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COMPRESSION AND SHEAR BUCKLING PERFORMANCE OF INFINITELY LONG AND FINITE LENGTH PLATES WITH BENDING-TWISTING COUPLING.

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Presentation Contents

- Background, motivation and context.

- Design Heuristics:
  o ply angle,
  o ply percentage and
  o ply contiguity.

- Overview of databases for symmetric and non-symmetric warp-free laminates ($B = 0$):
  o Bending-Twisting coupled laminates (balanced and symmetric?),
  o Extension-Shearing, Bending-Twisting coupled laminates (unbalanced and symmetric?).

- Lamination parameter point clouds are used to interrogate the effect of design heuristics on the design space. Each point represents a unique and practical design.

- Compression and shear buckling strength contours are mapped onto the design space, from which optimum designs can be readily identified.

- Conclusions.
Background

**Compression buckling strength** may be **overestimated** (unsafe) if the effects of **Bending-Twisting coupling** are ignored.

**Shear buckling strength** may be **underestimated** (over-designed) or overestimated if the effects of **Bending-Twisting coupling** are ignored.

In this study, the effect of **Bending-Twisting coupling** on the buckling strength of **infinitely long laminated plates with simply supported edges** is investigated….

….it complements an **extensive literature** on the subject, where the focus is primarily on **finite length plates**. Infinitely long plates represent useful lower-bound solutions for preliminary design.

With very few exceptions, the study of **Bending-Twisting coupling** effects has focussed entirely on symmetric designs….

….the relative buckling performance of non-symmetric designs are now revealed.
Laminate databases

Databases have been developed\textsuperscript{1,2}, which contain listings for UD material with standard (or non-standard) fibre angle orientations, i.e.:

\[
0, 90 \text{ and/or } \pm 45^\circ (= \pm \theta^\circ).
\]

The derivations involved the restrictions that \textit{each layer in the laminate}:

- has identical material properties;
- has identical thickness and;
- differs only by its orientation.

Databases contain both symmetric and non-symmetric \textit{warp-free} laminates ($B = 0$) for:

- \textbf{Bending-Twisting} coupled laminates\textsuperscript{1},
- and
- \textbf{Extension-Shearing, Bending-Twisting} coupled laminates\textsuperscript{2}.

Databases also contain lamination parameter information for standard ply angle designs.

Lamination Parameter Design Space for Interrogation of Bending-Twisting coupled designs.

Lamination parameters \((\xi^A, \xi^R, \xi^c)\) = \((\xi_1, \xi_2, \xi_3)\) are related to the associated extensional stiffnesses through the following expression:

\[
[A] = H \begin{bmatrix}
U_E + \xi^A U_\Delta + \xi^R U_R & U_E - 2U_G - \xi^A U_R & \xi^A U_\Delta / 2 + \xi^A U_R \\
U_E - 2U_G - \xi^A U_R & U_E - \xi^A U_\Delta + \xi^A U_R & \xi^A U_\Delta / 2 - \xi^A U_R \\
\xi^A U_\Delta / 2 + \xi^R U_R & \xi^A U_\Delta / 2 - \xi^A U_R & U_G - \xi^A U_R
\end{bmatrix}
\]

\[\begin{align*}
A_{xx} & A_{xy} A_{xs} \\
A_{xy} & A_{yy} A_{ys} \\
A_{xs} & A_{ys} A_{ss}
\end{align*}\]

Note that \(\xi^A_{Re} = 0\) for standard ply orientations, hence the \([A]\) matrix is described by a three dimensional lamination parameter coordinate for all designs considered here.

The laminate invariant properties are defined in terms of the reduced stiffnesses, \(Q_{ij}\):

\[U_E = \left(3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}\right) / 8\]
\[U_G = \left(Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66}\right) / 8\]
\[U_\Delta = \left(Q_{11} - Q_{22}\right) / 2\]
\[U_R = \left(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}\right) / 8\]

\((U_E \equiv U_1, U_G \equiv U_5, U_\Delta \equiv U_2, U_R \equiv U_3)\)
Lamination Parameter Design Space for Interrogation of *Bending-Twisting* coupled designs.

