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MODELLING OF TRAILING EDGE SEPARATION ON ARBITARY TWO-DIMENSIONAL AEROFOILS IN INCOMPRESSIBLE FLOW USING AN INVISCID FLOW ALGORITHM.

BY

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RANKINE BUILDING　　JAMES WATT BUILDING

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A B S T R A C T

An algorithm for estimating the lift, moment and pressure distribution on arbitary two dimensional aerofoils in incompressible flow is presented.

The procedure uses an inviscid analysis of the physics of the real flow, which invokes the application of a linear vortex panel model.

The separated wake geometry is determined iteratively, starting from an initial assumption. A boundary layer analysis is not performed, hence the upper surface separation point is a necessary input to the algorithm. Lower surface separation is assumed to occur at the trailing edge.

A selection of results and comparison with experimental data is presented. The scatter in the calculated and experimental data values is attributed mainly to the lack of boundary layer displacement and compressibility effects.

A fortran code listing of the algorithm is given in the Appendix.

A C K N O W L E D G E M E N T S

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CONTENTS

NOMENCLATURE

A_{ij}, B_{ij}	-	elements of the influence coefficients.
C_{ij}	-	influence coefficient.
C_L	-	lift coefficient.
C_M	-	moment coefficient.
C_p	-	pressure coefficient.
D_{ij}	-	element of influence coefficient of free shear layer.
F.S.L.	-	free shear layer.
F1, F2	-	initial angles at which the free shear layers leave the aerofoil.
L	-	length of a panel.
$\vec{n}$	-	normal vector.
S	-	surface distance.
$\vec{t}$	-	tangental vector.
V	-	velocity.
WL, WH	-	wake length, wake height respectively.
WF, (W.L.F.)		wake(length)factor
α	-	angle of attack.
β	-	angle from horizontal of line joining separation points.
$\Delta\theta$	-	correction angle.
ε	-	mean of free shear layer starting angles.
γ	-	vorticity.

SUBSCRIPTS.

C	-	control point.
i	-	i^{th} term.
j	-	j^{th} term.
int	-	intersection point.
n	-	normal component
Sep	-	separation condition.
∞	-	free stream condition.

1. INTRODUCTION.

Determination of aerofoil lift and moment values are important in all aspects of flight vehicle design, both fixed wing and rotary. Often, flight vehicles are required to be operated at extremes of their flight envelope, where significant amounts of boundary layer separation may be present on the aerodynamic surfaces. Full computational analysis of aerofoil flowfields is limited by the present inability to compute the effects of separation. An exact analysis is possible via the time-dependent solution to the full Navier-Stokes equations, however, the computational power does not yet exist to achieve this. Engineers who require quick and accurate predictions, must resort to alternative analysis, usually panel models, of which one is presented here. It is assumed that the reader is familiar with the general features of such panel models.

The effects of separation occurring at the trailing edge of an aerofoil are indicated in Figures (1.2) and (1.3).

Generally, the separation is progressive, starting at the trailing edge and moving forward with increasing angle of attack, until the aerofoil stalls. At low angles of attack where the boundary layer is thin and little if any separation is present, a potential flow analysis may suffice to determine the flowfield. At higher angles, the boundary layer thickens, and departures from the linear CL VS α behaviour appear. Here, analysis of the boundary layer and incorporation of associated displacement effects within a viscous - inviscid interation procedure will be necessary for accurate predictions, e.g. Ref. (4).

When, however, separation begins to occur, the resulting flowfield as a whole, is affected by the separated wake. The general features of the flowfield with separation are shown in Figure (1.1). Experimental evidence has shown that the separated wake closes quickly downstream of the trailing edge as a consequence of the strong entrainment process brought about by rotation in the free shear layers.

An analysis which does not model the separation, cannot be expected to give realistic predictions. Clearly this is unacceptable.

It should be pointed out that all aerofoils do not exhibit gradual trailing edge separation, and stall may occur sharply due to abrupt separation close to the leading edge or abrupt forward movement of trailing edge separation. In these cases, departures from the linear CL-α behaviour will be minimal up to the stall, and can be adequately predicted by accounting for the boundary layer only.

In this report, an algorithm, based on two reported methods by Maskew et al (2) and Henderson (3) is developed to extend the panel method of Leishman and Galbraith (1), to the treatment of aerofoils with trailing edge separation.

Generally, the algorithm as presented contains the essential features of Ref. (2) but also, significant numerical differences are apparent. A review of other methods used to predict separation effects and further details on the development of the present work is given by Hanna in Ref. (6).

The algorithm is shown to be successful in producing aerofoil pressure distribution which compare well with experiment even in the absence of boundary layer displacement corrections. Input for the upper surface separation point can be obtained from experiment or boundary layer calculations. Ideally of course, any boundary layer calculation must be incorporated within the present algorithm as an inner iteration loop. This work is hoped to be incorporated in the near future.

2. BASIC POTENTIAL FLOW METHOD.

The panel method used by Leishman and Galbraith (1) was extended for the treatment of separated trailing edge flows. For completeness, a brief summary of the potential flow method is included here.

The aerofoil geometry is replaced by an inscribed polygon, i.e. panels, on which is placed a linear variation of surface vorticity. The vorticity varies continuously along each panel and is piecewise continuous at the panel corners. (See Figure (2.1).).

Each panel has a control point situated at the panel mid-point. The condition of flow tangency is applied at each control point. Considering such a point, the total velocity has "induced" contributions from every panel around the contour plus that from the free-stream. Since this total velocity must be tangential to the control point the scalar product with the normal vector must be zero. This condition applies at each control point, and can be represented by the integral equation.

$$\int_C C\,\gamma(s)\,ds + \vec{V}_\infty \cdot \vec{n} = 0 \qquad (2.1)$$

Thus, the contributions from all the panels give a linear equation in N + 1 unknown vorticity values γ_i. The process is repeated for each control point around the N panel contour, giving N equations in N + 1 unknown vorticity values. An additional equation comes from the classical Kutta condition - by specifying that the net vorticity at the upper and lower trailing edges is zero.

The square set of equations in terms of N + 1 unknowns are thus amenable to solution.

A detailed mathematical analysis of the influence coefficient evaluation is contained in Appendix 1 (A).

After solution, the surface velocity can be directly obtained from:

$$V = \left| \gamma(s) \right| \tag{2.2}$$

3. METHOD WITH SEPARATED FLOW MODELLING.

In analysing the case of flow with separation, the following assumptions were made:

(i) The free shear layers do not have significant thickness and can be represented as streamlines across which there exists a jump in velocity.

(ii) The wake region does not have any significant vorticity and has constant total pressure.

3.1 The Modified Vorticity Distribution.

The flowfield is constructed in a similar way as the potential flow case with "induced" velocities associated with the vorticity distribution on both the aerofoil surface and free shear layers, being added to the free-stream velocity, and the condition of flow tangency applied at the control points. In this case, the representative integral equation is:

$$\int_C C\,\gamma(s)\,ds + \gamma_{N+1}\left\{\int_L C\,ds - \int_u C\,ds\right\} + \vec{V}_\infty \cdot \vec{n} = 0 \tag{3.1}$$

The general panel distribution is shown in Figure (3.1).

The upper and lower free shear layers are represented as panels of uniform vorticity. Downstream of the upper surface separation point, and on the upper surface trailing edge, the vorticity is set to zero. Vorticity values between these two points are obtained as part of the solution.

3.1.1. Influence Coefficients for the Free Shear Layers.

The mathematical analysis is contained in Appendix 1 (B) (i). The free shear layers are assumed to have constant vorticity, and thus, the evaluation of the influence coefficients are greatly simplified. (See Figure (3.2)). The total influence coefficient for the whole shear layer is obtained by simple addition of the individual panel contributions.

3.1.2 Panels upon which Separation Occurs.

(a) Upper surface separation.

The upper surface separation panel is shown in Figure (3.3). The complete panel is dealt with by recognising the three types of basic panel present. (See Appendix 1 (B)).

The total influence coefficient due to the vorticity value at separation (γ_{sep}) is obtained along with the "A_{ij}" contribution due to γ_j and the "B_{ij}" due to γ_{j+1}.

(b) Lower surface separation.

The lower surface separation panel is shown in Figure (3.4). Separation occurs at the aerofoil trailing edge. Therefore, the contribution of the free shear layer to the influence coefficient of γ_{j+1}, can be obtained by simple addition.

The assumption of zero vorticity on the upper surface trailing edge is really only physically realistic for aerofoils with sharp trailing edges, however, the application of the algorithm to blunt aerofoils such as the NACA 00XX series has encountered no problems.

3.1.3 The Kutta Condition.

The classic Kutta condition of zero vorticity at the aerofoil trailing edge cannot be applied with separation present. From experimental evidence, it is found that the separated free shear layers close quickly downstream of the trailing edge to give a narrow wake region with little vorticity. (See Fig. (3.5)). Hence, a Kutta condition for separated flow can be applied - that the net vorticity at the upper and lower separation points is zero. This implies that there is zero vorticity convected into the far wake.

$$\gamma_{sep} + \gamma_{N+1} = 0 \qquad (3.2)$$

3.2 Initial Wake Geometry.

The position of the vortex sheets representing the free shear layers are not known a priori, and hence must be obtained iteratively starting from an initial assumption. An extensive investigation into the wake starting geometry was undertaken during the development phase, and is reported in Ref. (6).

Basically, it was found that the flowfield is quite sensitive to the wake geometry and that parabolic curves which intersect just downstream of the aerofoil trailing edge, provide a sensible starting approximation. The closer the initial shape to the actual flow streamlines, the fewer iterations that will be required for a final solution. The method reported by Maskew et al was eventally adapted.

In this method, the vortex sheets are specified by two parabolic curves intersecting at a point (X_{int} , Y_{int}) downstream, in addition to the separation point co-ordinates and the angle at which the sheets leave the aerofoil surface. A full analysis of this method is given in Appendix 2.

3.3 Iterated Wake Geometry.

Using the initial wake approximation, a solution can be generated. Unless this initial approximation is close to the streamlines, the normal velocity on the vortex sheets will not be zero. A new wake can be obtained from the following analysis:

(i) Control points at the mid-points of each F.S.L. panel are determined.

(ii) The velocity vector, $\vec{V_i}$, at each control point on the F.S.L. is computed.

(iii) A local F.S.L. panel correction angle is computed from the equation:

$$\Delta\theta_i = \sin^{-1}\left(\frac{\vec{V_i}\cdot\vec{n_i}}{|\gamma_{sep}|}\right) \tag{3.3}$$

(iv) Each F.S.L. panel is rotated by the corresponding local correction angle .

(v) The new positions for the F.S.L.'s are found by adding each "corrected" panel to the end of the preceding one in a downstream direction. (See Figure (3.6)).

Generally, when using this analysis, the new wake shape will be found not to have a common intersection point downstream. Henderson (3) has used a wake closure function, but has reported little effect on the solution. Therefore, no wake closure has been imposed in the present algorithm.

3.4 Influence Coefficients of the Upper Separation Panel when it Contains the Considered Control Point.

The evaluation of this influence coefficient occurs only once during the execution of a solution. When it does occur, the preceding analysis in 3.1.1 (a) is invalid, and a special "near field" solution must be used. The mathematics are contained in Appendix 3. Note that three "partial" influence coefficients due to γ_j, γ_{j+1}, and γ_{sep} are obtained. The contribution of the free vortex sheet is linearly added to the γ_{sep} influence coefficient.

3.5 The Influence Matrix and Solution.

From the foregoing description, it appears as though an extra unknown (γ_{sep}) has been created, giving N + 2 unknowns and only N + 1 equations. The specification of the upper surface trailing edge value (γ_1) being set to zero effectively removes an unknown to square the matrix. The influence coefficients due to γ_{sep} are placed into the blank column of the matrix.

The vorticity solution matrix is determined in the present algorithm using a Gauss-Jordan elimination technique.

From the vorticity matrix, the surface velocity is given by

$$V = \left| \gamma(s) \right| \tag{3.4}$$

and in particular, at the control points

$$V_i = \left| \frac{\gamma_i + \gamma_{i+1}}{2} \right| \tag{3.5}$$

and the pressure coefficient

$$C_{p_i} = 1 - \left| \frac{V_i}{V_\infty} \right|^2 \tag{3.6}$$

In the wake region, the pressure coefficient values are assumed to take the values at separation

i.e. $$C_{p_i} = 1 - \left| \frac{\gamma_{sep}}{V_\infty} \right|^2 \tag{3.7}$$

4. RESULTS.

A selection of results for the G.A. (W) - 1 and Wortmann FX 66 - 17 A II - 182 aerofoil sections are presented.

The aerofoil geometry and experimental aerodynamic characteristics are reported by McGhee (7) and Somers (8) for the G.A. (W) - 1 and Wortmann aerofoils respectively. Separation point data is inferred from these works.

Examples of the iterated pressure distributions and force coefficients are compared with experiment. Examples of the iterated wake geometry are also shown.

5. DISCUSSION.

The algorithm in the presented format may be considered as the inviscid part of a viscous-inviscid interaction procedure. The general outline of such a procedure would include the following steps:

(i) Potential flow analysis of aerofoil geometry.

(ii) Boundary layer analysis based on (i).

(iii) Displacement of the surface of the local boundary layer displacement thickness up to the separation point.

(iv) Application of the present algorithm.

(v) Boundary layer analysis based on the final pressure distribution from (iv).

(vi) Repetition of steps (iii), (iv) and (v) until the separation point has converged.

Advantages of the algorithm are, that it is versitile enough to handle many aerofoil geometries, and the velocity distribution on the aerofoil surface may be obtained from the solution matrix directly. The computer program requires approximately 1.5 minutes on the GEC 4070, to compute a single solution, the majority of the time being devoted to the solution of the simultaneous equations.

The placement of the influence coefficient due to the separation vorticity value (γ sep) in the influence matrix is conveniently dealt with by noting that the vorticity on the upper surface trailing edge has been set to zero. This leaves a blank column in the influence matrix into which the values due to γ sep are placed. The influence matrix values are effectively constant for a given aerofoil geometry (all constants except for first column if separation point is fixed), hence the time for a single solution can be shortened by selective replacement of the influence coefficients within the matrix. The current algorithm recalculates the entire matrix for each iteration although a selective replacement algorithm is currently under development.

The algorithm is strictly mathematically correct only for sharp edged aerofoils, although it has been found that the application to "blunt" edged aerofoils can be done with no apparent problems. Furthermore, application to excessively thin or cusped aerofoils should be considered with caution, due to the tendency for the influence matrix to become singular under these circumstances. Generally, the user will be guided by the eroneous results for non-applicable cases. If separation is found to occur on the first panel, then a potential flow solution is performed.

The present algorithm has been applied to the G.A. (W) - 1 aerofoil, which appears to be the standard section for testing algorithms that model trailing edge separation. (2), (3). Results for angles of attack of 18.25°, 19.06°, 20.05° and 21.14° are shown in comparison with experimental data at a Reynolds number of 6.3×10^6 and Mach number of 0.15. Very good agreement between the pressure distribution is found, although no boundary layer displacement effects or compressibility corrections have been applied to the calculated data.

For angles of attack below that of the stall, the wake shape takes about 3 or 4 iterations for convergence. Above stall, at least 6 iterations have been found necessary. The convergence rate is slower than that reported by Maskew et al (2), however, details of their wake integration procedure were not available to the Authors at the time of writing.

An additional comparison of the algorithm when applied to the Wortmann FX 66 - 17 A II - 182 aerofoil at 12.14° is shown. Again, good agreement is found.

6. CONCLUSIONS AND RECOMMENDATIONS.

(i) The algorithm has been demonstrated as satisfactory in producing realistic pressure distributions about aerofoils with trailing edge separation.

(ii) The pressure distribution has been found sensitive to the wake geometry and the technique discussed by Maskew [2] has been adopted for the initial shape.

(iii) The pressure distribution, and in particular, the peak suction, has been found sensitive to the separation point.

(iv) Separation point values have been inferred from experiment, although for general application, inclusion of the algorithm within a viscous-inviscid interaction procedure must be done.

(v) A version of the algorithm with selective influence coefficient replacement due to the changing wake and separation point is currently under development.

(vi) Examination of wake convergence characteristics is currently under consideration.

(vii) Compressibility corrections can be applied for subcritical Mach numbers.

(viii) Panelisation techniques apply to the present algorithm as in Ref. (1). Solution time is approximately proportional to the cube power of the number of panels, however as a minimum, 40 to 50 panels is recommended for realistic solution.

APPENDIX 1 (A).

