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A Meta-analysis of coal mining induced subsidence data and implications for their use in the carbon industry

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Abstract
Many empirical subsidence estimation tools exist worldwide but are designed and calibrated for specific coalfields. This paper presents an universal tool for the estimation of maximum subsidence \( S_{\text{Max}} \). The subsidence tool is based on pooling and meta-analysis of empirical data from a number of different countries and coalfields. The key factors influencing \( S_{\text{Max}} \) are the void dimensions and the mechanical competency of the overburden. These factors are used to estimate subsidence using the empirical equation \( S_{\text{Max}} = \frac{c}{1+10^{(-a((W/D)-b))}}*m \), where \( W \) is the width of the void, \( D \) the depth, \( m \) the effective void thickness, and \( a, b, c \) are parameters related to the mechanical competency of the overburden. This universal empirical method was validated against historical data from United Kingdom and Australia. The method also provided \( S_{\text{Max}} \) estimations for underground coal gasification (UCG) projects, that were inline with those from numerical modelling under certain conditions. This tool would likely be most useful when investigating areas, where there are little or no historical data of subsidence and mining. Such areas are most likely to be targeted by UCG schemes.

Keywords
Subidence, UCG, estimation, meta-analysis, data.
1.0 Introduction

The challenge of subsidence

Areas with a legacy of coal mining are very familiar with the damaging effects of surface subsidence. Subsidence can cause damage to utilities (e.g. Holla 1988), structures, water bodies (Booth 2002, and Dumbleton 2002), and agricultural land (Darmody et al. 1989). Surface subsidence is caused by the eventual collapse of the roof strata over a mined volume (Figure 1); filling the mined area with collapsed material. The beds overlying the collapse flex resulting in a zone of net extension where the beds sag and crack. Higher, there is a zone of net compression (colloquially referred to as the pressure arch), which is typically overlain by another zone of net extension towards the surface (Booth 2002 and Dumbleton 2002).

Figure 1. Schematic cross-section showing the impacts of void collapse around a gasification borehole, forming goaf and overlying zones of extensional and compressional deformation. The angle of draw delimits the outer edge of the zone of strata deformation and the inflection angle the locations of half maximum subsidence on the surface. Figure edited after Younger 2011. $S_{max}$ is defined as the point of maximum surface subsidence, $W$ as the width of the excavated zone, $D$ as the depth of the excavated zone.

Increasing sophistication in the planning and execution of coal mining post second world war resulted in the development of empirical methods to predict (e.g. Marr 1957) and mitigate the effects of subsidence (Marr 1965 and Orchard 1964). The national coal board of the UK (NCB 1975) is the earliest attempt at a standard prediction method for a specific coal region. NCB (1975) was later followed by empirical methods in Australia (Holla 1987 and 2000), USA (Dunrud 1984), India (Sanexa et al. 1989), and many other locations. These empirical methods were possible in areas with a history of extensive mining providing sufficient data from which to create robust correlations.
between mining parameters and the resulting subsidence; and were thus location specific. For example, the UK techniques were applied directly to Australia but often overestimated subsidence (Holla 2000), due to higher mechanical competency of the overburden of Australian coal mines compared to British.

Although developed large economies, such as U.S.A. and China, are increasingly meeting their energy needs from natural gas and renewables rather than coal (Ren et al. 201), demand for coal and hence coal mining remains resilient. Beyond conventional coal mining, underground coal gasification (UCG) has long been suggested as an answer to safety issues (Liu et al. 2015), how to access unmineable stranded assets, and as a way to directly couple coal exploitation with carbon capture (Younger 2011). The process of UCG leaves voids underground of similar expanse as shortwall mining, resulting in surface subsidence (Derbin et al. 2015). Potash mining can also result in voids of similar scale to coal mining, i.e. tens of metres wide and hundreds of metres long (Chrzanowski et al. 1997). The UCG and potash examples demonstrate that even if conventional mining declines in the future, the is continued need to better characterise and de-risk estimation of surface subsidence.

The ratio between the width and depth (W:D) of a void left after the coal had been extracted, called a panel, is used as a key parameter for predicting maximum subsidence in many empirical prediction tools, including the UK (NCB 1975), Australia (Holla 1987 and 2000), and U.S.A. (Karmis et al. 1984). These empirical methods converged on a sigmoidal relationship between maximum subsidence and W:D, which were presented graphically by NCB (1975) and Holla (2000). Alternatively Karmis et al. (1987) presented an upper bound of maximum subsidence with the following equation.

\[ S_{\text{Max}} = [0.61 - (0.05/(W/h-0.07))] \times m \]  

**Equation 1**

These sigmoidal relationships offer the conceptual model, that below W:D of 0.5 there is initially little increase in subsidence associated with wider panel. Then as W:D ratio increases there is a significant increase in subsidence (usually between 0.5 and 1.0 but varies by coal basin or region), these W:D ratios are know as sub-critical extraction widths. With increasing width, there comes a point where subsidence maxes out and no longer increases, known as the critical extraction width and typically found in W:D values of 1.5 to 2.0. Increases in W:D ratio beyond the critical width (known as supercritical extraction) does not results in higher magnitudes of subsidence. The magnitude of subsidence at this critical width varies significantly between coal basins, resulting in an \( S_{\text{Max}} \) of 90% of extraction thickness in the UK compared with only around 60% in Australia.

