Treading a fine line: (Im)possibilities for Nash implementation with partially-honest individuals*

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January 31, 2018

Abstract

This paper investigates the robustness of Dutta and Sen's (2012) Theorem 1 to weaker notions of truth-telling. It models individual i's honesty standard as a profile of (possibly non-empty) collections of ordered pairs of outcomes, one for each member of society, over which individual i feels truth-telling concerns. Individual i is honest provided that she states her true preferences as well as rankings (not necessarily complete) of outcomes that are consistent with the true preferences of individuals in her honesty standard. Under this notion of honesty, we offer a condition, called $\mathcal{S}(N)$-*partial-honesty monotonicity*, which is necessary for Nash implementation when there are partially-honest agents. In an independent domain of preferences, we show that this condition is equivalent to Maskin monotonicity provided that honesty means stating the orderings of individuals (in a honesty standard) truthfully and individuals' honesty standards are non-connected.

JEL classification: C72; D71; D82.

Keywords: Nash implementation; partial-honesty; non-connected honesty standards; independent domain; Maskin monotonicity.

*Special thanks go to Luis Corchón and Hans Peters, whose comments and suggestions have led to substantial improvements in the paper. We are also grateful to an associate editor and a referee of this journal for their helpful and thoughtful comments and suggestions. They are not responsible for any remaining deficiencies.

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1. Introduction

The implementation problem is the problem of designing a mechanism or game form with the property that for each profile of participants' preferences, the equilibrium outcomes of the mechanism played with those preferences coincide with the recommendations that a given social choice rule (SCR) would prescribe for that profile. If that mechanism design exercise can be accomplished, the SCR is said to be implementable. The fundamental paper on implementation in Nash equilibrium is thanks to Maskin (1999; circulated since 1977), who proves that any SCR that can be Nash implemented satisfies a remarkably strong invariance condition, now widely referred to as Maskin monotonicity. Moreover, he shows that when the mechanism designer faces at least three individuals, a SCR is Nash implementable if it is Maskin monotonic and satisfies the condition of no veto-power, subsequently, Maskin's theorem. Maskin (1999) obtains his original result by means of a mechanism that requires each individual to report, besides two auxiliary data, the whole description of the state. In a preference model, this means that each participant is asked to report preferences that members of the society have (preference profile).

Since Maskin's theorem, economists have also been interested in understanding how to circumvent the limitations imposed by Maskin monotonicity by exploring the possibilities offered by approximate (as opposed to exact) implementation (Matsushima, 1988; Abreu and Sen, 1991), as well as by implementation in refinements of Nash equilibrium (Moore and Repullo, 1988; Abreu and Sen, 1990; Palfrey and Srivastava, 1991; Jackson, 1992) and by repeated implementation (Kalai and Ledyard, 1998; Lee and Sabourian, 2011; Mezzetti and Renou, 2016). One additional way around those limitations is offered by implementation with partially-honest individuals.

A partially-honest individual is an individual who deceives the mechanism designer when the truth poses some obstacle to her material well-being. Thus, she does not deceive when the truth is equally efficacious. Simply put, a partially-honest individual follows the maxim, "Do not lie if you do not have to" to serve her material interest.

In a general environment, a seminal paper on Nash implementation problems involving partially-honest individuals is Dutta and Sen (2012), whose Theorem 1 (p. 157) shows that for implementation problems involving at least three individuals and in which there is at least one partially-honest individual, the Nash implementability is assured by no veto-power. Similar positive results are uncovered in other environments by Matsushima (2008a,b), Kartik and Tercieux (2012), Kartik et al. (2014), Lombardi and Yoshihara (2016b,c), Saporiti (2014) and Ortner (2015). Thus, there are far fewer limitations for Nash implementation when there are partially-honest individuals.[1]

[1] A pioneering work on the impact of decency constraints on Nash implementation problems is Corchón and Herrero (2004). These authors propose restrictions on sets of strategies available to agents that depend

As in Maskin's (1999) original result, Dutta and Sen's (2012) Theorem 1 uses a mechanism that asks participants to report, among two auxiliary data, the whole preference profile. Moreover, according to Dutta and Sen's (2012) definition of honesty, a participant's play is honest if she plays a strategy choice which is veracious in its preference profile announcement component. In this paper, we consider weaker notions of honesty and then investigate the robustness of Dutta and Sen's (2012) Theorem 1 to these notions of truth-telling.

By taking as given any mechanism, denoted by $\Gamma \equiv (M, g)$, and any individual i's truth-telling correspondence, denoted by T_i^{Γ}, the paper shows that any SCR that can be Nash implemented with partially-honest individuals by the pair $\left\langle \Gamma, \left(T_i^{\Gamma}\right)_{i \in N} \right\rangle$ satisfies a variant of Maskin monotonicity, called *partial-honesty monotonicity* with respect to $\left\langle \Gamma, \left(T_i^{\Gamma}\right)_{i \in N} \right\rangle$. Before describing the condition, we need additional notation. For each individual i, let $S_i\left(\theta'; x, \theta\right)$ denote the set of *truthful outcomes* for individual i at the state θ' when the state moves from θ to θ' and x is selected by the SCR F at θ, that is, $x \in F(\theta)$. Thus, the partial-honesty monotonicity condition prescribes that *if* x is selected by F at θ but is not selected by F at θ' and, moreover, preferences around x are monotonically changed from θ to θ' (that is, whenever $x R_i(\theta) x'$, one has that $x R_i(\theta') x'$), *then* there is at least one partially-honest individual h whose set of truthful outcomes $S_h\left(\theta'; x, \theta\right)$ has the following properties: first, it is contained in the weak lower contour set of $R_h(\theta)$ at x; and second, there is a truthful outcome $z \in S_h\left(\theta'; x, \theta\right)$ which is indifferent to x at her ordering $R_h(\theta')$. Finally, *if*, in addition, there exists a truthful outcome $z' \in S_h\left(\theta'; x, \theta\right)$ that is indifferent to x at her ordering $R_h(\theta)$, *then* individual h's set of truthful strategy choices for θ differs from her set of truthful strategy choices for θ', that is, $T_h^{\Gamma}(\theta) \neq T_h^{\Gamma}(\theta')$.

Every SCR is partial-honesty monotonic when it assumed that for each mechanism Γ, individual i's truthful strategy choices for the state θ are different from those for the state θ' whenever $\theta \neq \theta'$. This is consistent with Theorem 1 of Dutta and Sen (2012), according to which the partially-honest Nash implementability is assured by no veto-power when to be honest means to report the true state of the world. This implies that if one would like to derive a Maskin monotonicity-type condition as a necessary condition that imposes restrictions on the class of implementable SCRs, it is bound to give a weaker meaning to the notion of honesty.

Since the main goal of this study is to offer notions of honesty that are weaker than that employed by Dutta and Sen (2012) and then to investigate the robustness of their result to these notions, we model *individual i's honesty standard*, denoted by $\mathcal{S}(i)$, as a profile of (possibly non-empty) collections of ordered pairs of outcomes, one for each member of

on the state of the world. They refer to these strategies as decent strategies and study Nash implementation problems in decent strategies. For a particular formulation of decent strategies, they are also able to circumvent the limitations imposed by Maskin monotonicity.

society, over which individual i feels truth-telling concerns. We write $\mathcal{S}(N) \equiv (\mathcal{S}(i))_{i \in N}$ for a typical honesty standard of society.

This notion of individual i's honesty standard is flexible enough to allow the individual i's collection for individual j to be empty. Our interpretation is that in this case individual i does not have any truth-telling concern about individual j. We also adopt the view that individual i concerns herself with at least her own self; that is, her own collection of ordered pairs is not empty. Also, we require that the collection over which individual i feels truth-telling concerns about herself has the property that she is able to reveal truthfully her own complete ranking of outcomes.

Thus, an individual i is *honest* provided that she states her true preferences as well as rankings (not necessarily complete) of outcomes that are consistent with the true preferences of individuals in her honesty standard. It is worth emphasizing that this notion of truth-telling encompasses, as a special case, that of Dutta and Sen (2012).

With these notions of honesty and honesty standards, the paper shows that any SCR that can be Nash implemented with partially-honest individuals satisfies a variant of Maskin monotonicity, called $\mathcal{S}(N)$-*partial-honesty monotonicity.* The idea of this axiom is quite intuitive. If x is one of the outcomes selected by a given SCR at state θ but is not selected when there is a monotonic change of preferences around x from θ to θ', then the rankings of outcomes in the honesty standard of a partially-honest individual has been altered by this monotonic change. This condition implies partial-honesty monotonicity - indeed, they are equivalent under some qualifications. Furthermore, it is trivially satisfied when partially-honest individuals concern themselves with the announcement of the whole preference profile as in Dutta and Sen (2012). However, it can have more bite when weaker notions of honesty are considered.

Indeed, in section 5, a specific type of an individual honesty standard is considered, which is modeled as a subset of individuals involved in an implementation problem. Our interpretation is that participant i concerns herself with the truth-telling of individuals in her honesty standard when she plays a strategy choice. Also, this definition endorses the view that an individual concerns herself with at least her own self. Thus, an individual i is truthful provided that she states the true preferences of individuals in her honesty standard. Moreover, we consider what we call *non-connected honesty standards.* Simply put, individual honesty standards are connected if some participant is in the honest standard of every other participant. When that is not the case, we call them non-connected honesty standards. In other words, they are non-connected if every participant is excluded from the honesty standard of another participant.

