\tilde{\gamma} = \cos\tilde{\gamma}_* + \frac{\overline{\rho}_0 \lambda C_D}{2}\exp(-h/\varepsilon) \tag{34}$$

Having obtained the uniformly valid composite solution, it is noted that a problem occurs near $\gamma = 0$. As the inner solution differs from the composite solution by terms of order ε, $\cos\tilde{\gamma}$ may reach unity before the composite solution. In this event the condition $\cos\tilde{\gamma}>1$ arises near $\gamma = 0$ rendering a solution for u (eqn.(32)) impossible. To avoid this the constant $\tilde{\gamma}_*$ is evaluated from the inner solution (eqn.(21)) by setting $\cos\tilde{\gamma} = 1$ at $h = h_{\min}$ i.e.

$$\cos\tilde{\gamma}_* = 1 - \frac{\overline{\rho}_0 \lambda C_D}{2}\exp(-h_{\min}/\varepsilon) \tag{35}$$

where $h_{\min}$ is defined as the value of h at which the composite $\cos\gamma(\omega)$ becomes unity. It should be noted at this point that the evaluation of $h_{\min}$ from eqn.(33) requires prior evaluation of the constants u_* and ω_* This concern is addressed more fully later.

3.4. Exit Trajectory

The evaluation of the trajectory expressions does not assume either a negative or positive flight path angle and consequently the relations are equally applicable to both the atmospheric entry and exit portions of the skip, though obviously the initial conditions for each will be different. Recognising that the initial conditions for the exit are the final conditions from the entry it should therefore be possible to obtain the exit trajectory constants in terms of the entry constants.

Given that the flight path angle will take a positive value over the exit trajectory it is logical to assume that the constant $\tilde{\gamma}_*$ will also take a positive value. From eqn. it can be seen that the inner solution for the flight path angle is symmetrical about $h_{\min}$ and consequently

$$\tilde{\gamma}_{*_e} = -\tilde{\gamma}_* \tag{36}$$

where the sub-subscript e denotes an exit trajectory constant.

Equating the two sets of composite solutions at $h = h_{\min}$ the remaining exit constants are found as,

$$u_{*_e} = (u_* + 2)\exp(4\tilde{\gamma}_*/\lambda) - 2 \tag{37}$$

$$\omega_{*_e} = \left(1 - \frac{\bar{\rho}_0 \lambda C_D}{2} \exp(-h_{\min}/\varepsilon)\right)\sqrt{u_{*_e}(1+h_{\min})^2 + 2(1+h_{\min})} \tag{38}$$

4. Guidance

From the initial lunar return trajectory the vehicle is required to lose sufficient velocity that the resultant elliptical orbit has an apogee altitude as close as possible to some target altitude. It may not be possible for the vehicle achieve the ΔV necessary to attain the desired apogee on a simple skip and so a control is implemented at the bottom of the skip to maintain that altitude until a release condition is satisfied and the vehicle rolls to the pull-up bank angle. The control used to track this altitude is a non-linear transformation controller implemented via the vehicle bank angle.

The release condition referred to above is the prediction, using the analytic relations developed, that the desired apogee (within acceptable tolerances) would be achieved should the vehicle roll to a pre-determined "pull-up" bank angle. This pull-up value is chosen that the in-plane lift is not saturated, leaving some control authority for the exit leg.

Control over the exit trajectory is again implemented using non-linear transformations via the bank angle. The analytic predictions made in determining the pull-up point are used to provide a reference trajectory and the control is implemented to guide the vehicle along this path towards apogee. Once at apogee, the vehicles propulsive systems would circularise the orbit and proceed from there to rendezvous with the station.

Although in this study the control laws are implemented by banking the vehicle, it should be noted that aerodynamic control can also be achieved by modulating the angle of attack. This approach would seem more suitable for manoeuvres requiring zero plane change as no out of plane forces are created. Implementation would appear to be more difficult, however, as the large pitching moments generated at hypersonic velocities would probably restrict the range of maintainable angles for a gliding vehicle lacking aerodynamic control surfaces. Gas jet control is possible but maintaining an off-trim pitch angle would require large amounts of fuel, far more than control of the bank angle which would also give access to the entire range of in-plane lift, from maximum outward to maximum inward[6]. There is also the added advantage of simplicity in the control law. Whereas a change in angle of attack will affect both C_L and C_D, a change in bank angle affects neither, rather it redirects the lift vector. It is also considered that the ability to control, to some degree, the plane of vehicle motion would be useful in correcting small changes in plane where none were desired.

4.1. Non-linear transformation controller for constant altitude.[7]

The control used here was developed using non-linear transformations. As stated above the controller is implemented via the bank angle, this has the effect of redirecting the lift vector. It is assumed that the desired out-of-plane motion is achieved through periodic roll-reversals and so the variable λ is modified such that it becomes the in-plane lift-to-drag ratio, i.e.

$$\lambda = \frac{C_L \cos\sigma}{C_D} \tag{39}$$

This definition will be used for the remainder of the paper.