Numerical modelling of coal mining related subsidence had used a variety of different approaches, such as elastic or rigid block (O’Connor and Dowding 1990, Choi and Coulthard 1990). As computational power and experience progressed numerical models showed good agreement with empirical data and models (Coulthard 1995, Alejano et al. 1999) but with specific limitations for each approach. Numerical modelling supports risk assessment for site specific issues such as effects of faults (Otto et al. 2016). However numerical models can be time consuming to set up and require sufficient data to properly calibrate material properties. The advantage of empirical models remains in initial characterisation and subsidence risk assessment, as they are able to quickly and simply provide subsidence estimates. Influence function methods can offer prediction of subsidence curves and zone of influence on the surface (Karmis et al. 1990) and can be tuned to site specific conditions (Ren and Buckeridge 2010). Probability-integral method has been used for mining subsidence
estimations (Jianjun et al. 2012, Zhang et al. 2009). But the influence function methods require known subsidence data such as $S_{\text{Max}}$ to create the subsidence influence profiles (Karmis et al. 1990), and probability-integral methods are also dependent on reliability of input parameters (Zhang et al. 2009). In the U.S.A., Karmis et al. 1987 proposed using a factor related to the competency of the overburden, to allow tuning of empirical predictions to different coal basins. However, despite the wealth of data available on a global basis, such methods have mainly been used for subsidence prediction at the region they were developed. A subsidence prediction method that can be applied without dependence on the region of interest, would be of particular importance for potential future coal exploitation techniques, such as underground coal gasification, where lack of experience means there is not sufficient data for any area, around which an empirical subsidence method could be based.

Here, we investigate the development and formation of a universal empirical subsidence ($S_{\text{Max}}$) prediction model based on the collection of representative regional data across the world and using a novel analytical approach. In particular, a meta-analytical approach is used for the initial treatment of the collected data, followed by a statistical optimal fitting approach to extract useful trends between the interconnected parameters and allow for quantification of the subsidence predictions.

2.0 Methods and Results

2.1 Data Pooling and Meta-Analysis

Meta-analysis techniques have been used to compare collated subsidence studies and data, such techniques were originally developed and widely used in fields such as medical research and social science (Schmidt and Hunter 2014). In these disciplines, meta analysis techniques systematic protocols to compare tens to hundreds of studies and data, e.g. Biondi-Zoccai et al. (2006) began by investigating 612 studies but eventually used just six for use in a meta-analysis investigating the effect of aspirin use for risk of coronary artery disease. Another need was for systematic merging of studies with thousands of data, e.g. Bischoff-Ferrari et al. (2005) investigating 19,114 data on bone fracture prevention with vitamin D supplement. In the geosciences, attempts at meta-analysis have been rarely employed but for some issues such as salt marshes (Shephard et al. 2011) and soil carbon storage (Guo and Gifford 2002) or high level comparisons of life cycle analysis of carbon capture and storage (Schreiber et al. 2012).

To implement this method, an extended literature search for regional subsidence data, with a wide geographical span, has been conducted. 59 publications were identified which could have data, of which 23 publications contained some subsidence data (these publications are listed in supporting material). Publications were not used if they did not contain the raw data which would be required for further analysis. Orchard and Allen (1970), and Aynsley and Hewitt (1960), and Orchard (1964) presented tables from which data were extracted, however the others studies only provided the data in graphs which had to be extracted digitally.

Factors which influence $S_{\text{Max}}$ were already combined on the axes of many of these graphs, e.g. the ratio between panel depth and width was shown but not the separate information (known as width/depth ratio or $W:D$ for the remainder of this paper). Therefore, it was not clear whether the data were for a shortwall panel that is shallow, or a deep longwall panel, as such factors relating to
effect on subsidence from a similar W:D but different depths are not investigated in this study, but are discussed in section 3.1 and 3.2.

In addition to individual studies some countries have produced national or regional specific subsidence prediction manuals (NCB 1975, Holla 1987). These manuals pooled data from a country or particular coal basin and produced empirical predictive tools based on those data. Where present these manuals provided information on the approach to subsidence by different countries and sometimes contained subsidence data. The UK manual (NCB 1975) does not contain any data but referenced literature, of which some contained extractable data.

Despite these issues, fourteen different studies contained comparable information about the effect of W:D on $S_{\text{Max}}$. Five of these studies separated depth and width data, allowing direct comparison of the effect of depth. Only two studies (both from U.S.A.) had information about the influence of the mechanical properties of the overburden on subsidence, however mechanical property information could still be gained from geological reports into the coal basins where other subsidence data existed.

Correlation coefficients were calculated for the effect on subsidence of W:D for each of the fourteen studies containing suitable data and the five studies for depth, including 0.95 confidence intervals. The correlation coefficients were then plotted alongside each other to allow comparison between the studies (presented in section 3.2). These correlation coefficients assume a linear relationship between panel geometry or depth and subsidence. The relationship between panel geometry and subsidence is generally considered to follow a sigmoidal curve (e.g. Holla 2000). However, the linear relationship assumption will still indicate positive and negative correlations and strength of relationship.

The data were not actively filtered as common meta-analysis methods suggest avoiding filtering unless necessary as it adds author bias to the data (Schmidt and Hunter 2014). Typically any bias is already balanced by using a range of data from different studies and authors. The exception is for the data from India (Saxena et al. 1990) where around half the data were stowed. Such a high proportion of data from stowed panels significantly affected the accuracy of the subsidence correlation.

