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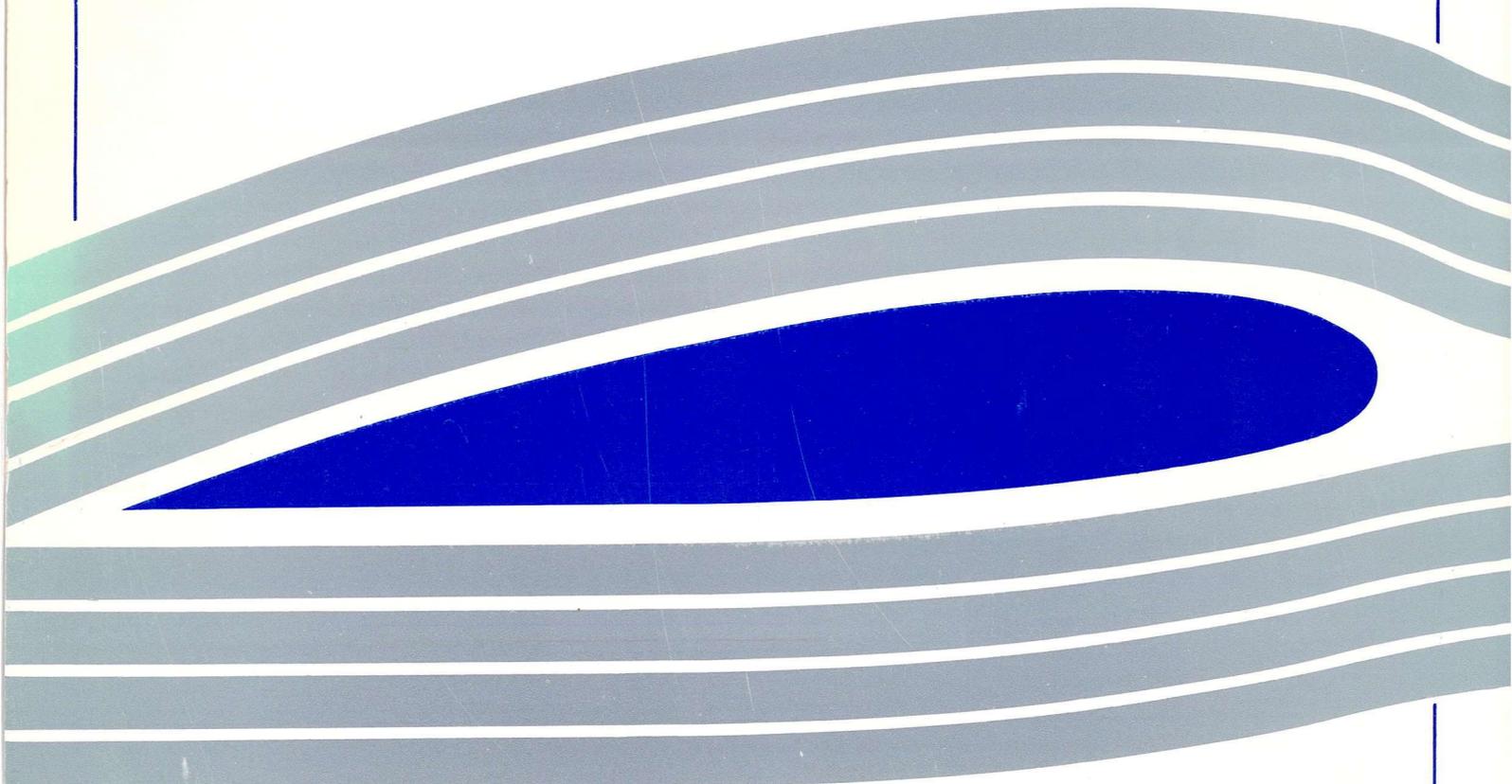
A prescribed wake model for horizontal axis wind turbine incorporating yaw and dynamic inflow