VORTEX PANEL METHOD

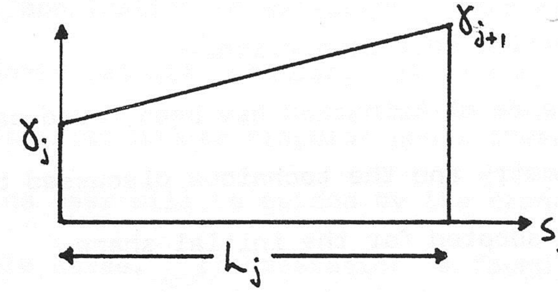


FIGURE A (1.1)

The vorticity at any point along the panel is given by:

$$\gamma_S = \gamma_j + \left(\frac{\gamma_{j+1} - \gamma_j}{L_j} \right) S_j \qquad A(1.1)$$

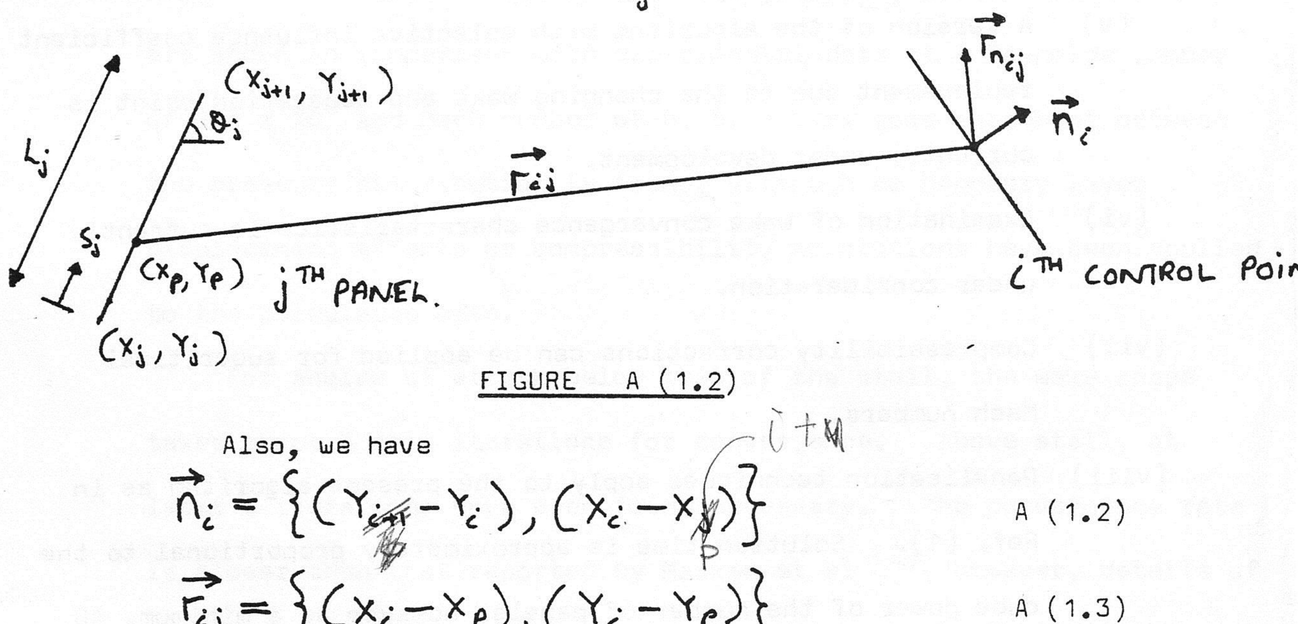


FIGURE A (1.2)

Also, we have

$i+M$

$$\vec{n}_i = \left\{ (Y_{i+1} - Y_i), (X_i - X_p) \right\} \qquad A\ (1.2)$$

$$\vec{r}_{ij} = \left\{ (X_i - X_p), (Y_i - Y_p) \right\} \qquad A\ (1.3)$$

$$\vec{r}_{n_{ij}} = \left\{ (Y_p - Y_i), (X_i - X_p) \right\} \qquad A\ (1.4)$$

$$r_{ij}^2 = (X_i - X_p)^2 + (Y_i - Y_p)^2 \qquad A\ (1.5)$$

$$\left. \begin{aligned} X_p &= X_j + \left(\frac{X_{j+1} - X_j}{L_j} \right) S_j \\ Y_p &= Y_j + \left(\frac{Y_{j+1} - Y_j}{L_j} \right) S_j \end{aligned} \right\} \qquad A\ (1.6)$$

$$\vec{V}_{n_{ij}} = \frac{1}{2\pi} \int_0^{L_j} \frac{\gamma_s}{|r_{ij}|^2} \left(\vec{r}_{n_{ij}} \cdot \vec{n}_i \right) ds_j \qquad \text{A (1.7)}$$

Suitable substitution leads to:

$$A_{ij}\gamma_j + B_{ij}\gamma_{j+1} = \frac{1}{2\pi L_j} \Big\{ \left(I_1 \gamma_j + I_2 \gamma_{j+1} \right) \underset{\sim}{i} + \left(I_3 \gamma_j + I_4 \gamma_{j+1} \right) \underset{\sim}{j} \Big\} \cdot \vec{n}_i \qquad \text{A (1.8)}$$

where,

$$I_1 = \int_0^{L_j} \left\{ L_j (y_j - y_i) + (y_{j+1} - 2y_j - y_i) s_j - \left(\frac{y_{j+1} - y_j}{L_j} \right) s_j^2 \right\} \frac{ds_j}{|r_{ij}|^2}$$

$$I_2 = \int_0^{L_j} \left\{ (y_j - y_i) s_j + \left(\frac{y_{j+1} - y_j}{L_j} \right) s_j^2 \right\} \frac{ds_j}{|r_{ij}|^2}$$

$$I_3 = \int_0^{L_j} \left\{ (x_i - x_j) L_j - (x_{j+1} - 2x_j + x_i) s_j + \left(\frac{x_{j+1} - x_j}{L_j} \right) s_j^2 \right\} \frac{ds_j}{|r_{ij}|^2}$$

$$I_4 = \int_0^{L_j} \left\{ (x_i - x_j) s_j - \left(\frac{x_{j+1} - x_j}{L_j} \right) s_j^2 \right\} \frac{ds_j}{|r_{ij}|^2}$$

$I_1 - I_4$ are evaluated using standard integral solutions.

If this is repeated around the aerofoil contour for each panel :

$$\vec{V}_{n_i} = \frac{1}{2\pi} \sum_{j=1}^{N} C_{ij}\, \gamma_j \qquad \text{A (1.9)}$$

where,

$$\begin{aligned} C_{ij} &= A_{ij} + B_{ij-1} \quad (2 \leq j \leq N) \\ & \qquad\qquad\qquad\quad (1 \leq i \leq N) \\ C_{i1} &= A_{i1} \\ C_{iN+1} &= B_{iN} \end{aligned} \qquad \text{A (1.10)}$$

C_{ij} are the influence coefficients. Thus for each control point the following must hold:

$$\frac{1}{2\pi} \sum_{j=1}^{N} C_{ij}\, \gamma_j + \vec{V}_{\infty} \cdot \vec{n}_i = 0$$

A (1.11)

This represents a set of N linear equations containing N + 1 unknowns. i.e. $\gamma_1 \rightarrow \gamma_{N+1}$. An additional equation comes from the Kutta condition :

$$\gamma_1 + \gamma_{N+1} = 0 \qquad \text{A (1.12)}$$

APPENDIX 1 (B).

PANEL TYPES.

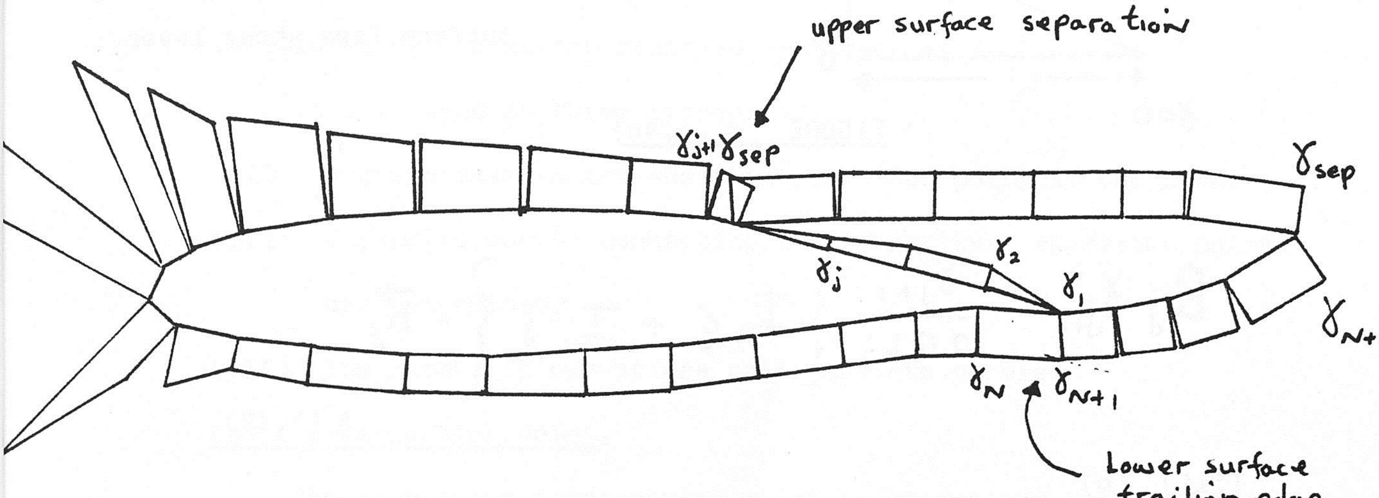
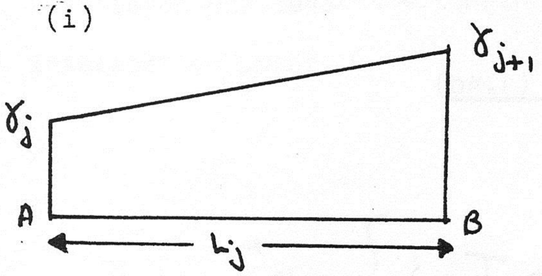


FIGURE A (1.3)

In the model with trailing edge separation, three types of basic panels are used.

(i)



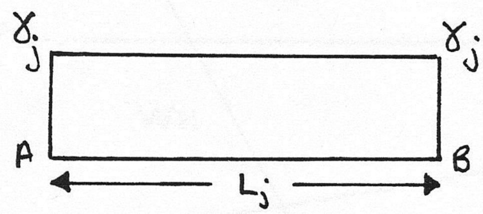
Typical aerofoil panel.

FIGURE A (1.4a)

$$A_{ij}\gamma_j + B_{ij}\gamma_{j+1} = \frac{1}{2\pi L_j}\left\{\left(I_1\gamma_j + I_2\gamma_{j+1}\right)\underset{\sim}{i} + \left(I_3\gamma_j + I_4\gamma_{j+1}\right)\underset{\sim}{j}\right\}\cdot\vec{n}_i$$

A (1.15)

(ii)

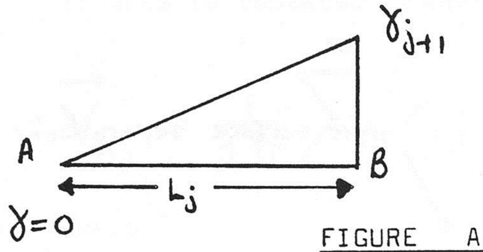


Free shear layer panel.

FIGURE A (1.4b).

A (1.16)

(iii) (a)



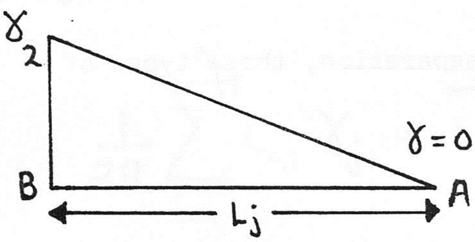
Part panel aft of upper surface free shear layer.

<u>FIGURE A (1.4c)</u>

$$B_{ij}\,\gamma_{j+1} = \frac{\gamma_{j+1}}{2\pi l_j}\left\{ I_2 \underset{\sim}{i} + I_4 \underset{\sim}{j} \right\} \cdot \vec{n}_i$$

A (1.17)

(iii) (b)



Panel at upper surface trailing edge.

<u>FIGURE A (1.4d)</u>

$$A_{ij}\,\gamma_j = \frac{\gamma_j}{2\pi l_j}\left\{ I_1 \underset{\sim}{i} + I_3 \underset{\sim}{j} \right\} \cdot \vec{n}_i$$

A (1.18)

APPENDIX 2.

METHOD FOR THE CALCULATION OF THE INITIAL WAKE GEOMETRY.

For each angle of attack, the starting wake position is calculated by the method reported by Maskew et al [2].

This is done in three stages:

(i) A downstream vortex sheet intersection point is computed.

(ii) Parabolic curves connecting the respective separation points are determined.

(iii) The parabolic curves are split up into panels.

(a) Intersection Point.

The downstream intersection point is determined from a "Wake Length Factor" (W.L.F.), which is assumed constant for a given aerofoil. The W.L.F. is used with the separation points and angle of attack to compute the length of the wake and thereby the downstream intersection point.

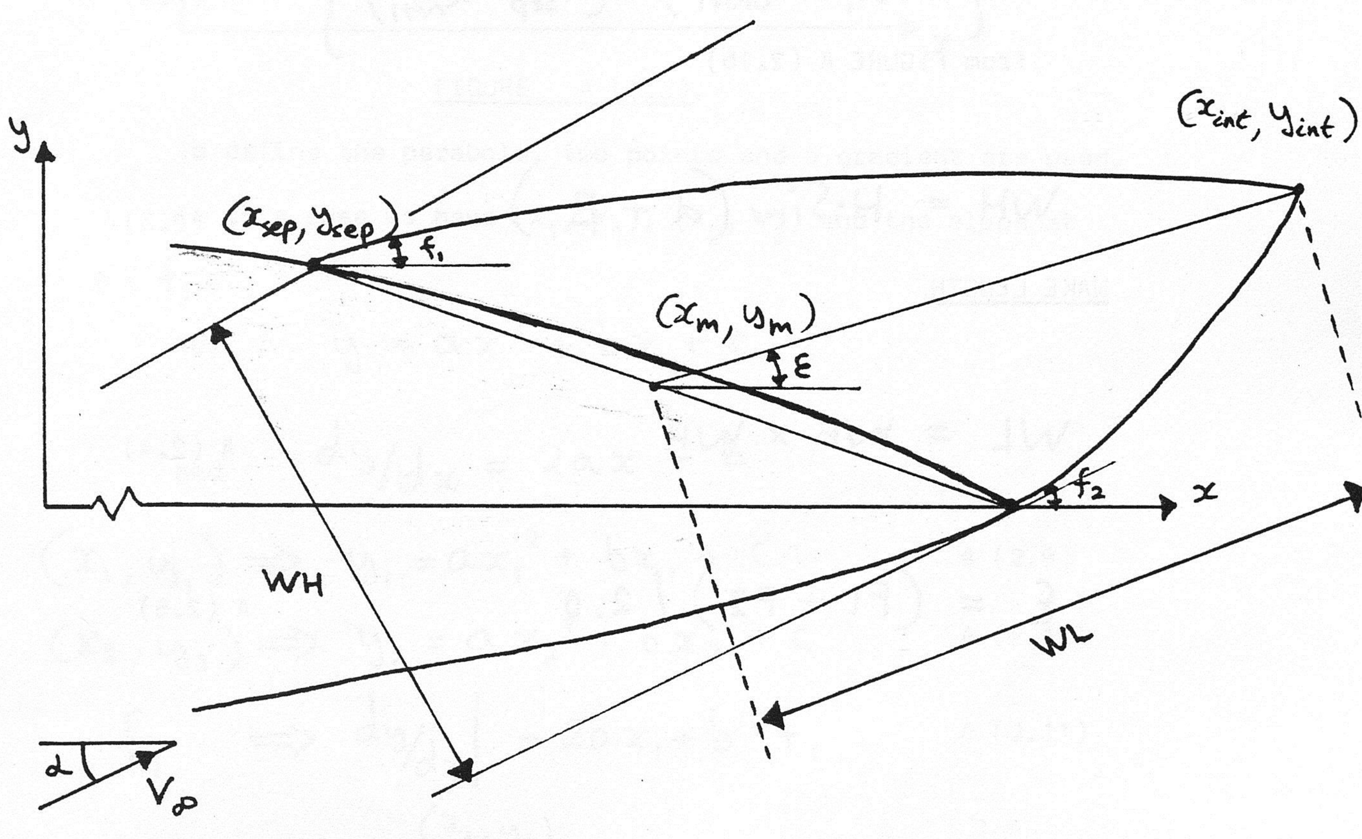


FIGURE A (2.1a).

α = angle of incidence.

WL = wake length.

WH = wake height.

(X_m, Y_m) = mid-point of separated region.

ε = mean wake direction, bisects angle between initial and lower curves.

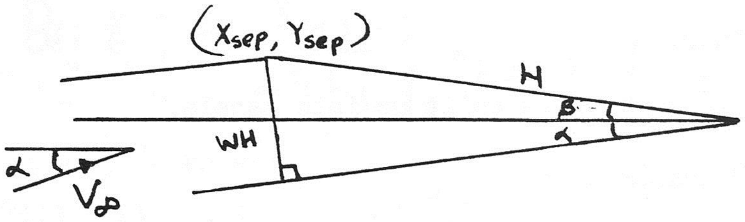


FIGURE A (2.1b).

WAKE HEIGHT.

$$\beta = \tan^{-1}\left(\frac{y_{sep} - y_{N+1}}{x_{N+1} - x_{sep}}\right) \qquad \text{A (2.1)}$$

$$H = \left\{(y_{sep} - y_{N+1})^2 + (x_{sep} - x_{N+1})^2\right\}^{1/2} \qquad \text{A (2.2)}$$

from FIGURE A (2.1b)

$$WH = H \cdot \sin(\alpha + \beta) \qquad \text{A (2.3)}$$

WAKE LENGTH

$$WL = WF \times WH \qquad \text{A (2.4)}$$

$$\varepsilon = (F1 + F2)/2.0 \qquad \text{A (2.5)}$$

W.F. = Wake factor (W.L.F.).

F1, F2 = Angles at which vortex sheets leave the aerofoil.

Intersection point.

$$x_m = (x_{sep} + x_{N+1})/2 \; ; \; y_m = (y_{sep} + y_{N+1})/2 \qquad \text{A (2.6)}$$

$$Wh_x = Wh \cdot \cos \varepsilon \; ; \; Wh_y = WL \cdot \sin \varepsilon \qquad \text{A (2.7)}$$

$$x_{int} = Wh_x + x_m \; ; \; y_{int} = Wh_y + y_m \qquad \text{A (2.8)}$$

(b) Parabolic Curves.

Consider an arbitary parabolic curve (Fig. A (2.2)).

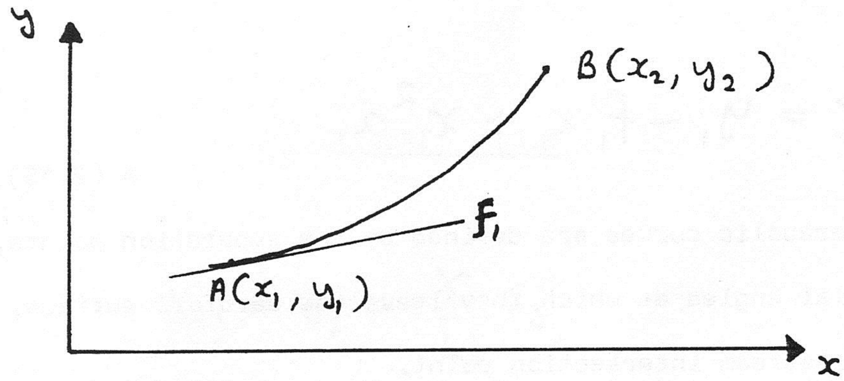


FIGURE A (2.2).