Figure 2 shows the Correlation Coefficient (Pearson’s R-value) for each of the fourteen studies with suitable data. Ogden and Orchard 1959, Holla 2000, and Aynsley and Hewitt 1961 show the strongest positive correlations, and also have relatively small CI. Seven other studies show less strong positive correlation but additionally have CI which only range within the positive correlation. Six have a positive correlation coefficient, but despite this show a small portion of the confidence intervals in the negative range. Only Orchard (1964) shows a negative correlation between W:D and subsidence. However approximately one third of the confidence interval for this study lies within the positive correlation region. The data in Orchard (1964) are all of very low W:D. Overall W:D has a positive correlation with subsidence.
Correlations between the panel width/depth ratio ($W:D$) and normalised subsidence for each of the reviewed studies. Correlation coefficient was calculated assuming a linear relationship between $W/h$ and $S\%$. A value towards 1 indicates a strong positive correlation, a value towards -1 indicates a strong correlation, and a value around 0 indicates a weak or non-existent correlation. Error bars show the 0.95 confidence intervals. Asterix (*) indicates which studies had tables from which data could be directly extracted.

The correlation coefficients between depth of void ($D$) and magnitude of subsidence are shown on Figure 3. Of the five studies, two show a positive correlation and three show a negative correlation. One of those showing a negative correlation (Aynsley and Hewitt 1961) shows a large portion of its CI is within the positive region. One of the positive correlation studies (Orchard and Allen 1970) has a small portion of its CI range in the negative region. Therefore, although individual studies indicate either positive or negative correlation, there is no overall trend.
Correlations between extracted panel depth and normalised subsidence from the reviewed publications. Correlation coefficient was calculated assuming a linear relationship between depth and $S\%$. Error bars on data show the 0.95 confidence intervals.

2.3 Curve fitting
The meta-analysis was used to determine the general influences that key factors have on subsidence. This next step, is to investigate the commonalities of the subsidence data and trends between the different countries and coal regions. The data from the fourteen studies showing the ratio between panel width and depth ($W:D$) influence on subsidence are presented in Figure 4. There are several consistencies between different countries and studies as well as important differences. Many data, in Figure 4, have both low $W:D$ and $S_{\text{Max}}(\%)$ and thus do not elucidate overall relationship trends. But generally, the relationship between $W:D$ and subsidence follow sigmoidal curves in a number of different geological settings, i.e. Australia (Holla 2000), UK (Aynsley and Hewitt 1961), and USA (Gray and Bruhn 1984). The sigmoidal curves suggest an initial low increase in $S_{\text{Max}}(\%)$ with increasing $W:D$ before a rise, which occurs at a $W:D$ value of 0.4 for Holla 2000 but much lower for the UK studies. Then with increasing $W:D$ subsidence continues to increase before levelling off and not showing any more increase of subsidence for increasing $W:D$. For the UK this levelling off is around 80-90% of extracted thickness (King and Whetton 1958 and Ogden and Orchard 1959), but for Australian studies it is around 60% (Holla 2000 and Holla 1987).
Figure 4. Pooled data showing the influence of extraction panel geometry on subsidence. X-axis shows the ratio between the width and the depth of the extracted panel (W:D). Y-axis shows subsidence normalised to the thickness of extracted material. Data split between regions for clarity and shown by icons. Best fit curves use the formula \( \frac{c}{1+10^{a((W:D)-b)}} \), where “a” defines steepness, “b” the mid-point of curve on x-axis, “c” the limit of the curve.

Sigmoidal curves were initially fitted to the data from the fourteen studies containing information about the relationship between W:D and subsidence, using the following equation.

\[
S_{Max} \% = \frac{c}{1+10^{a((W:D)-b)}}
\]  

**Equation 2**

where \( S_{Max} \% \) is the ratio of the maximum subsidence to extraction thickness expressed as a percentage; \( W \) is the excavated panel width, \( D \) is the depth to the excavated panel, \( a, b, \) and \( c \) are parameters, which define the shape of the sigmoidal curve.

Although there are several options for choosing a sigmoidal type of curve to fit the data available, it was found that the three parameter sigmoidal model (equation 2) not only performed better, but also provided a more intuitive and versatile parameterisation. This is because the parameters are clearly responsible for specific characteristics of the curves: with \( a \) governing the steepness of the curve, \( b \) controlling the mid-point of the curve on the x-axis, and \( c \) is directly linked to the upper limit of the curve. Thus one or two parameter sigmoidal models were avoided. Nonlinear model fitting with the available sparse data was performed, via the use of Mathematica (Wolfram 1991), to
produce best fits of the sigmoidal curves (parameterised by equation 2) to the respective data, for each study.

The assigned parameters to the sigmoidal curve for each study are shown in Table 1. The data in some studies were concentrated at either the lower or higher values of panel’s with to depth ratio (W:D), so a best fit curve did not provide confident estimates of all three of the parameters (a, b, or c). Five studies had sufficient data spread that parameter a could be confidently assigned to the sigmoidal curve (Aynsley and Hewitt, Holla 1987, Holla 2000, King and Whetton 1958, and Ogden and Orchard 1959). The same five studies also reliably constrained parameter b, due to both parameters being determined in the same section of the sigmoidal curve. Parameter c was reliably constrained by eight studies in total (Dunrud 1984, Gray and Bruhn 1984, Holla 1987, Holla 2000, Karmis 1984a, Karmis 1984b, King and Whetton 1958, and Ogden and Orchard 1959).