In an independent domain of preferences, where the set of the profiles of participants' preferences takes the structure of the Cartesian product of individual preferences, we show that $\mathcal{S}(N)$-partial-honesty monotonicity is equivalent to Maskin monotonicity whenever there exists at least one partially-honest individual and individuals' honesty standards are non-connected. Thus, under those hypotheses, Maskin's theorem provides an almost complete characterization of SCRs that are Nash implementable in the society with partially-honest individuals.

The remainder of the paper is divided into five sections. Section 2 presents the theoretical framework and outlines the implementation model. Section 3 presents partial-honesty monotonicity, with the notions of truth-telling and of an honesty standard presented in subsection 3.1. Section 4 presents $\mathcal{S}(N)$-partial-honesty monotonicity, with the equivalence result offered in section 5. Section 6 concludes.

2. Preliminaries

2.1 Basic framework

We consider a finite set of individuals indexed by $i \in N = \{1, \cdots, n\}$, which we will refer to as a society. The set of outcomes available to individuals is X. The information held by the individuals is summarized in the concept of a state. Write Θ for the domain of possible states, with θ as a typical state. In the usual fashion, individual i's preferences in state θ are given by a complete and transitive binary relation, subsequently an ordering, $R_i(\theta)$ over the set X. The corresponding strict and indifference relations are denoted by $P_i(\theta)$ and $I_i(\theta)$, respectively. The preference profile in state θ is a list of orderings for individuals in N that are consistent with this state and is denoted by $R_N(\theta)$.

We assume that the mechanism designer does not know the true state. We assume, however, that there is complete information among the individuals in N and that the mechanism designer knows the preference domain consistent with the domain Θ. In this paper, sometimes we identify states with preference profiles.

2.2 Intrinsic preferences for honesty

An individual who has an intrinsic preference for truth-telling can be thought of as an individual who is torn by a fundamental conflict between her deeply and ingrained propensity to respond to material incentives and the desire to think of herself as an honest person. In this paper, the theoretical construct of the balancing act between those contradictory desires is based on two ideas.

First, the pair (Γ, θ) acts as a "context" for individuals' conflicts. The reason for this is that an individual who has an intrinsic preference for honesty can categorize her strategy choices as truthful or untruthful relative to the state θ and the mechanism Γ designed by the mechanism designer to govern the communication with individuals. That categorization can be captured by the following notion of truth-telling correspondence:

Definition 1 For each Γ and each individual $i \in N$, individual i's *truth-telling correspondence* is a (non-empty) correspondence $T_i^\Gamma : \Theta \twoheadrightarrow M_i$. Strategy choices in $T_i^\Gamma(\theta)$ will be referred to as truthful strategy choices for θ.

In the following, a pair of a mechanism Γ and a profile of the associated truth-telling correspondences $\left(T_i^\Gamma\right)_{i\in N}$ is called a mechanism with the truth-telling correspondences (a mechanism, for short), and is denoted by $\left\langle \Gamma, \left(T_i^\Gamma\right)_{i\in N} \right\rangle$.

Second, in modeling intrinsic preferences for honesty, we endorse the notion of partially-honest individuals introduced by Dutta and Sen (2012). First, a partially-honest individual is an individual who responds primarily to material incentives. Second, she strictly prefers to tell the truth whenever lying has no effect on her material well-being. That behavioral choice of a partially-honest individual can be modeled by introducing an individual's ordering over the strategy space M which contains the information of this individual's ordering over X, because that individual's preference between being truthful and being untruthful is contingent upon announcements made by other individuals as well as the outcome(s) obtained from them. By following standard conventions of orderings, write $\succsim_i^{T_i^\Gamma,\theta}$ for individual i's ordering over M in state θ whenever she is confronted with the mechanism $\left\langle \Gamma, \left(T_i^\Gamma\right)_{i\in N} \right\rangle$. Formally, our notion of a partially-honest individual is as follows:

Definition 2 For each $\left\langle \Gamma, \left(T_i^\Gamma\right)_{i\in N} \right\rangle$, individual $i \in N$ is *partially-honest* if for all $\theta \in \Theta$ individual i's intrinsic preference for honesty $\succsim_i^{T_i^\Gamma,\theta}$ on M satisfies the following properties: for all m_{-i} and all $m_i, m_i' \in M_i$ it holds that:

(i) If $m_i \in T_i^\Gamma(\theta)$, $m_i' \notin T_i^\Gamma(\theta)$ and $g(m) R_i(\theta) g(m_i', m_{-i})$, then $m \succ_i^{T_i^\Gamma,\theta} (m_i', m_{-i})$.

(ii) In all other cases, $m \succsim_i^{T_i^\Gamma,\theta} (m_i', m_{-i})$ if and only if $g(m) R_i(\theta) g(m_i', m_{-i})$.

An intrinsic preference for honesty of individual i is captured by the first part of the above definition, in that, for a given $\left\langle \Gamma, \left(T_i^\Gamma\right)_{i\in N} \right\rangle$ and state θ, individual i strictly prefers the strategy profile (m_i, m_{-i}) to (m_i', m_{-i}) provided that the outcome $g(m_i, m_{-i})$ is at least as good as $g(m_i', m_{-i})$ according to her ordering $R_i(\theta)$ and that m_i is truthful for θ and m_i' is not truthful for θ.

If individual i is *not* partially-honest, this individual cares for her material well-being associated with outcomes of the mechanism and nothing else. Then, individual i's ordering over M is just the transposition into space M of individual i's relative ranking of outcomes. More formally:

Definition 3 For each $\left\langle \Gamma, \left(T_i^\Gamma\right)_{i\in N}\right\rangle$, individual $i \in N$ is *not partially-honest* if for all $\theta \in \Theta$, individual i's intrinsic preference for honesty $\succcurlyeq_i^{T_i^\Gamma,\theta}$ on M satisfies the following property:

$$m \succcurlyeq_i^{T_i^\Gamma,\theta} m' \iff g(m)\, R_i(\theta)\, g(m'), \quad \text{for all } m, m' \in M.$$

2.3 Implementation problems

In formalizing the mechanism designer's problem with partially-honest individuals, we first introduce an informational assumption and discuss its implications for our analysis. It is:

Assumption 1 There exists at least one partially-honest individual in the society N.

Thus, in our setting, the mechanism designer only knows the set Θ as well as the fact that there is at least one partially-honest individual among the individuals, but she does not know either the true state or the identity (or identities) of the partially-honest individual(s). Indeed, the mechanism designer cannot exclude any member(s) of society from being partially-honest purely on the basis of Assumption 1. Therefore, the following considerations are in order from the viewpoint of the mechanism designer.

An environment is described by two parameters, (θ, H): a state θ and a conceivable set of partially-honest individuals H. We denote by H a typical conceivable set of partially-honest individuals in N, with h as a typical element, and by $\mathcal{H}$ the class of conceivable sets of partially-honest individuals.

When combined with an environment (θ, H), each tuple $\left\langle \Gamma, \left(T_i^\Gamma\right)_{i\in N}\right\rangle$ induces a strategic game $\left(\Gamma, \succcurlyeq^{\left(T_i^\Gamma\right)_{i\in N},\theta,H}\right)$, where:

$$\succcurlyeq^{\left(T_i^\Gamma\right)_{i\in N},\theta,H} \equiv \left(\succcurlyeq_i^{T_i^\Gamma,\theta}\right)_{i\in N}$$

is a profile of orderings over the strategy space M as formulated in Definition 2 and in Definition 3. Specifically, $\succcurlyeq_i^{T_i^\Gamma,\theta}$ is individual i's ordering over M as formulated in Definition 2 if individual i is in H, whereas it is the individual i's ordering over M as formulated in Definition 3 if individual i is not in H.

A (pure strategy) Nash equilibrium of the strategic game $\left(\Gamma, \succcurlyeq^{\left(T_i^\Gamma\right)_{i\in N},\theta,H}\right)$ is a strategy profile m such that for all $i \in N$, it holds that

$$m \succcurlyeq_i^{T_i^\Gamma,\theta} (m_i', m_{-i}), \quad \text{for all } m_i' \in M_i.$$

Write $NE\left(\Gamma, \succcurlyeq^{\left(T_i^\Gamma\right)_{i\in N},\theta,H}\right)$ for the set of Nash equilibrium strategies of the strategic game $\left(\Gamma, \succcurlyeq^{\left(T_i^\Gamma\right)_{i\in N},\theta,H}\right)$ and $NA\left(\Gamma, \succcurlyeq^{\left(T_i^\Gamma\right)_{i\in N},\theta,H}\right)$ for its corresponding set of Nash equilibrium outcomes.

The following definition is to formulate the designer's Nash implementation problem involving partially-honest individuals.