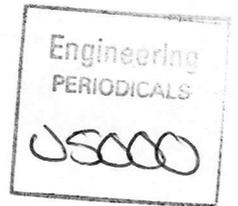
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R.A.McD. Galbraith, F.N. Coton,  
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Final contractor report for SERC/EPSRC contract number GR/H82105

December 1995

# **A PRESCRIBED WAKE MODEL FOR HORIZONTAL AXIS WIND TURBINES INCORPORATING YAW AND DYNAMIC INFLOW**

by

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**Final contractor report for SERC/EPSRC contract number GR/H82105**

## **ABSTRACT**

The following report summarises a three year research programme in the Department of Aerospace Engineering, University of Glasgow to develop a comprehensive prescribed wake aerodynamic model for horizontal axis wind turbines (HAWTs) capable of considering yawed and yawing flow. The original motivation for the work lay in the extensive and successful use of similar methodologies for vertical axis wind turbines and in the helicopter field. It was considered that the developed scheme would be suitable for design applications; running in minutes rather than hours. The approach used was similar to that of Coton et al.(1994) for vertical axis wind turbine applications where the blade was represented as a lifting line and a lattice of shed and trailing vortex filaments comprised the wake. The spatial and temporal development of the wake was pre-assigned using prescription functions developed in the current study and the induced flow at the blades was calculated by application of the Biot-Savart equation. Initially, the model was validated against free wake and field data for steady axial flow but was then extended to consider yawed flow. As a final stage, the unsteady aerofoil performance scheme of Lieshman and Beddoes (1989) was incorporated to account for the dynamic changes in blade loading around the azimuth in yawed flow. The resulting model has been compared to wind tunnel test data and field data supplied by NREL from the combined experiment. The work has resulted in a comprehensive aerodynamic model which is currently the focus of a follow-on EPSRC study (GR/K14995) which will extend its capabilities further.

## **OBJECTIVES**

The primary objective of the work was to develop a 'fast' prescribed wake model capable of assessing unsteady aerodynamic performance in yawed flow. The developed scheme should have run times appropriate to design calculations.

## **METHODOLOGY**

The starting point for the work was an existing prescribed wake vortex model for vertical axis wind turbines. In this study, the basic philosophy used in the existing model was applied to the geometric configuration of a horizontal axis machine. A pseudo free-wake approach, described in detail below, was adopted to determine prescription functions both for head-on and yawed flow. Once these had been established, the method was coupled to the unsteady aerodynamic model of Lieshman and Beddoes to give predictions of unsteady blade loads in yawed flow.

## **VARIATION FROM THE ORIGINAL PROPOSAL**

It was originally proposed that the study, should attempt to include dynamic inflow effects within the model. After consultation with colleagues in the U.K., Europe and the U.S. it became evident that dynamic inflow was not as significant an issue for wind turbines as it had appeared during the preparation of the original proposal. Of more concern was the development of a high quality, design speed, prediction scheme for steady yawed flow since, in the vast majority of cases, yawing flow may be considered to be quasi-steady. This prompted a change of emphasis in the study and, although dynamic inflow modelling was examined and an implementation strategy developed, the project focused solely on the development of a high quality method for steady yawed flow.

It was originally proposed that extensive use should be made of test data during the development of the model. In fact, very little appropriate data could be identified. This was particularly true of reliable wind turbine wake measurements in yawed flow. As a result, the EPSRC has funded a three-year follow-on project to measure yawed flow wake structures in a wind tunnel using particle image velocimetry. The project is a collaboration between Glasgow University and Heriot-Watt University and will involve both wake measurements and

unsteady aerodynamic measurements on the blades. As part of this study, the present model will be modified to include a representation of the wind tunnel walls thus dealing with the obvious constraint effect.

## PHASING OF THE WORK

The work was carried out in three phases, i.e.

- (1) Development of prescribed wake model for steady head-on flow
- (2) Extension of the above to yawed flow using steady blade aerodynamics
- (3) Inclusion of unsteady blade aerodynamics via the Liewman and Beddoes model

The achievements made with respect to these phases is given in the following sections.

