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Minimizing Wireless Resource Consumption for Packetized Predictive Control in Real-Time Cyber Physical Systems

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Abstract—In real-time cyber physical systems, *packetized predictive control* (PPC) is an effective solution to conduct robust control over unreliable wireless links. However, the conventional PPC design considers the control and communication systems independently, which does not guarantee the minimum wireless resource consumption. In this paper, we investigate the tight interaction between control and communication. In particular, we propose a communication-control co-design method to optimize the predication length of PPC, then the system can achieve the minimum wireless resource consumption. Our results demonstrate the advantages of the co-design, which is expected to obtain a fully integrated system with good overall performance for real-time wireless control.

I. INTRODUCTION

In future wireless communication systems, a significant number of “things”, e.g., sensors and actuators, are expected to be connected, which leads to the cyber physical system and allows us to interact with the physical world in a real-time fashion [1]-[4], e.g., tactile internet, industrial automation, self-driving vehicles, remote surgery, and smart grid. Therefore, it is critical to provide *ultra high-reliable and low-latency communication* (URLLC), e.g., the end-to-end latency requirement is as low as 1 ms [5] and the reliability is more than 99.999% [6]. This will cost huge wireless resource consumption and becomes the main technical challenge.

One of the most effective solutions is to conduct robust control under unreliable wireless links, e.g., *packetized predictive control* (PPC) [7]-[9]. Then, the required communication performance can be significantly reduced. In PPC, a remote controller not only estimates the past state of a plant, i.e., control target, but also predicts the future state. This allows the remote controller to obtain a sequence of control commands and send them to an actuator via a wireless communication packet. Once the actuator receives the packet, it executes the current command and caches the future ones. This gives the PPC system the robustness to handle unreliable wireless links. For example, if a wireless packet is lost or does not arrive on time, the actuator can autonomously execute the cached commands in the previous packet to continue a control task.

Therefore, the communication performance required by PPC can be effectively reduced.

Currently, there are many contributions discussing the PPC from two perspectives. One is to maximize the control performance under a certain communication performance [10][11]. The other is to minimize the communication requirement under a certain control performance [7][12]. They assume that the wireless resource consumption monotonously increases as the communication requirement grows, i.e., more wireless traffic requires more wireless resource consumption, which is true in conventional cyber systems. However, this assumption may not be valid in cyber physical systems. This is mainly because the overall goal of the control and wireless systems is to finish a control task rather than deliver bit stream. As a result, to achieve a certain control performance, reducing the communication performance requirement may not lead to decreasing the wireless resource consumption.

In this paper, we consider the real-time cyber physical system with short packet wireless communication. We discuss the tight interaction between control and wireless systems, and investigate the relationship between PPC and wireless resource consumption. In particular, we propose a communication-control co-design method to optimize the prediction length of PPC, which does not generate the minimum wireless traffic but can achieve the minimum wireless resource consumption. This result is very unique compared with the one in conventional wireless communication systems. It also indicates the importance of the co-design, which is expected to obtain a fully integrated wireless control system.

The rest of this paper is organized as follows: In Section II, we provide system model and formulate the optimization problem that captures the relationship between control and communication systems. In Section III, we solve the optimization problem, which obtains the optimal prediction length of the PPC controller that minimizes the wireless resource consumption. In Section IV, we provide simulation results to verify our co-design method. Finally, Section V concludes the paper.

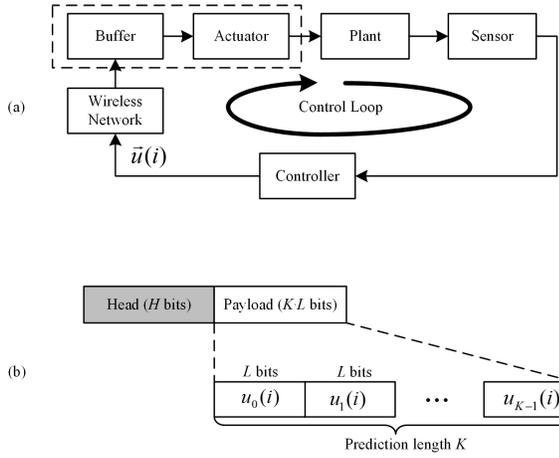


Fig. 1. System model: (a) a typical system diagram of PPC; (b) packet structure.

II. SYSTEM MODEL

Fig. 1-(a) provides a typical system diagram of PPC, where the controller-actuator link is wireless. In the control loop, the sensor first observes the state of the plant and sends its observations to the controller via a wired communication link. The controller estimates the state of the plant and obtains a sequence of K control commands for the actuator, where $k = 0$ and $k > 0$ indicate the control command(s) for the current time slot i and future time slots $i + k$, respectively. Then, the wireless communication network delivers the packet $\vec{u}(i)$ to the buffer in the current time slot i , where $\vec{u}(i)$ consists of K commands and can be expressed as $\vec{u}(i) = [u_0(i), u_1(i), \dots, u_{K-1}(i)]$. Once the buffer receives the packet $\vec{u}(i)$ successfully, it gives the current command $u_0(i)$ to the actuator for execution and caches $K - 1$ future commands. In the following time slot $i + k$, where $k > 0$, if the wireless communication fails, the buffer will send its cached control command to the actuator for execution. Therefore, the buffer actually acts as a safeguard to against packet-loss and delay in the wireless communication link [10][13].