The control is required to track a given altitude, r_c, and so successive time derivatives of r are taken until the control σ appears explicitly in the relations. This occurs in the second derivative of r so a pseudocontrol P is defined as

$$P \equiv \frac{d^2 r}{dt^2} = -\frac{\rho V^2 S C_D}{2m}\sin\gamma - \frac{\rho V^2 S C_L}{2m}\cos\sigma\cos\gamma - \frac{V^2}{r}\cos^2\gamma - \frac{\mu}{r^2} \tag{40}$$

Stability is ensured by evaluating the pseudocontrol in terms of the altitude error and error rate, viz.,

$$P = -\phi_1(r - r_c) - \phi_2\left(\frac{dr}{dt} - \frac{dr_c}{dt}\right) \tag{41}$$

where the reference climb rate, $\frac{dr_c}{dt}$, will be zero for a constant reference altitude. The feedback gains ϕ_1 and ϕ_2 are chosen to produce the desired vehicle response in following the reference altitude. Rearranging eqn.(40) the command bank angle is found to be

$$\cos\sigma = -\frac{2m}{\rho V^2 S C_L \cos\gamma}\left[\frac{V^2}{r}\cos^2\gamma + \frac{\mu}{r^2} + \frac{\rho V^2 S C_D}{2m}\sin\gamma + P\right] \tag{42}$$

4.2. Apogee Targeting

Throughout the entry and constant altitude phases of the motion it is possible to predict the apogee the vehicle will achieve for a particular value of $\cos\sigma$., using eqns.(32)&(33). In order to apply these equations, however, it is necessary to first evaluate the trajectory constants u_*, ω_* and $\tilde{\gamma}_*$.

During exoatmospheric flight the initial conditions are used to evaluate the two outer constants directly, and prediction of the minimum altitude yields $\tilde{\gamma}_*$. The procedure becomes a little more complicated during the atmospheric phase of the motion.

A problem arises when considering the modified constant $\tilde{\gamma}_*$. Remembering that the modification of this constant is required to avoid ω

taking a value greater than unity near the bottom of the skip, it is essential for accurate evaluation of the exit trajectory constants. It has been noted that evaluation of $\tilde{\gamma}_*$ requires prior knowledge of u_* and ω_*, and, as the motion is now within the inner region these outer constants must be evaluated from their inner equivalents which are in turn evaluated from the current vehicle state.

The problem is that one of the inner constants, $\tilde{\omega}_*$, is the cosine of the very constant that is being modified. Obviously one cannot be changed without the other, thankfully elimination of both from the minimum altitude evaluation is possible. Eqn.(33) is used to express both $\tilde{\gamma}_*$ and $\tilde{\omega}_*$ in terms of the minimum altitude. The inner velocity constant, $\tilde{u}_*$, may be evaluated with impunity, as it is depends only on the current velocity and flight path angle. The outer constants are now obtained through the matching process and substitution of the resultant expressions in $h_{\min}$ into eqn.(33) with $h = h_{\min}$ yields

$$1 = \frac{\bar{\rho}_0 \lambda C_D}{2}\exp(-h_{\min}/\varepsilon) + \frac{\left(1-\frac{\bar{\rho}_0 \lambda C_D}{2}\exp(-h_{\min}/\varepsilon)\right)\sqrt{\tilde{u}_*\exp\left(-2\cos^{-1}\left(1-\frac{\bar{\rho}_0 \lambda C_D}{2}\exp(-h_{\min}/\varepsilon)\right)\Big/\lambda\right)}}{\sqrt{2(1+h_{\min})+(1+h_{\min})^2\left(-2+\tilde{u}_*\exp\left(-2\cos^{-1}\left(1-\frac{\bar{\rho}_0 \lambda C_D}{2}\exp(-h_{\min}/\varepsilon)\right)\Big/\lambda\right)\right)}} \qquad (43)$$

which is then solved for $h_{\min}$ using a Newton-Raphson iterative solver as before.

For computational simplicity, during the constant altitude portion of the motion the trajectory constants are evaluated using the assumption that for flight path angles very close to zero (± 0.1 degrees) ω may be taken as unity, implicitly defining the current altitude as the minimum altitude. Having so defined the minimum altitude the exit trajectory constants are readily evaluated from eqns.(20),(21),(28),(29)&(35).

In predicting the apogee, rather than iterate to a solution as is done for the minimum altitude evaluation the predictions are used to evaluate the trajectory variables at a point outwith the atmosphere. The values of the trajectory variables at this exo-atmospheric point are then used as the initial conditions for the purely Keplerian motion to apogee. Using the outer solution expressions alone then it can be shown that the resultant apogee, h_a, is given by

$$h_a = -\frac{\left(1+\sqrt{1+u_{*_{exo}}\,\omega_{*_{exo}}}\right)}{u_{*_{exo}}} - 1 \qquad (44)$$

where $u_{*_{exo}}$ and $\omega_{*_{exo}}$ are the outer constants obtained from eqns.(26)&(27) using the values of u and ω predicted at the exoatmospheric point.

In this manner the apogee predictions are constantly updated and the pull-up control value achieved when the predicted apogee lies within a specified tolerance of the target altitude.

4.3. Trajectory Tracking

The trajectory constants evaluated in predicting the point for the pull-up manoeuvre are now used to determine the ideal velocity and flight path angle at any point along the vehicle's trajectory. This data is then used as reference data for a linear feedback controller designed to guide the vehicle along the path by the analytic relations. As the trajectory data is generated analytically in-flight the need for either storage of pre-planned trajectory data or repeated numerical integration of the trajectory is removed. The control is devised as though it were following an altitude plan. As will be seen this allows account to be taken of errors in both velocity and flight path angle.

The altitude error y is defined as

$$y = r - r_{ref}(t) \tag{45}$$

and consecutive time derivatives are taken until the control σ appears explicitly. This occurs in the second derivative which then defines a pseudo-control.

$$\dot{y} = V\sin\gamma - \dot{r}_{ref}(t) \tag{46}$$

$$\ddot{y} = -\frac{\rho V^2 SC_D}{2m}\sin\gamma - \frac{\rho V^2 SC_L}{2m}\cos\sigma\cos\gamma - \frac{V^2}{r}\cos^2\gamma - \frac{\mu}{r^2} - \ddot{r}_{ref}(t) \tag{47}$$

In order to assure stability the pseudo-control is evaluated from the altitude error and climb rate error as

$$\ddot{y} = -\alpha_1\dot{y} - \alpha_2 y \tag{48}$$

It is noted at this time that as the reference trajectory is altitude driven that at any given point the altitude error is always zero and consequently the pseudo control is defined solely in terms of the climb rate error, i.e.

$$\ddot{y} = -\alpha\dot{y} \tag{49}$$

where the feedback gain α is chosen to achieve the desired vehicle response.