To define the parabola, two points and a gradient are used. In this case we have (X_1, Y_1), (X_2, Y_2) and the slope at A (f_1).

Now $$y = ax^2 + bx + c$$

and $$dy/dx = 2ax + b$$

$$(x_1, y_1) \Rightarrow y_1 = ax_1^2 + bx_1 + c \qquad \text{A (2.9)}$$

$$(x_2, y_2) \Rightarrow y_2 = ax_2^2 + bx_2 + c \qquad \text{A (2.10)}$$

$$f_1 \Rightarrow \left. dy/dx \right|_{(x_1, y_1)} = 2ax_1 + b = f_1 \qquad \text{A (2.11)}$$

Defining;

$$x_{21} = x_2 - x_1 \quad ; \quad y_{21} = y_2 - y_1$$

A (2.12)

Then,

$$a = \frac{y_{21} - f_1 x_{21}}{x_{21}^2}$$

A (2.13)

$$b = f_1 - 2a x_1$$

A (2.14)

$$c = y_1 - f_1 x_1 + x_1^2 a$$

A (2.15)

The parabolic curves are defined by the separation points, their initial angles at which they leave the aerofoil surface, and the downstream intersection point.

The starting angles F1 and F2, are defined as the bisector of the local surface tangent and the angle of attack.

(c) Panelisation.

Using the equations that specify the free vortex sheets, they can now be split into "panels". The curves are always fairly "flat" i.e. they lie almost parallel to the y - axis, and consequently a simple panelisation method can be used, viz:

(i) Determine from X_{21}, the X - step values.

$$X \text{ step} = X_{21} / N_W$$

A (2.16)

where N_W = number of panels required.

(ii) Compute the X and Y values for the panels from :

$$x_i = x_s + (i-1) \cdot X_{step} \qquad \text{A (2.17)}$$

$$y_i = a x_i^2 + b x_i + c \qquad \text{A (2.18)}$$

APPENDIX 3.

THE INFLUENCE COEFFICIENTS OF A PANEL ON WHICH SEPARATION OCCURS AND WHICH CONTAINS THE CONSIDERED CONTROL POINT.

$$\vec{V}_{jj} = \int_0^{L_j} \frac{\gamma(s)\, ds_j}{2\pi |r|}$$

$$r = s_j - s_c$$

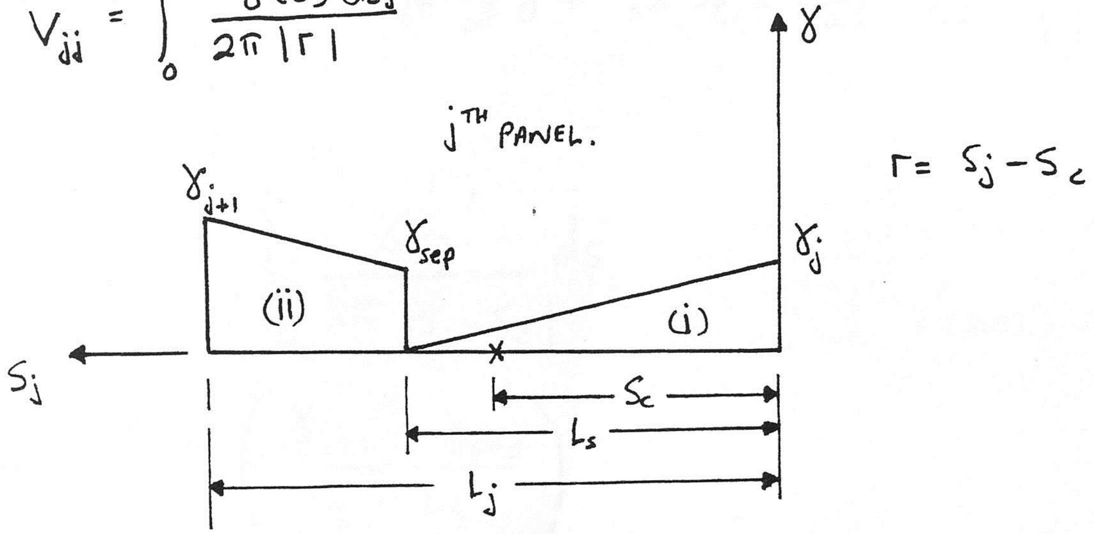


FIGURE A(3.1)

(i)

$$\gamma_{(i)} = \gamma_j - \frac{\gamma_j}{L_j} s_j \qquad \text{A (3.1)}$$

(ii)

$$\gamma_{(ii)} = \gamma_{sep} + \left(\frac{\gamma_{j+1} - \gamma_{sep}}{L_j - L_s} \right) (s_j - L_s) \qquad \text{A (3.2)}$$

for (i)

$$\vec{V}_{jj_{(i)}} = \int_0^{L_s} \frac{\left(\gamma_j - \left(\frac{\gamma_j}{L_s} \right) s_j \right)}{2\pi |s_j - s_c|} ds_j \qquad \text{A (3.3)}$$

$$= \frac{1}{2\pi} \left[\int_0^{L_s} \frac{\gamma_j}{|s_j - s_c|} ds_j - \int_0^{L_s} \frac{\gamma_j s_j}{L_s |s_j - s_c|} ds_j \right]$$

$$= \frac{1}{2\pi}\left\{\left[\gamma_j \ln|s_j - s_c|\right]_0^{L_s} - \left[\frac{\gamma_j}{L_s}\left(s_j + s_c \ln|s_j - s_c|\right)\right]_0^{L_s}\right\}$$

$$\vec{V}_{jj_{(i)}} = \frac{\gamma_j}{2\pi}\left\{\ln\left|\frac{L_s - s_c}{s_c}\right| - 1 - \frac{s_c}{L_s}\ln\left|\frac{L_s - s_c}{s_c}\right|\right\} \qquad A(3.4)$$

For (ii)

$$\vec{V}_{jj_{(ii)}} = \int_{L_s}^{L_j} \frac{\gamma_{(ii)}}{2\pi|s_j - s_c|}\, ds_j \qquad A\ (3.5)$$

LET $A = \left(\dfrac{\gamma_{j+1} - \gamma_{sep}}{L_j - L_s}\right)$

THEN $\gamma_{(ii)} = \gamma_{sep} + A(s_j - L_s)$

$$\vec{V}_{jj_{(ii)}} = \frac{1}{2\pi}\int_{L_s}^{L_j}\left(\frac{\gamma_{sep}}{|s_j - s_c|} + \frac{A(s_j - L_s)}{|s_j - s_c|}\right) ds_j$$

$$= \frac{1}{2\pi}\left[\gamma_{sep} \ln|s_j - s_c| + A\left(s_j + (s_c - L_s)\ln|s_j - s_c|\right)\right]_{L_s}^{L_j}$$

LET $B = \left|\dfrac{L_j - s_c}{L_s - s_c}\right|$

$$\vec{V}_{jj_{(ii)}} = \frac{1}{2\pi}\left\{\gamma_{sep} \ln B + A\left[(L_j - L_s) + (s_c - L_s)\ln B\right]\right\} \qquad A\ (3.6)$$

$$\text{Now} \quad \vec{V}_{jj_T} = \vec{V}_{jj_{(i)}} + \vec{V}_{jj_{(ii)}} \qquad A\ (3.7)$$

$$= \frac{1}{2\pi}\left\{\int_0^{L_s} \frac{\gamma_{(i)}}{|s_j - s_c|}\, ds_j + \int_{L_s}^{L_j} \frac{\gamma_{(ii)}}{|s_j - s_c|}\, ds_j\right.$$

$$\vec{V}_{jj_T} = \frac{\gamma_j}{2\pi}\left\{ \ln\left|\frac{L_s - S_c}{S_c}\right|\left(1 - \frac{S_c}{L_s}\right) - 1 \right\}$$

$$+ \frac{\gamma_{sep}}{2\pi}\left\{ \ln\left|\frac{L_j - S_c}{L_s - S_c}\right|\left(1 - \left[\frac{S_c - L_s}{L_j - L_s}\right]\right) - 1 \right\}$$

$$+ \frac{\gamma_{j+1}}{2\pi}\left\{ 1 + \left(\frac{S_c - L_s}{L_j - L_s}\right)\ln\left|\frac{L_j - S_c}{L_s - S_c}\right| \right\}$$

$$\vec{V}_{jj_T} = \frac{\gamma_j}{2\pi}\left\{ \ln\left|\frac{L_s}{S_c} - 1\right|\left(1 - \frac{S_c}{L_s}\right) - 1 \right\}$$

$$+ \frac{\gamma_{sep}}{2\pi}\left\{ \left(\frac{S_c - L_s}{L_j - L_s}\right)\ln\left|\frac{L_j - S_c}{L_s - S_c}\right| - 1 \right\}$$

$$+ \frac{\gamma_{j+1}}{2\pi}\left\{ \left(\frac{S_c - L_s}{L_j - L_s}\right)\ln\left|\frac{L_j - S_c}{L_s - S_c}\right| + 1 \right\}$$

A (3.8)

$$\Rightarrow \vec{V}_{jj_T} = A_{jj}\,\gamma_j + B_{jj}\,\gamma_{j+1} + D_{jj}\,\gamma_{sep} \qquad \text{A (3.9)}$$

APPENDIX 4.

CALCULATION OF LIFT AND MOMENT COEFFICIENTS.

From the discrete pressure distribution around the aerofoil, the lift and moment coefficients can be obtained using the trapesium rule. Generally, a large number of sata points are produced, (2N + 1 points for an N panel contour) so the C_L and C_M values are comparatively accurate using this technique.

$$C_N = \int_0^1 \Delta C_p \, d(x/c) \qquad \text{A (4.1)}$$

$$C_L = C_N \cos\alpha \qquad \text{A (4.2)}$$

$$C_{m_{1/4}} = \int_0^1 \Delta C_p \left(x/c - 1/4\right) d(x/c) \qquad \text{A (4.3)}$$

APPENDIX 5.

LISTING OF THE COMPUTER CODE.

A listing of the computer program is given in FORTRAN on the following pages.

The program has been designed to run on a GEC 4070 computer but a prospective user should have little difficulty in adapting the program to run on other computers.

The sub-program COFPRI is used to output the aerofoil pressure distribution at the panel corner and control points for immediate plotting or additional processing. This sub-program can be easily modified to the users requirements.

The variable ITMAX limits the number of wake shape itterations allowed for each angle of attack. In the program, ITMAX has been set to take the value 4, however for aerofoils close to, or above the stall angle, this should be changed to 6 or 7.

```
Listing of file .AEROSEP1 by LF 3 on 21 JUN 1982 at 15:20:42
File created on MON 21 JUN 1982  :  file last modified on MON 21 JUN 1982

C***********************************************************************
C  PROGRAM TO ESTIMATE THE INCOMPRESSIBLE  FLOW ABOUT AN ARBITARY
C  2-D AEROFOIL WITH BOUNDARY LAYER TRAILING EDGE SEPARATION.
C***********************************************************************
C  WRITTEN BY J.G.LEISHMAN
C  WITH ADDITIONAL CONTRIBUTIONS BY J.HANNA & R.GALBRAITH
C  LAST MODIFIED BY J.G.LEISHMAN -- 18/6/82
C************************************************************************
C  DEPARTMENT OF AERONAUTICS AND FLUID MECHANICS.
C  UNIVERSITY OF GLASGOW.
C  MAY 1982
C***********************************************************************


C DIMENSION ARRAYS AND INITIALIZE VARIABLES.................
      REAL A1(80,80),B1(80,80),A(80,80),NORM(80),VT(80),CP(80)
      REAL X(80),Y(80),ALEN(80),SOL(80)
      REAL ALPHA(30),THETA(80),LTEMP
      REAL XC(80),YC(80),ALN(80),AMN(80)

C DIMENSION ARRAYS FOR WAKE SHAPE....................
      REAL XUP(30),YUP(30),ALENUP(30),XLOW(30),YLOW(30),ALENLO(30)
      REAL THETAU(30),THETAL(30),DTHETU(30),DTHETL(30)
      REAL MIDX,MIDY,MWANG

C  OTHER DATA.....................................
      PIE=4.0*ATAN(1.0)
      ITMAX=4

C  READ IN AEROFOIL GEOMETRY (UPPER T.E. TO LOWER T.E.)
      READ(1,*) N,(X(M),Y(M),M=1,N+1)

C  READ IN ANGLES OF ATTACK (IN DEGREES)
      READ(3,*) NA,(ALPHA(M),M=1,NA)

C  CALCULATE CONTROL POINTS AND POLYGONAL GEOMETRY.
      DO 10 M=1,N
      ALEN(M)=SQRT(((Y(M+1)-Y(M))**2)+((X(M+1)-X(M))**2))
      XC(M)=(X(M+1)+X(M))/2.00
      YC(M)=(Y(M+1)+Y(M))/2.00
      ALN(M)=(Y(M)-Y(M+1))/ALEN(M)
      AMN(M)=(X(M+1)-X(M))/ALEN(M)
      THETA(M)=ATAN((Y(M+1)-Y(M))/(X(M+1)-X(M)))
   10 CONTINUE

C  START COMPUTATION FOR EACH ANGLE OF ATTACK HERE......
      DO 2 NANG=1,NA
      ALPHA(NANG)=ALPHA(NANG)*PIE/180.0
      IWK=0
```

Listing of file .AEROSEP1 by LF 3 on 21 JUN 1982 at 15:20:42

```
C  READ IN INITIAL WAKE GEOMETRY
C  SEPARATION X VALUE, AEROFOIL WAKE FACTOR, NUMBER OF PANELS ON
C  THE FREE SHEAR LAYERS
      READ(5,*)XS,WF,NU,NL

C  FIND SEPARATION PANEL AND COMPUTE ASSOCIATED PARAMETERS
      DO 1 K=1,N+1
      IF(X(K).LT.XS)GOTO4
    1 CONTINUE
    4 NSEP=K-1
      IF(NSEP.EQ.1) GOTO 8000

C  CALCULATE SEPARATION PT.ORDINATE & ANGLES AT WHICH F.S.L.'S
C  LEAVE AEROFOIL SURFACE
      YS=(XS-X(NSEP))*TAN(THETA(NSEP))+Y(NSEP)
      F1=TAN((ALPHA(NANG)+THETA(NSEP))/2.00)
      F2=TAN((ALPHA(NANG)+THETA(N))/2.00)

C  CALCULATE LENGTHS OF SEPARATION PANEL FORE AND AFT OF
C  SEPARATION POINT.
      DX=X(K)-XS
      DY=Y(K)-YS
      AL1=SQRT((DX**2)+(DY**2))
      DX=XS-X(K-1)
      DY=YS-Y(K-1)
      AL2=SQRT((DX**2)+(DY**2))

C  CALCULATE DOWNSTREAM WAKE INTERSECTION PT......
      HORZ=X(N+1)-XS
      VERT=YS-Y(N+1)
      BETA=ATAN(VERT/HORZ)
      DIST=SQRT((HORZ**2)+(VERT**2))

C  CALCULATE WAKE HEIGHT AND WAKE LENGTH.
      WH=DIST*SIN(ALPHA(NANG)+BETA)
      WL=WH*WF
      MIDX=0.5*(X(1)+XS)
      MIDY=0.5*(Y(1)+YS)
      MWANG=(F1+F2)/2.00
      WLX=WL*COS(MWANG)
      WLY=WL*SIN(MWANG)

C  CALCULATE DOWNSTREAM WAKE INTERSECTION PT.
      XI=MIDX+WLX
      YI=MIDY+WLY

C UPPER FREE SHEAR LAYER ..........................
      X21=XI-XS
      Y21=YI-YS
```

Listing of file .AEROSEP1 by LF 3 on 21 JUN 1982 at 15:20:42

```
      P1=(Y21-F1*X21)/(X21**2)
      P2=F1-(2.0*XS*P1)
      P3=YS-(F1*XS)+((XS**2)*P1)
      CONU=X21/NU
      DO 120 K=1,NU+1
      XUP(K)=CONU*(K-1)+XS
      YUP(K)=P1*(XUP(K)**2)+P2*XUP(K)+P3
  120 CONTINUE
      DO 140 K=1,NU
      ALENUP(K)=SQRT(((XUP(K+1)-XUP(K))**2)+((YUP(K+1)-YUP(K))**2))
  140 CONTINUE
C LOWER FREE SHEAR LAYER ...............................
      X21=XI-X(N+1)
      Y21=YI-Y(N+1)
      P1=(Y21-F2*X21)/(X21**2)
      P2=F2-2.0*X(N+1)*P1
      P3=Y(N+1)-(F2*X(N+1))+(X(N+1)**2)*P1
      CONU=X21/NL
      DO 160 K=1,NL+1
      XLOW(K)=CONU*(K-1)+X(N+1)
      YLOW(K)=P1*(XLOW(K)**2)+P2*XLOW(K)+P3
  160 CONTINUE
      DO 180 K=1,NL
      ALENLO(K)=SQRT(((XLOW(K+1)-XLOW(K))**2)+((YLOW(K+1)-YLOW(K))**2))
  180 CONTINUE
C NOW SETUP INFLUENCE COEFFICIENTS FOR LINEAR VORTEX SHEETS
C ON BOTH AEROFOIL SURFACE AND THE FREE SHEAR LAYERS...........
 8000 DO 200 MM=1,N
      DO 300 M=1,N
      IF(M.NE.(NSEP))GOTO5
      IF(NSEP.EQ.1) GOTO 5
      IF(M.EQ.MM)GOTO 7000
      LTEMP=ALEN(M)
      ALEN(M)=AL1
      XTEMP=X(M)
      X(M)=XS
      YTEMP=Y(M)
      Y(M)=YS
    5 IF(M.EQ.MM) GOTO 350
      CALL VELVEC(X(M),Y(M),X(M+1),Y(M+1),XC(MM),YC(MM),ALEN(M)
     1,P11,P12,P13,P14)
      A1(MM,M)=(ALN(MM)*P11)-(AMN(MM)*P13)
      B1(MM,M)=(ALN(MM)*P12)-(AMN(MM)*P14)
      IF(NSEP.EQ.1) GOTO 300
      IF(M.NE.NSEP) GOTO300

C  INFLUENCE COEFFICIENT FOR UPPER F.S.L.
      COEFF=0.0
```