### PHASE 1

#### Basic model description

Prescribed wake modelling has, for many years, been seen as a viable alternative to more intensive free wake methods in providing aerodynamic performance estimates for helicopter rotors. This approach was successfully applied by Basuno et al. (1992) to vertical axis wind turbines and later extended by Coton et al (1994) to address unsteady aerodynamic effects. The fundamental difference between a prescribed wake method and free wake models lies in the manner in which the wake geometry is obtained. In free vortex models the wake shape is created by consideration of self-induced velocities and can either be built up gradually over a number of time steps or obtained by a large number of successive gross distortions of the entire wake geometry. Both of these techniques can be very time consuming and the former has the added problem of producing unrepresentative wake shapes in the early stages of the calculation. In the present study, a prescribed wake model, which uses simple geometry prescription functions derived from consideration of momentum theory, has been developed for horizontal axis wind turbine applications. In this model the wake geometry is, therefore, generated without regard to the wake self-induced velocities by free vortex type models. Rather, the fully developed wake shape is obtained purely by consideration of induced velocities at the turbine blades. In this way the wake geometry can be viewed as an input to the problem rather than part of the solution.

Precise details of the computational implementation of the model can be gleaned from Robison et al. (1994a, 1994b, 1995a) and Galbraith et al. (1995) but the main features are summarised here.

In order to provide a detailed assessment of the variation of aerodynamic coefficients across the blade span, each blade is divided into a number of spanwise elements which are considered to be aerodynamically independent. The blade division is carried out in a manner which yields a finer distribution of elements at the blade tip, so giving a more accurate representation of the rapidly changing aerodynamic conditions associated with this region. A uniform loading is assumed on each blade element and is evaluated at the quarter chord of the element mid-span. In the case of steady axial flow, blade aerodynamic coefficients may be assumed to remain constant around the blade azimuth and the forces acting on each blade element may, therefore, be evaluated using aerofoil data from 2-D static tests. The spanwise blade loading distribution is represented by a series of straight-line vortex filaments which lie along the quarter chord line of each blade element, the strengths of which are evaluated by application of the Kutta-Joukowski theorem.

By representing the turbine blades in this manner, a discontinuity in spanwise bound circulation is created at each element boundary. This is redressed by the introduction of trailed vortex filaments whose strengths correspond to the differences in bound circulation between adjacent blade elements. Similarly, changes in blade incidence with time would require the introduction of shed vorticity into the wake to account for temporal changes in bound circulation. At this stage, however, only head-on flow was under consideration and so the wake was composed solely of trailed vorticity which was modelled as a discretised series of sequential, finite, straight-line vortex filaments, forming a piece-wise continual helical line which extends downstream from the blade trailing edge.

The flow passing through the swept area of a wind turbine experiences a deceleration in the streamwise direction and, from continuity, a corresponding expansion in the cross-stream direction. The axial deceleration of the flow is not an instantaneous phenomenon but begins ahead of the turbine and continues until the flow reaches a new equilibrium condition in the far field. Consequently, the wake may be thought of as consisting of two distinct regions; the near wake in which large scale changes in wake geometry occur and the far wake which represents the far field equilibrium condition of the flow.

In the present scheme, the near wake geometry is defined by simple prescription functions which map the wake induced velocities from the turbine blades to the far field equilibrium condition. The physical extent of the near wake is dictated by a cut-off criterion which is expressed as a universal non-dimensional time constant. This strategy allows the extent of the near wake to vary with operating conditions, yielding a more flexible criterion than linking the cut-off to a length scale.