Definition 4 Let Assumption 1 hold. Let the tuple $\left\langle \Gamma, \left(T_i^\Gamma\right)_{i\in N}\right\rangle$ be given. Then, a SCR $F : \Theta \twoheadrightarrow X$ is partially-honestly Nash implementable by $\left\langle \Gamma, \left(T_i^\Gamma\right)_{i\in N}\right\rangle$ if it holds that

$$F(\theta) = NA\left(\Gamma, \succcurlyeq^{\left(T_i^\Gamma\right)_{i\in N},\theta,H}\right) \quad \text{for every pair } (\theta, H) \in \Theta \times \mathcal{H}.$$

Moreover, whenever there exists such a tuple, we say that the SCR F is partially-honestly Nash implementable.

The objective of the mechanism designer is thus to design a mechanism whose Nash equilibrium outcomes coincide with $F(\theta)$ for each state θ as well as each set H. Note that there is no distinction between the above formulation and the standard Nash implementation problem as long as Assumption 1 is discarded.

3. Partial-honesty monotonicity with respect to $\left\langle \Gamma, \left(T_i^\Gamma\right)_{i\in N}\right\rangle$

In his seminal paper, Maskin (1999) shows that only Maskin monotonic SCRs are Nash implementable. Maskin monotonicity says that if an outcome x is F-optimal at the state θ, and this x does not strictly fall in preference for anyone when the state is changed to θ', then x must remain an F-optimal outcome at θ'. An equivalent statement of Maskin monotonicity stated above follows the reasoning that if x is F-optimal at θ but not F-optimal at θ', then the outcome x must have fallen strictly in someone's ordering at the state θ' in order to break the Nash equilibrium via some deviation. Therefore, there must exist some (outcome-)preference reversal if an equilibrium strategy profile at θ is to be broken at θ'. Formally, given a state θ, an individual i, and an outcome $x \in X$, the *weak lower contour set of* $R_i(\theta)$ *at* x is $L_i(\theta, x) \equiv \{x' \in X | x R_i(\theta) x'\}$. Then:

Definition 5 A SCR $F : \Theta \twoheadrightarrow X$ is *Maskin monotonic* provided that for all $x \in X$ and all $\theta, \theta' \in \Theta$, if $x \in F(\theta)$ and $L_i(\theta, x) \subseteq L_i(\theta', x)$ for all $i \in N$, then $x \in F(\theta')$.

When there are partially-honest individuals, Maskin monotonicity is not a necessary condition for implementation. This is because though the strategy profile m is a Nash equilibrium for $\left(\Gamma, \succcurlyeq^{\left(T_i^\Gamma\right)_{i\in N},\theta,H}\right)$ and (outcome-)preferences of each individual i change from

$R_i(\theta)$ to $R_i(\theta')$ in a monotonic way around $g(m)$ (that is, whenever $g(m) R_i(\theta) x'$, one has that $g(m) R_i(\theta') x'$), this m may fail to be a Nash equilibrium for $\left(\Gamma, \succsim^{(T_i^\Gamma)_{i\in N}, \theta', H}\right)$. This can happen when at least for one partially-honest individual h it is true that the message profile m falls with respect to any other profile (m_h', m_{-h}) in her intrinsic preference for honesty.

The key to our analysis is to identify the appropriate notion of monotonicity when there are partially-honest individuals, which can be stated as follows: Given a state θ, an individual i, and an outcome $x \in X$, the *indifferent contour set of* $R_i(\theta)$ *at* x is $I_i(\theta, x) \equiv \{x' \in X | x I_i(\theta) x'\}$. Therefore:

Definition 6 Let $\left\langle \Gamma, (T_i^\Gamma)_{i\in N} \right\rangle$ be given. A SCR $F : \Theta \to X$ is *partial-honesty monotonic with respect to* $\left\langle \Gamma, (T_i^\Gamma)_{i\in N} \right\rangle$ provided that for all $H \in \mathcal{H}$ and all $\theta, \theta' \in \Theta$, if $x \in F(\theta) \backslash F(\theta')$ and $L_i(\theta, x) \subseteq L_i(\theta', x)$ for each $i \in N$, then for at least one $h \in H$ there exists a (non-empty) set $S_h(\theta'; x, \theta) \subseteq L_h(\theta, x)$ such that $S_h(\theta'; x, \theta) \cap I_h(\theta', x) \neq \varnothing$ holds, and:

$$S_h(\theta'; x, \theta) \cap I_h(\theta, x) \neq \varnothing \implies T_h^\Gamma(\theta) \neq T_h^\Gamma(\theta').$$

Let us first give an intuitive explanation of the set $S_i(\theta'; x, \theta)$. Suppose that F is partially-honestly Nash implementable by $\left\langle \Gamma, (T_i^\Gamma)_{i\in N} \right\rangle$. Suppose that $x = g(m)$ is F-optimal at θ, that is, $x \in F(\theta)$. Whilst the set $g(M_i, m_{-i})$ represents the set of outcomes that individual i can generate by varying her own strategy, keeping the other individuals' equilibrium strategy choices fixed at m_{-i}, the set $S_i(\theta'; x, \theta) = g\left(T_i^\Gamma(\theta'), m_{-i}\right)$ represents the set of outcomes that this individual can attain by playing truthful strategy choices for θ' when the state moves from θ to θ', keeping the other individuals' equilibrium strategy choices fixed at m_{-i}. Given this idea of the set of $S_i(\theta'; x, \theta)$, we refer to elements of $S_i(\theta'; x, \theta)$ as *truthful outcomes* for individual i at the state θ' when the state moves from θ to θ' and x is an F-optimal outcome at θ.

Thus, partial-honesty monotonicity with respect to $\left\langle \Gamma, (T_i^\Gamma)_{i\in N} \right\rangle$ prescribes that for each conceivable set of partially-honest individuals, H, if there exists an outcome x that is F-optimal at θ but not F-optimal at θ' and, moreover, preferences of each individual i change from $R_i(\theta)$ to $R_i(\theta')$ in a monotonic way around x, then only a partially-honest individual in the given conceivable set H can break the Nash equilibrium via a unilateral deviation. Therefore, at least for one partially-honest individual $h (\in H)$ it is true that her set of truthful outcomes at the state θ' when the state moves from θ to θ', $S_h(\theta'; x, \theta)$, is a subset of the weak lower contour set of $R_h(\theta)$ at x, and that there exists a truthful outcome $z \in S_h(\theta'; x, \theta)$ that is equally good to x, according to her ordering $R_h(\theta')$. Finally, if, in addition, there exists a truthful outcome $z' \in S_h(\theta'; x, \theta)$ that is equally good to x, according to her ordering $R_h(\theta)$, then individual h's set of truthful strategy choices for θ differs from her set of truthful strategy choices for θ', that is, $T_h^\Gamma(\theta) \neq T_h^\Gamma(\theta')$.

For any given $\left\langle \Gamma, \left(T_i^\Gamma\right)_{i\in N}\right\rangle$, partial-honesty monotonicity with respect to $\left\langle \Gamma, \left(T_i^\Gamma\right)_{i\in N}\right\rangle$ is a weaker requirement than Maskin monotonicity, with the two concepts being equivalent under some qualifications (to be discussed in section 5). Our first main result is that this condition is necessary for a SCR to be partially-honestly Nash implementable by $\left\langle \Gamma, \left(T_i^\Gamma\right)_{i\in N}\right\rangle$.

Theorem 1 Let Assumption 1 and $\left\langle \Gamma, \left(T_i^\Gamma\right)_{i\in N}\right\rangle$ be given. A SCR $F : \Theta \twoheadrightarrow X$ is *partial-honesty monotonic with respect to* $\left\langle \Gamma, \left(T_i^\Gamma\right)_{i\in N}\right\rangle$ if it is *partially-honestly Nash implementable by* $\left\langle \Gamma, \left(T_i^\Gamma\right)_{i\in N}\right\rangle$.

Proof. Let Assumption 1 and $\left\langle \Gamma, \left(T_i^\Gamma\right)_{i\in N}\right\rangle$ be given. Suppose that a SCR $F : \Theta \twoheadrightarrow X$ is partially-honestly Nash implementable by $\left\langle \Gamma, \left(T_i^\Gamma\right)_{i\in N}\right\rangle$. Thus, it holds that $F\left(\bar{\theta}\right) = NA\left(\Gamma, \succcurlyeq^{\left(T_i^\Gamma\right)_{i\in N}, \bar{\theta}, H}\right)$ for every pair $\left(\bar{\theta}, H\right) \in \Theta \times \mathcal{H}$. Fix any $(\theta, H) \in \Theta \times \mathcal{H}$ such that $x \in F(\theta)$. Thus, there is $m \in NE\left(\Gamma, \succcurlyeq^{\left(T_i^\Gamma\right)_{i\in N}, \theta, H}\right)$ such that $g(m) = x$.