Given that there are limits to the control which can be applied judicious choice of α is required. This is discussed further in the implementation section of this paper.

The command bank angle may now be obtained from eqns.(47)&(48) as

$$\cos\sigma_c = -\frac{2m}{\rho V^2 SC_L\cos\gamma}\left[\frac{V^2}{r}\cos^2\gamma + \frac{\mu}{r^2} + \frac{\rho V^2 SC_D}{2m}\sin\gamma + \ddot{y} + \ddot{r}_{ref}(t)\right] \tag{50}$$

As stated before, the reference trajectory is altitude driven and so the description of the reference variables as functions of time is somewhat misleading. The reference radial acceleration, $\ddot{r}_{ref}(t)$, for example, is found from the values of the trajectory variables for the altitude at which the vehicle finds itself at time t.

$$\ddot{r}_{ref}(t) = -\frac{\mu}{r^2} + \frac{\rho V_{ref}^2 S C_L}{2m}\cos\sigma_{ref}\cos\gamma_{ref} - \frac{\rho V_{ref}^2 S C_D}{2m}\sin\gamma_{ref} - \frac{V_{ref}^2}{r}\cos^2\gamma_{ref} \quad (51)$$

The reference climb rate, $\dot{r}_{ref}(t)$, required for determination of the pseudocontrol, is also found in this manner as

$$\dot{r}_{ref}(t) = V_{ref}\sin\gamma_{ref} \quad (52)$$

5. Implementation

Matched asymptotic expansions have been shown in the past to yield good results in comparison to numerical simulations. Nothing new is added to the solutions and so the justification for their use is taken as proven. It is the way in which they are implemented which is crucial to this study.

As has been said, the initial entry phase is envisaged as monitored rather than guided. During this portion of the motion matched asymptotic expansions are used to predict the apogee which would result if the vehicle were rolled to the pull-up control value. In this way allowance is made for extreme variation in the atmospheric conditions. Should the vehicle achieve the release condition during this phase pull-up would be effected at that point. In the worst case scenario the vehicle would be unable to achieve the desired altitude and an abort to ground would be initiated.

In implementing the control algorithm an altitude of 100km was used for final prediction of the minimum altitude. Some iteration is needed to obtain the minimum altitude so the choice of 100km altitude is arbitrarily made to ensure the on-board computer has time to complete the calculations. For this reason the results presented here are given in terms of the flight path angle at 100km altitude, referred to here as the "initial" flight path angle. There will be some disparity between this value and the entry angle. However, as there will inevitably be some error in the anticipated atmospheric conditions on entry, precise estimation of the flight path angle at the start of the first control phase would not be possible. Given this the choice of control gains is made such that a good degree of accuracy was maintained over a range of initial flight path angles rather than choosing a gain to precisely achieve the target altitude for one particular flight path angle. It was felt that this approach would be a more realistic test of the control algorithm.

Given the fast dynamics of the system, the data sample speed will be important for both state vector update and determination of the pull-up point. Ideally, pull-up would be performed at the exact instant when the desired apogee is predicted. However, sample rates and computing speed make this a pipe-dream for now, and so pull-up is achieved when the predicted apogee lies within an acceptable range of the target. A tolerance of ±1.5km was chosen for this work, with an elapse time of one-tenth of a second between state vector updates and a first order filter applied to the control output to compensate for this and smooth out the control time history. The filter takes the form

$$\Delta\sigma = \left(\sigma_c - \sigma_{ref}\right)\exp(-\varsigma t) \tag{53}$$

so that the final control demand is

$$\sigma_d = \sigma_{ref} + \Delta\sigma \tag{54}$$

The performance of the routine was checked on a trans-atmospheric vehicle simulation running with an exponential atmosphere and then a model of the 1962 U.S. standard atmosphere[8], representing the ideal atmosphere used by the on-board computer and the "real" atmosphere, respectively. Runs were carried out using the analytic apogee targeting system both with and without the trajectory tracking routine.

The comparison between the two runs for an exponential atmosphere with an initial flight path angle of -6° (fig.3) shows a small overshoot for both with the trajectory tracking reducing the apogee error by over 25%. Fig.4 shows the absolute apogee error over a range of initial flight path angles from -5° to -8°, with a similar improvement in apogee error for each initial angle when the trajectory tracking is used.

The aforementioned fast dynamics of the system leave it susceptible to perturbations. It was intended that the relatively deep pass into the atmosphere and the use of a relatively high lift-to-drag ratio would exaggerate the differences between the ideal exponential atmosphere and the "real" atmosphere, again providing a more realistic test of the guidance.

Fig.5 shows the results for a pass through a model of the 1962 U.S. standard atmosphere. As expected the different density profile introduces a new source of error to analytic predictions and without tracking of the predicted path the final apogee is in error by 36km. Using the trajectory tracking, however, this error is reduced by almost 86%. From fig.6 it can be seen that the trajectory tracking produces sizeable reductions in apogee error over the same range of initial flight path angles as before.

The absolute disparity between the apogees achieved for the two atmosphere models (fig.7) gives a guide to the robustness of the guidance algorithm. Once again, the trajectory tracking shows considerable improvement in consistency over the apogee targeting alone.