Simple momentum theory dictates that the axial induced velocity in the far field is twice that at the turbine blades. When considered in more detail, however, it would be unreasonable to expect this relationship to hold for the variation in streamwise velocity of a particular vortex filament. Indeed, in the tip region, where large spanwise gradients in flow conditions exist, significant departures from the basic momentum relationship could be expected. For this reason, the far field induced velocity is defined in terms of the on-blade conditions using a deficiency function whose value varies along the span. Once the near-wake cut-off criterion has been satisfied, flow conditions in the far wake are assumed to be constant resulting in a cylindrical, axisymmetric flow field.

The complex nature of the flow field means that the parameters used to define the wake development are, to a great extent, inter-dependent. Thus, determination of appropriate prescription parameters is problematic if detailed information on the wake structure for a variety of different tip-speed ratios is not available. This was found to be a problem in the present study as comprehensive and reliable test data were not available to assist in the code development. As indicated in the original proposal, this could be overcome by the use of a free-wake model to provide the required information but the run times incurred by such schemes limited this approach. A novel and more acceptable technique was, therefore, developed during this study. This involved the use of a hybrid prescribed/free wake scheme to determine the optimum solution for the induced velocity functions, near wake cut-off criterion and far wake deficiency function. In this technique, an imaginary high resolution grid, defining a series of node points, was positioned behind the turbine. This allowed the induced velocity components to be calculated at each node point and subsequently compared to those prescribed. By starting from an initial approximation based on momentum theory, the information from these node points could be used in an iterative loop to provide feedback on the suitability of the current wake shape. This iterative process was continued until the feedback from the grid nodes matched the prescribed velocity distribution throughout the wake over a wide range of tip speed ratios.

It was found that the most appropriate geometry was obtained when the radial induced velocity variation in the near wake is expressed as a quadratic function of time. The corresponding axial induced velocity is defined as three sub-regions in each of which the velocity development is a linear function of time. These relationships are applied over the entire near wake region which is assumed to reach the far wake state after a fixed non-dimensional time has elapsed. The corresponding spanwise variation of the far wake deficiency function is described in the references.

As in previous studies for the prescribed wake model of vertical axis wind turbines, the above geometry prescription functions were found to be relatively insensitive to realistic changes in blade planform, solidity and, consequently, the number of blades.

The numerical implementation of the model is similar to that of Basuno et al. (1992) although, in this case, a first order Newton scheme is used to provide a converged solution for the blade loadings for a given wake geometry. The overall calculation procedure is outlined in Fig. 1.

### **Validation**

At the end of Phase 1, model validation for head on flow conditions was carried out and is discussed in detail by Robison et al. (1995b). As an example of this, Fig. 2 shows predictions of generated power for the MOD-1 turbine compared with field measurements and numerical predictions obtained from the free wake method of Afje and Keith (1986). The predicted values from the prescribed wake method compare well with these results over the entire range of operating conditions. The largest differences between the free and prescribed wake predictions can be seen to occur at low wind velocities, where the wake structure is highly concentrated. This introduces the possibility of blade-vortex and vortex-vortex interactions which may compromise a solution. It is pertinent to note, however, that a free vortex method will be more sensitive to these effects than a prescribed wake scheme.

Another example of the predictive capability of the model is given in Fig. 3 by comparison with field data for the LM17 rotor on a VESTAS 55kW turbine. Once again agreement is very good except, in this case, at very high wind speeds. Here, however, the low tip-speed ratio results in very high incidence values on the blade, producing deep stall and strong three-dimensionality on affected blade sections. In this case, the 2-D blade section data used in the calculation may be inappropriate.

## PHASE 2

The direct consequence of a yawed onset flow is that the blades experience an azimuthal incidence variation. Thus, in the present model, the bound circulation on a given blade element changes as the blade rotates and this necessitates the introduction of shed vorticity elements in the wake structure. Another obvious consequence of yawed flow is that the wake structure is now skewed with respect to the turbine.

As in the previous phase, the hybrid free/prescribed wake method was used to provide information on convection rates throughout the wake for a range of operating conditions including yaw angles up to 60 degrees. It was found that the principle effect on the wake structure occurred through the loading imbalance on the disc. This, in turn, affected the convection rates within the wake via the previously developed functions for head-on flow. As a result of the calculations in this phase, no changes to the functions were made.