Consider any state $\theta' \in \Theta$ such that

$$\text{for all } i \in N \text{ and all } x' \in X : xR_i(\theta)\, x' \implies xR_i(\theta')\, x'. \tag{1}$$

If there exists an individual $i \in N$ such that $g(m_i', m_{-i})\, P_i(\theta')\, g(m)$, then, from (1),

$$g(m_i', m_{-i})\, P_i(\theta)\, g(m),$$

a contradiction of the fact that $m \in NE\left(\Gamma, \succcurlyeq^{\left(T_i^\Gamma\right)_{i\in N}, \theta, H}\right)$. Therefore, we conclude that

$$\text{for all } i \in N \text{ and all } m_i' \in M_i : g(m)\, R_i(\theta')\, g(m_i', m_{-i}). \tag{2}$$

Suppose that $x \notin F(\theta')$. Then, the strategy profile m is not a Nash equilibrium for $\left(\Gamma, \succcurlyeq^{\left(T_i^\Gamma\right)_{i\in N}, \theta', H}\right)$; that is, there exists an individual $i \in N$ who can find a strategy choice $m_i' \in M_i$ such that $(m_i', m_{-i}) \succ_i^{T_i^\Gamma, \theta'} m$. Given that (2) holds, it must be the case that $i \in H$. From part (i) of Definition 2 we conclude, therefore, that

$$m_i \notin T_i^\Gamma(\theta') \text{ and } m_i' \in T_i^\Gamma(\theta') \tag{3}$$

and that

$$g(m_i', m_{-i})\, R_i(\theta')\, g(m). \tag{4}$$

For this $i \in H$, let us define the set $S_i(\theta'; x, \theta)$ by

$$S_i(\theta'; x, \theta) = g\left(T_i^{\Gamma}(\theta'), m_{-i}\right). \tag{5}$$

It is plain that this set is not empty - since $T_i^{\Gamma}(\theta')$ is not empty - and that $S_i(\theta'; x, \theta) \subseteq L_i(\theta, x)$. Moreover, by (2) and (4), individual i is indifferent between $g(m)$ and $g(m_i', m_{-i})$, and by (3) and (5), $g(m_i', m_{-i})$ is an element of $S_i(\theta'; x, \theta)$. Thus, $g(m_i', m_{-i}) \in S_i(\theta'; x, \theta) \cap I_i(\theta', x)$, which implies $S_i(\theta'; x, \theta) \cap I_i(\theta', x) \neq \varnothing$.

To show the remaining property, we use a proof by contrapositive here. Assume that $T_i^{\Gamma}(\theta') = T_i^{\Gamma}(\theta)$. Suppose that there is a $w \in X$ such that $w \in S_i(\theta'; x, \theta) \cap I_i(\theta, x)$. Thus, by definition of $S_i(\theta'; x, \theta)$ in (5), it follows that there exists $m_i'' \in T_i^{\Gamma}(\theta') = T_i^{\Gamma}(\theta)$ such that $g(m_i'', m_{-i}) = w$ and that

$$g(m_i'', m_{-i})\, I_i(\theta)\, g(m). \tag{6}$$

Furthermore, since $T_i^{\Gamma}(\theta') = T_i^{\Gamma}(\theta)$, it follows from (3) that $m_i \notin T_i^{\Gamma}(\theta)$. Given that (6) holds, from part (i) of Definition 2 we conclude, therefore, that m is not a Nash equilibrium for $\left(\Gamma, \succcurlyeq^{\left(T_i^{\Gamma}\right)_{i\in N}, \theta, H}\right)$, which is a contradiction. Then, the intersection $S_i(\theta'; x, \theta) \cap I_i(\theta, x)$ needs to be empty, and so F is partial-honesty monotonic. ■

It is worth emphasizing that partial-honesty monotonicity with respect to $\left\langle \Gamma, \left(T_i^{\Gamma}\right)_{i\in N} \right\rangle$ does not impose any restriction on the class of SCRs if $\left\langle \Gamma, \left(T_i^{\Gamma}\right)_{i\in N} \right\rangle$ is such that $T_i^{\Gamma}(\theta) \neq T_i^{\Gamma}(\theta')$ for each individual i and each pair of states θ and θ', with $\theta \neq \theta'$.[2] This is consistent with Theorem 1 of Dutta and Sen (2012), according to which the partially-honest Nash implementability is assured by no veto-power when to be honest means to report the true state of the world.

Corollary 1 Let $\left\langle \Gamma, \left(T_i^{\Gamma}\right)_{i\in N} \right\rangle$ be given. Suppose that $\left\langle \Gamma, \left(T_i^{\Gamma}\right)_{i\in N} \right\rangle$ is such that $T_i^{\Gamma}(\theta) \neq T_i^{\Gamma}(\theta')$ for all $i \in N$ and all $\theta, \theta' \in \Theta$, with $\theta \neq \theta'$. Then, every SCR $F : \Theta \rightarrow X$ is partial-honesty monotonic with respect to $\left\langle \Gamma, \left(T_i^{\Gamma}\right)_{i\in N} \right\rangle$.

However, the condition may be stringent when for each individual i, the value of the correspondence T_i^{Γ} is constrained by a weaker notion of honesty. This will be the subject of what follows below.

3.1 Truth-telling and honesty standards

Thus, let us first formalize our notions of truth-telling as well as individuals' honesty standards. Let the family $\mathcal{X}$ have as elements all non-empty subsets of the space $X \times X$ as

[2] Note that for any $\theta, \theta' \in \Theta$ with $x \in F(\theta) \backslash F(\theta')$, the set $S_i(\theta'; x, \theta)$ can be defined as $S_i(\theta'; x, \theta) = L_i(\theta, x)$ for each $i \in N$.

well as the set whose element is the empty set. As usual, let us denote by $\mathcal{X}^n$ the n-fold Cartesian product of the family $\mathcal{X}$.

An *honesty standard of individual* i, denoted by $\mathcal{S}(i) \equiv (\mathcal{S}_j(i))_{j \in N}$, is an element of $\mathcal{X}^n$ (that is, $\mathcal{S}(i) \in \mathcal{X}^n$). Whilst our interpretation of the set $\mathcal{S}_j(i) = \{\varnothing^j\}$ is that individual i does not have any truth-telling concern about individual j, our interpretation of the set $\mathcal{S}_j(i) \neq \{\varnothing^j\}$ is that individual i concerns herself about individual j and the set $\mathcal{S}_j(i)$ represents the collection of ordered pairs over which this i feels truth-telling concerns - when she plays a strategy choice. An *honesty standard of society* is a list of honesty standards for all members of society. Write $\mathcal{S}(N) \equiv (\mathcal{S}(i))_{i \in N}$ for a typical honesty standard of society.

We adopt the view that individual i concerns herself with at least her own self; that is, $\mathcal{S}_i(i) \neq \{\varnothing^i\}$. Moreover, the collection $\mathcal{S}_i(i)$ of ordered pairs over which she feels truth-telling concerns has the property that this i is able to reveal truthfully her complete ranking of outcomes; formally, we adopt the view that $\mathcal{S}(i)$ is an honesty standard of individual i provided that

$$\mathcal{S}_i(i) \cap R_i(\theta) = R_i(\theta) \quad \text{for all } \theta \in \Theta. \tag{7}$$

Our interpretation of these requirements is that individual i, to view herself as an honest person, has at least to concern herself with the truth-telling of her own preference ordering. This also means that individual i may display an honesty standard which allows her to hide partially or totally other individuals' rankings over outcomes without that being harmful to her self view as an honest person.

Let us observe that this formulation of an honesty standard satisfies important properties. First, it does not depend on the current state of the world. Second, it is also independent of the social objectives that society or its representatives want to achieve. Last but not least, our formulation of honesty standards do not hinge on the existence of any mechanism.

We are now in a position to state our notion of truth-telling. Formally, for a given state θ and individual i's honesty standard $\mathcal{S}(i)$, to save notation we write $R_N(\theta) \cap \mathcal{S}(i)$ for $R_j(\theta) \cap \mathcal{S}_j(i)$ for each individual j. Thus:

Definition 7 For each Γ and each individual $i \in N$ with an honesty standard $\mathcal{S}(i)$ satisfying the requirement in (7), individual i's *truth-telling correspondence* is a (non-empty) correspondence $T_i^\Gamma(\cdot; \mathcal{S}(i)) : \Theta \twoheadrightarrow M_i$ with the property that for any two states θ and θ', it holds that

$$T_i^\Gamma(\theta; \mathcal{S}(i)) = T_i^\Gamma(\theta'; \mathcal{S}(i)) \iff R_N(\theta) \cap \mathcal{S}(i) = R_N(\theta') \cap \mathcal{S}(i). \tag{8}$$

Strategy choices in $T_i^\Gamma(\theta; \mathcal{S}(i))$ will be referred to as truthful strategy choices for θ according to $\mathcal{S}(i)$.

According to the above definition, in a state θ, every truthful strategy choice of individual i is to encode information of individuals' rankings of outcomes that are consistent with the profile of individuals' orderings at the state θ. Moreover, if in two different states, say θ and θ', it holds that for each individual j, the set of ordered pairs in $\mathcal{S}_j(i)$ that are consistent with individual j's ordering at θ is identical to the set of ordered pairs that are consistent with individual j's ordering at θ' (that is, $R_j(\theta) \cap \mathcal{S}_j(i) = R_j(\theta') \cap \mathcal{S}_j(i)$), then the sets of individual i's truthful strategy choices for those two states need to be identical according to her honesty standard $\mathcal{S}(i)$.