Lamination parameters \((\xi^D_\Delta, \xi^D_R, \xi^D_{\Delta c}) = (\xi_9, \xi_{10}, \xi_{11})\) are related to the associated *bending* stiffnesses through the following expression:

\[
[D] = \frac{H^3}{12} \begin{bmatrix}
U_E + \xi_D^D U_\Delta + \xi_R^D U_R & U_E - 2U_G - \xi_R^D U_R & \xi_{\Delta c}^D U_\Delta / 2 + \xi_{R c}^D U_R \\
U_E - 2U_G - \xi_R^D U_R & U_E - \xi_D^D U_\Delta + \xi_R^D U_R & \xi_{\Delta c}^D U_\Delta / 2 - \xi_{R c}^D U_R \\
\xi_{\Delta c}^D U_\Delta / 2 + \xi_{R c}^D U_R & \xi_{\Delta c}^D U_\Delta / 2 - \xi_{R c}^D U_R & U_G - \xi_R^D U_R
\end{bmatrix}
\]

(3)

\[
[D] = \begin{bmatrix}
D_{xx} & D_{xy} & D_{xs} \\
D_{xy} & D_{yy} & D_{ys} \\
D_{xs} & D_{ys} & D_{ss}
\end{bmatrix}
\]

Note that \(\xi_{R c}^D = 0\) for standard ply orientations, hence the \([D]\) matrix is described by a *three dimensional lamination parameter coordinate* for all designs considered here.
Interpretation of Lamination Parameter Design Spaces: Ply percentages.

Lamination parameter coordinates can be illustrated as orthographic projections of *Extensional Stiffness* and related to ply percentages.

![Lamination parameter design space](image)

Figure 1: Lamination parameter design space with ply percentage mapping for: (a) orthotropic stiffness \((\xi_A^A, \xi_R^A)\) and; (b) anisotropic stiffness \((\xi_{\Delta c}^A)\) relating to differing angle-ply percentages. The 10% design rule constraint is also illustrated.
Point clouds of **lamination parameter coordinates** from the databases can be illustrated as **isometric projections** for **extensional stiffness**:

![Isometric view of lamination parameter design spaces](image)

Figure 2: Isometric view of lamination parameter design spaces for **extensional stiffness**, with **10% rule applied**, corresponding to:

(a) **Symmetric** and 
(b) **Non-symmetric Bending-Twisting coupled** (balanced) laminates with up to 18 plies and;

(c) **Symmetric** and 
(d) **Non-symmetric Extension-Shearing Bending-Twisting coupled** (unbalanced) laminates with up to 18 plies.
Point clouds of **lamination parameter coordinates** from the databases can be illustrated as **isometric projections** for **bending stiffness**:

![Point clouds of lamination parameter coordinates](image)

Figure 3 - Isometric view of lamination parameter design spaces for **bending stiffness**, with 10% rule applied, corresponding to:

(a) **Symmetric** and
(b) **Non-symmetric Bending-Twisting coupled** (balanced) laminates with up to 18 plies and;

(c) **Symmetric** and
(d) **Non-symmetric Extension-Shearing Bending-Twisting coupled** (unbalanced) laminates with up to 18 plies.
For higher fidelity interpretation, the point clouds of **lamination parameter coordinates** can be illustrated as a three view orthographic projections of *Extensional Stiffness*:

Figure 3: Lamination parameter design spaces for **symmetric Extension-Shearing Bending-Twisting coupled laminates** with $7 \leq n \leq 18$, with 10\% rule and ply contiguity constraints ($\leq 3$) applied.
For higher fidelity interpretation, the point clouds of lamination parameter coordinates can be illustrated as a three view orthographic projections of *Extensional Stiffness*:

Figure 4: Lamination parameter design spaces for **Non-symmetric Extension-Shearing Bending-Twisting coupled laminates** with $7 \leq n \leq 18$, with 10% rule and ply contiguity constraints ($\leq 3$) applied.
For higher fidelity interpretation, the point clouds of lamination parameter coordinates can be illustrated as a three view orthographic projections of *Bending Stiffness*:

Figure 5: Lamination parameter design spaces for symmetric *Extension-Shearing Bending-Twisting coupled laminates* with $7 \leq n \leq 18$, with 10% rule and ply contiguity constraints ($\leq 3$) applied.
For higher fidelity interpretation, the point clouds of lamination parameter coordinates can be illustrated as a three view orthographic projections of Bending Stiffness:

![Diagram showing lamination parameter design spaces for Non-symmetric Extension-Shearing Bending-Twisting coupled laminates.](image)

Figure 6: Lamination parameter design spaces for **Non-symmetric Extension-Shearing Bending-Twisting coupled laminates** with $7 \leq n \leq 18$, with 10% rule and ply contiguity constraints ($\leq 3$) applied.
Table 1 – **Effect of ply contiguity constraints on the 10% design rule** design space for: (a) Symmetric and; (b) Non-symmetric *Extension-Shearing Bending-Twisting* coupled laminates.

<table>
<thead>
<tr>
<th>n</th>
<th>1</th>
<th>≤2</th>
<th>≤3</th>
<th>10%</th>
<th>1</th>
<th>≤2</th>
<th>≤3</th>
<th>10%</th>
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<td>-</td>
<td>-</td>
<td>2</td>
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<td>8</td>
<td>-</td>
<td>8</td>
</tr>
<tr>
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<td>-</td>
<td>34</td>
<td>-</td>
<td>36</td>
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<tr>
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<td>-</td>
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<td>1,896</td>
<td>62,632</td>
<td>102,178</td>
<td>114,638</td>
</tr>
</tbody>
</table>
Table 2 – **Effect of ply contiguity constraints on the 10\% design rule** design space for: (a) Symmetric and; (b) Non-symmetric *Bending-Twisting coupled laminates*.

<table>
<thead>
<tr>
<th></th>
<th>(a) Symmetric laminates</th>
<th>(b) Non-symmetric laminates</th>
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</thead>
<tbody>
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<td>-</td>
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Closed form buckling equations for \textit{infinitely long plates} in shear and compression.

An \textbf{exact closed form solution}, necessary to handle the vast number of database designs, can be used to assess the \textbf{compression buckling strength}:

\[
N_{x,\infty} = \pi^2 \left[ D_{xx} \left( \frac{1}{\lambda} \right)^2 + 2 \left( D_{xy} + 2D_{ss} \right) \frac{1}{b^2} + D_{yy} \left( \frac{1}{b^4} \right) \lambda^2 \right]
\]  \hspace{1cm} (4)

…but only for uncoupled designs!

An \textbf{approximate closed form solution} can also be represented by a 2 dimensional, 4\textsuperscript{th} order polynomial and can be solved against the exact closed form buckling solution of Eqn. (1) from equally spaced points across the lamination parameter design space \((\xi_D^D, \xi_R^D)\):

\[
k_{\infty} = c_1 + c_2 \xi_D^D + c_3 \xi_R^D + c_4 \left( \xi_D^D \right)^2 + c_5 \left( \xi_R^D \right)^2 + c_6 \xi_D^D \xi_R^D + c_7 \left( \xi_D^D \right)^3 + c_8 \left( \xi_R^D \right)^3 + c_9 \xi_D^D \left( \xi_R^D \right)^2 + c_{10} \left( \xi_D^D \right)^2 \xi_R^D + c_{11} \left( \xi_D^D \right)^4 + c_{12} \left( \xi_R^D \right)^4 + c_{13} \xi_D^D \left( \xi_D^D \right)^3 + c_{14} \left( \xi_D^D \right)^2 \left( \xi_R^D \right)^2 + c_{15} \left( \xi_D^D \right)^3 \xi_R^D
\]

\hspace{1cm} (5)

For compression buckling, \(k_{\infty} = k_{x,\infty}\) and is defined by:

\[
k_{x,\infty} = \frac{N_{x,\infty} b^2}{\pi^2 D_{iso}}
\]

\hspace{1cm} (6)

For IM7/8552 carbon-fiber/epoxy material….
Closed form buckling equations for infinitely long plates in shear and compression.