A typical wake structure resulting from the prescribed wake scheme for a yawed flow of 30 degrees is shown in Fig. 4. Unfortunately little reliable data were available to assess the accuracy of the method at this stage. Additionally, any comparison would be subject to the limitations of the steady aerodynamic input supplied to the model. One qualitative comparison was, however, made with wind tunnel measurements made on the WG500 rotor in the CARDIC wind tunnel in China. The comparison is shown in Fig. 5 where it should be noted that the wind tunnel results are subject to wall interference and blockage effects and corrected data were only available for the axial flow case. Nevertheless, the axial flow prediction is close to the corrected value and the trend of the predicted yaw variation mirrors that measured in the experiment.

## PHASE 3.

The final stage in the code development was the inclusion of the unsteady aerodynamic model of Leishman and Beddoes (1989). The main features of this technique may be summarised as follows: (1) Unsteady effects during attached flow conditions are simulated by the superposition of indicial aerodynamic responses. (2) Nonlinearities in aerofoil behaviour, related to small amounts of trailing edge separation, are represented using a Kirchhoff flow model. The movement of the unsteady flow separation point position is related to the static separation position via a deficiency function. (3) The onset of leading-edge separation (and hence, dynamic stall) is identified using a criterion based on the attainment of a critical local leading-edge pressure which is further related to the normal force. For unsteady conditions, a lag in the normal force response to changes in angle of attack and a lag in the leading edge pressure response with respect to normal force have to be taken into consideration. (4) The induced vortex force and the associated pitching moment are represented empirically in a time dependent manner during dynamic stall.

The unsteady aerodynamic model requires information such as the reduced pitching rates, the local relative velocity and the instantaneous angle of attack which the turbine blade experiences. In the unsteady prescribed wake method this information is determined by initial application of the prescribed wake scheme using static aerodynamic characteristics. This stage of the calculation procedure also produces the wake geometry for the subsequent dynamic calculation. On completion of this stage, the unsteady aerodynamic model is used iteratively with the wake model to obtain a more accurate estimate of the turbine performance. As the stability of the indicial approach can be sensitive to the calculation time step, a final series of iterations are conducted with an increased number of azimuthal steps.

In modelling the performance of an aerofoil in unsteady flow, the technique described above implicitly includes the induced effect of the shed wake structure downstream of the aerofoil. Similarly, the wake structure generated in the prescribed wake method contains filaments of shed vorticity and their induced effect on the blade flowfield is calculated directly by application of the Biot-Savart relationship. Consequently, coupling of the two schemes is hindered by this duplicative effect. As shown by Coton et al (1994), this can be overcome by neglecting the shed wake terms from the prescribed wake model and calculating the induced effect via the unsteady aerofoil performance scheme. In this case, only the influence of the wake element on the blade element from which it is shed is removed from the prescribed wake model. Thus, in a two bladed system, the induced effect which the wake from the blade one has on blade two, and vice-versa, is calculated directly by the prescribed wake method. The effect on a blade of the vorticity trailed from that blade and of shed elements produced by other sections of the blade are also calculated by the prescribed wake model.

To date, very little detailed information on unsteady blade loads has been available to validate the final version of the method. This is the prime motivation for the follow-on programme discussed earlier. Nevertheless, earlier this year, data were made available by NREL in the U.S. for yawed flow cases on their combined downwind rotor experiment. Figure 6. shows the prediction of the model compared with data from the

NREL combined experiment for a small yaw angle of ten degrees. In this figure, the effect of tower shadow is clearly visible in the field data but is not, as yet, included in the predicted results. Whilst the local effect of tower shadow is obvious, the global effect is uncertain. Nevertheless, the magnitude of the normal forces and angles of attack are well predicted in this example. Further validation using these data is still on-going for both steady yawed flow and low frequency yawing flow in the follow-on project. In addition, a simple tower-shadow model is being added to facilitate comparison.