Listing of file .AEROSEP1 by LF 3 on 21 JUN 1982 at 15:20:42

```
      DO 401 K=1,NU
      CALL VELVEC(XUP(K),YUP(K),XUP(K+1),YUP(K+1),XC(MM),YC(MM),
     1ALENUP(K),P11,P12,P13,P14)
      COEFF=COEFF+((P11+P12)*ALN(MM)-(P13+P14)*AMN(MM))
  401 CONTINUE
      A(MM,1)=A1(MM,M)+COEFF

C  REINSTATE TEMPORARY VALUES.........................
      ALEN(M)=LTEMP
      X(M)=XTEMP
      Y(M)=YTEMP

C  INFLUENCE COEFFICIENT FOR REMAINDER OF SEPARATION PANEL........
      CALL VELVEC(X(M),Y(M),XS,YS,XC(MM),YC(MM),AL2,
     1P11,P12,P13,P14)
      A1(MM,M)=P11*AMN(MM)-P13*AMN(MM)
      GOTO 300
  350 A1(MM,M)=-1.00
      B1(MM,M)=1.00
      GOTO 300
 7000 IF(NSEP.EQ.1) GOTO 350
      SC=ALEN(M)/2.00

C  INFLUENCE COEFFICIENT FOR UPPER F.S.L.
      COEFF=0.0
      DO 402 K=1,NU
      CALL VELVEC(XUP(K),YUP(K),XUP(K+1),YUP(K+1),XC(MM),YC(MM)
     1,ALENUP(K),P11,P12,P13,P14)
      COEFF=COEFF+((P11+P12)*ALN(MM)-(P13+P14)*AMN(MM))
  402 CONTINUE
      A1(MM,1)=(1.0-((SC-AL2)/(ALEN(M)-AL2)))*ALOG(ABS((ALEN(M)
     1-SC)/(AL2-SC)))-1.0
      A(MM,1)=A1(MM,1)+COEFF
      A1(MM,M)=(1.0-(SC/AL2))*ALOG(ABS((AL2-SC)/SC))-1.0
      B1(MM,M)=((SC-AL2)/(ALEN(M)-AL2))*ALOG(ABS((ALEN(M)-SC)/
     1(AL2-SC)))+1.0
  300 CONTINUE
  200 CONTINUE

C  GENERATE INFLUENCE COEFFICIENT MATRIX..............
      DO 500 MM=1,N
      IF(NSEP.EQ.1) A(MM,1)=A1(MM,1)
      DO 550 M=2,N
      A(MM,M)=A1(MM,M)+B1(MM,M-1)
  550 CONTINUE
      IF(NSEP.EQ.1) A(MM,N+1)=B1(MM,N)
      IF(NSEP.EQ.1) GOTO 500

C  INFLUENCE COEFFICIENT FOR LOWER F.S.L.
      COEFF=0.0
```

Listing of file .AFROSEP1 by LF 3 on 21 JUN 1982 at 15:20:42

```
      DO 403 K=1,NL
      CALL VELVEC(XLOW(K),YLOW(K),XLOW(K+1),YLOW(K+1),XC(MM),YC(MM)
     1,ALENLO(K),P11,P12,P13,P14)
      COEFF=COEFF+((P11+P12)*ALN(MM)-(P13+P14)*AMN(MM))
  403 CONTINUE
      A(MM,N+1)=B1(MM,N)+COEFF
  500 CONTINUE

C IMPLEMENT THE KUTTA-JOUKOWSKI CONDITION FOR SEPARATED FLOW BY
C SPECIFYING THAT THE VORTICITY VALUES AT THE UPPER AND LOWER
C SEPARATION POINTS BE EQUAL AND OPPOSITE..........................
      DO 650 M=1,N+1
      A(N+1,M)=0.0
  650 CONTINUE
      A(N+1,1)=1.0
      A(N+1,N+1)=1.0

C COMPUTE RHS OF EQUATIONS BY FINDING DOT PRODUCT OF THE ONSET
C FLOW VECTOR WITH THE POLYGONAL PANEL NORMAL VECTORS.............
      VELX=COS(ALPHA(NANG))
      VELY=SIN(ALPHA(NANG))
      DO 850 MM=1,N
      NORM(MM)=-2.0*PIE*((VELX*ALN(MM))+(VELY*AMN(MM)))
  850 CONTINUE
      NORM(N+1)=0.0


C CALL SUBROUTINE FOR SOLUTION OF LINEAR SIMULTANEOUS EQUATIONS.....
      CALL GAUSSJ(N+1,A,NORM,SOL)
C .........TO FIND THE POLYGONAL VORTICITY DISTRIBUTION.

C TANGENTIAL VELOCITIES AND PRESSURE COEFFICIENTS......
      CALL COFPRI(X,Y,SOL,N,NSEP,XS)

C WAKE GEOMETRY PRINT OUT.....................
   83 WRITE(8,*) NU+1
      DO 901 J=1,NU+1
      WRITE(8,*) XUP(J),YUP(J)
  901 CONTINUE
      WRITE(8,*) NL+1
      DO 902 J=1,NL+1
      WRITE(8,*) XLOW(J),YLOW(J)
  902 CONTINUE
      IWK=IWK+1
      IF(IWK.EQ.ITMAX) GOTO 2
      IF(NSEP.EQ.1) GOTO 2

C  CORRECTION ANGLES FOR NEW WAKE GEOMETRY........
C    UPPER ONE...........................
      DO 710 I=1,NU
```

Listing of file .AEROSEP1 by LF 3 on 21 JUN 1982 at 15:20:42

```
      THETAU(I)=ATAN((YUP(I+1)-YUP(I))/(XUP(I+1)-XUP(I)))
      ALNW=(YUP(I)-YUP(I+1))/ALENUP(I)
      AMNW=(XUP(I+1)-XUP(I))/ALENUP(I)
      XP=(XUP(I)+XUP(I+1))/2.0
      YP=(YUP(I)+YUP(I+1))/2.0
      CALL VELCOM(XS,YS,X,Y,ALEN,SOL,NSEP,V1,V2,XP,YP,I,0
     1,XUP,YUP,ALENUP,XLOW,YLOW,ALENLO,N,NU,NL,AL1,AL2)
      V1=V1+VELX
      V2=V2+VELY
      DTHETU(I)=ASIN(((V1*ALNW)+(V2*AMNW))/(ABS(SOL(1))))
  710 CONTINUE

C     LOWER ONE......................
      DO 720 I=1,NL
      THETAL(I)=ATAN((YLOW(I+1)-YLOW(I))/(XLOW(I+1)-XLOW(I)))
      ALNW=(YLOW(I)-YLOW(I+1))/ALENLO(I)
      AMNW=(XLOW(I+1)-XLOW(I))/ALENLO(I)
      XP=(XLOW(I)+XLOW(I+1))/2.0
      YP=(YLOW(I)+YLOW(I+1))/2.0
      CALL VELCOM(XS,YS,X,Y,ALEN,SOL,NSEP,V1,V2,XP,YP,0,I
     1,XUP,YUP,ALENUP,XLOW,YLOW,ALENLO,N,NU,NL,AL1,AL2)
      V1=V1+VELX
      V2=V2+VELY
      DTHETL(I)=ASIN(((V1*ALNW)+(V2*AMNW))/(ABS(SOL(N+1))))
  720 CONTINUE

C   NEW WAKE GEOMETRY FOR UPPER F.S.L.
      DO 730 I=1,NU
      XUP(I+1)=XUP(I)+(ALENUP(I)*COS(THETAU(I)+DTHETU(I)))
      YUP(I+1)=YUP(I)+(ALENUP(I)*SIN(THETAU(I)+DTHETU(I)))
  730 CONTINUE

C   NEW WAKE GEOMETRY FOR LOWER F.S.L.
      DO 740 I=1,NL
      XLOW(I+1)=XLOW(I)+(ALENLO(I)*COS(THETAL(I)+DTHETL(I)))
      YLOW(I+1)=YLOW(I)+(ALENLO(I)*SIN(THETAL(I)+DTHETL(I)))
  740 CONTINUE

C   RETURN FOR ANALYSIS WITH NEW WAKE......
      GOTO 8000

C   NEXT ANGLE OF ATTACK.....
    2 CONTINUE
      STOP
      END

C   SUBROUTINE <VELVEC>
C   COMPUTATION OF THE INDUCED VELOCITY VECTOR AT A POINT (X3,Y3)
C   DUE TO PANEL (X1,Y1),(X2,Y2).
      SUBROUTINE VELVEC(X1,Y1,X2,Y2,X3,Y3,ALEN,P11,P12,P13,P14)
```

Listing of file .AEROSEP1 by LF 3 on 21 JUN 1982 at 15:20:42

```
      P1=(Y3-Y1)*ALEN
      P2=Y2-Y1+Y3-Y1
      P3=(Y2-Y1)/ALEN
      P4=Y3-Y1
      P5=(Y2-Y1)/ALEN
      P6=(X3-X1)*ALEN
      P7=X2-X1+X3-X1
      P8=(X2-X1)/ALEN
      P9=X3-X1
      P10=(X2-X1)/ALEN
      O=1.0
      P=(X3-X1)*(X2-X1)
      Q=(Y3-Y1)*(Y2-Y1)
      R=-2.0*(P+Q)/ALEN
      S=(X3-X1)**2
      T=(Y3-Y1)**2
      U=S+T
      CALL STINT1(O,R,U,ALEN,AINT1)
      CALL STINT2(O,R,U,ALEN,AINT1,AINT2)
      CALL STINT3(O,R,U,ALEN,AINT1,AINT3)
      P11=((P1*AINT1)-(P2*AINT2)+(P3*AINT3))/ALEN
      P12=((P4*AINT2)-(P5*AINT3))/ALEN
      P13=((P6*AINT1)-(P7*AINT2)+(P8*AINT3))/ALEN
      P14=((P9*AINT2)-(P10*AINT3))/ALEN
      RETURN
      END

C   SUBROUTINE <VELCOM>
C   COMPUTATION OF THE INDUCED VELOCITY VECTOR AT A POINT (XP,YP)
C   DUE TO THE ENTIRE POLYGONAL VORTICITY DISTRIBUTION.
      SUBROUTINE VELCOM(XS,YS,X,Y,ALEN,SOL,NSEP,V1,V2,XP,YP,I1,I2
     1,XUP,YUP,ALENUP,XLOW,YLOW,ALENLO,N,NU,NL,AL1,AL2)
      REAL X(80),Y(80),ALEN(80),SOL(80)
      REAL XUP(30),YUP(30),ALENUP(30),XLOW(30),YLOW(30),ALENLO(30)
      PIE=4.0*ATAN(1.0)
      V1=0.0
      V2=0.0
      CALL VELVEC(X(1),Y(1),X(2),Y(2),XP,YP,ALEN(1),P1,P2,P3,P4)
      V1=P2*SOL(2)
      V2=P4*SOL(2)
      DO 20 I=2,NSEP-1
      CALL VELVEC(X(I),Y(I),X(I+1),Y(I+1),XP,YP,ALEN(I),P1,P2,P3,P4)
      V1=V1+(P1*SOL(I)+P2*SOL(I+1))
      V2=V2+(P3*SOL(I)+P4*SOL(I+1))
   20 CONTINUE
      CALL VELVEC(X(NSEP),Y(NSEP),XS,YS,XP,YP,AL2,P1,P2,P3,P4)
      V1=V1+(P1*SOL(NSEP))
      V2=V2+(P3*SOL(NSEP))
      CALL VELVEC(XS,YS,X(NSEP+1),Y(NSEP+1),XP,YP,AL1,P1,P2,P3,P4)
```

Listing of file .AEROSEP1 by LF 3 on 21 JUN 1982 at 15:20:42

```
      V1=V1+(P1*SOL(1)+P2*SOL(NSEP+1))
      V2=V2+(P3*SOL(1)+P4*SOL(NSEP+1))
      DO 40 I=NSEP+1,N
      CALL VELVEC(X(I),Y(I),X(I+1),Y(I+1),XP,YP,ALEN(I),P1,P2,P3,P4)
      V1=V1+(P1*SOL(I)+P2*SOL(I+1))
      V2=V2+(P3*SOL(I)+P4*SOL(I+1))
   40 CONTINUE
      DO 50 I=1,NU
      IF(I.EQ.I1) GOTO 50
      CALL VELVEC(XUP(I),YUP(I),XUP(I+1),YUP(I+1),XP,YP,ALENUP(I)
     1,P1,P2,P3,P4)
      V1=V1+((P1+P2)*SOL(1))
      V2=V2+((P3+P4)*SOL(1))
   50 CONTINUE
      DO 60 I=1,NL
      IF(I.EQ.I2) GOTO 60
      CALL VELVEC(XLOW(I),YLOW(I),XLOW(I+1),YLOW(I+1),XP,YP,ALENLO(I)
     1,P1,P2,P3,P4)
      V1=V1+((P1+P2)*SOL(N+1))
      V2=V2+((P3+P4)*SOL(N+1))
   60 CONTINUE
      V1=V1/(2*PIE)
      V2=-V2/(2*PIE)
      RETURN
      END

C   SUBROUTINE <GAUSSJ>
C   WRITTEN BY R.GALBRAITH -- NOVEMBER 1979
C   SOLTION OF A SET OF LINEAR SIMULTANEOUS EQUATIONS USING A
C   GAUSS-JORDAN ELIMINATION TECHNIQUE.
      SUBROUTINE GAUSSJ(N,A,B,C)
      REAL A(80,80),B(80),C(80),AM(80,80),BM(80,80)
      REAL TEMP
      K=N
      DO 100 I=1,K
      DO 101 J=1,K+1
      IF(J.EQ.K+1) GOTO 102
      AM(I,J)=A(I,J)
      GOTO 101
  102 CONTINUE
      AM(I,J)=B(I)
  101 CONTINUE
  100 CONTINUE
    5 IF(AM(1,1))11,6,11
    6 DO 9 I=2,K
      IF(A(I,1))7,9,7
    7 DO 8 J=1,K+1
      TEMP=AM(I,J)
      AM(I,J)=AM(1,J)
    8 AM(1,J)=TEMP
```

Listing of file .AEROSEP1 by LF 3 on 21 JUN 1982 at 15:20:42

```
      GOTO 11
    9 CONTINUE
      WRITE(2,10)
   10 FORMAT(" ? FAIL ERROR -- NO UNIQUE SOLUTION IN SUB <GAUSSJ>")
      GOTO 18
   11 DO 12 J=2,K+1
      DO 12 I=2,N
   12 BM(I-1,J-1)=AM(I,J)-AM(1,J)*AM(I,1)/AM(1,1)
      DO 13 J=2,K+1
   13 BM(N,J-1)=AM(1,J)/AM(1,1)
      K=K-1
      DO 14 J=1,K+1
      DO 14 I=1,N
   14 AM(I,J)=BM(I,J)
      IF(K)5,16,5
   16 DO 17 I=1,N
      C(I)=AM(I,1)
   17 CONTINUE
   18 CONTINUE
      RETURN
      END

C   SUBROUTINE <STINT1>
C   SOLUTION OF STANDARD INTEGRAL TYPE 1
      SUBROUTINE STINT1(A,B,C,ALIM2,AINT)
      REAL A,B,C,ALIM2,AINT,AA,BB,CC,DD
      AA=(4.00*C)-(B**2)
      IF(AA.EQ.0.0) GOTO 12
      IF(AA.LT.0.0) GOTO 14
      BB=2.0/SQRT(AA)
      CC=((2.0*ALIM2)+B)/SQRT(AA)
      DD=B/SQRT(AA)
      AINT=BB*(ATAN(CC)-ATAN(DD))
      GOTO 20
   12 AINT=(2.0*ALIM2)/((2.0*C)+(ALIM2*B))
      GOTO 20
   14 AA=-AA
      BB=1.0/SQRT(AA)
      CC=(2.0*ALIM2)+B-SQRT(AA)
      DD=(2.0*ALIM2)+B+SQRT(AA)
      EE=B-SQRT(AA)
      FF=B+SQRT(AA)
      AINT=BB*ALOG((CC*FF)/(DD*EE))
   20 RETURN
      END

C   SUBROUTINE <STINT2>
C   SOLUTION OF STANDARD INTEGRAL TYPE 2
      SUBROUTINE STINT2(A,B,C,ALIM2,AINT1,AINT)
      REAL A,B,C,ALIM2,AINT1,AINT,AA,BB
```

Listing of file .AEROSEP1 by LF 3 on 21 JUN 1982 at 15:20:42

```
      AA=(C+(B*ALIM2)+(ALIM2**2))/C
      BB=0.50*ALOG(AA)
      AINT=BB-((B/2)*AINT1)
      RETURN
      END

C  SUBROUTINE <STINT3>
C  SOLUTION OF STANDARD INTEGRAL TYPE 3
      SUBROUTINE STINT3(A,B,C,ALIM2,AINT1,AINT)
      REAL A,B,C,ALIM2,AINT1,AINT,AA,BB
      AA=(C+(B*ALIM2)+(ALIM2**2))/C
      BB=((B**2)-(2*C))/2.0
      AINT=ALIM2-((B/2)*ALOG(AA))+(BB*AINT1)
      RETURN
      END


C  SUBROUTINE <COFPRI>
C  PRINT OUT OF AEROFOIL PRESSURE DISTRIBUTION
      SUBROUTINE COFPRI(X,Y,SOL,N,NSEP,XS)
      REAL X(80),Y(80),SOL(80)
      WRITE(6,*) (2*N)+1
      DO 10 I=1,N
      VT=ABS(SOL(I))
      CP=1.0-(VT**2)
      IF(I.LE.NSEP) CP=1.0-(SOL(1)**2)
      WRITE(6,*) X(I),CP
      IF(I.EQ.NSEP) WRITE(6,*) XS,1.0-(SOL(1)**2)
      IF(I.EQ.NSEP) GOTO 10
      VT=ABS(SOL(I)+SOL(I+1))/2.0
      CP=1.0-(VT**2)
      IF(I.LT.NSEP) CP=1.0-(SOL(1)**2)
      WRITE(6,*) (X(I)+X(I+1))/2.0,CP
   10 CONTINUE
      WRITE(6,*) X(N+1),(1.0-SOL(N+1)**2)
      RETURN
      END
```

END OF PRINT: AEUG01 T006 SPQR0006 AEROSEP1

FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF

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LIST OF FIGURES.