\[
k_{x,\infty} = 4.000 - 1.049 \xi_D - 1.217 (\xi_D)^2 + 0.340 \xi_D (\xi_D)^2 - 0.360 (\xi_D)^4 - 0.034 (\xi_D)^2 (\xi_D)^2
\]  

\text{(7)}

For uncoupled laminates in shear (or Bending-Twisting coupled laminates in compression or shear), an exact infinite strip analysis\(^3\) is used to generate buckling factors from which the polynomial coefficients of Eqn. (2) can be solved, where \(k_\infty = k_{s,\infty}\) and is defined by:

\[
k_{s,\infty} = \frac{N_{s,\infty} b^2}{\pi^2 D_{iso}}
\]

\text{(8)}

giving:

\[
k_{s,\infty} = 5.336 - 2.914 \xi_D - 0.518 \xi_D - 1.303 (\xi_D)^2 - 0.213 (\xi_D)^2 + 1.048 \xi_D \xi_D
\]

\[- 0.236 (\xi_D)^3 + 0.031 (\xi_D)^3 - 0.197 \xi_D (\xi_D)^2 + 0.405 (\xi_D)^2 \xi_D - 0.443 (\xi_D)^4
\]

\[- 0.001 (\xi_D)^4 + 0.022 \xi_D (\xi_D)^3 - 0.185 (\xi_D)^2 (\xi_D)^2 + 0.472 (\xi_D)^3 \xi_D
\]

\text{(9)}

These equations facilitate the mapping of buckling factor contours…..

\text{---}

Interpretation of Lamination Parameter Design Spaces.

Buckling factor contour mapping on cross-sections through the design space

Figure 7: (a) Three-dimensional representation of the feasible design space indicating the positions through which two dimensional cross-sections have been taken. Positive shear load and positive fibre orientation are defined in the thumbnail sketch. Sections representing fully uncoupled laminates in bending, correspond to: (b) compression buckling contours, $k_x, \infty (N_x b^2 / \pi^2 D_{iso})$ and; (c) positive/negative shear buckling contours, $k_s, \infty (N_s b^2 / \pi^2 D_{iso})$. 

Stability of Structures 15th Symposium, Zakopane, 17-21 September, 201
Compression and shear buckling performance of Bending-Twisting coupled laminated plates

Interpretation of Lamination Parameter Design Spaces.

Buckling contour mapping on cross-sections

Figure 8: Compression buckling factor contours, $k_{x,\infty} = N_b b^2/\pi^2 D_{iso}$, for: (a) $\xi_{\Delta c}^D = 0.1$ and; (b) $\xi_{\Delta c}^D = 0.3$, representing Bending-Twisting coupled laminates.
Interpretation of Lamination Parameter Design Spaces.

Buckling contour mapping on cross-sections

Figure 9: Positive shear buckling factor contours, $k_{s,\infty} = N_s b^2 / \pi^2 D_{iso}$, for: (a) $\xi_{\Delta c} = 0.1$ and; (b) $\xi_{\Delta c} = 0.3$, representing Bending-Twisting coupled laminates.
Interpretation of Lamination Parameter Design Spaces.

Buckling contour mapping on cross-sections

Figure 10: **Negative shear buckling** factor contours, \( k_{s, \infty} = \frac{N_s b^2}{\pi^2 D_{iso}} \), for: (a) \( \frac{\Delta_c^D}{\Delta_c} = 0.1 \) and; (b) \( \frac{\Delta_c^D}{\Delta_c} = 0.3 \), representing **Bending-Twisting** coupled laminates.
Interpretation of Lamination Parameter Design Spaces.

Buckling factor contour mapping on surfaces of the design space

Figure 11: Lamination parameter design space surface contours for Compression buckling factor, $k_{x,\infty}$ ($= N_3 b^2/\pi^2 D_{iso}$), corresponding to 3rd angle orthographic projections of: (a) Rear (sloping) face with; (b) Left (sloping) face.
Interpretation of Lamination Parameter Design Spaces.