Percentage hard rock (HR%) has previously been used to estimate the resistance of the overburden to subsidence (e.g. Karmis et al. 1984). HR% is the ratio of competent layers (typically and simply defined as sandstone and limestone) to incompetent layers (e.g. shale and coal), however no universal definition exists. Two studies used in the curve fitting already had associated HR% values (Karmis et al. 1984, Dunrud 1984). For studies without existing HR%, then estimates were made using stratigraphic columns of the respective coal basins. A lower HR% was applied to the UK studies, due to the extensive faulting of UK coal basins compared to the others. Table 1. Parameters for the best fit curves shown in Figure 4. N is the number of data from each study that the best fit curve is based on. HR% is the estimated percentage of hard (mechanically competent rock) above the seams for each study, (*) represents those parameters where there was not enough confidence in the data to use during the analysis.

<table>
<thead>
<tr>
<th>Study</th>
<th>Country</th>
<th>N</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>HR%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aynsley and Hewitt 1960</td>
<td>UK</td>
<td>40</td>
<td>2.4</td>
<td>0.58</td>
<td>82.5*</td>
<td>20</td>
</tr>
<tr>
<td>Dunrud 1984</td>
<td>USA</td>
<td>13</td>
<td>6.4*</td>
<td>0.61*</td>
<td>63.1</td>
<td>n/a</td>
</tr>
<tr>
<td>Gray and Bruhn 1984</td>
<td>USA</td>
<td>16</td>
<td>3*</td>
<td>0.5*</td>
<td>60</td>
<td>35</td>
</tr>
<tr>
<td>Holla 1987</td>
<td>Australia</td>
<td>40</td>
<td>2.2</td>
<td>1.01</td>
<td>49</td>
<td>53</td>
</tr>
<tr>
<td>Holla 2000</td>
<td>Australia</td>
<td>34</td>
<td>4</td>
<td>0.62</td>
<td>56.0</td>
<td>63</td>
</tr>
<tr>
<td>Karmis 1984a</td>
<td>USA</td>
<td>18</td>
<td>3*</td>
<td>0.4*</td>
<td>69</td>
<td>21</td>
</tr>
<tr>
<td>Karmis 1984b</td>
<td>USA</td>
<td>18</td>
<td>7.7*</td>
<td>0.41*</td>
<td>55</td>
<td>41</td>
</tr>
<tr>
<td>King and Whetton 1958</td>
<td>UK</td>
<td>19</td>
<td>2.1</td>
<td>0.53</td>
<td>84.8</td>
<td>20</td>
</tr>
<tr>
<td>Ogden and Orchard 1959</td>
<td>UK</td>
<td>27</td>
<td>1.9</td>
<td>0.56</td>
<td>90.3</td>
<td>20</td>
</tr>
<tr>
<td>Saxena and Singh 1989</td>
<td>India</td>
<td>12</td>
<td>4*</td>
<td>1*</td>
<td>40*</td>
<td>60</td>
</tr>
<tr>
<td>Waddington and Kay 1995</td>
<td>Australia</td>
<td>10</td>
<td>3*</td>
<td>0.75*</td>
<td>60*</td>
<td>63</td>
</tr>
<tr>
<td>Wardell and Webster 1957</td>
<td>UK</td>
<td>4</td>
<td>2.3*</td>
<td>0.34*</td>
<td>45.5*</td>
<td>20</td>
</tr>
</tbody>
</table>
The parameters \( a, b \) and \( c \), which had been reliably and satisfactorily constrained, were then used to investigate the influence of the mechanical properties of the overburden on subsidence. A metric for the mechanical competency of the overburden (\( HR\% \)) was developed for each study area, following the method of Karmis (1984), where \( HR\% \) is the percentage of mechanically competent layers (defined as sandstone and limestone) to the overburden depth (including other layers as well, such as shale and coal). Two studies used in the curve fitting already had associated \( HR\% \) values (Karmis et al. 1984, Dunrud 1984). For studies without existing \( HR\% \), then estimates were made using stratigraphic columns of the respective coal basins. A lower \( HR\% \) was applied to the UK studies, due to the extensive faulting of UK coal basins compared to other countries. Karmis (1984) presented subsidence data alongside \( HR\% \) for each of the data. For use in this study we have grouped each of the data, from Karmis (1984), into two groups: the first is those data with \( HR\% \) less than 30 hereafter labelled as “Karmis 1984a”, and the second group is those data with \( HR\% \) of more than 30, hereafter labelled as “Karmis 1984b”.