#### DYNAMIC INFLOW

As discussed above, dynamic inflow modelling has not been included in the present scheme. A strategy for this was identified which utilised the unsteady wake method of Beddoes (1989). Such a scheme would interface well with the existing code and would adequately account for changes in wake structure associated with dynamic inflow. This may be reconsidered for the follow-on study if time allows.

#### CONCLUSIONS

A fully unsteady prescribed wake model has been developed for steady yawed flow on horizontal axis wind turbines. The model has been demonstrated to compare well with field data for both head-on flow and steady yawed flow. Additional validation is currently underway and a follow-on study should help to refine the model further.

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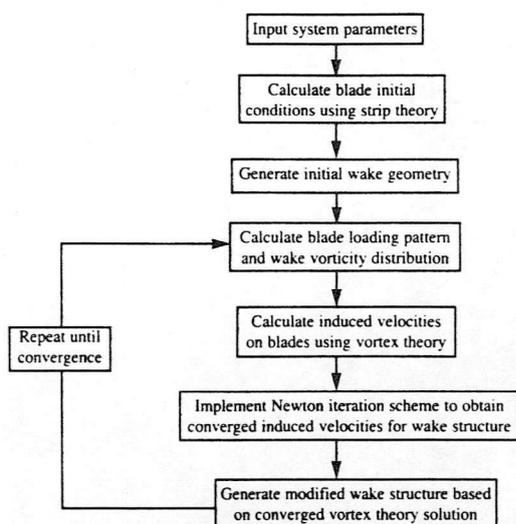