AEROFOIL FLOWFIELD AND BASIS OF THE MATHEMATICAL FLOW MODEL. (Taken from Ref.2)

REGION I: A region external to the boundary layer where viscous stresses are negligible and can be assumed a potential flow region.

REGION 2: The boundary layer on the aerofoil surface where there are significant viscous stresses.

REGION 3: The free shear layers formed by the separating boundary layers.

REGION 4: The wake, which has insignificant vorticity and viscous stresses.

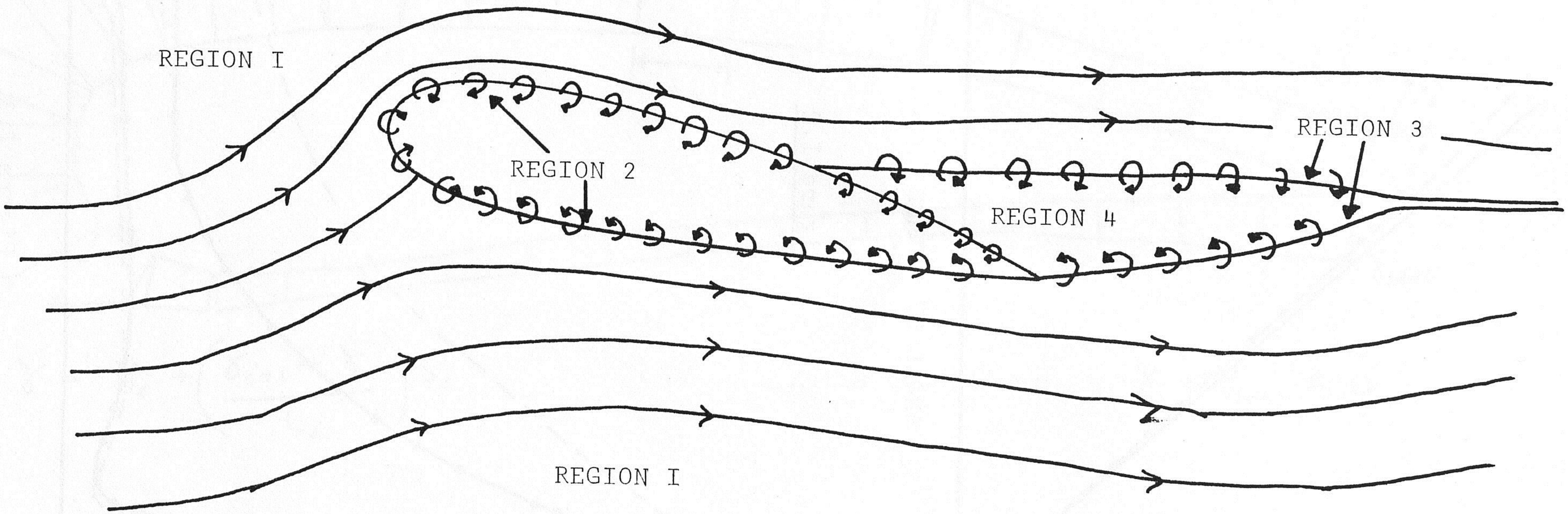


Figure I.I

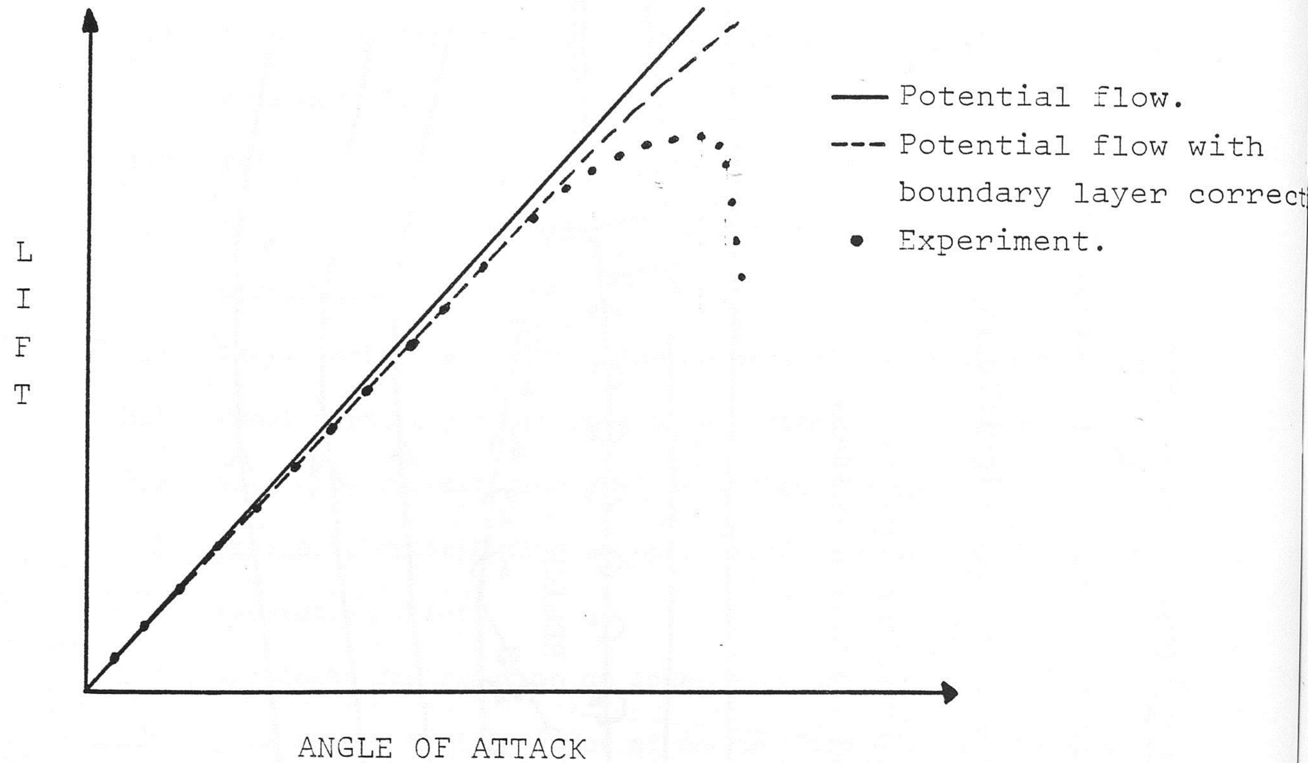


Figure I.2

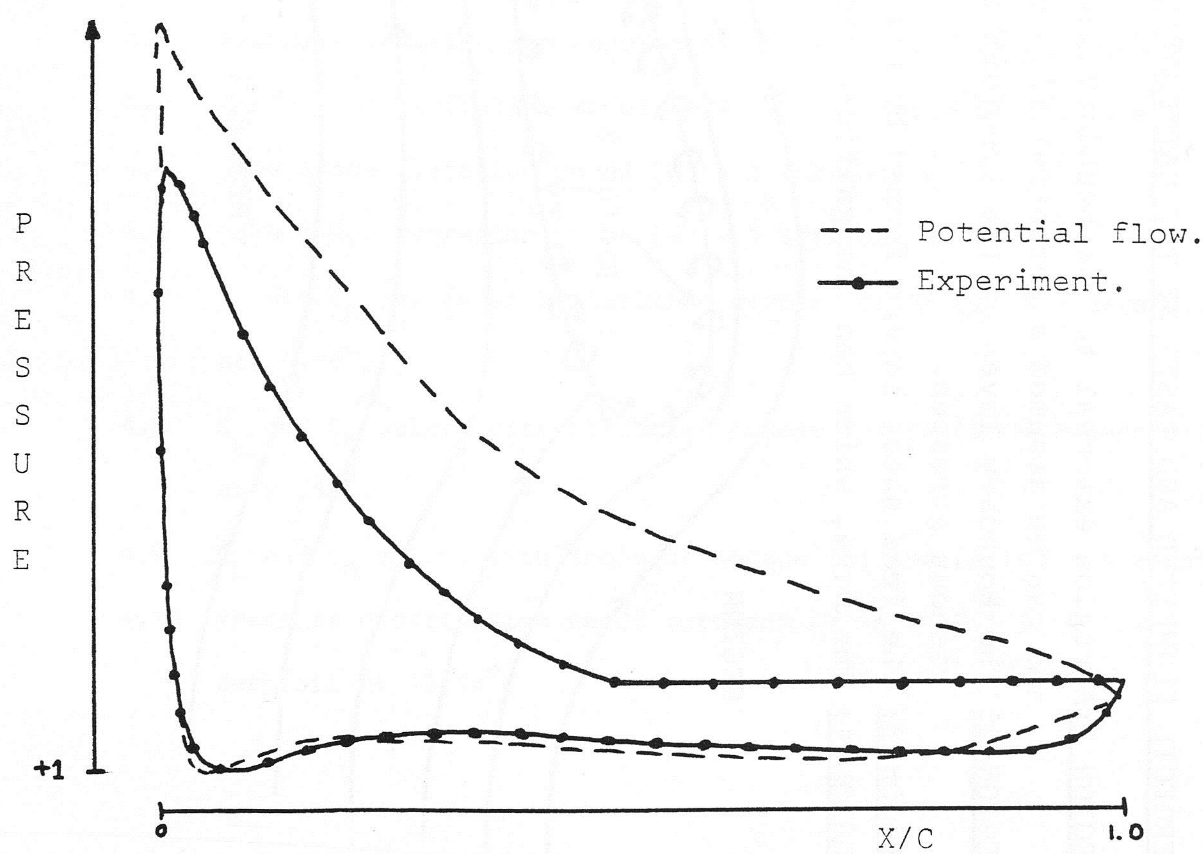
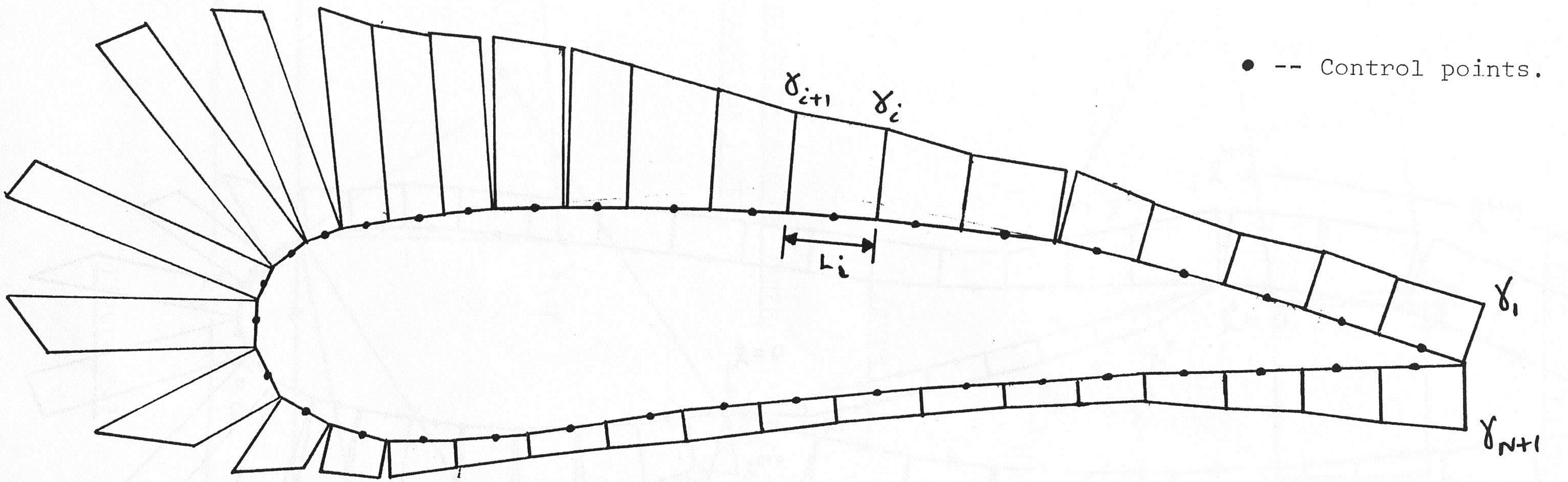


Figure I.3

PANEL VORTICITY DISTRIBUTION ON AEROFOIL IN POTENTIAL FLOW



$$\gamma = \gamma_i + \left(\frac{\gamma_{i+1} - \gamma_i}{L_i}\right) s_i$$

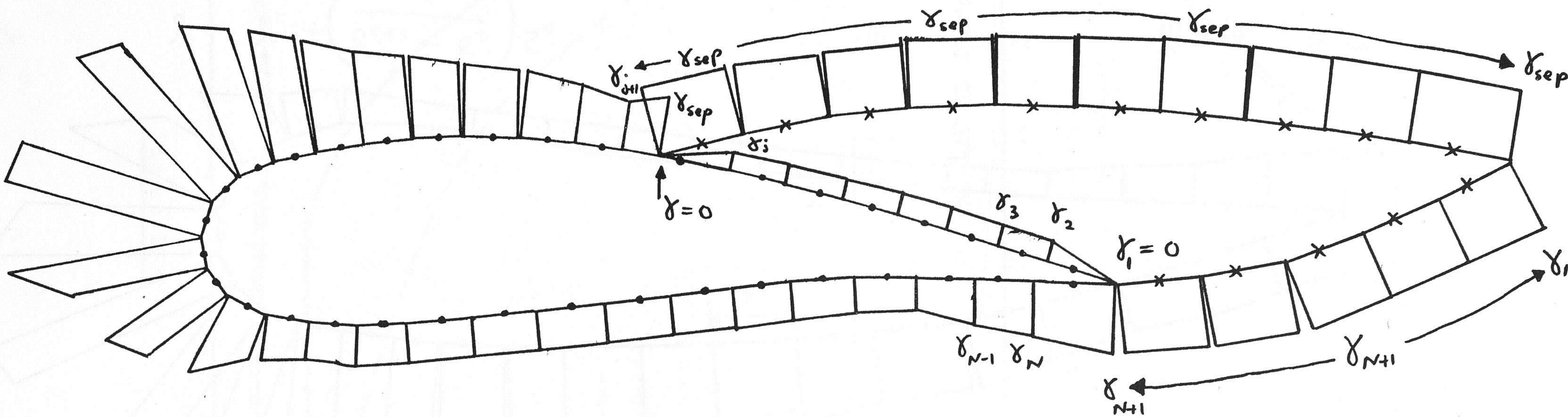
Kutta condition: $\gamma_1 + \gamma_{N+1} = 0$

Figure 2.I

PANEL VORTICITY DISTRIBUTION ON AEROFOIL WITH TRAILING EDGE SEPARATION.

• -- Control points.

X -- Wake iteration control points.



Kutta condition: $\gamma_{sep} + \gamma_{N+1} = 0$

Figure 3.I

VORTICITY DISTRIBUTION ON FREE VORTEX SHEETS.

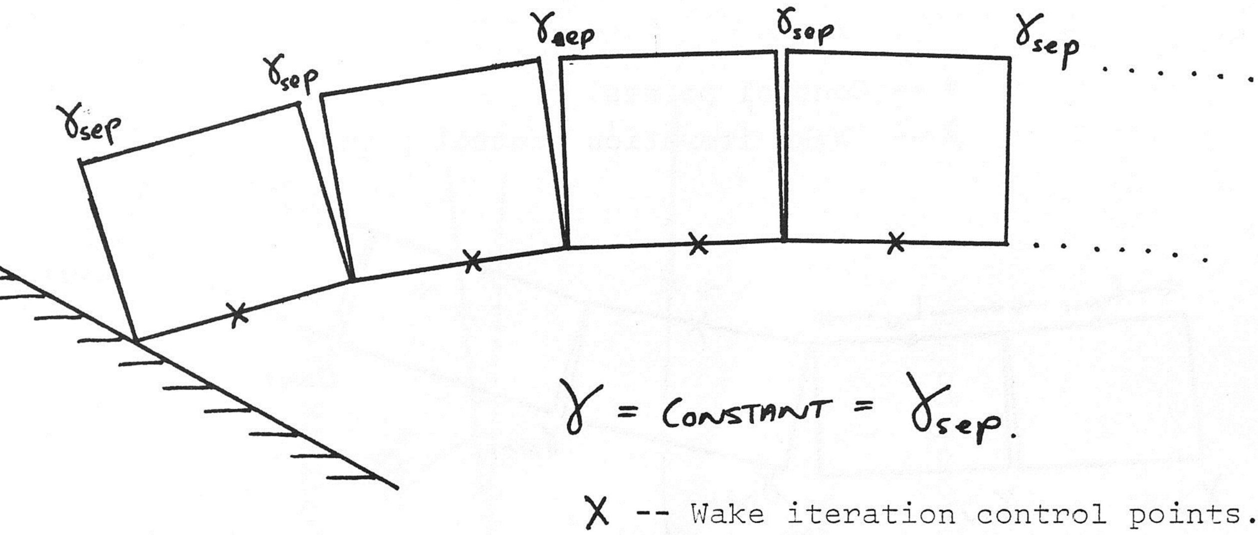


Figure 3.2

VORTICITY DISTRIBUTION ON UPPER
SURFACE SEPARATION PANEL.