Buckling factor contour mapping on surfaces of the design space

Figure 11: Lamination parameter design space surface contours for Compression buckling factor, $k_{x,\infty}$ ($= N_3 b^2 / \pi^2 D_{Iso}$), corresponding to 3rd angle orthographic projections of: (c) Front (sloping) face and; (d) Right (sloping) face.
Interpretation of Lamination Parameter Design Spaces.

Buckling factor contour mapping on surfaces of the design space

Figure 12: Lamination parameter design space surface contours for Positive Shear buckling factor, \( k_{s,\infty} \) (\( = N_s b^2 / \pi^2 D_{iso} \)), corresponding to 3\(^{rd}\) angle orthographic projections of: (a) Rear (sloping) face with; (b) Left (sloping) face.
Interpretation of Lamination Parameter Design Spaces.

Buckling factor contour mapping on surfaces of the design space

Figure 12: Lamination parameter design space surface contours for Positive Shear buckling factor, \( k_{s,\infty} (= N_s b^2/\pi^2 D_{iso}) \), corresponding to 3\(^{rd}\) angle orthographic projections of: (c) **Front (sloping) face** and; (d) **Right (sloping) face**.

The **global optimum** shear buckling factor corresponds to \( k_{s,\infty} = 9.06 \) @ \((\xi_D, \xi_R, \xi_{\Delta c}) = (-0.18, -0.64, -0.82)\). A practical and near optimum design, \([+45/90/+45/90/+45/-45/0]_s\), leads to \( k_{s,\infty} = 8.98 \).
Closed form buckling equations for finite length plates in shear and compression.

An exact closed form solution, necessary to handle the vast number of database designs, can be used to assess the compression buckling strength:

\[ N_x = \pi^2 \left[ D_{xx} \left( \frac{m}{a} \right)^2 + 2 \left( D_{xy} + 2D_{ss} \right) \frac{1}{b^2} + D_{yy} \left( \frac{1}{b^4} \right) \left( \frac{a}{m} \right)^2 \right] \]  

(10)

…but only for uncoupled designs!

For orthotropic laminates, the following polynomial can be solved against the exact closed form buckling solution from equally spaced points across the lamination parameter design space:

\[ k_x = c_1 + c_2 \xi_D^2 + c_3 \xi_R^2 + c_4 \left( \xi_D^2 \right)^2 + c_5 \left( \xi_D^2 \right)^2 + c_6 \xi_D^2 \xi_R^2 + c_7 \left( \xi_R^3 \right)^3 + c_8 \left( \xi_D^3 \right)^3 + c_9 \xi_D^2 \left( \xi_D^2 \right)^2 \]

\[ + c_{10} \left( \xi_D^2 \right)^2 \xi_D^2 + c_{11} \left( \xi_D^2 \right)^4 + c_{12} \left( \xi_R^4 \right)^4 + c_{13} \xi_D^2 \left( \xi_R^3 \right)^3 + c_{14} \left( \xi_R^2 \right)^2 \left( \xi_D^2 \right)^2 + c_{15} \left( \xi_D^3 \right)^3 \xi_R^2 \]  

(11)

In this case, \( k_x \) is defined by:

\[ k_x = \frac{N_x b^2}{\pi^2 D_{iso}} \]  

(12)

By contrast to the infinite plate results investigated previously, mode changes now complicate the contour maps of finite length plates - no longer continuous across the laminate design space.

Stability of Structures 15th Symposium, Zakopane, 17-21 September, 201
Interpretation of Lamination Parameter Design Spaces.

Buckling contour mapping on cross-sections

Figure 13: Compression buckling contours of uncoupled laminates (i.e. $\xi_{\Delta c} = 0.0$), $k_x (= N_x b^2 / \pi^2 D_{iso})$, for: (a) $a/b = 1.0$; (b) $a/b = 1.5$ and; (c) $a/b = 2.0$.

Parabolic bounds\(^4\), shown in (a) are applicable to non-standard ply angle combinations $\pm 0^\circ$, $0^\circ$ and $90^\circ$.


$Stability$ of $Structures$ $15^{th}$ $Symposium$, Zakopane, 17-21 September, 201
Interpretation of through Garland curves.

Figure 14: Garland curves for $\xi_{\Delta c} = 0$ (solid lines) and 0.5 (broken lines).
Interpretation of Lamination Parameter Design Spaces.