The above definition of truth-telling imposes a mild restriction on the class of truth-telling correspondences and, perhaps more interestingly, it represents a minimal notion of honesty that one can formulate in our general environment. It is vital to emphasize here that our notion of veracity encompasses, as a special case, that of Dutta and Sen (2012) when each individual i's honesty standard $\mathcal{S}(i)$ is such that $\mathcal{S}_j(i) \cap R_j(\theta) = R_j(\theta)$ for every individual j and every state θ.

4. $\mathcal{S}(N)$-*partial-honesty monotonicity*

In this section, we discuss a condition, called $\mathcal{S}(N)$-*partial-honesty monotonicity*. We show that for any given honesty standard of society summarized in $\mathcal{S}(N)$, this condition is equivalent to partial-honesty monotonicity when for each mechanism Γ, individual i's truth-telling correspondence T_i^{Γ} is consistent with the conditions in Definition 7. We obtain as a corollary of this equivalence result that $\mathcal{S}(N)$-partial-honesty monotonicity is a necessary condition for the partially-honest Nash implementation when the honesty standard of society is prescribed by $\mathcal{S}(N)$.

Our variant of Maskin monotonicity for Nash implementation problems involving partially-honest individuals when the standard of honesty in a society is represented by $\mathcal{S}(N)$ can be formulated as follows:

Definition 8 A SCR $F : \Theta \twoheadrightarrow X$ is $\mathcal{S}(N)$-*partial-honesty monotonic given the standard* $\mathcal{S}(N)$ (satisfying the requirement in (7) for each individual i) provided that for all $x \in X$, all $H \in \mathcal{H}$ and all $\theta, \theta' \in \Theta$, if $x \in F(\theta) \backslash F(\theta')$ and $L_i(\theta, x) \subseteq L_i(\theta', x)$ for all $i \in N$, then there exists at least one $h \in H$ such that $R_N(\theta) \cap \mathcal{S}(h) \neq R_N(\theta') \cap \mathcal{S}(h)$.

This says that if x is F-optimal at θ but not F-optimal at θ' and, moreover, there is a monotonic change of preferences around x from θ to θ' (that is, whenever $xR_i(\theta)x'$, one has that $xR_i(\theta')x'$), then the rankings of outcomes in the honesty standard of a partially-honest individual h has been altered by this monotonic change (that is, $R_N(\theta) \cap \mathcal{S}(h) \neq R_N(\theta') \cap \mathcal{S}(h)$). Stated in the contrapositive, this says that if x is F-optimal at θ and there is a monotonic change of preferences around x from θ to θ' and, moreover, the rankings of outcomes in the honesty standard of every partially-honest individual h in H has not been altered by this monotonic change, then x must continue to be one of the outcomes selected by F at the state θ'.[3]

Remark 1 Note that if each individual i's honesty standard $\mathcal{S}(i)$ is such that $\mathcal{S}_j(i) \cap R_j(\theta) = R_j(\theta)$, for every individual j and every state θ, and if x is F-optimal at θ but not F-optimal at θ' and it happens that the lower contour sets of preferences at x are nested for every agent across the two environments, then one has that $R_N(\theta) \neq R_N(\theta')$. Thus, any SCR is $\mathcal{S}(N)$-partial-honesty monotonic whenever the honesty standard of society is that studied by Dutta and Sen (2012).

Note that the above definitions of partially-honest individuals (that is, Definition 2) as well as of partially-honest Nash implementation can be easily adapted to the environments with honesty standards. Now, a pair $\left\langle \Gamma, \left(T_i^\Gamma\right)_{i\in N}\right\rangle$ and an environment with honesty standards $(\theta, \mathcal{S}(N), H)$ induce a strategic game $\left(\Gamma, \succcurlyeq^{\left(T_i^\Gamma\right)_{i\in N}, \theta, \mathcal{S}(N), H}\right)$, where:

$$\succcurlyeq^{\left(T_i^\Gamma\right)_{i\in N}, \theta, \mathcal{S}(N), H} \equiv \left(\succcurlyeq_i^{T_i^\Gamma, \theta, \mathcal{S}(i)}\right)_{i\in N}$$

is a profile of orderings over the strategy space M as formulated in Definition 2 and in Definition 3 where the truth-telling correspondence of individual i is that provided in Definition 7. Also, note that the notion of implementation of Definition 4 can easily be adapted to the environments with honesty standards - where every individual's truth-telling correspondence meets the requirements of Definition 7.

Our second main result can be stated as follows:

Theorem 2 Let the *honesty standard of society* be summarized in $\mathcal{S}(N)$, where every individual honesty standard satisfies the requirement in (7). For each mechanism Γ, let the truth-telling correspondence T_i^Γ of individual i be consistent with the conditions in Definition 7. Then, for any $\left\langle \Gamma, \left(T_i^\Gamma\right)_{i\in N}\right\rangle$, *partial-honesty monotonicity with respect to* $\left\langle \Gamma, \left(T_i^\Gamma\right)_{i\in N}\right\rangle$ is equivalent to $\mathcal{S}(N)$-*partial-honesty monotonicity.*

Proof. Let the premises hold. Assume that the SCR $F : \Theta \twoheadrightarrow X$ is $\mathcal{S}(N)$-partial-honesty monotonic. We show that for any $\left\langle \Gamma, \left(T_i^\Gamma\right)_{i\in N}\right\rangle$, it is partial-honesty monotonic with respect to $\left\langle \Gamma, \left(T_i^\Gamma\right)_{i\in N}\right\rangle$, too. For each individual i, for each θ such that $x \in F(\theta)$ and each θ', let us define the set $S_i(\theta'; x, \theta)$ by $S_i(\theta'; x, \theta) = L_i(\theta, x)$. Thus, for each individual i, $S_i(\theta'; x, \theta) \cap I_i(\theta', x) \neq \varnothing$, and $x \in S_i(\theta'; x, \theta) \cap I_i(\theta, x)$ holds for all θ and θ' with $x \in F(\theta)$.

[3]Note that Remark 1 could also be seen as a direct consequence of Corollary 1 when one sets $S_i(\theta'; x, \theta) = L_i(\theta, x)$ and $T_i^\Gamma(\theta) = \mathcal{S}(i) \cap R_N(\theta) = R_N(\theta)$ for each individual i.

Fix any $H \in \mathcal{H}$. Take any θ and θ' such that $x \in F(\theta) \backslash F(\theta')$. Furthermore, let us suppose that $L_i(\theta, x) \subseteq L_i(\theta', x)$ for each individual i. Since F is $\mathcal{S}(N)$-partial-honesty monotonic, it implies that $R_N(\theta) \cap \mathcal{S}(h) \neq R_N(\theta') \cap \mathcal{S}(h)$ for at least one $h \in H$. This means that $T_h^{\Gamma}(\theta; \mathcal{S}(h)) \neq T_h^{\Gamma}(\theta'; \mathcal{S}(h))$ for at least one $h \in H$, as we sought. Thus, for any $\left\langle \Gamma, \left(T_i^{\Gamma}\right)_{i \in N} \right\rangle$, F is partial-honesty monotonic with respect to $\left\langle \Gamma, \left(T_i^{\Gamma}\right)_{i \in N} \right\rangle$.

To show the converse relation, assume that F is partial-honesty monotonic with respect to $\left\langle \Gamma, \left(T_i^{\Gamma}\right)_{i \in N} \right\rangle$. Fix any H. Take any θ and θ' such that $x \in F(\theta) \backslash F(\theta')$ and suppose that $L_i(\theta, x) \subseteq L_i(\theta', x)$ for each individual i. Suppose that F does not satisfy $\mathcal{S}(N)$-partial-honest monotonicity. That is, suppose that, for any $i \in H$, $R_N(\theta) \cap \mathcal{S}(i) = R_N(\theta') \cap \mathcal{S}(i)$. Since the truth-telling correspondences $\left(T_i^{\Gamma}\right)_{i \in N}$ are consistent with the conditions in Definition 7, it follows that $T_i^{\Gamma}(\theta; \mathcal{S}(i)) = T_i^{\Gamma}(\theta'; \mathcal{S}(i))$ for each $i \in H$. Partial-honesty monotonicity with respect to $\left\langle \Gamma, \left(T_i^{\Gamma}\right)_{i \in N} \right\rangle$ implies that for at least one $h \in H$, it holds that $S_h(\theta'; x, \theta) \cap I_h(\theta', x)$ is not empty. Since $T_i^{\Gamma}(\theta; \mathcal{S}(i)) = T_i^{\Gamma}(\theta'; \mathcal{S}(i))$, partial-honesty monotonicity with respect to $\left\langle \Gamma, \left(T_i^{\Gamma}\right)_{i \in N} \right\rangle$ also implies that $S_h(\theta'; x, \theta) \cap I_h(\theta, x)$ is empty. However, since $R_N(\theta) \cap \mathcal{S}(h) = R_N(\theta') \cap \mathcal{S}(h)$, requirement (7) implies that $R_h(\theta) = R_h(\theta')$, and so $I_h(\theta', x) = I_h(\theta, x)$. Since $S_h(\theta'; x, \theta) \cap I_h(\theta', x)$ is not empty, it follows that $S_h(\theta'; x, \theta) \cap I_h(\theta, x)$ is not empty, which is a contradiction. Thus, F satisfies $\mathcal{S}(N)$-partial-honest monotonicity. ■

As a corollary of this theorem, we obtain that $\mathcal{S}(N)$-partial-honest monotonicity is necessary for partially-honest Nash implementation. This is because if x is F-optimal at θ but not F-optimal at θ' and, moreover, the outcome x has not fallen strictly in any individual's ordering at the state θ', then only a partially-honest individual in the given conceivable set H can break the Nash equilibrium via a unilateral deviation. Therefore, there must exist a partially-honest individual $h \in H$ whose equilibrium strategy to attain x at $(\theta, \mathcal{S}(N), H)$ is not a truthful strategy choice at $(\theta', \mathcal{S}(N), H)$. This means that $R_N(\theta) \cap \mathcal{S}(h) \neq R_N(\theta') \cap \mathcal{S}(h)$, according to Definition 7.