The estimates for \( HR\% \) (Table 1) range from a low of 20% to the highest of 63%. The parameters, \( a \), \( b \), and \( c \) (Table 1) with sufficient confidence were plotted against \( HR\% \) (Figure 5). For parameter \( a \), a linear positive trend line could be plotted within the confidence intervals of all five studies (Figure 5a) and is shown in equation 3.

\[
a = 1.47 + 0.03*HR \quad \text{Equation 3}
\]

This implies that equation 3, which gives the best fit relationship between parameters \( a \) and \( HR\% \) (Figure 5a), is a good predictor for modelling purposes. A linear trend could be also fitted for parameter \( b \) using Mathematica, but this does not go through the CI ranges for two out of five of the data. Therefore, there is no observable relationship between parameters \( b \) and \( HR\% \). Actually, it could be assumed with good confidence for most of the available data, that parameter \( b \) does not vary with \( HR\% \) (e.g. could be kept constant at a value of \( b=0.55 \), Figure 5b). Parameter \( c \) has a negative exponential fit with \( HR\% \). The data indicate a rapid decrease in parameter \( c \) with only a small increase in \( HR\% \) above 20%, but the trends reaches a minimum at a value of parameter \( c \) of approximately 50.

\[
c = 50.03 + 168.45 \ e^{-0.10\left(-4.37 + HR\right)} \quad \text{Equation 4}
\]

Equation 4 demonstrates the dependence of parameter \( c \) to the percentage of hard rock (\( HR\% \)) as shown on Figure 5-parameter c.
Figure 5. Plots of the relationship between HR% and parameters $a$, $b$, and $c$ (from equation 2) used to produce the best fit curves modelling $S_{\text{Max}}\%$, shown on Figure 4 and Table 1. The solid lines show the resulting...
regression line, enveloped by the upper and lower CI of the regression lines calculated using upper and lower CIs of the data.

3.0 Discussion

3.1 Attempt at using relationships to create empirical model for predicting maximum subsidence

In modern mining and underground coal gasification test sites, subsidence and other effects can be investigated using numerical modelling (e.g. Otto et al. 2016 and Li et al. 2016). These 3D numerical models allow useful investigation of different scenarios, informing the later operations. However the models can be time consuming to set up and computationally expensive. A simple model based on empirical relationships could be of value by offering a method of quickly investigating the magnitude of effect that key design parameters have over subsidence.

In this section we explore how we can use the identified relationships, to make a first attempt at a simple empirical tool to estimate maximum subsidence from a proposed development. The data are currently inadequate to tune a precise empirical prediction model of similar accuracy to those produced for specific regions (e.g. Holla 2000 or NCB 1975). Therefore the feasibility of an empirical subsidence estimation method will be explored here (then validated in section 4.3) which could act as a foundation for a predictive model. Equation 2 models a sigmoidal relationship between W:D and subsidence based on data from twelve different studies (Figure 4). The sigmoidal fit of equation 2 is determined by three parameters (a, b, and c). Two of these parameters (a and c) showed some correlation with the percentage of hard rock, HR%, of the overburden (Figure 5, equations 3 and 4). However only five studies had sufficient range of data to compare HR% with the fitted value of a and b, whereas c was compared with eight studies. Although parameter b showed no correlation with HR%, all five studies had b values between 0.5 and 1 which potentially describes upper and lower bounds for parameter b.

The W:D ratio does not differentiate between a shallow shortwall panel and a deep longwall panel. This study could not investigate differences that these two scenarios may have on subsidence, as the data the analysis were based on do not differentiate between shallow and deep panels of the same W:D ratio. Further work, looking specifically at this issue, could better constrain parameter b. Note, that section 3.2. of the discussion compares two examples with the same W:D but different panel depths.

Equation 2 is plotted graphically in Figure 6, the sigmoidal fit represents the positive correlation between panel W:D and maximum subsidence that was identified in section 3. Below a W:D value of 0.5, there is only small increase in subsidence with increasing W:D. This initial small increase in subsidence represents widths which are small enough for the overburden to effectively bridge across and not fully collapse. There is a steep gradient between W:D of between 0.5 and 1.5. This steep gradient represents void approaching critical width where the roof strata can no longer support the weight of the overburden leading to collapse. Beyond W:D of 1.5, maximum subsidence only shows little increases for higher W:D. This flattening out of the graphs represents voids that are of super-critical width (Figure 6) where any increase in width does not increase maximum subsidence.
Figure 6. Graphical representation of equation 1, showing the relationship between void width to depth ratio (W:D) and subsidence. Y-axis is normalised for thickness of void, so the predicted subsidence would be a percentage of the thickness as given by the y-axis. These graphs are based on equation 2 but for a given value of parameter $b$. Increasing the Hard rock % (HR%) decreases the predicted subsidence for a given width/depth ratio (W:D).

Figure 6 shows seven projections of equation 2, for different values of HR%, which in turn modifies parameters $a$ and $c$, according to equations 3 and 4, respectively. For any given value of W:D ratio, a higher HR% reduces the estimated maximum subsidence. For example, a void with a W:D value of 2.5 and HR% of 60% would have half the estimated subsidence compared with an HR% of 17%. Furthermore, the effect of HR% is more pronounced at lower values of panel W:D. For example, a void with W:D of 0.4 has approximately a quarter of predicted subsidence when HR% is 60% (estimated max subsidence of 11%) than when HR% is 17% (estimates maximum subsidence of 2.4%).

It should be noted, that HR% is effectively averages the mechanical properties of the overlying strata. Local considerations such as a particularly competent rock immediately overlying the excavated panel could lead to different subsidence relationships than if there were incompetent rock in the roof zone; both scenarios could have identical HR% values. Further numerical modelling may be required to thoroughly explore this issue at specific sites, e.g., Otto et al. (2016) discussed in section 3.2.