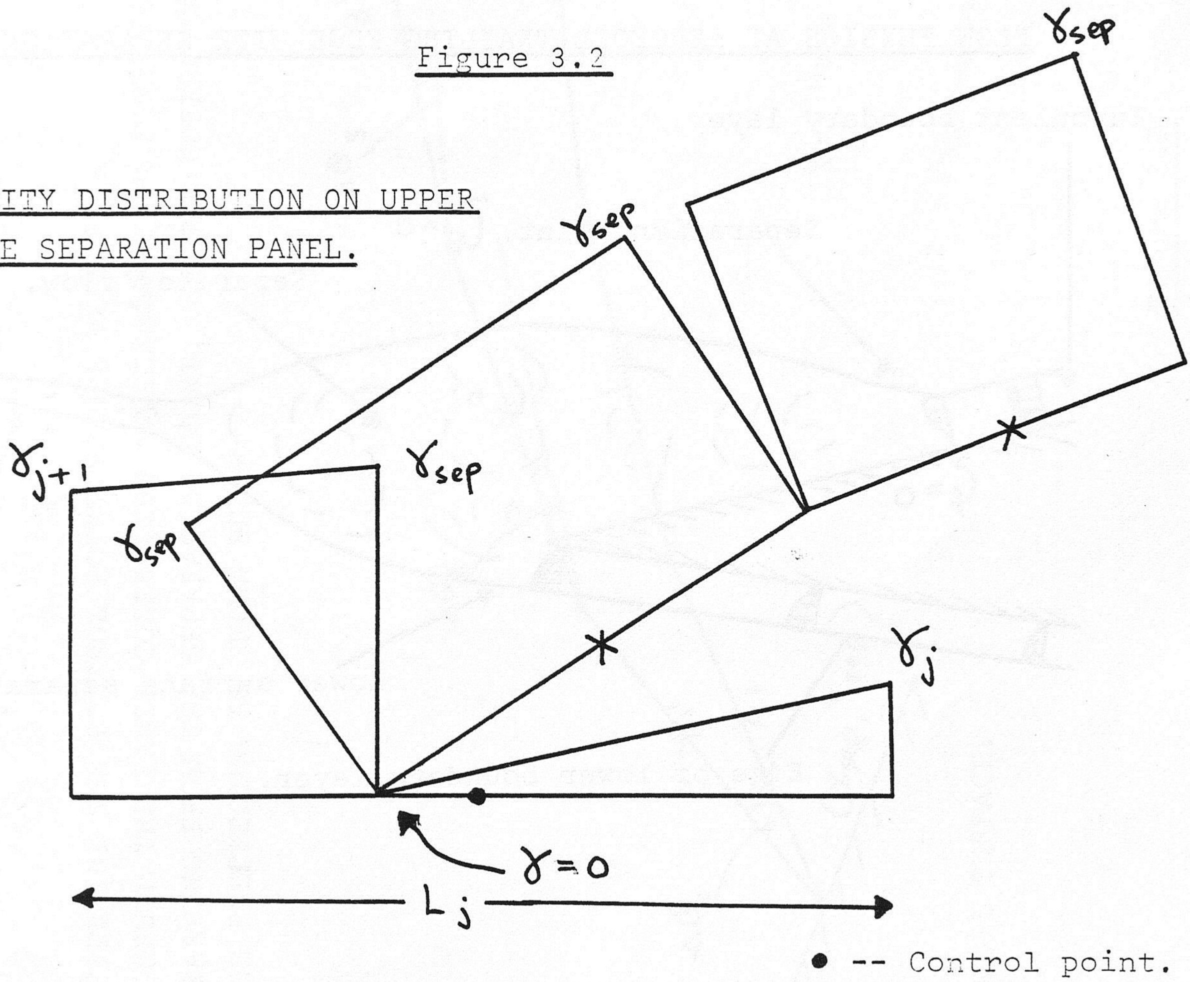


Figure 3.3

VORTICITY DISTRIBUTION ON LOWER SURFACE TRAILING EDGE.

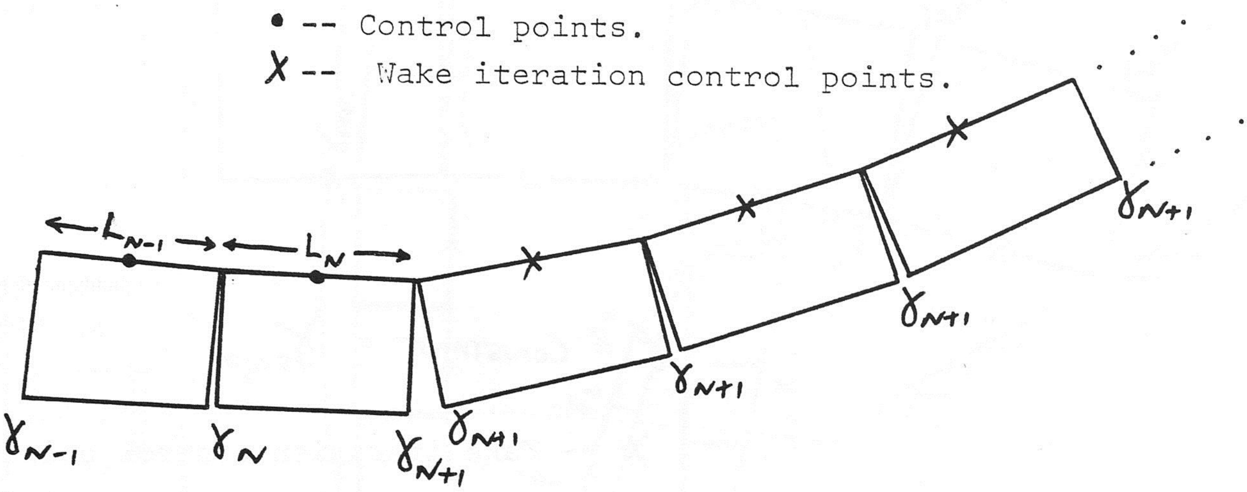


Figure 3.4

FLOW PHYSICS AT AEROFOIL TRAILING EDGE WITH SEPARATION.

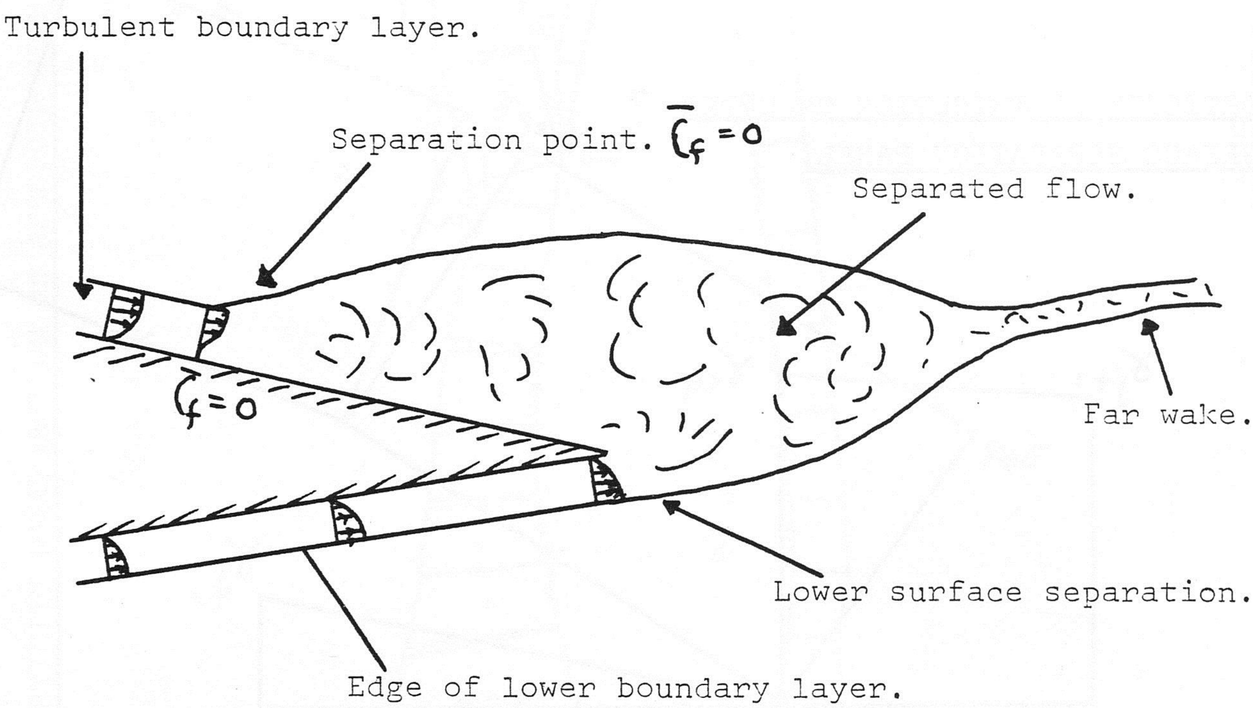


Figure 3.5

PROCEEDURE FOR WAKE ITERATION GEOMETRY.

I. DETERMINE THE CONTROL POINT NORMAL VECTORS.

2. COMPUTE THE PANEL CORRECTION ANGLES, $\Delta\theta_i$, FROM EQUATION (3.3)

3. STARTING FROM THE SEPARATION POINT, ADD THE CORRECTED PANEL ORIENTATION TO THE ONE PRECEEDING AS SHOWN BELOW:

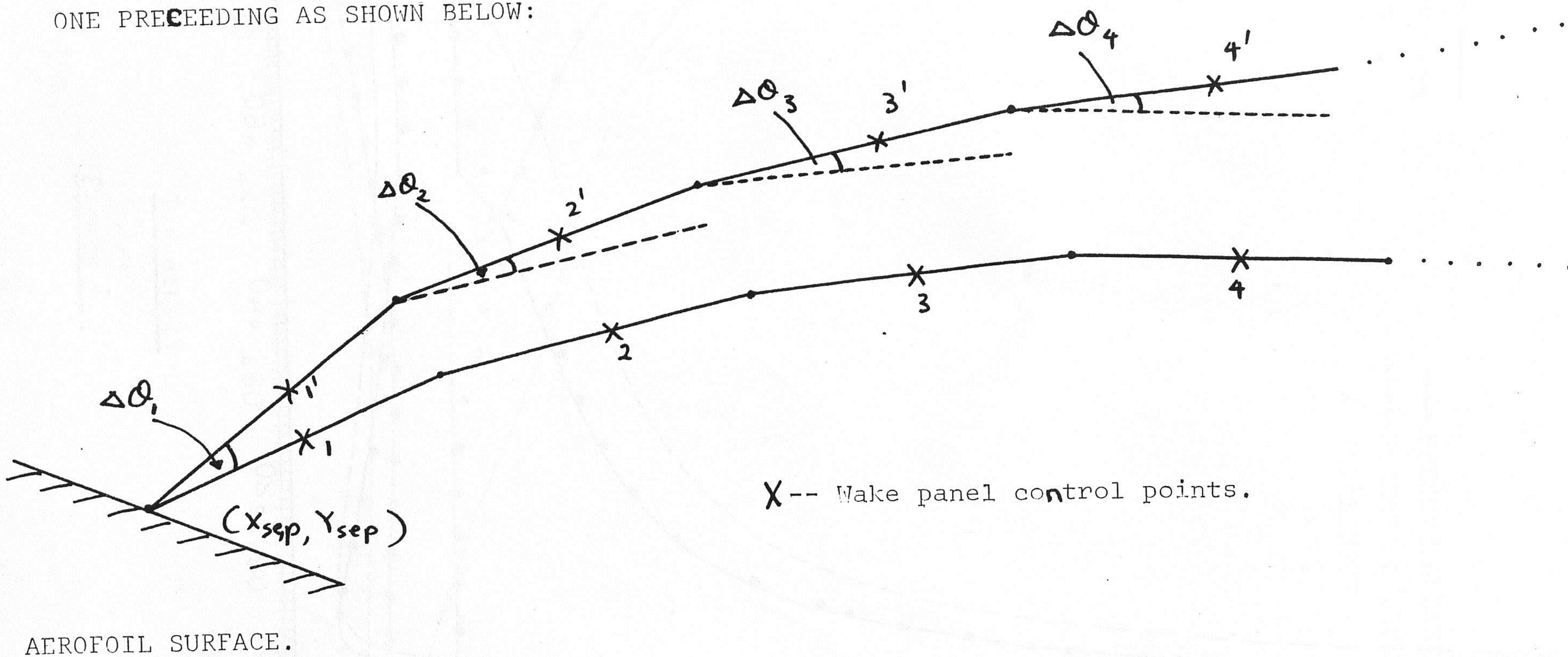
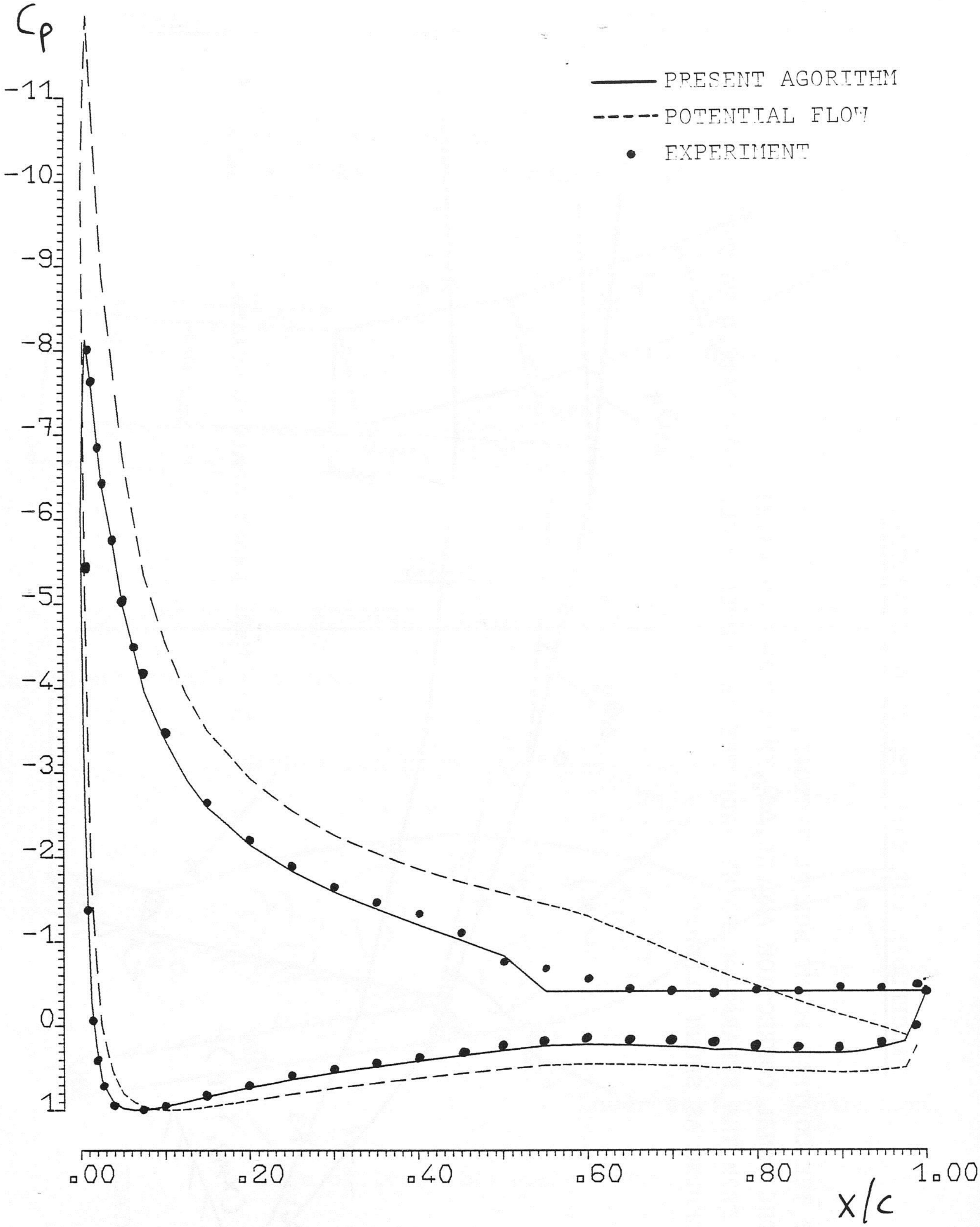


Figure 3.6



C_p
-11
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1
PRESENT AGORITHM
POTENTIAL FLOW
EXPERIMENT
.00
.20
.40
.60
.80
1.00
X/C

Figure 4.I

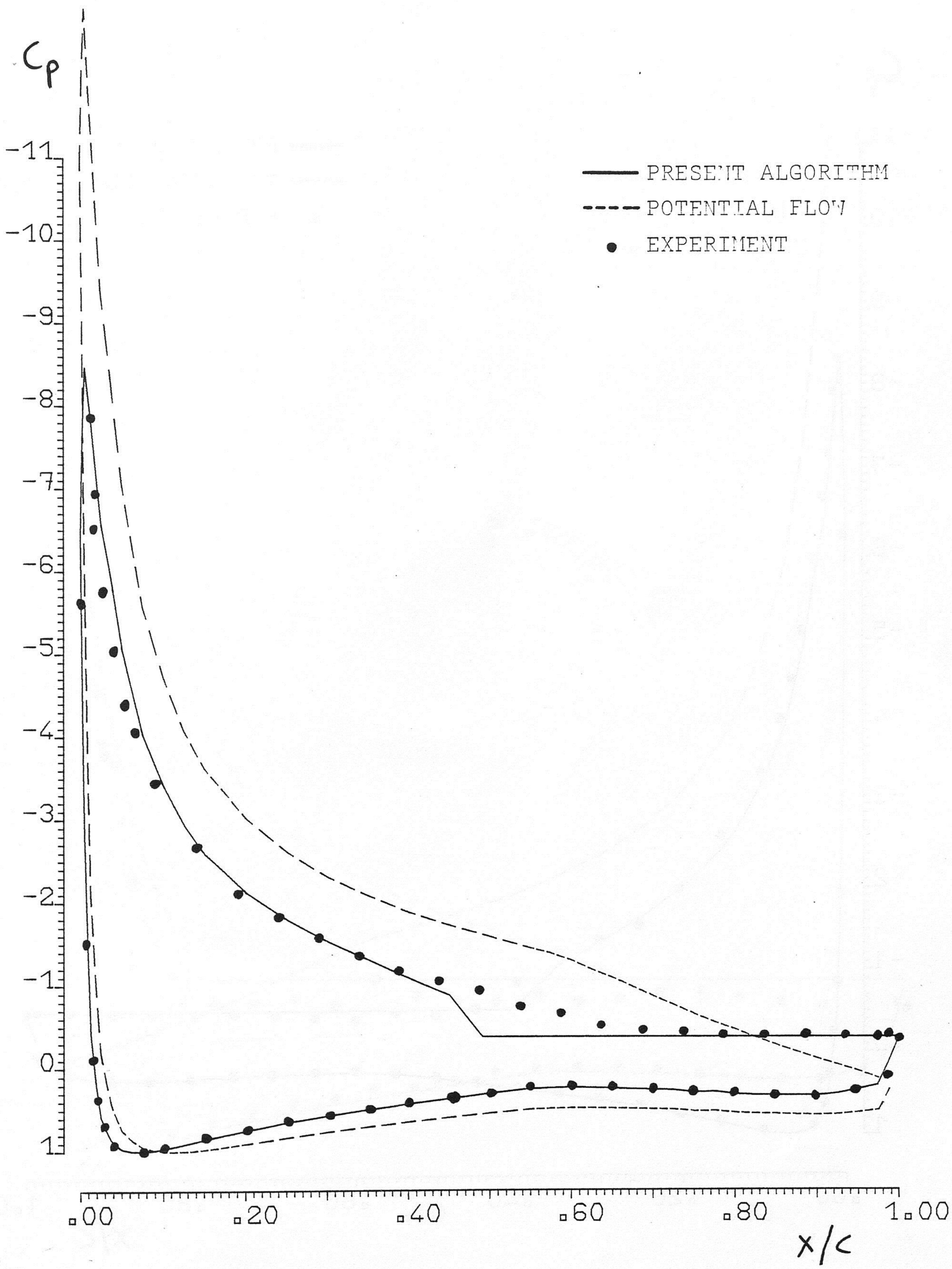

Cp
-11
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1
PRESENT ALGORITHM
POTENTIAL FLOW
EXPERIMENT
.00
.20
.40
.60
.80
1.00
x/c