Corollary 2 Let Assumption 1 be given. Let the *honesty standard of society* be summarized in $\mathcal{S}(N)$, where every individual honesty standard satisfies the requirement in (7). For each mechanism Γ, let the truth-telling correspondence T_i^{Γ} of individual i be consistent with the conditions in Definition 7. Then, a SCR $F : \Theta \twoheadrightarrow X$ is $\mathcal{S}(N)$-*partial-honesty monotonic given the standard* $\mathcal{S}(N)$ if it is *partially-honestly Nash implementable.*

5. Equivalence result

The classic paper on Nash implementation theory is Maskin (1999), which shows that where the mechanism designer faces a society involving at least three individuals, a SCR is Nash implementable if it is Maskin monotonic and satisfies the auxiliary condition of no veto-power.[4]

The condition of no veto-power says that if an outcome is at the top of the preferences of all individuals but possibly one, then it should be chosen irrespective of the preferences of the remaining individual; that individual cannot veto it. Formally:[5]

Definition 9 A SCR $F : \Theta \twoheadrightarrow X$ satisfies *no veto-power* provided that for all $\theta \in \Theta$ and all $x \in X$, if

$$|\{i \in N | X \subseteq L_i (\theta, x)\}| \geq n - 1,$$

then $x \in F(\theta)$.

Proposition 1 (Maskin's Theorem, 1999) If $n \geq 3$ and $F : \Theta \twoheadrightarrow X$ is a SCR satisfying *Maskin monotonicity* and *no veto-power*, then it is *Nash implementable*.

In a general environment such as that considered here, a seminal paper on Nash implementation problems involving partially-honest individuals is Dutta and Sen (2012). It shows that for Nash implementation problems involving at least three individuals and in which there is at least one partially-honest individual, the Nash implementability is assured by no veto-power (Dutta and Sen, 2012; p. 157). From the perspective of this paper, Dutta-Sen's notion of truth-telling and their Theorem 1 can be formally restated as follows.

We have already mentioned that our notion of truth-telling encompasses, as a special case, that of Dutta and Sen (2012) provided that each individual i's honesty standard $\mathcal{S}(i)$ is such that $\mathcal{S}_j(i) \cap R_j(\theta) = R_j(\theta)$ for every individual j and every state θ. As a generalization of Dutta and Sen's (2012) honesty standard, let us consider a specific type of an honesty standard $\mathcal{S}(i)$ of each individual i such that:

$$\text{for any } j \in N,\ \mathcal{S}_j(i) \neq \left\{\varnothing^j\right\} \text{ implies } \mathcal{S}_j(i) \cap R_j(\theta) = R_j(\theta) \text{ for every state } \theta.$$

Such a type of honesty standard ensures the existence of a subgroup of society, denoted by $S(i)$, such that $j \in S(i)$ if and only if $\mathcal{S}_j(i) \cap R_j(\theta) = R_j(\theta)$ for every state θ. In this section, we focus our attention to this type of honesty standards for all individuals. To ease notation, in what follows we can denote an honesty standard of individual i by $S(i)$. Thus, given a state θ, $R_{S(i)}(\theta)$ is a list of orderings consistent with θ for individuals in the honesty standard $S(i)$ of individual i. Our interpretation is that participant i concerns herself with the truth-telling of preferences of individuals in her honesty standard when she

[4] Moore and Repullo (1990), Dutta and Sen (1991), Sjöström (1991) and Lombardi and Yoshihara (2013) refined Maskin's theorem by providing necessary and sufficient conditions for an SCR to be implementable in (pure strategies) Nash equilibrium. For an introduction to the theory of implementation see Jackson (2001), Maskin and Sjöström (2002) and Serrano (2004).

[5] For any finite set S, $|S|$ denotes the cardinality of S.

plays a strategy choice. To capture the requirement in (7), our definition endorses the view that an individual concerns herself with at least her own self; that is, $i \in S(i)$.

Furthermore, given that in Dutta-Sen's Theorem 1 the mechanism designer knows the honesty standard of society, denoted by $S(N) \equiv (S(i))_{i \in N}$, we also need the following information assumption in order to state their result from the perspective of this paper.

Assumption 2 The mechanism designer knows the honesty standard of the society N.

Therefore:

Proposition 2 (Dutta-Sen's Theorem 1, 2012) Let Assumption 1 and Assumption 2 be given. Let the *honesty standard of society* be summarized in $\overline{S}(N)$, where $\overline{S}(i) \equiv N$ for all $i \in N$. If $n \geq 3$ and $F : \Theta \twoheadrightarrow X$ is a SCR satisfying *no veto-power*, then it is *partially-honestly Nash implementable.*

It follows from Corollary 1 and Theorem 2 that any SCR is $\bar{S}(N)$-partial-honesty monotonic whenever the honesty standard of society is such that every individual considers truthful only messages that encode the whole truth about preferences of individuals in society, that is, $\overline{S}(i) = N$ for all $i \in N$.

That is a particular kind of honesty standards of individuals but there is no reason to restrict attention to such standards. Thus, in what follows, we are interested in understanding the kind of honesty standards of individuals which would make it impossible for the mechanism designer to circumvent the limitations imposed by Maskin monotonicity. To this end, let us introduce the following notion of standards of honesty of a society.

Definition 10 Given a society N involving at least two individuals, an honesty standard of this society is said to be *non-connected* if and only if for all $i \in N$, $i \notin S(j)$ for some $j \in N$.

Given that the honesty standard of individual i includes the individual herself, by definition of $S(i)$, the honesty standard of society is non-connected whenever every one of its members is excluded from the honesty standard of another member of the society. Simply put, members of a society do not concern themselves with the same individual.

It is self-evident that the kind of honesty standards in Dutta-Sen's theorem are not non-connected because every individual of the society is interested in telling the truth about the whole society. As another example of honesty standards of a society that are not non-connected, consider a three-individual society where individual 1 concerns herself with herself and with individual 2 (that is, $S(1) = \{1, 2\}$), individual 2 concerns herself with everyone (that is, $S(2) = \{1, 2, 3\}$) and, finally, individual 3 concerns herself with herself and with individual 1 (that is, $S(3) = \{1, 3\}$). The honesty standard of this three-individual society is not non-connected because everyone concerns themselves with individual 1.

Moreover, it is not necessarily true that every non-connected honesty standard of society implies that every individual honesty standard be of the form $S(i) \neq N$, as we demonstrate with the next example. Consider a three-individual society where individual 1 is concerned only with herself (that is, $S(1) = \{1\}$), individual 2 with everyone (that is, $S(2) = \{1, 2, 3\}$) and individual 3 with herself and with individual 2 (that is, $S(3) = \{2, 3\}$). The honesty standard of this society is non-connected given that individual 2 and individual 3 are both excluded from the honesty standard of individual 1 and individual 1 is excluded from the honesty standard of individual 3.

As is the case here, the above definition is a requirement for the honesty standard of a society that is sufficient for $S(N)$-partial-honesty monotonicity to be equivalent to Maskin monotonicity when two further assumptions are satisfied. The first assumption requires that the family $\mathcal{H}$ includes singletons. This requirement is innocuous given that the mechanism designer cannot exclude any individual from being partially-honest purely on the basis of Assumption 1.

The second requirement is that the set of states Θ takes the structure of the Cartesian product of allowable independent characteristics for individuals. More formally, the domain Θ is said to be *independent* if it takes the form

$$\Theta = \prod_{i \in N} \Theta_i,$$

where Θ_i is the domain of allowable independent characteristics for individual i, with θ_i as a typical element. A typical example of an independent domain is that each Θ_i simply represents the domain of the preference orderings over X of individual i and so the domain of the profiles of all individuals' preference orderings on X has the structure of the Cartesian product. In such a case, in a state $\theta = (\theta_i)_{i \in N}$, individual i's preference ordering over X depends solely on individual i's independent characteristic θ_i rather than on the profile θ.

The above requirements lead to the following conclusion:

Theorem 3 Let N be a society involving at least two individuals, Θ be an independent domain and $\mathcal{H}$ include singletons. Suppose that the honesty standard of the society, denoted by $S(N)$, is non-connected. Then, $S(N)$-*partial-honesty monotonicity* is equivalent to *Maskin monotonicity*.