For UCG, the void thickness could not actually be measured, but a representative thickness could be estimated using cavity growth modelling (e.g., Luo et al. 2009) or parameterised with a characteristic feature. For example, Li et al. (2015) estimate it to be 0.55 of coal seam thickness. Thus the value of effective void thickness is introduced by means of modifying equation 2 as follows:

$$S_{\text{Max}} = \frac{c}{(1+10^{a((W:D)-b)})} \times \text{effective void thickness} \quad \text{Equation 5}$$
Where $S_{\text{Max}}$ is maximum subsidence, $c$ is from equation 3, $a$ is from equation 2, and $b$ is constrained between 0.5 and 1.0. Equation 5 shows a proposed model for the prediction of the maximum subsidence, based on experience from longwall conventional coal mining subsidence. Upper and lower bound estimates for maximum subsidence can be made using the upper and lower bound trend lines from Figure 5.

With the inclusion of void thickness, then the output of equation 5 is an estimate of maximum subsidence. This subsidence estimate is based on the following four factors: void depth ($D$), void width ($W$), hard rock percentage ($HR\%$), and thickness of the void. All of these factors would be available in the feasibility stage of any mining or UCG project. Equation 5 could provide the opportunity to quickly test the effect of various scenarios on surface subsidence, prior to developing required 3D numerical modelling.

The method discussed in this paper has not sought to explore different extraction types, longwall vs shortwall or room and pillar (for which cases data availability is even more limited), thus almost all of the data presented are from longwall extraction. Holla (2000) presents data differentiated between longwall and non-longwall extraction and found a significant difference between the equivalent of parameter $b$ (equation 5), with the non-longwall parameter $b$ lower. However Holla (2000) does not state which extraction types were considered non-longwall. A further study would need to specifically seek out and collect data from different mining techniques in order to compare the effect on the proposed subsidence parameters in equation 5.

### 3.2 Validation of proposed empirical subsidence estimation method

To test the validity of the proposed empirical estimation method against what could be the future of coal exploitation (i.e. UCG), the results of equation 4 are first compared with data from two of the studies used with vastly different $HR\%$ (i.e. Aynsley and Hewitt 1960 and Holla 2000), then equation 4 is compared with results from two numerical 3D mechanical models. One a proposed UCG site in Poland (Otto et al. 2016), the other a test UCG site in China (Li et al. 2016). It would be preferable to test the empirical model against past measurements of subsidence caused by UCG operations. Derbin et al. 2015 present subsidence data from Soviet era UCG operations, however most of these data were from very shallow mines (less than 100m deep) and operated from 1930-1960 so unlikely to be representative of the deeper modern UCG designs. There are important differences between UCG and conventional longwall mining. The UCG voids are much smaller than longwall, UCG voids are expected to have widths of around 30m (Couch 2009). Additionally the rocks surrounding UCG voids will be affected by the high temperatures within the reactor zone affecting mechanical properties (Hattema et al. 1998), but these affects are so near-field to the reactor that they are unlikely to influence overall subsidence behaviour (Otto and Kempka 2015).

Figure 7 shows a comparison between estimated subsidence (from equation 5) and the data from two studies, to ensure the empirical model provides estimations of reasonable fit to its underlying data. An $HR\%$ of 20% was used for Aynsley and Hewitt (1960), and 63% for Holla (2000), to calculate the values of parameters $a$ and $c$. Parameter $b$ had to be selected from table 1.

The subsidence estimations from equation 5 are closely aligned with the best fit lines (Figure 7), suggesting that the empirical model can replicate subsidence data and that the equations 3 and 4 provide good estimations for parameters $a$ and $c$. However, the subsidence estimation is sensitive to parameter $b$, which could not yet be constrained to $HR\%$. 

Otto et al. (2016) developed a thermo-mechanical 3D numerical model of the geology and potential gasification chambers for a proposed UCG site in Poland. This model investigated parameters such as, stress on pillars, thermal response of rock, but also it provided an estimate of $S_{\text{Max}}$ (of 5.5cm). Li et al. (2016) used FLAC3D to model a UCG experiment consisting of four panels. Li et al. (2016) recorded $S_{\text{Max}}$ as 0.7cm after gasification of a single chamber within the numerical model; after four adjacent panels the total subsidence was modelled as 2.99cm. Li et al. (2016) reported that modelled subsidence was similar to measured subsidence but didn’t provide the actual figure for surface subsidence.

The UCG design parameters (Table 2) from Otto et al. (2016) and Li et al. (2016) were then used as inputs for Equation 5. The estimated values of subsidence from equation 5 are then shown in Table 3. Since parameter $b$ is not constrained a range of values were used between 0.5 and 1 to compare what affect changing values of this parameter has on the estimated value of subsidence.
Table 2. Key subsidence factors used in numerical models by Otto et al. 2016 and Li et al. 2015.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Void width (m)</td>
<td>30</td>
<td>16</td>
</tr>
<tr>
<td>Void depth (m)</td>
<td>488</td>
<td>266</td>
</tr>
<tr>
<td>Width/depth ratio</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>Effective void thickness</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>Hard rock %</td>
<td>36%</td>
<td>53%</td>
</tr>
<tr>
<td>3D numerical predicted max</td>
<td></td>
<td></td>
</tr>
<tr>
<td>surface subsidence (cm)</td>
<td>5.5</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Both the width and depth of the gasification chambers in Li et al. (2016) are approximately half that of Otto et al. 2016. These dimensions result in the two studies having the same value of W:D, despite the panels being different depths. These two studies also provide a useful example of the issue of W:D not differentiating between shallow and deep panels; raised in the introduction and section 3.1. However Otto et al. (2016) has a HR% of 36%, which is lower than the 53% of Li et al. (2016).