6. Conclusions

It has been demonstrated that lunar return to a space station can be achieved using fuel only to circularise and thence achieve rendezvous from orbital radii within 10km of the target altitude. Feedback linearisation has been used to guide the vehicle along a constant altitude path until the appropriate ΔV has been decremented. Release from level flight is determined by the analytic prediction of the exit trajectory and the resultant apogee. These predictions are then used to analytically generate reference trajectory data in-flight negating the need for data storage or numerical integration. Using this data, a second period of feedback linearisation guidance is initiated which guides the vehicle along the predicted path. This approach artificially improves the predictions obtained from the matched asymptotic solutions by using the feedback guidance to counter the effects not modelled in the predictions. Further improvement of the analytic model will obviously improve the guidance algorithm by including some of these effects. The inherent simplicity of the scheme lends itself well to implementation on the limited computer resources available in terms of power, storage space and robustness.

Acknowledgements

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Figure Captions

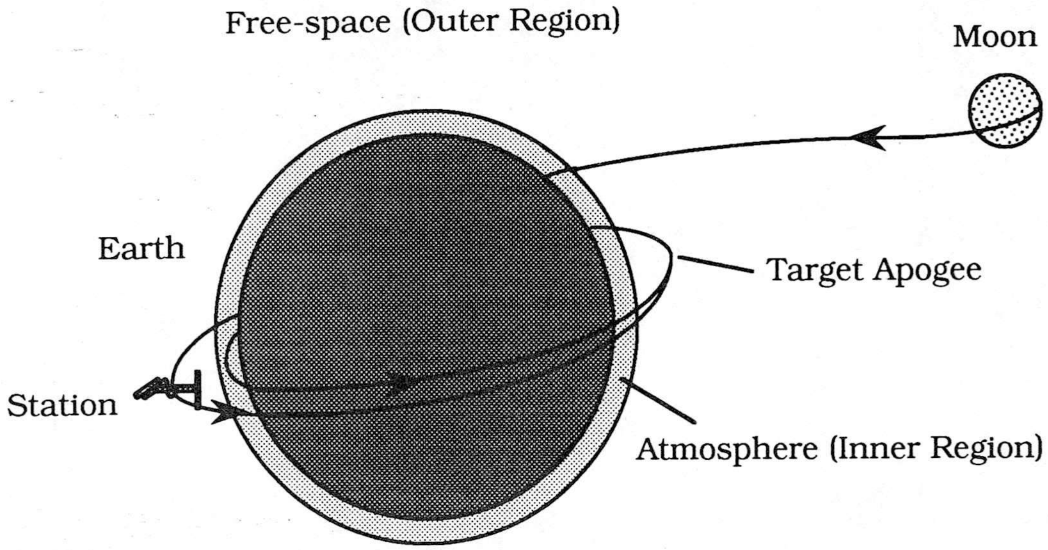


Fig. 1 Schematic of a Lunar Return Trajectory

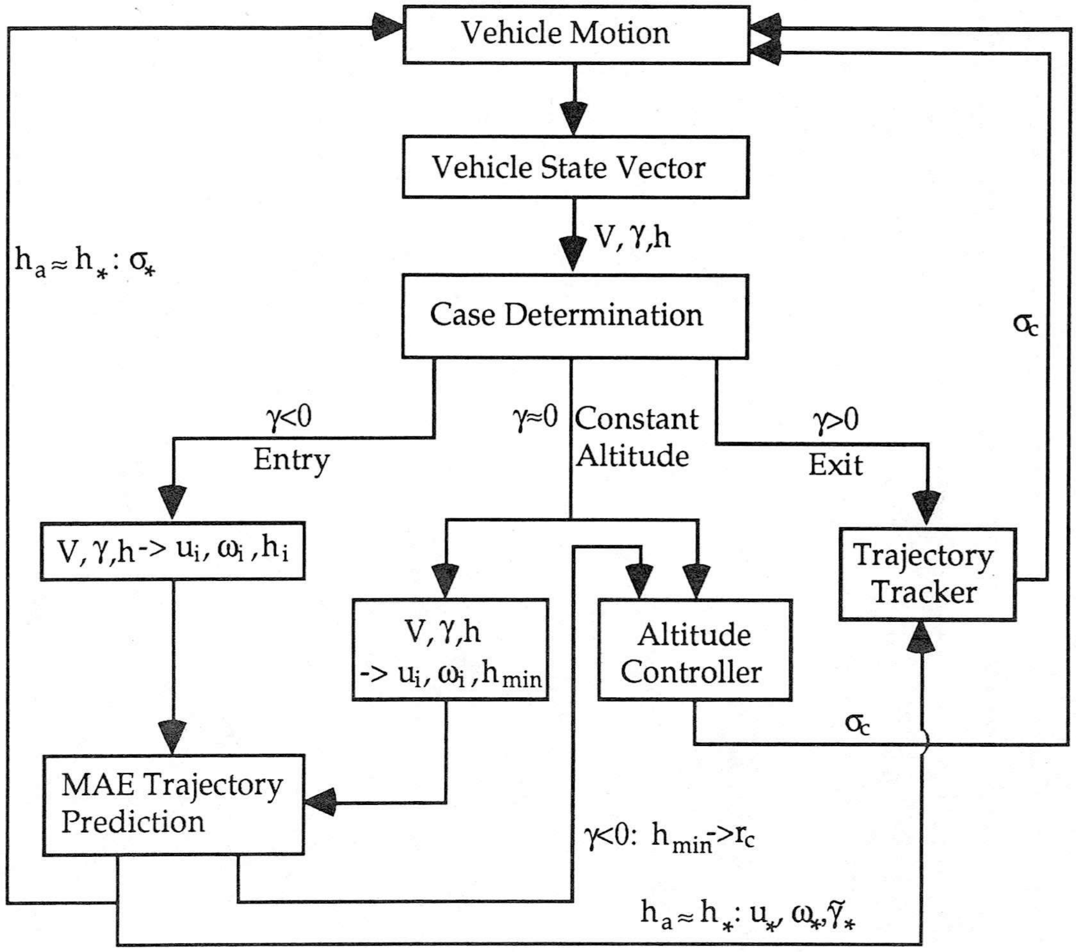
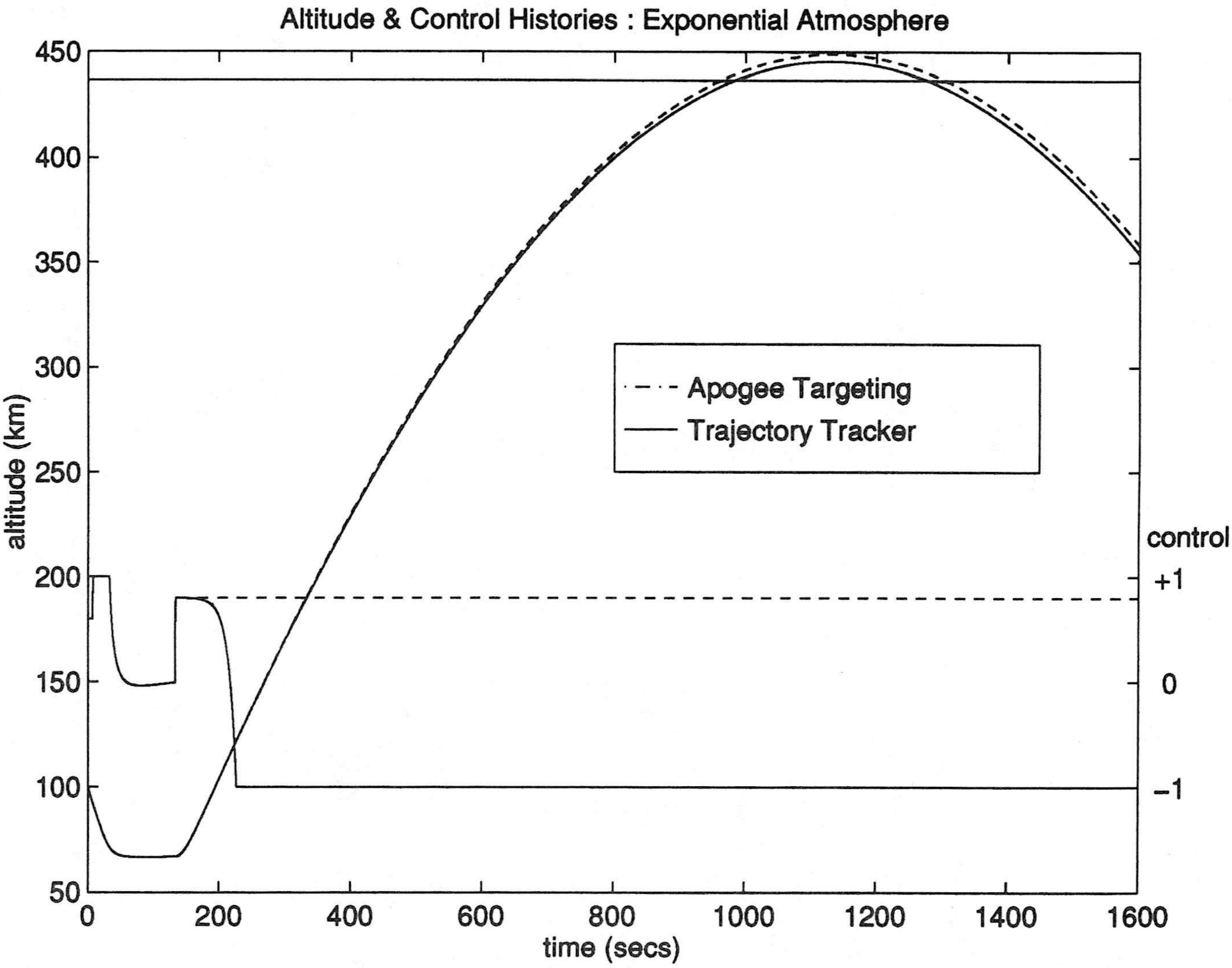
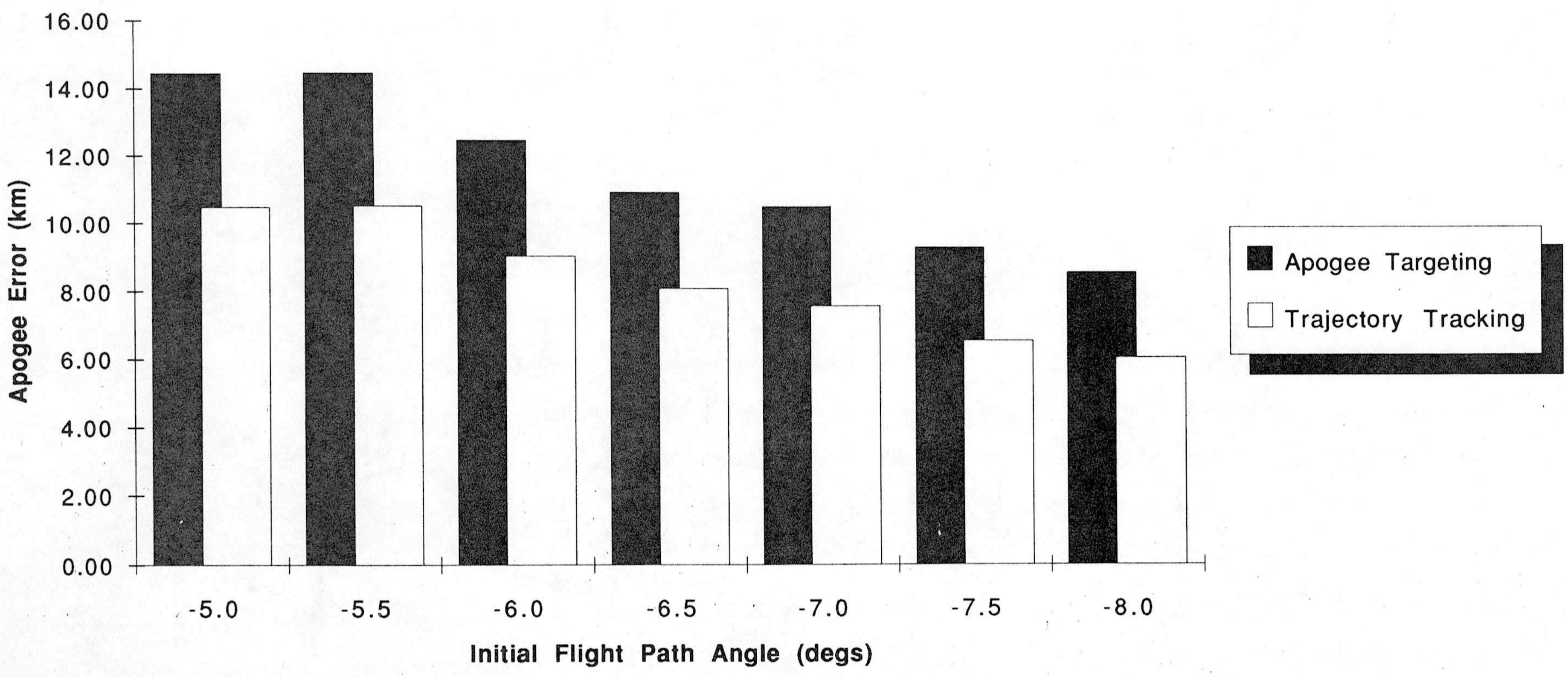
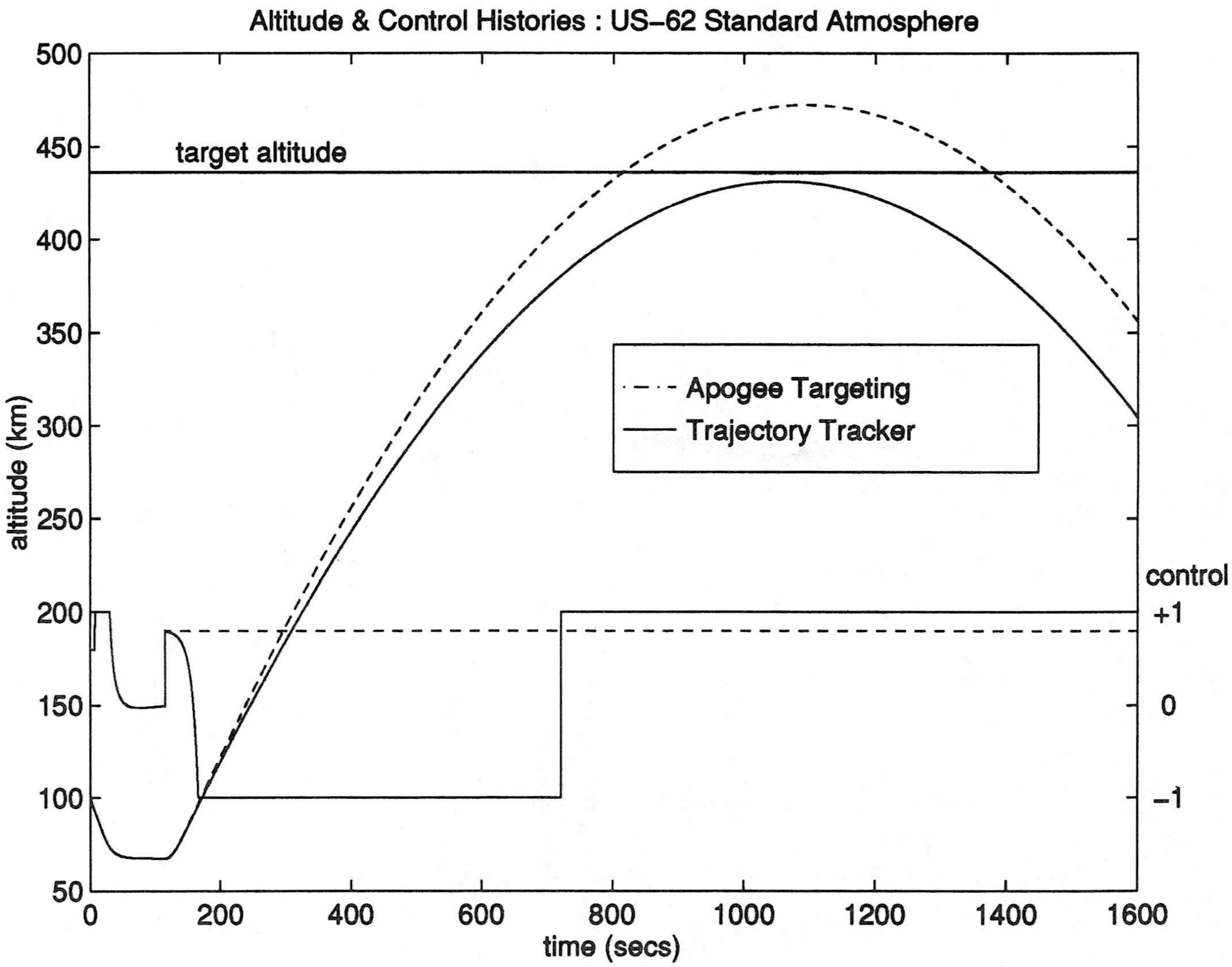