Figure 4.2

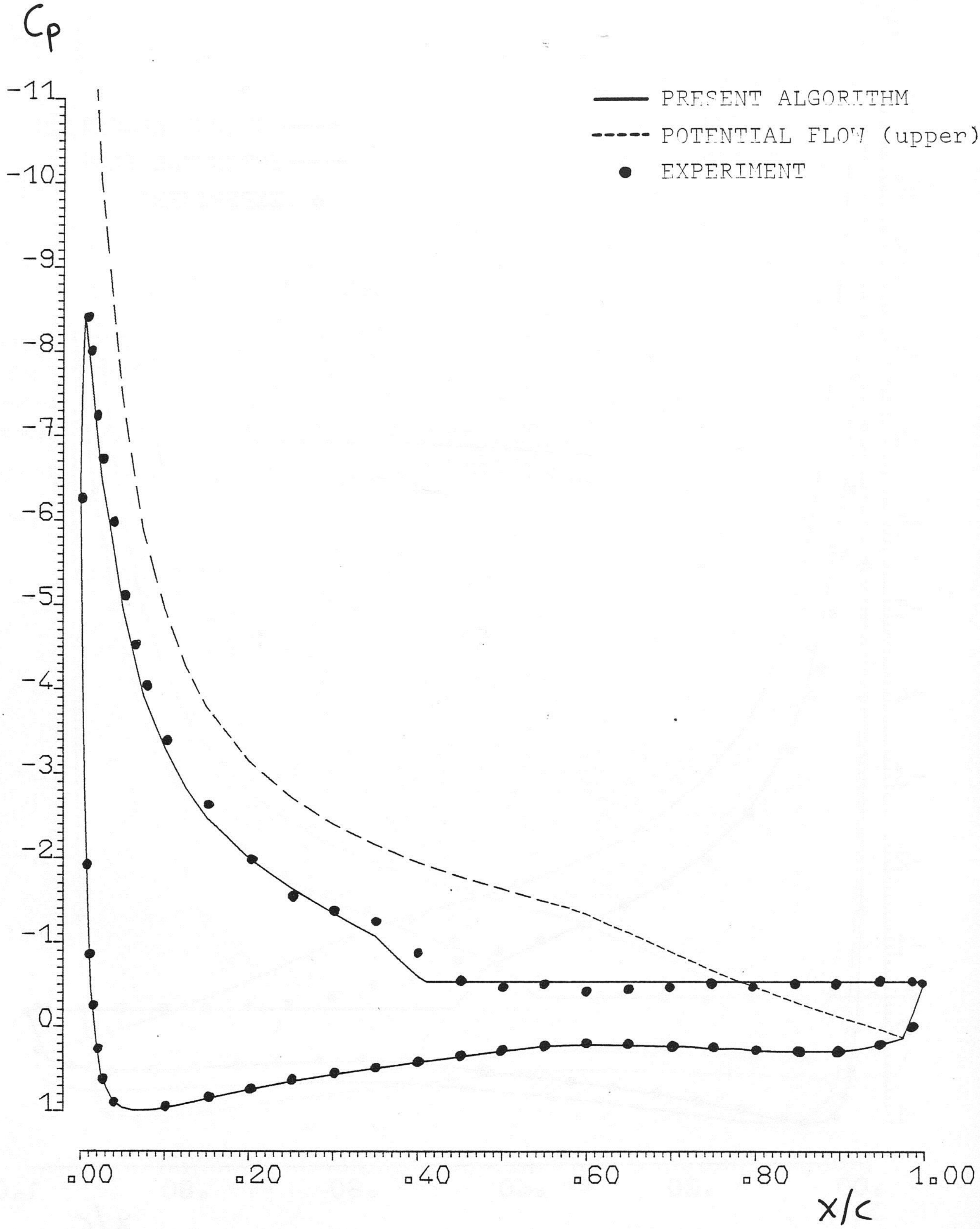

Cp
-11
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1
PRESENT ALGORITHM
POTENTIAL FLOW (upper)
EXPERIMENT
.00
.20
.40
.60
.80
1.00
x/c

Figure 4.3

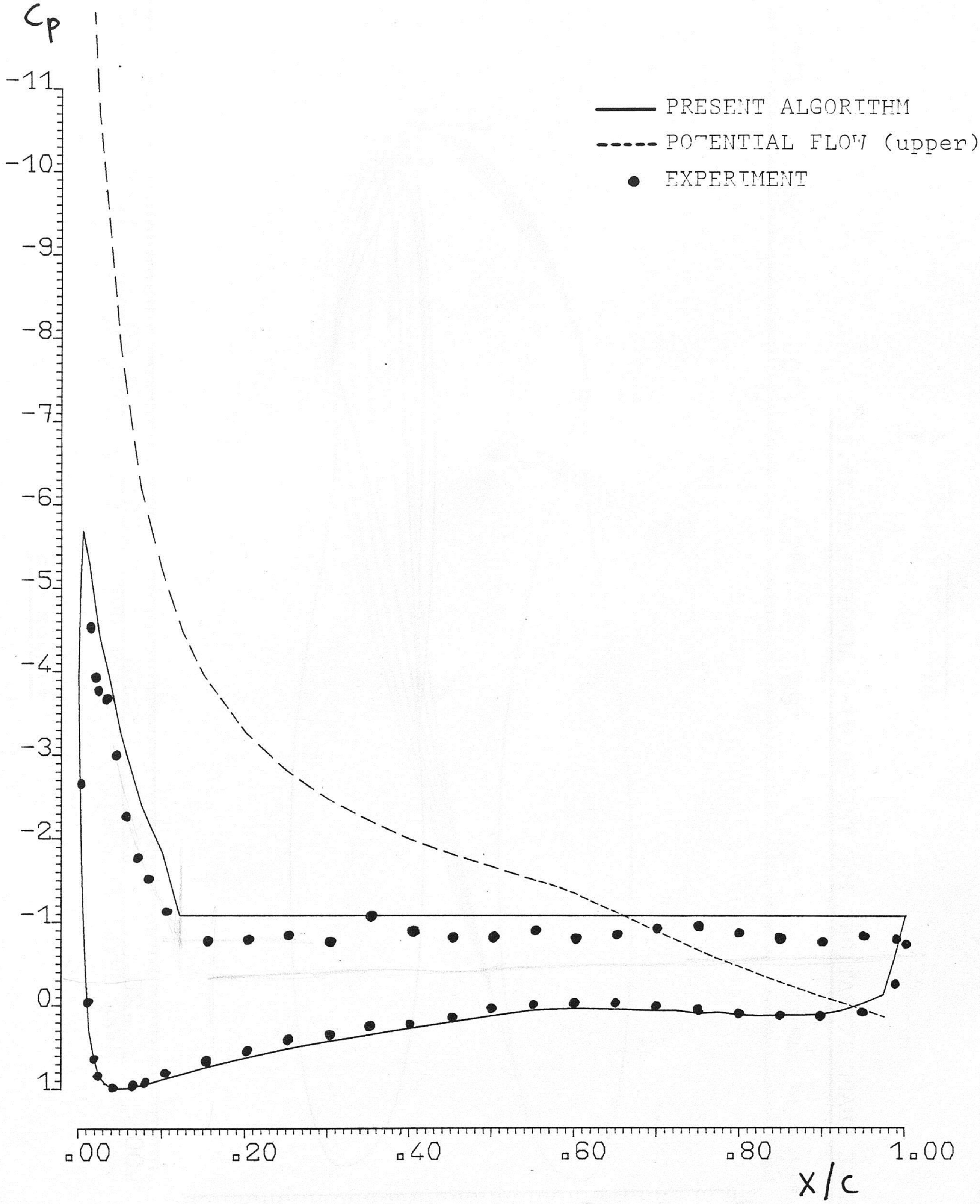

C_p
-11
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
1
PRESENT ALGORITHM
POTENTIAL FLOW (upper)
EXPERIMENT
.00
.20
.40
.60
.80
1.00
X/c

Figure 4.4

WAKE SHAPE ITERATIONS FOR THE GA(W)-I AEROFOIL AT I8.25°

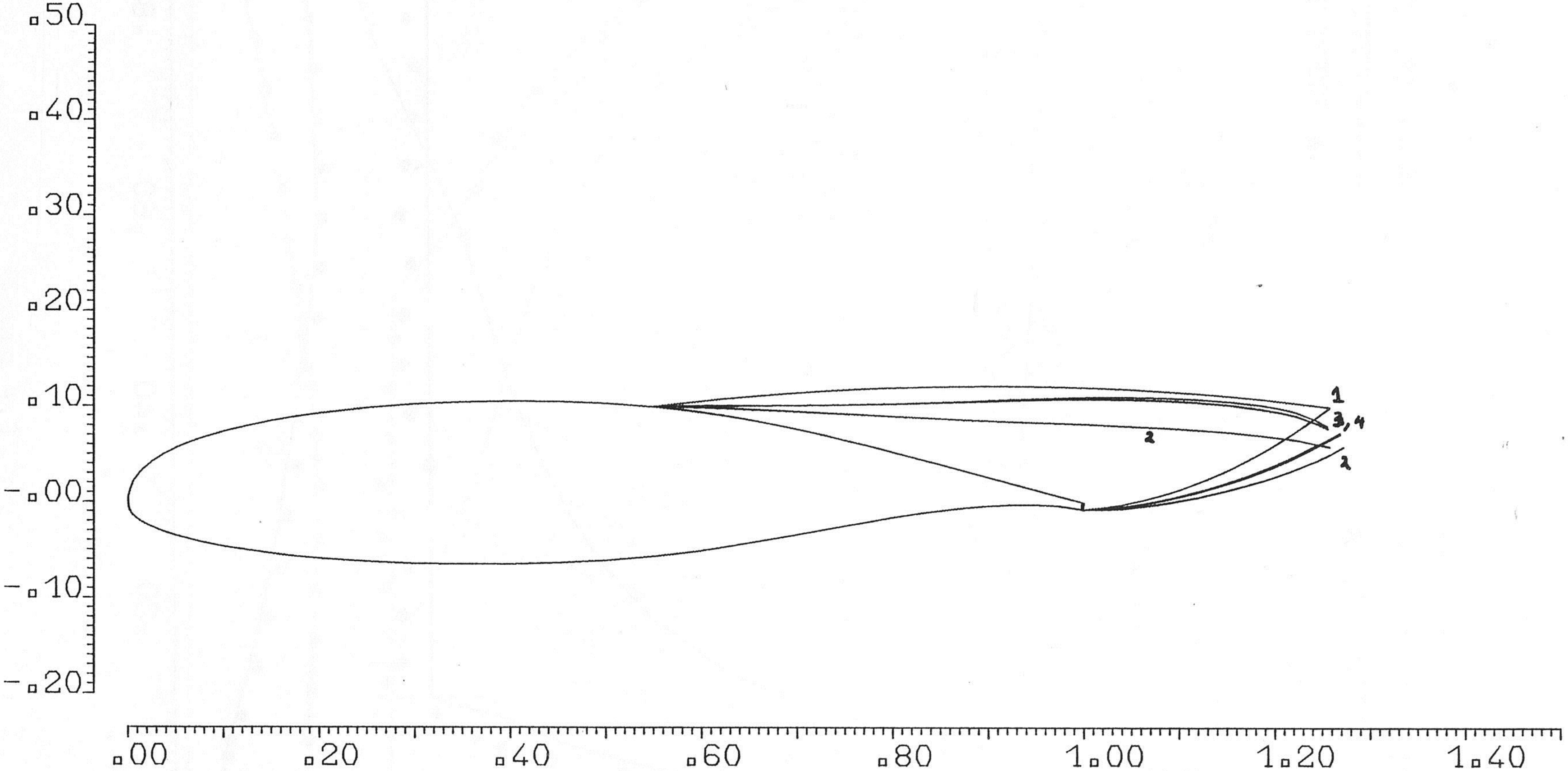


Figure 4.5

WAKE SHAPE ITERATIONS FOR THE GA(W)-1 AEROFOIL AT 21.14

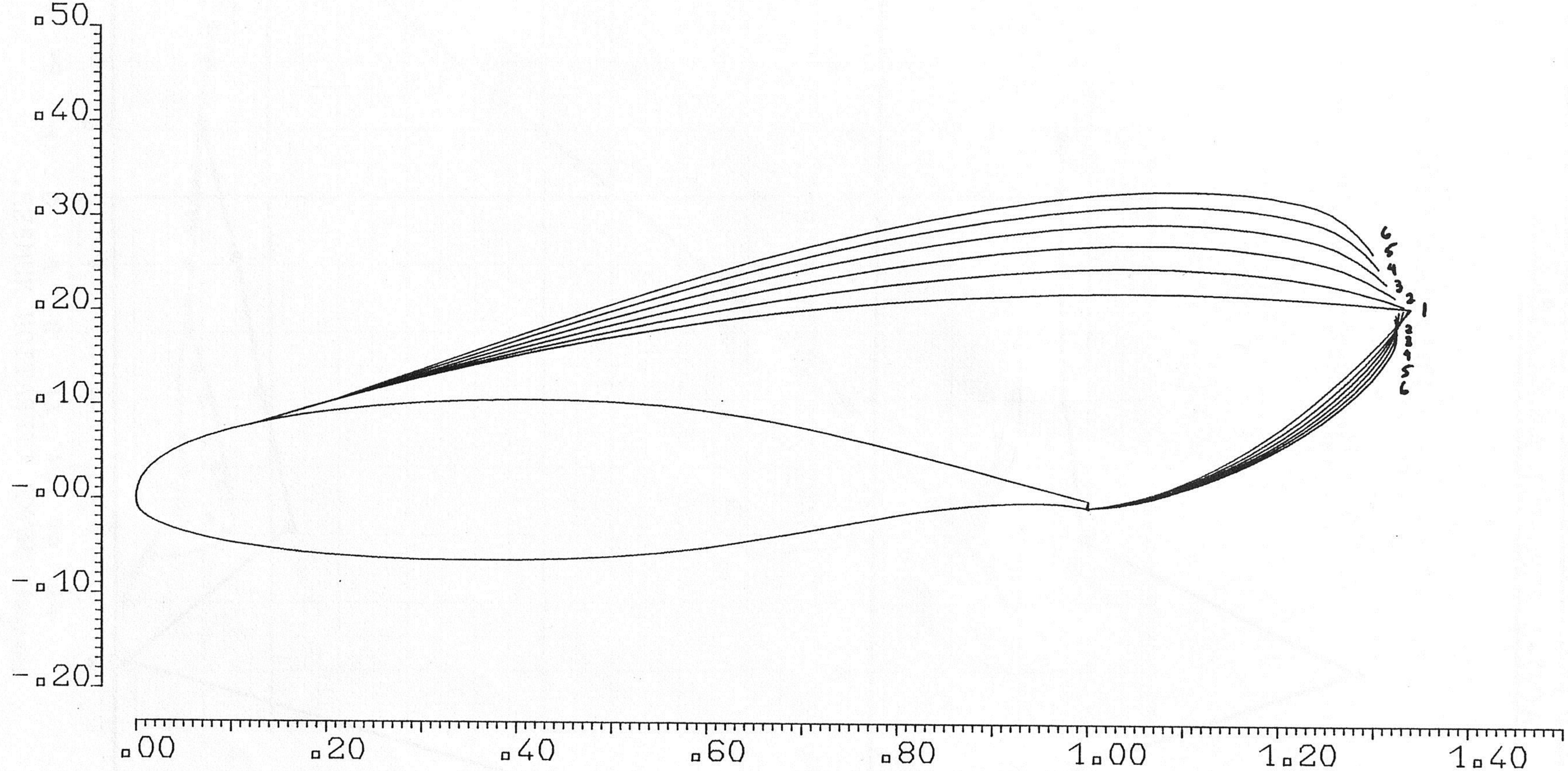
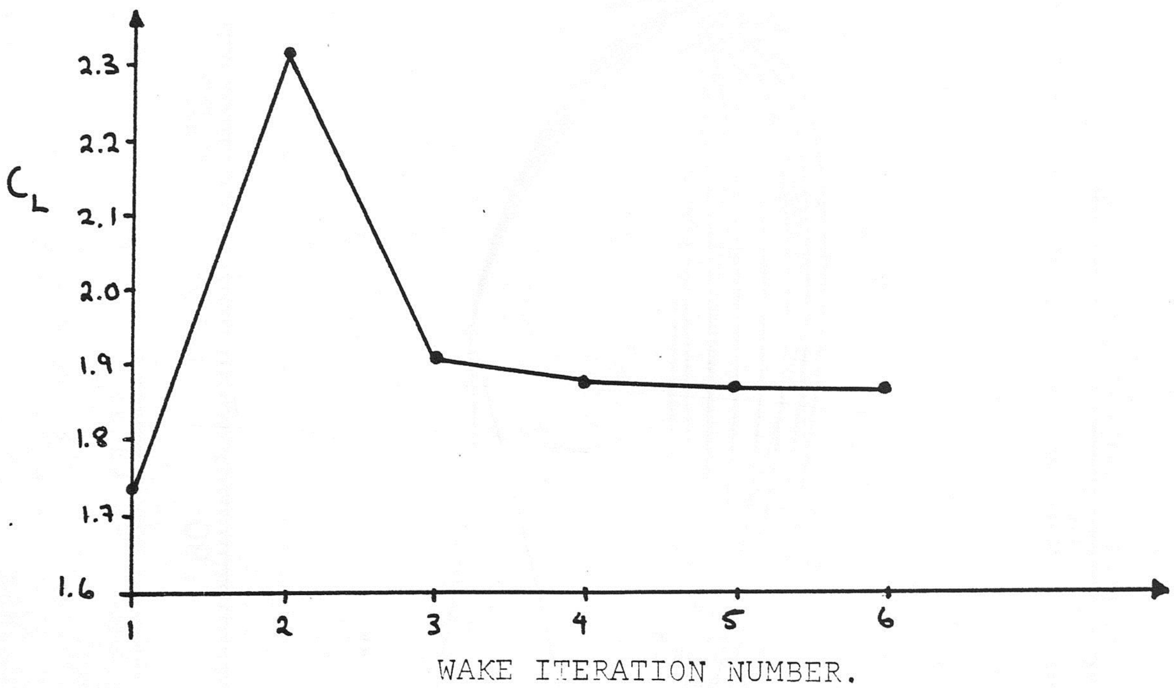