Table 3. Values of subsidence predicted by equation 5, when inputs are used from Otto et al. 2016 and Li et al. 2015. Values presented in bold font, are those where numerical model predicted subsidence agrees with the empirical model presented in this paper. Otto et al. (2016) predicted maximum surface subsidence as 5.5cm, Li et al. (2015) predicted maximum surface subsidence as 0.7cm.

<table>
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<tr>
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<tbody>
<tr>
<td></td>
<td>Subsidence lower bound estimate (cm)</td>
<td>Subsidence best fit estimate (cm)</td>
</tr>
<tr>
<td>0.55</td>
<td>16</td>
<td>24</td>
</tr>
<tr>
<td>0.60</td>
<td>12</td>
<td>18</td>
</tr>
<tr>
<td>0.65</td>
<td>9.1</td>
<td>14</td>
</tr>
<tr>
<td>0.70</td>
<td>6.3</td>
<td>10</td>
</tr>
<tr>
<td>0.75</td>
<td>4.2</td>
<td>7.9</td>
</tr>
<tr>
<td>0.80</td>
<td>2.4</td>
<td>5.9</td>
</tr>
<tr>
<td>0.85</td>
<td>0.99</td>
<td>4.4</td>
</tr>
<tr>
<td>0.90</td>
<td>0</td>
<td>3.3</td>
</tr>
<tr>
<td>0.95</td>
<td>0</td>
<td>2.5</td>
</tr>
<tr>
<td>1.00</td>
<td>0</td>
<td>1.8</td>
</tr>
</tbody>
</table>

The estimates of maximum surface subsidence (from equation 5) lie between 0 and 24 cm when using the parameters from Otto et al. (2016), and between 0 and 4.8 cm using Li et al. (2016), as shown on table 3. There are significant changes in subsidence estimation when b is altered, from 24 cm when b is set at 0.55 and 1.8 cm when b is set at 1.00 for the parameters from Otto et al. (2016). This is also true for the parameters from Li et al. (2016), which estimates subsidence at 4.8 cm when b is 0.55 and 0.21 cm when b is at 1.00.

When b was set at 0.75 or greater the subsidence range estimated by equation 5 encompasses the subsidence prediction from both Li et al. (2016) and Otto et al. (2016). Whereas any value for
parameter $b$ under 0.7 overestimates the subsidence when compared with the results from the numerical modelling.

Figure 8 shows the effect of the parameters (from table 2) on the proposed subsidence estimation method, by showing a zoomed-in version of the graph in Figure 6. In the example shown on Figure 8 the maximum surface subsidence of Otto et al. (2016) and Li et al. (2016) are approximately 0.4% and 0.1% of the extracted thickness, respectively. The magnitude difference between the estimations from the empirical models is in line with those predicted by the numerical modelling studies. Otto et al. (2016) predicts a maximum surface subsidence of 5.5cm for 8m of extracted thickness, and Li et al. (2016) predicts a maximum surface subsidence of 0.7cm for 3m of extracted thickness. Once corrected for the difference between the extracted thicknesses (i.e. the 8m being significantly more than 3m leading to higher subsidence), the predicted $S_{\text{Max}}$ of Otto et al. (2016) is 2-3 times greater than that of Li et al. (2016).

![Figure 8](image)

**Figure 8.** Zoomed in section of graph in Figure 6, showing area of interest for comparison with UCG numerical modelling studies. Panel width/depth ratio ($W:D$) is now shown only between 0.00 and 0.20. Open and closed circles show locations of Otto et al. 2016 and Li et al. 2016 on the subsidence curve.

### 3.3 Utility of the developed model and modelling framework

The subsidence estimates of the simple empirical model (equation 5) fall within the ranges provided by the complicated site specific numerical modelling, for certain values of parameter $b$. Such a positive result suggests the empirical technique could be useful for quick approximate estimates of subsidence sensitivity to specific parameters at proposed UCG sites. The empirical technique cannot, act as a replacement for robust site specific subsidence modelling. However, its use is for a simple, low computational cost, method of quickly exploring the effect on subsidence from possible
expected UCG gasification chambers and local geology. In particular, it could be used as an initial site design tool to test the sensitivity of different design parameters (e.g. depth of seam) on surface subsidence. This would help regulators and UCG engineers gain quick scenarios for how to mitigate and manage subsidence during any UCG developments.

Despite the initial positive view when comparing the empirical model to the numerical predictions, the empirical estimation tool still needs improved. Three parameters (a, b, and c) define the shape of the empirical relationships between gasification chamber geometry and subsidence. The parameters relationship with overburden mechanical properties is currently poorly constrained. C defines the upper limit of subsidence and is the best constrained parameter. A defines the steepness of the transition from low subsidence to high subsidence with increasing W:D and is only currently constrained by five studies. B defines how quickly the onset of significant subsidence begins with increasing W:D, there is currently no relationship between b and HR%. More subsidence data, from a greater range of countries needs to be added to the data pooling in order to increase the confidence in the relationship between the parameters and HR%. In particular data is required that would help strengthen the trends for parameter a vs hard rock. This study uses averaged HR% with no consideration of distance of competent strata to the panel (e.g. roof rock over the void), and does not differentiate between panels of similar W:D, which could be of significantly different depths. Further work focussing on these issues could be key to further constraining parameter b.