Fig.2 Guidance Scheme Implementation


Altitude & Control Histories : Exponential Atmosphere
altitude (km)
time (secs)
control
+1
0
-1
Apogee Targeting
Trajectory Tracker
450
400
350
300
250
200
150
100
50
0
200
400
600
800
1000
1200
1400
1600

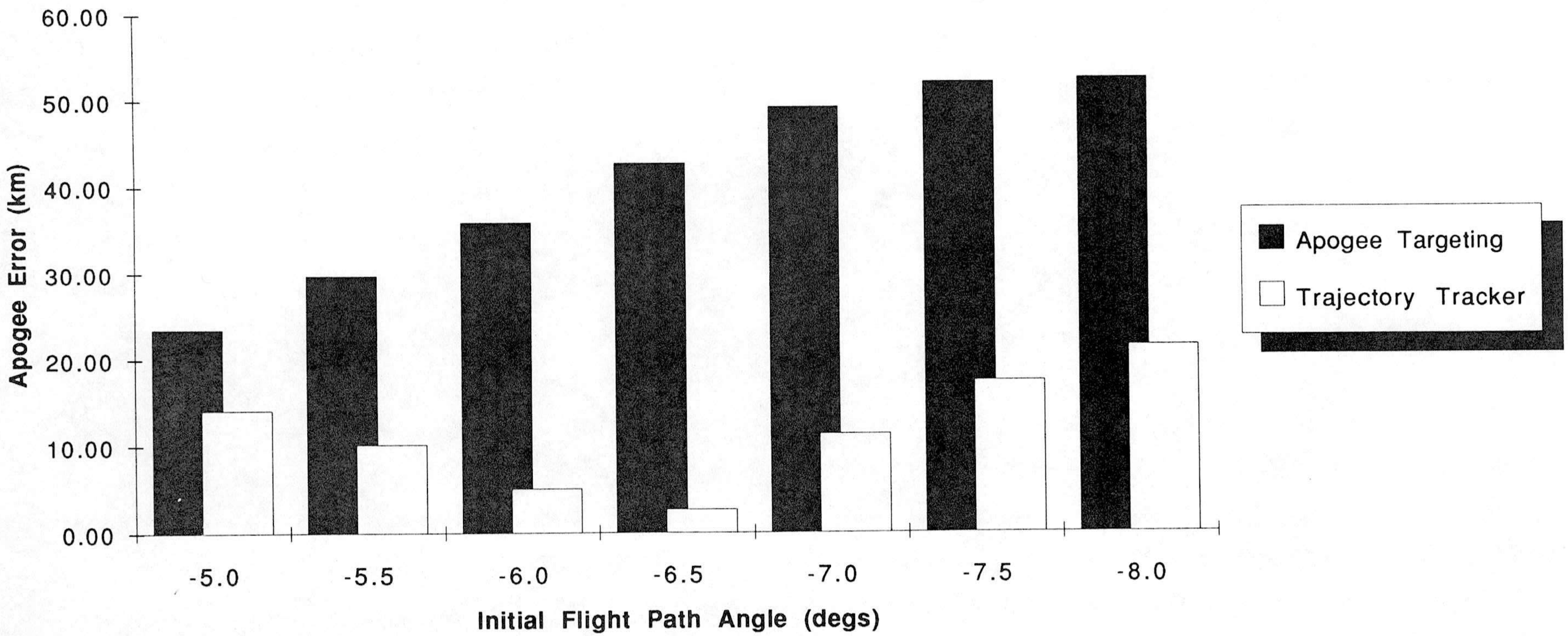
Absolute Apogee Error : Exponential Atmosphere