Buckling contour mapping on cross-sections

Figure 15: Compression buckling contours of Bending-Twisting coupled laminates (i.e. $\xi^D = 0.5$), $k_x$ ($= N_x b^2/\pi^2 D_{Iso}$), for: (a) $a/b = 1.0$; (b) $a/b = 1.5$ and (c) $a/b = 2.0$. 

Stability of Structures 15th Symposium, Zakopane, 17-21 September, 201
Interpretation of Lamination Parameter Design Spaces.

Buckling factor contour mapping on surfaces of the design space

![Figure 16: Lamination parameter design space surface contours for Compression buckling factor, \( k_x \), \( (= N_x b^2 / \pi^2 D_{iso}) \), with aspect ratio \( a/b = 1.5 \), corresponding to 3\textsuperscript{rd} angle orthographic projections of: (a) Rear (sloping) face with; (b) Left (sloping) face.]
Interpretation of Lamination Parameter Design Spaces.

Buckling factor contour mapping on surfaces of the design space

Figure 16: Lamination parameter design space surface contours for Compression buckling factor, \( k_x \), \( = N_x b^2 / \pi^2 D_{iso} \), with aspect ratio \( a/b = 1.5 \), corresponding to 3rd angle orthographic projections of: (c) Front (sloping) face and; (d) Right (sloping) face.
Interpretation of Lamination Parameter Design Spaces.

Buckling factor contour mapping on surfaces of the design space

Figure 17: Lamination parameter design space surface contours for Positive Shear buckling factor, $k_s$, (=$N_s b^2/\pi^2D_{iso}$), with aspect ratio $a/b = 2.0$, corresponding to 3rd angle orthographic projections of: (a) Rear (sloping) face with; (b) Left (sloping) face.
Interpretation of Lamination Parameter Design Spaces.

Buckling factor contour mapping on surfaces of the design space

Figure 17: Lamination parameter design space surface contours for Positive Shear buckling factor, $k_s$, ($= N_s b^2 / \pi^2 D_{iso}$), with aspect ratio $a/b = 2.0$, corresponding to 3$^{rd}$ angle orthographic projections of: (c) Front (sloping) face and; (d) Right (sloping) face.

For $a/b = 1.0$ and $1.5$, the global optimum is found at the bottom of feasible region $(\xi_D^\Delta, \xi_D^R, \xi_D^{\Delta c}) = (0.0, -1.0, 1.0)$ as would be expect for compression buckling, but for higher aspect ratios, $a/b = 2.0$, $k_s = 11.15$ @ $(\xi_D^\Delta, \xi_D^R, \xi_D^{\Delta c}) = (-0.05, -0.9, 0.95)$
Concluding Remarks

- The reduced design space, resulting from the application of the 10% rule, has been shown to virtually match the application of the common design constraint of limiting the number of contiguous plies, i.e. adjacent plies with the same orientation, to a maximum of 3.

No significant impact has been observed on the size of the lamination design space for bending stiffness as a result of the combined effect of the 10% rule and limiting the number of contiguous plies to a maximum of 3.

- New insights have been given for compression and shear buckling strength for infinitely long plates, through the superposition of contour maps onto the lamination parameter design space for composite laminates with Bending-Twisting coupling.

- The contour maps demonstrate the added complexity associated with laminate selection from a design space in which compression buckling strength is a non-continuous function. This is due to mode changes that are dependent both on bending stiffness properties (or lamination parameter coordinate) and plate aspect ratio.

- The contour maps demonstrate the degrading effect on the buckling strength resulting from Bending-Twisting coupling by simple inspection. Inspection also reveals the non-intuitive location for optimum shear buckling.
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Additional interpretations through Garland curves for continuous vs finite length plates.

Compression buckling mode shape comparisons for the infinitely long plate with simply supported edges, corresponding to Pseudo Quasi-Homogeneous Quasi-Isotropic laminates with: (a) \( \xi_{11} = 0.0 \) and (b) \( \xi_{11} = 0.5 \). Plate width \( b \) intervals are indicated along the panel edge.