Underground coal gasification projects are arguably furthest on in China (Yang et al. 2008), Poland (Mocek et al. 2016), and South Africa (Sharifovich and Varma 2009). However, no data from these countries are used in the current empirical subsidence estimation, with the exception of some few isolated data from Poland. Accessibility and language barriers prevented data from many more countries being used in this study. Subsidence data from conventional mining could be used to weight the empirical model to local conditions. Section 4.3 also shows how the empirical model can be useful in estimating subsidence, even in such areas which haven’t contributed to the underlying data.

The empirical model presented in this paper focusses on maximum surface subsidence. Other subsidence factors are vital for properly managing potential environmental risk arising from UCG development. Strain in overlying strata is a key component to ensure that the gasification chamber is removed enough from key aquifers so there is minimal risk of flooding of the gasification chamber (Younger 2011). There is also no constraint in the empirical model over the surface area affected by subsidence. Constraining the affected area would help give information over how near UCG operations could be sited to strategic infrastructure, critical water courses, or sensitive farmland. A similar data-pooling approach could also provide simple empirical methods of estimating these factors prior to the more involved numerical modelling.

3.4 The utility of the meta-analysis for subsidence prediction and engineering geology applications
Since meta-analysis techniques are so seldom employed for geoscience applications, the usefulness of the technique is discussed here.

The fourteen studies showing subsidence correlated with W:D are comparable in size to previous meta-analysis work, e.g. Biondi-Zoccai et al. (2006) screened down to six studies. However a significant challenge to involving more subsidence data in the meta-analysis was the inconsistent
reporting standards for subsidence. Most studies reporting subsidence did not present the underlying data from which they have based their conclusions, meaning values had to be read from graphs or could not be accessed at all. The data collected from these reports has been presented along with this study to aid any future subsidence work.

For comparing the W:D influence on subsidence, then the meta-analysis style visualisation (Figures 2 and 4) provided greater clarity of the overall and individual study trends. Figure 2 showed that there was a general positive correlation between panel W:D and subsidence. However the trends for individual studies may have been masked by the collective data. The individual study trends could then be visualised in Figure 4. Most studies followed the positive relationship between panel W:D with the only exception being Orchard (1964). Figure 2 showed that Orchard (1964) only had data from the very smallest of the W:D. Such a poor spread of data means that it is not clear if this trend would have continued for larger W:D in this study. By comparing the two Figures (2 and 4) it is clearer that the negative correlation shown in Orchard (1964) should not be considered a reasonable representation of a broader trend.

The relationship between depth and subsidence is positive on two studies (Orchard 1964 and Orchard and Allen 1970) but negative one three others (Aynsley and Hewitt 1960, Karmis et al 1984, and Wardell and Webster 1957), as shown on Figure 3. The two studies with positive correlations had depth data ranging from 100 to 800m (Figure 3), whereas the three studies with negative correlations were confined to shallower depths of between 10-500m. Using such correlations assumes that other factors (such as panel width) are constant, however that is unlikely to be the case. In Orchard and Allen (1970) deeper panels were generally wider; when depth was less that 400m then panel width was tightly clustered around 50m, but for depths greater than 400m then panel widths ranged between 50m and 200m. Orchard and Allen (1970) claim that for two panels with the same W:D, then the deeper one will lead to more subsidence, which was also suggested by K. Wardell during a question when Orchard and Allen (1970) presented their paper.

“Not a simple function of W:D ratio….depth in itself probably an independent parameter” K. Wardell.

This effect is shown in Figure 9, where there the data from shallower mines tends to have lower subsidence than the data from deeper mines for equivalent W:D. However, data from Aynsley and Hewitt (1961) (Figure 9) do not show such a clear effect. More data is required for further investigation of how depth could be treated independently from W:D.
Figure 9. Data from Orchard and Allen 1970 (a) and Aynsley and Hewitt (1960) of panel width/depth ratio (W:D) relationship with subsidence. Data have been split into those from mines less than 300m depth and those mines of greater than 300m depth.

The meta-analysis techniques have allowed visualisation of the difference and strength of the relationships that W:D and depth have to subsidence. The techniques helped both when investigating trends of individual studies and also when assessing the collated studies as a whole. The principles of these meta-analysis techniques would be appropriate to use for any other engineering-geology applications in which many studies need to be compared.

4.0 Conclusions
An empirical data driven method for estimating subsidence from coal mining, namely the meta-analysis technique, has been presented and its utility in generalising the prediction of surface subsidence modelling to various engineering geology applications has been demonstrated. The developed model is likely to be best employed as a preliminary estimation tool for subsidence, in areas where there is little or no previous mining or subsidence information or for new subsurface void generation (similar to underground coal mining) techniques.

Following the relevant literature, the empirically developed subsidence estimation tool showed reasonable fit to historical data from coal regions in U.K. and Australia. The tool also proved to match $S_{max}$ predictions from numerical modelling of subsidence from UCG schemes, for specific scenarios.

The framework presented here can allow for extension and improvement of the developed model, if more data are made available. Better data of the individual different key subsidence factors (such as
W or D) would allow more sophisticated multivariate statistical analysis than what can be achieved with the current data.
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