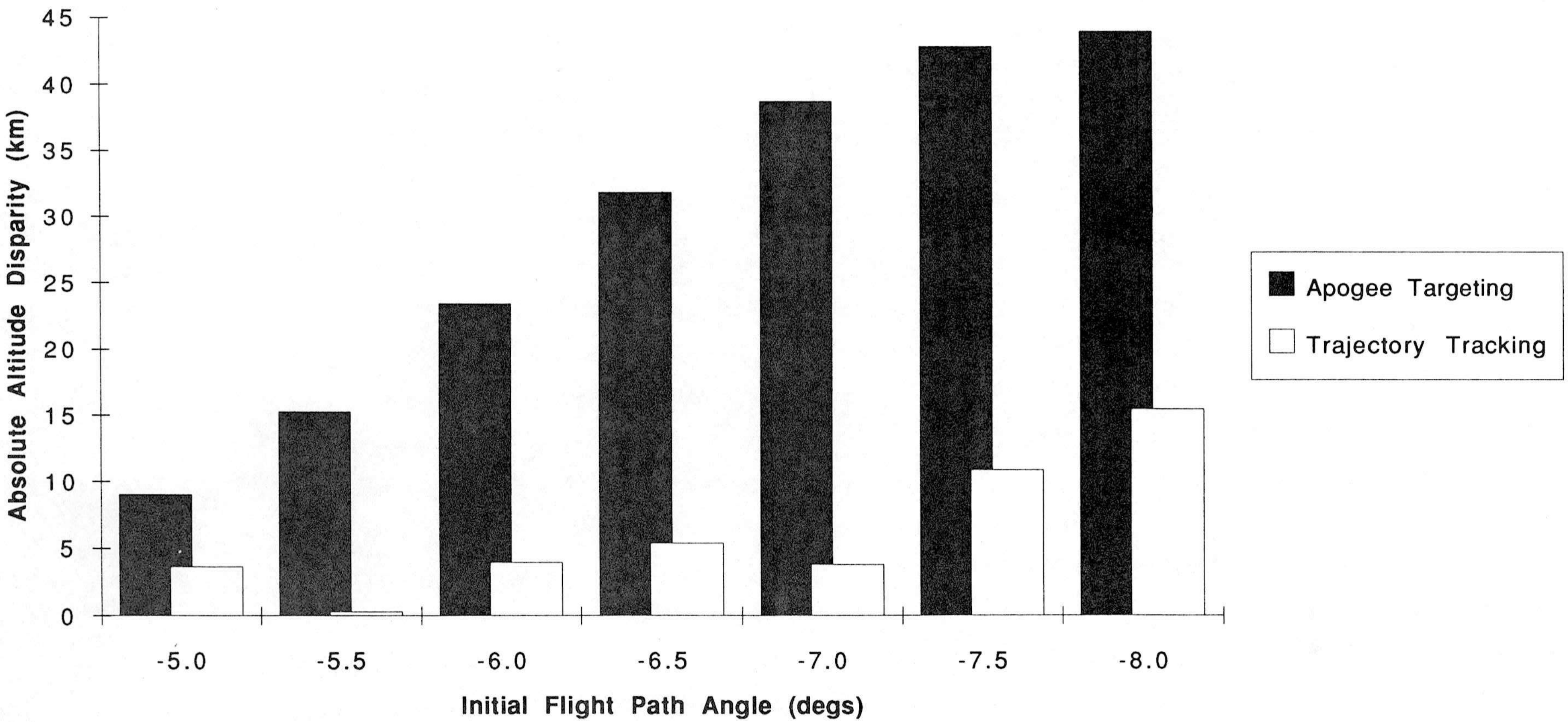


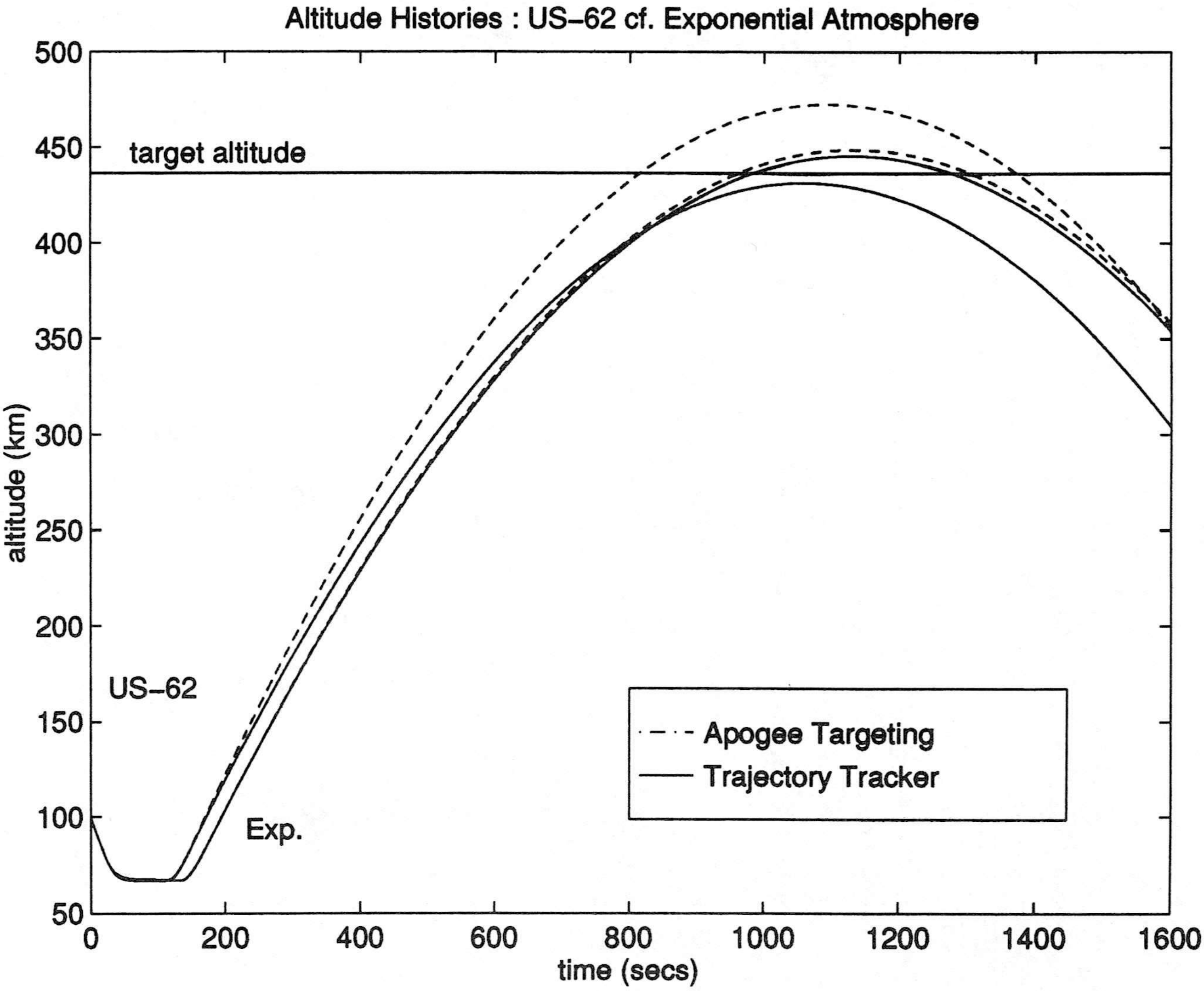
Altitude & Control Histories : US–62 Standard Atmosphere
target altitude
Apogee Targeting
Trajectory Tracker
altitude (km)
time (secs)
control
+1
0
–1
500
450
400
350
300
250
200
150
100
50
0
200
400
600
800
1000
1200
1400
1600

Absolute Apogee Error : US-62 Atmosphere



Apogee Disparity : "Real vs. Ideal"




Altitude Histories : US–62 cf. Exponential Atmosphere
target altitude
altitude (km)
time (secs)
US–62
Exp.
Apogee Targeting
Trajectory Tracker
500
450
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