Proof. Let $n \geq 2$, Θ be an independent domain and $\mathcal{H}$ include singletons. Let $S(N)$ be a non-connected honesty standard of N. One can see that Maskin monotonicity implies $S(N)$-partial-honesty monotonicity.

For the converse, consider any SCR $F : \Theta \twoheadrightarrow X$ satisfying $S(N)$-partial-honesty monotonicity. Consider any $x \in X$ and any state $\theta \in \Theta$ such that x is an F-optimal outcome at θ. Moreover, consider any state θ' such that individuals' preferences change in a Maskin monotonic way around x from θ to θ', that is,

$$\text{for all } i \in N \text{ and all } x' \in X : xR_i(\theta)\, x' \implies xR_i(\theta')\, x'.$$

We show that x remains F-optimal at θ'.

If characteristics of individuals in the honesty standard of individual $i \in N$ are identical in the two states, that is, $R_{S(i)}(\theta) = R_{S(i)}(\theta')$, $S(N)$-partial-honesty monotonicity for the case $H = \{i\}$ assures that x is still F-optimal at θ'. Thus, let us consider the case $R_{S(i)}(\theta) \neq R_{S(i)}(\theta')$ for every individual $i \in N$.

To economize notation, for any subset K of N, write K_C for the complement of K in N. Therefore, for any non-empty subset K of N, we can write any non-trivial combination of the states θ and θ' as $\left(\theta_K, \theta'_{K_C}\right)$, where it is understood that θ_K is a list of characteristics of individuals in K at the state θ and θ'_{K_C} is a list of characteristics of individuals in K_C at θ'. Note that any state that results by that combination is available in Θ because of its product structure.

Given that the honesty standard of society is non-connected, there must be an individual $j(1) \in N$ who does not concern herself with the whole society, that is, $S(j(1)) \neq N$. Consider the state

$$\left(\theta_{K(1)}, \theta'_{K(1)_C}\right) \text{ where } K(1) \equiv S(j(1)),$$

and call it θ^1. By construction, individuals' preferences change in a Maskin monotonic way around x from θ to θ^1 and, moreover, $\theta_{K(1)} = \theta^1_{K(1)}$. $S(N)$-partial-honesty monotonicity for the case $H = \{j(1)\}$ assures that the x remains an F-optimal outcome at θ^1.

If there is an individual $i \in N \setminus \{j(1)\}$ who is not concerned with any of the individuals in the honesty standard of individual $j(1)$, that is, the intersection $S(i) \cap S(j(1))$ is empty, then $S(N)$-partial-honesty monotonicity for the case $H = \{i\}$ assures that x is still F-optimal at θ'. This is because, by construction, individuals' preferences change in a Maskin monotonic way around x from θ^1 to θ' and $\theta^1_{S(i)} = \theta'_{S(i)}$.

Thus, consider any individual $j(2) \in N \setminus \{j(1)\}$, and denote by $K(2)$ the set of individuals with whom individual $j(1)$ and individual $j(2)$ are jointly concerned according to their individual honesty standards. Furthermore, consider the state

$$\left(\theta_{K(2)}, \theta'_{K(2)_C}\right) \text{ where } K(2) \equiv K(1) \cap S(j(2)),$$

and call it θ^2. By construction, individuals' preferences change in a Maskin monotonic way around x from θ^1 to θ^2 and, moreover, $\theta^1_{S(j(2))} = \theta^2_{S(j(2))}$. $S(N)$-partial-honesty monotonicity for the case $H = \{j(2)\}$ assures that x remains an F-optimal outcome at θ^2.

If there is an individual $i \in N \backslash \{j(1), j(2)\}$ who is not concerned with any of the individuals with whom individuals $j(1)$ and $j(2)$ are jointly concerned, $S(N)$-partial-honesty monotonicity for the case $H = \{i\}$ assures that x is also F-optimal at θ'. This is because, by construction, individuals' preferences change in a Maskin monotonic way around x from θ^2 to θ' and $\theta^2_{S(i)} = \theta'_{S(i)}$.

Thus, consider any individual $j(3) \in N \backslash \{j(1), j(2)\}$, and denote by $K(3)$ the set of individuals with whom individuals $j(1)$, $j(2)$ and $j(3)$ are jointly concerned according to their individual honesty standards. Furthermore, consider the state

$$\left(\theta_{K(3)}, \theta'_{K(3)_C}\right) \text{ where } K(3) \equiv K(2) \cap S(j(3)),$$

and call it θ^3. By construction, individuals' preferences change in a Maskin monotonic way around x from θ^2 to θ^3 and, moreover, $\theta^2_{S(j(3))} = \theta^3_{S(j(3))}$. $S(N)$-partial-honesty monotonicity for the case $H = \{j(3)\}$ assures that x remains an F-optimal outcome at θ^3.

As above, if there is an individual $i \in N \backslash \{j(1), j(2), j(3)\}$ who is not concerned with any of the individuals with whom individuals $j(1)$, $j(2)$ and $j(3)$ are jointly concerned, $S(N)$-partial-honesty monotonicity for the case $H = \{i\}$ assures that x remains also F-optimal at θ', because, by construction, individuals' preferences change in a Maskin monotonic way around x from θ^3 to θ' and $\theta^3_{S(i)} = \theta'_{S(i)}$. And so on.

Since the society N is a finite set and the above iterative reasoning is based on its cardinality, we are left to show that it must stop at most after $n-1$ iterations.

To this end, suppose that we have reached the start of the $n-1$th iteration. Thus, consider any individual $j(n-1) \in N$, with $j(n-1) \neq j(r)$ for $r = 1, \cdots, n-2$, and denote by $K(n-1)$ the set of individuals with whom individuals $j(1)$, $j(2), \cdots, j(n-2)$ and $j(n-1)$ are jointly concerned according to their individual honesty standards. Furthermore, consider the state

$$\left(\theta_{K(n-1)}, \theta'_{K(n-1)_C}\right) \text{ where } K(n-1) \equiv K(n-2) \cap S(j(n-1)),$$

and call it θ^{n-1}. As above, by construction, individuals' preferences change in a Maskin monotonic way around x from $\theta^{n-2} \equiv \left(\theta_{K(n-2)}, \theta'_{K(n-2)_C}\right)$ to θ^{n-1} and, moreover, $\theta^{n-2}_{S(j(n-1))} = \theta^{n-1}_{S(j(n-1))}$. $S(N)$-partial-honesty monotonicity for the case $H = \{j(n-1)\}$ assures that x is an F-optimal outcome at θ^{n-1}.

At this stage there is only one individual in N who is left to be considered. Call her $j(n)$. Suppose that this individual is concerned with one of the individuals for whom individuals $j(1)$, $j(2), \cdots, j(n-2)$ and $j(n-1)$ are jointly concerned. In other words, suppose that the intersection $K(n-1) \cap S(j(n))$ is non-empty. Then, the whole society concerns itself with one of its member, and this contradicts the fact that the honesty standard of society is non-connected. Therefore, it must be the case that individual $j(n)$ is not concerned with any of the individuals with whom individuals $j(1)$, $j(2), \cdots, j(n-2)$ and $j(n-1)$ are jointly concerned according to their individual honesty standards. $S(N)$-partial-honesty monotonicity for the case $H = \{j(n)\}$ assures that x remains also F-optimal at θ' given that, by construction, individuals' preferences change in a Maskin monotonic way around x from θ^{n-1} to θ' and $\theta^{n-1}_{S(j(n))} = \theta'_{S(j(n))}$.

The iterative reasoning would stop at the rth ($< n-1$) iteration if there were an individual $i \in N \setminus \{j(1), \cdots, j(r)\}$ who was not concerned with any of the individuals in $K(r)$, that is, if the intersection $S(i) \cap K(r)$ were empty. If that were the case, then the desired conclusion could be obtained by invoking $S(N)$-partial-honesty monotonicity for $H = \{i\}$ because, by construction, it would hold that individuals' preferences change in a Maskin monotonic way around x from θ^r to θ' and that $\theta^r_{S(i)} = \theta'_{S(i)}$. ■

Each of the requirements of Theorem 3 is indispensable. This can be seen as follows:

Consider a two-individual society where Θ is the set of states and X is the set of outcomes available to individuals. Let $S(i)$ be the honesty standard of individual $i = 1, 2$. Consider an outcome x and a state θ such that x is an F-optimal outcome at θ. Consider any other state θ' such that individuals' preferences change in a Maskin monotonic way around x from θ to θ'. Maskin monotonicity says that x must continue to be an F-optimal outcome at θ'. To avoid trivialities, let us focus on the case that $\theta \neq \theta'$, which means that $R_N(\theta) \neq R_N(\theta')$, given that we identify states with preference profiles.

If every individual were concerned with the whole society, we could never invoke (the contrapositive of) $S(N)$-partial-honesty monotonicity to conclude that x should remain F-optimal at θ' because $R_N(\theta) \neq R_N(\theta')$. Furthermore, consider the case that individual 1 concerns herself with only herself, that is, $S(1) = \{1\}$, while individual 2 concerns herself with the whole society, that is, $S(2) = \{1, 2\}$. Reasoning such as the one just used shows that $S(N)$-partial-honesty monotonicity cannot be invoked if $R_1(\theta) \neq R_1(\theta')$. The argument for honesty standards of the form $S(1) = \{1, 2\}$ and $S(2) = \{2\}$ is symmetric. Thus, the only case left to be considered is the one in which everyone concerns themselves with only themselves, that is, $S(i) = \{i\}$ for $i = 1, 2$. In this situation, the honesty standard of society is reduced to the non-connected one. Note that the standards considered earlier were not non-connected.