Fig. 1. Prescribed wake calculation scheme

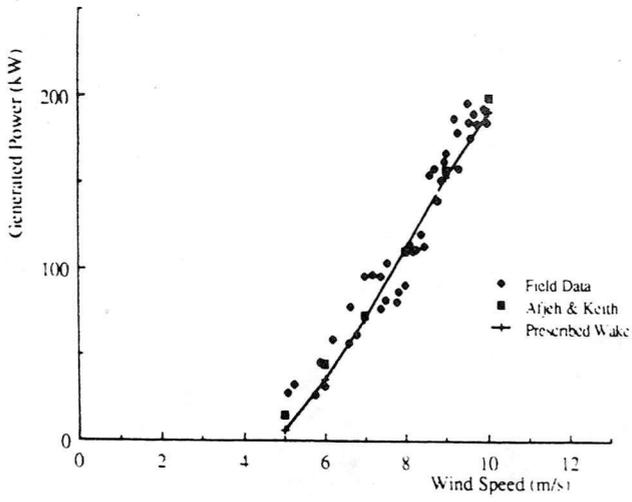


Fig. 2. Power curve for the MOD-1 turbine

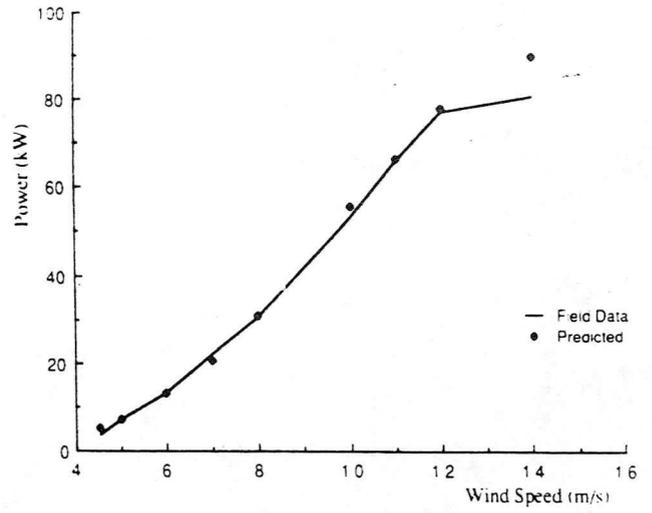


Fig. 3. Power curve for the VESTAS 55kW turbine

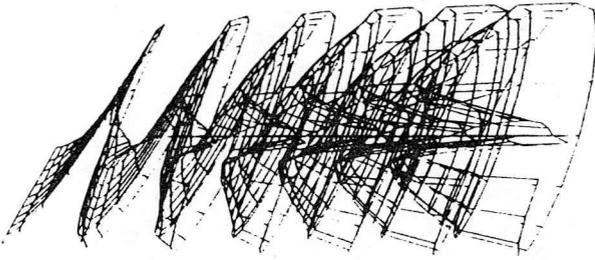


Fig. 4. Typical wake structure for 30° yaw

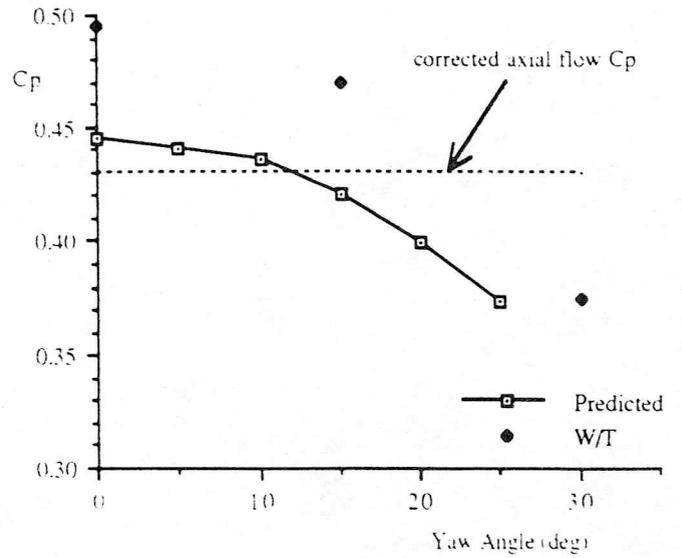
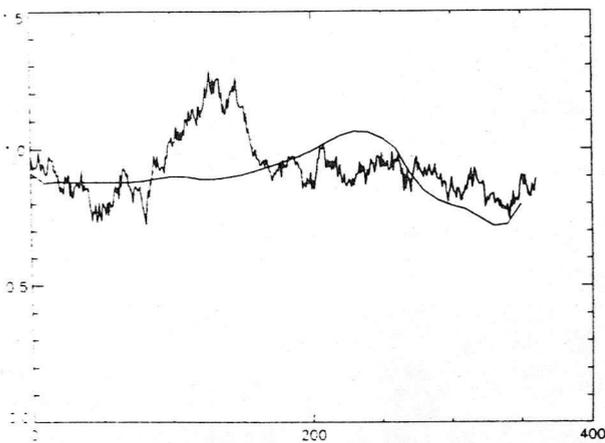
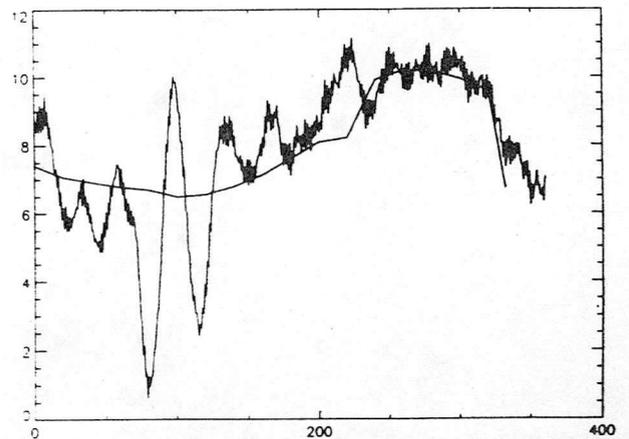


Fig. 5. Cp variation with yaw for WG 500 turbine



Normal force coefficient at 67% span



Angle of attack at 80% span

Fig. 6. Comparisons of predictions with NREL field data (10° yaw)