Figure 4.6

LIFT AND MOMENT COEFFICIENT VALUES WITH ITERATION NUMBER FOR THE GA(W)-I AEROFOIL AT ±8.25°



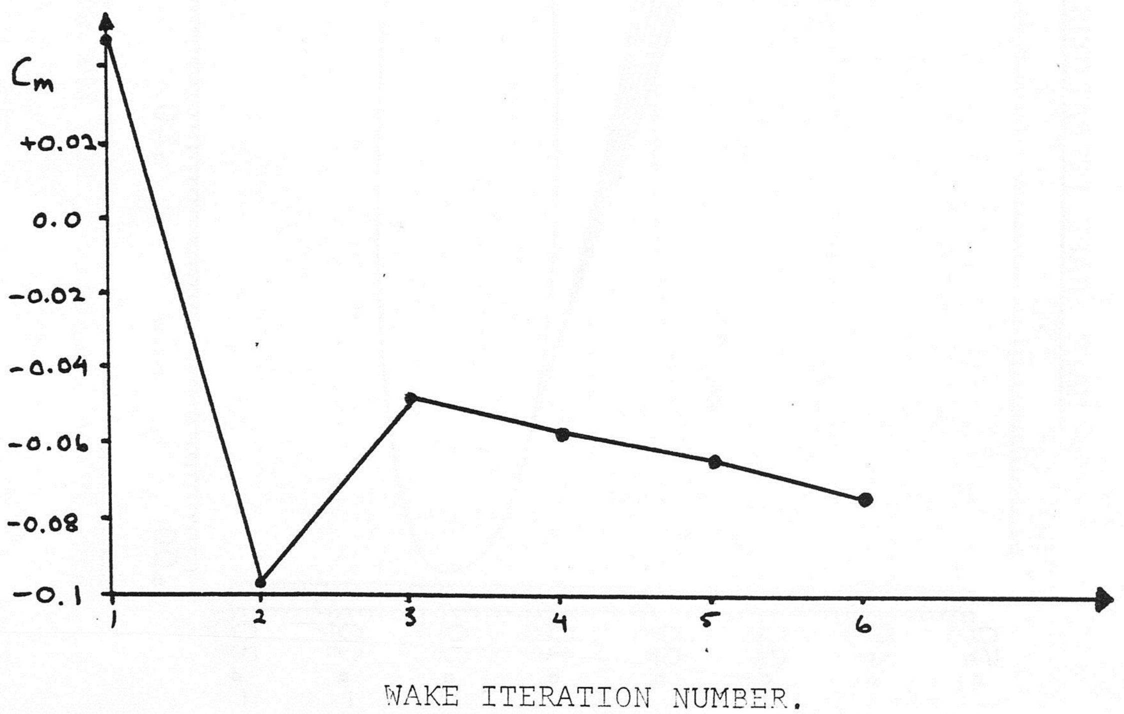


Figure 4.7

COMPARISON OF EXPERIMENTAL AND THEORETICAL AERODYNAMIC COEFFICIENTS.

---- POTENTIAL FLOW.
—×— MODEL OF REF. 4
—— PRESENT ALGORITHM.

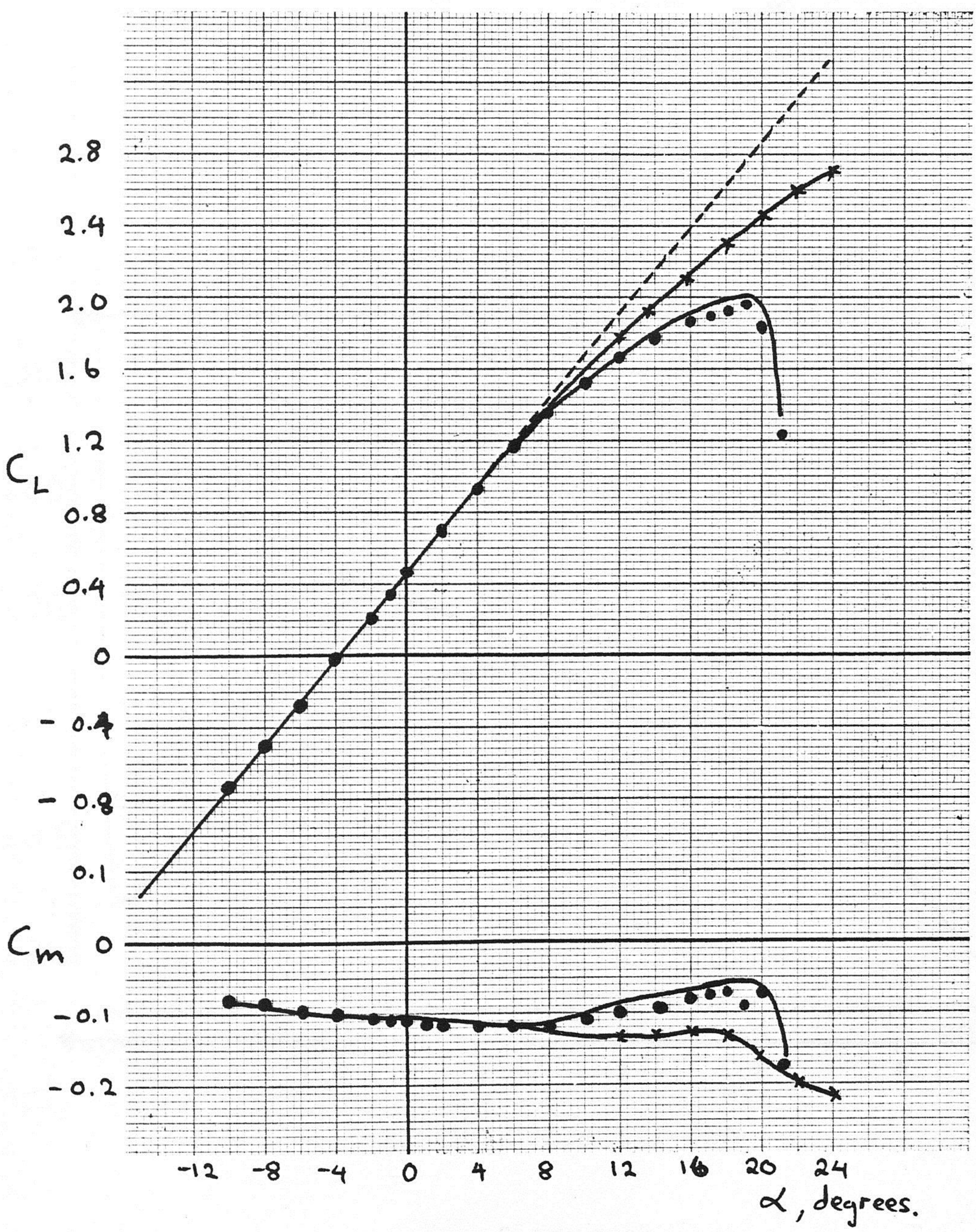
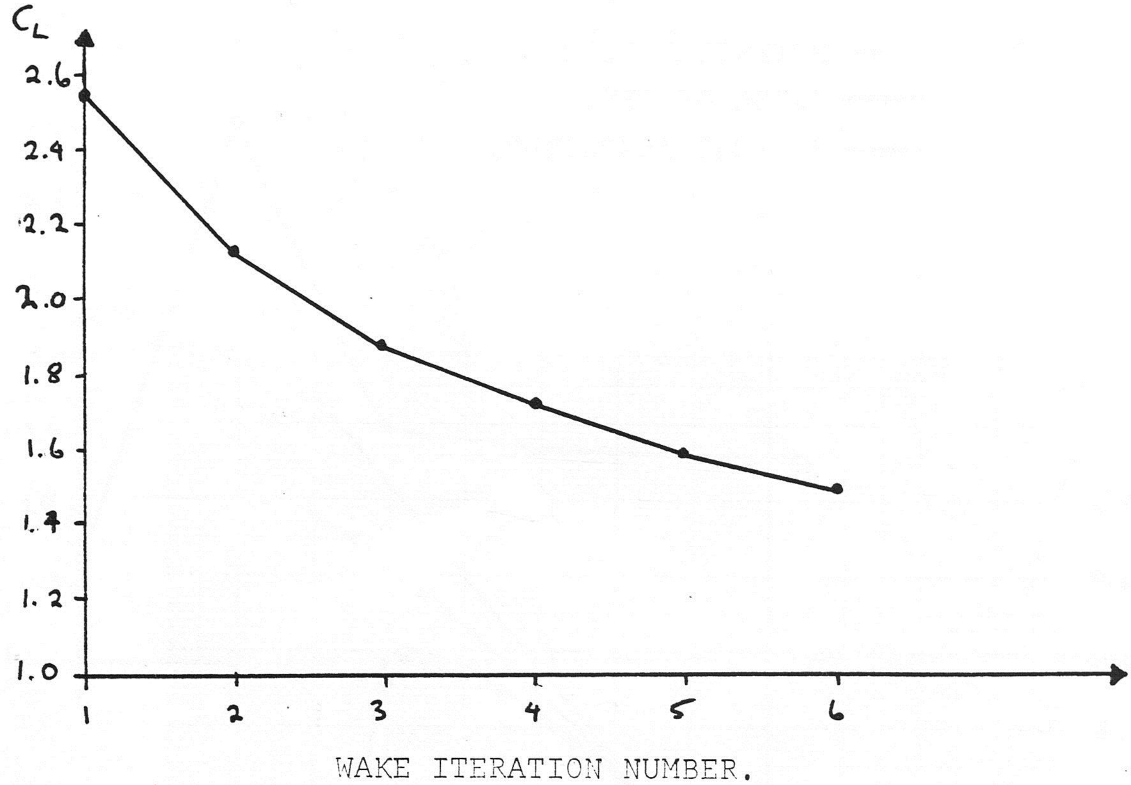


Figure 4.9

LIFT AND MOMENT COEFFICIENT VALUES WITH WAKE ITERATION NUMBER FOR THE GA(W)-I AEROFOIL AT 2I.I4



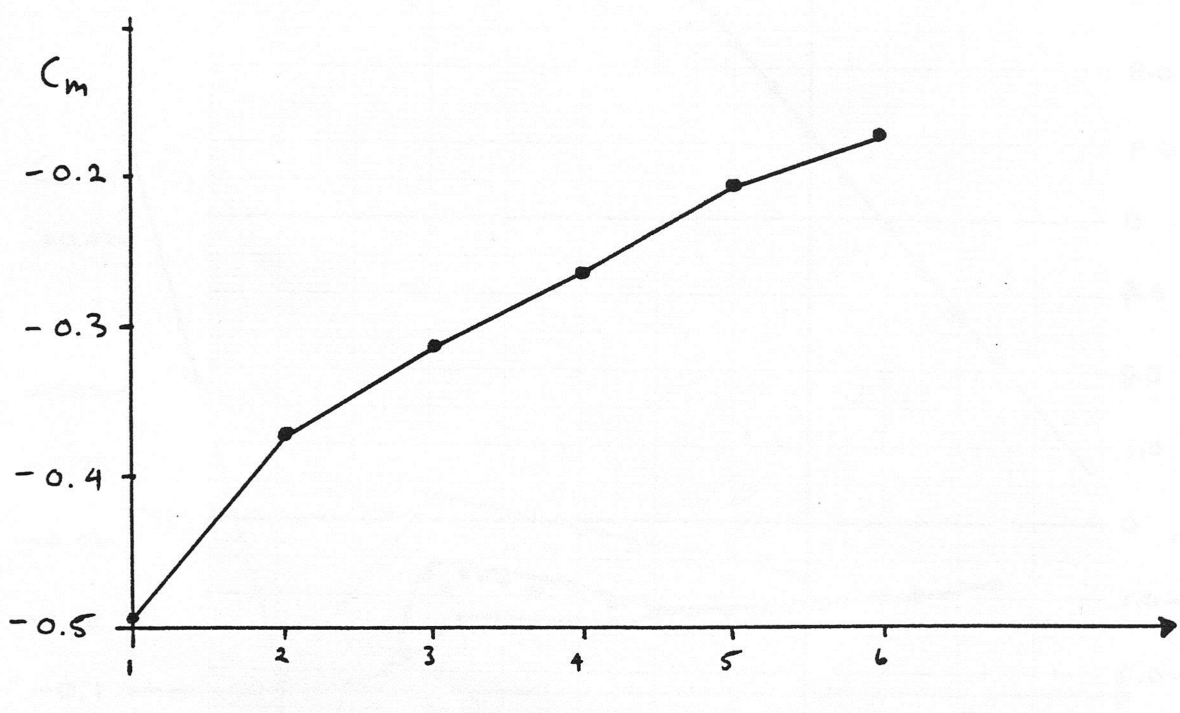
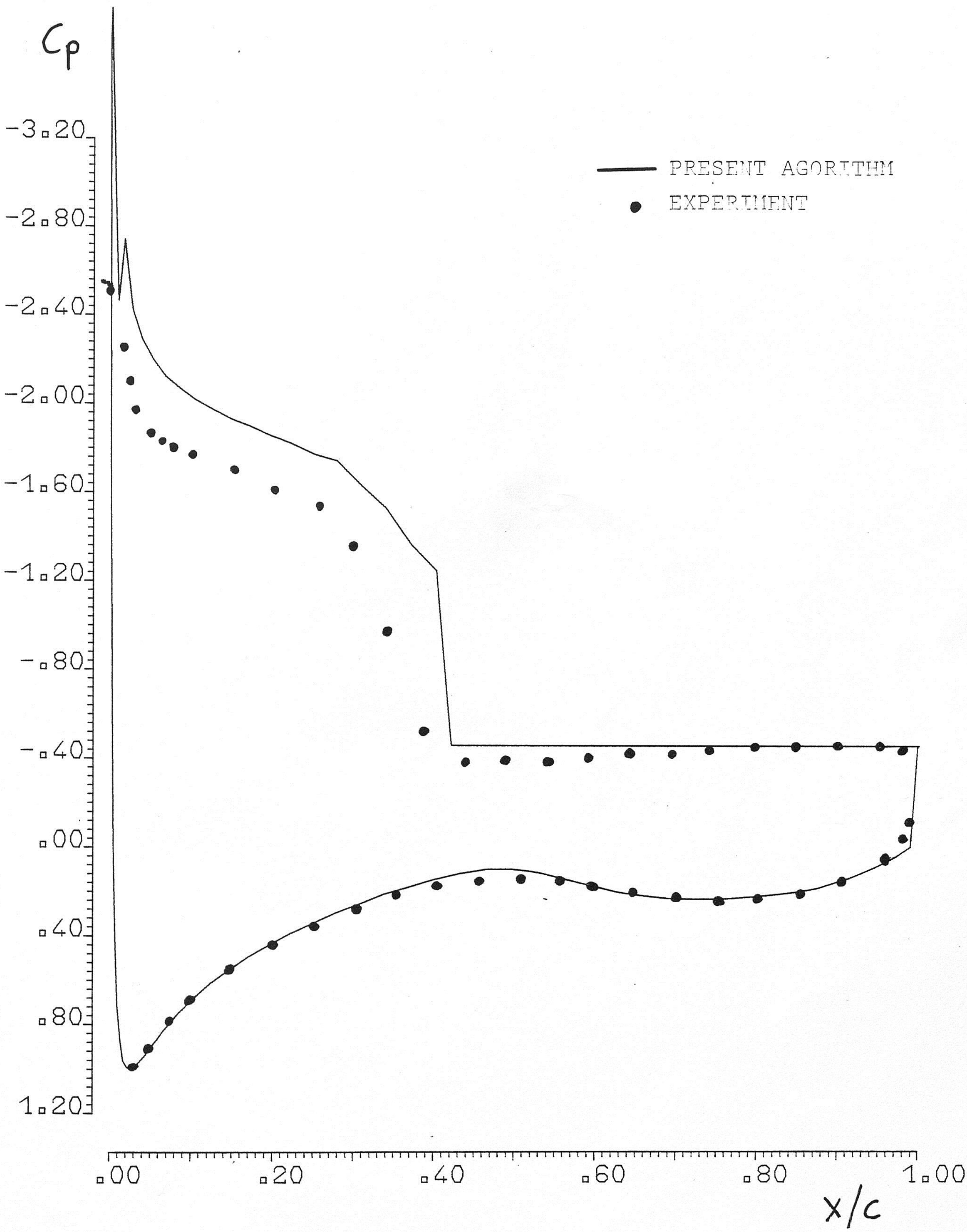


Figure 4.8



Cp
-3.20
-2.80
-2.40
-2.00
-1.60
-1.20
-.80
-.40
.00
.40
.80
1.20
PRESENT AGORITHM
EXPERIMENT
.00
.20
.40
.60
.80
1.00
X/c

Figure 4.IO