Suppose that preferences of individual 1 are identical in the two states, that is, $R_1(\theta) = R_1(\theta')$. To conclude that x should be F-optimal at θ' by invoking $S(N)$-partial-honesty monotonicity we need to find individual 1 in the family $\mathcal{H}$. The argument for the case $R_2(\theta) = R_2(\theta')$ is symmetric. Thus, if $R_i(\theta) = R_i(\theta')$ for one of the individuals, the requirement that the singleton $\{i\}$ is an element of $\mathcal{H}$ is needed for the completion of the argument.

Suppose that preferences of individuals are not the same in the two states, that is, $R_i(\theta) \neq R_i(\theta')$ for every individual i, though they have changed in a Maskin monotonic way around x from the state θ to θ'. In this case, one cannot directly reach the conclusion of Maskin monotonicity by invoking $S(N)$-partial-honesty monotonicity. One way to circumvent the problem is to be able to find a feasible state θ'' with the following properties: i) individuals' preferences change in a Maskin monotonic way around x from θ to θ'' and $R_i(\theta) = R_i(\theta'')$ for an individual i, and ii) individuals' preferences change in that way around x from θ'' to θ' and $R_j(\theta') = R_j(\theta'')$ for individual $j \neq i$. A domain Θ that assures the existence of such a state is the independent domain.

Even if one were able to find such a state θ'' by requiring an independent product structure of Θ, one could not invoke $S(N)$-partial-honesty monotonicity to conclude that x must continue to be an F-optimal outcome at θ' whenever the family $\mathcal{H}$ did not have the appropriate structure. This can be seen as in the following argument.

Suppose that Θ is an independent domain. Then, states take the form of profiles of individuals' characteristics, that is, $\theta = (\theta_1, \theta_2)$ and $\theta' = (\theta'_1, \theta'_2)$. Moreover, the characteristic of individual i in one state is independent from the characteristic of the other individual. That is, $R_i(\theta) = R_i(\theta_i)$ and $R_i(\theta') = R_i(\theta'_i)$ for every individual i. The product structure of Θ assures that the states (θ_1, θ'_2) and (θ'_1, θ_2) are both available and each of them has the properties summarized above.

Next, suppose that the family $\mathcal{H}$ has a structure given by $\{\{1\}, \{1,2\}\}$. One can invoke $S(N)$-partial-honesty monotonicity for $H = \{1\}$ to obtain that x is one of the outcomes chosen by the SCR F at (θ_1, θ'_2) when the state changes from θ to (θ_1, θ'_2), but he cannot conclude that x remains also F-optimal at θ' when it changes from (θ_1, θ'_2) to θ'. The reason is that $S(N)$-partial-honesty monotonicity cannot be invoked again for the case $H = \{2\}$ because the structure of the family $\mathcal{H}$ does not contemplate such a case. The argument for the case that $\mathcal{H}$ takes the form $\{\{2\}, \{1,2\}\}$ is symmetric.

In light of Corollary 2 and Maskin's theorem, the main implications of Theorem 3 can be formally stated as follows:

Corollary 3 Let N be a society involving at least two individuals, Θ be an independent domain and $\mathcal{H}$ include singletons. Suppose that the honesty standard $S(N)$ of the society is non-connected. Let Assumption 1 be given. Then, a SCR $F : \Theta \twoheadrightarrow X$ is *Maskin monotonic* if it is *partially-honestly Nash implementable.*

Corollary 4 Let N be a society involving at least three individuals, Θ be an independent domain and $\mathcal{H}$ include singletons. Suppose that the honesty standard $S(N)$ of the society is non-connected. Let Assumption 1 be given. Then, a SCR $F : \Theta \twoheadrightarrow X$ satisfying *no veto-power* is *partially-honestly Nash implementable* if and only if it is *Maskin monotonic.*

Remark 2 In a related but not identical setting, Kartik and Tercieux (2012) study Nash implementation problems where agents can choose to provide evidence as part of their strategies. In this setup, they show that any social choice function satisfying a weaker variant of Maskin monotonicity, called evidence-monotonicity, and no veto-power is Nash implementable. In an environment where there are partially-honest individuals, they show that even small intrinsic costs of lying create a substantial wedge between evidence-monotonicity and Maskin monotonicity, in the sense that every social choice function is evidence-monotonic. Under the assumptions of Theorem 3 and suitable specifications which resemble those of Example 2 in Kartik and Tercieux (2012; p. 333), one can show that this wedge disappears when participants are allowed/forced to produce partial evidence of the true state according to a non-connected (evidence) standard $S(N)$.[6]

6. Concluding remarks

The assumption that the mechanism designer knows the honesty standard of society is often not met in reality, although it may be plausible in societies with a small number of individuals in which the mechanism designer knows their sensitivity to honesty. Outside of cases like those, we view as more plausible the assumption that the mechanism designer only knows the types of honesty standards shared by individuals. Does the conclusion of Theorem 3 change in this case? The answer is no. After all, if individuals are honesty-sensitive, the mechanism designer can test for connectedness of their honesty standards. If the test fails, it would be in vain for him to attempt to Nash implement any SCR that is not Maskin monotonic. The reason for it is easy to identify: the fact that he solely knows that the honesty standard of society is non-connected can only make implementation harder than if the actual non-connected honesty standards of participants were known.

Theorem 3 is derived on the basis that in every state a strategy choice of an individual is truthful if it encodes information of individuals' preferences *consistent* with that state for members of society in her honesty standard. This implies that if we arrange agents in a

[6] To see it, let us suppose that individuals have separable preferences in the sense of Kartik and Tercieux (2012; p. 238). That is, suppose that each agent's (extended) preference ordering $R_i(\theta)$ over the outcome-evidence space $X \times E_i$ is represented by a utility function of the form $U_i(x, e_i, \theta) = u_i(a, \theta) - c_i(e_i, \theta)$, where $c_i(e_i, \theta)$ represents agent i's cost of producing evidence e_i. Fix any $S(N)$ and let the domain Θ be independent. For each individual i, let the evidence space be $E_i = \prod_{j \in S(i)} \Theta_j$. Fix any set H. For each $h \in H$, let the cost function be $c_h(\theta, \theta') = 0$ if $R_{S(h)}(\theta) = R_{S(h)}(\theta')$, otherwise, $c_h(\theta, \theta') = \varepsilon > 0$, where ε can be arbitrarily small. For each $i \notin H$, let $c_i(\theta, \theta') = 0$ for every θ and θ'. This structure implies that the set of the least-evidence cost for $h \in H$ given the pair (x, θ) is $E_h^{\ell}(x, \theta) = \{\theta_{S(h)}\}$ while it is $E_i^{\ell}(x, \theta) = E_i$ for every $i \notin H$. Let the evidence function of invididual $h \in H$ be $e_h^*(\theta) = \{R_{S(h)}(\theta)\}$ for every $\theta \in \Theta$. Under these specifications, one can now see from the proof of Theorem 3 that evidence-monotonicity (stated for each $H \in \mathcal{H}$) is equivalent to Maskin monotonicity.

directed circle and ask them to report their own preferences and those of their successors in the circle, and the honesty standard of every individual includes herself and her successors,[7] then this ‘simpler’ mechanism would impair the ability of the mechanism designer to escape the limitations imposed by Maskin monotonicity. Then, a natural question that arises immediately is: Under what conditions would the positive result of Dutta and Sen (2012) be restored? We answer this question in a companion paper (Lombardi and Yoshihara, 2016a) and it is as follows: The mechanism designer who knows that $\alpha(\geq 1)$ members of society have a taste for honesty can expect to do well if no participant has a veto-power by structuring communication with participants in a way that each of them reports her own preference and those of other $n - \alpha$ successors who are in her honesty standard.

Postlewaite and Schmeidler (1986), Palfrey and Srivastava (1989) and Jackson (1991) have shown that Maskin's theorem can be generalized to Bayesian environments. A necessary condition for Bayesian Nash implementation is Bayesian monotonicity. In a Bayesian environment involving at least three individuals, Bayesian monotonicity combined with no veto-power is sufficient for Bayesian Nash implementation provided that a necessary condition called closure and the Bayesian incentive compatibility condition are satisfied (Jackson, 1991). Although the implementation model developed in this paper needs to be modified to handle Bayesian environments, we believe a similar equivalence result holds in those environments for suitably defined communication schemes (on this point, see Lombardi and Yoshihara, 2013; section 5). This subject is left for future research.

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[7] In an environment in which knowledge is dispersed, how individuals will interact with the mechanism designer is a natural starting point when it comes to Nash implementing a SCR. A particular kind of communication is, as in Dutta and Sen's (2012) Theorem 1, to ask participants to report preferences of the entire society. However, there is no reason to restrict attention to such schemes.

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