Based on the above PPC model, the system can tolerate up to $K - 1$ contentious packet loss. If we define *control outage* as the case that the buffer is empty and the actuator does not know what to do next. Then, the system experiences the control outage only if the wireless system continuously occurs the packet loss up to K times.

Let p_e be the packet loss probability and p_s be control outage probability, then the communication system needs to satisfy the following inequality

$$p_e \leq p_s^{\frac{1}{K}}, \quad (1)$$

where we assume each packet transmission is independent for simplicity.

Since we consider the real-time wireless control in cyber physical systems, we adopt short packet wireless communication, where the packet structure is provided in Fig. 1-(b). Here, each packet has H bit head. The payload carries K control

commands, where the first one is for current time slot and others are for future time slots. Let L be the bit length of one control command, then the total number of bits in a packet is

$$N = H + KL. \quad (2)$$

Let R be the short packet communication capacity with unit bandwidth, then we can obtain the following expression from [14], i.e.,

$$R(n, p_e) = C - \sqrt{\frac{V}{n}} Q^{-1}(p_e) + o\left(\frac{\log_2 n}{n}\right), \quad (3)$$

where

$$C = \log_2(1 + \gamma), \quad (4)$$

$$V = \gamma \frac{2 + \gamma}{(1 + \gamma)^2} (\log_2 e)^2. \quad (5)$$

In (3), (4), and (5), the variable γ is the signal-to-noise ratio, C is the Shannon capacity with unit bandwidth, and V is a channel dispersion coefficient according to [15]. Here, $o(\log_2 n/n)$ represents the high order terms in short packet capacity and $Q^{-1}(\cdot)$ is the inverse Gaussian Q-function. In addition, since B is the system bandwidth and T is the transmission time of a packet, then $n = B \cdot T$ represents time-frequency resource used in each packet. As a result, we have

$$n = \frac{N}{R}, \quad (6)$$

where n ranges from 1 to a maximum value n_{max} .

III. PREDICTION LENGTH DESIGN FOR PPC: A COMMUNICATION-CONTROL CO-DESIGN METHOD

In principle, the control and communication systems tightly interact with each other, which determines the overall system performance. When we consider PPC, there is a basic trade-off between control and communication in terms of prediction length: If the prediction length K is too short, each packet becomes very small. Then, the wireless system needs to allocate a large amount of wireless resources to provide URLLC link, so that the control outage probability can be guaranteed. On the other hand, if the prediction length is too large, each packet becomes too big. This also requires a significant amount of wireless resources to deliver big packets. Thus, it is reasonable to jointly consider the control and communication systems to determine the prediction length, so that the wireless resource consumption can be minimized while the control outage probability can be guaranteed.

In the following, we first formulate the above problem as an optimization problem. Then, we develop a solution to obtain results.

A. Problem Formulation

Denote P_0 as the transmission power spectral density, then the signal-to-noise ratio at the actuator becomes

$$\gamma = \frac{P_0 B G}{N_0 B} = \frac{P_0 G}{N_0}, \quad (7)$$

where N_0 is the power spectral density of the *additive white Gaussian noise* (AWGN) and G is the wireless channel gain. Here, we assume that G is known to the system. Then, the wireless resource consumption can be obtained as follows

$$E = P_0BT = P_0n = P_0\frac{N}{R} = P_0\frac{H + kL}{R}, \quad (8)$$

which is the multiplication of the three terms that capture the essential resource elements of wireless communications, i.e., power, frequency bandwidth, and time.

According to [16], we have

$$o\left(\frac{\log_2 n}{n}\right) \approx \frac{\log_2 n}{2n} \quad (9)$$

and $p_e < 0.5$, then the expression (3) can be converted to

$$p_e \approx Q\left(\frac{nC - N + \frac{\log_2 n}{2}}{\sqrt{nV}}\right). \quad (10)$$

Combining (1), (4), (8), and (10), we formulate the following optimization problem:

$$\min_{K, P_0, n} : E = P_0BT = P_0n, \quad (11)$$

s.t.

$$p_e \leq p_s^{\frac{1}{K}},$$

$$p_e = Q\left(\frac{nC - N + \frac{\log_2 n}{2}}{\sqrt{nV}}\right),$$

$$C = \log_2(1 + \gamma),$$

$$V = \gamma \frac{2 + \gamma}{(1 + \gamma)^2} (\log_2 e)^2,$$

$$\gamma = \frac{P_0G}{N_0},$$

$$n \leq n_{max}.$$

The above optimization problem is to minimize the wireless resource consumption of each packet transmission in PPC, where the constraints mainly guarantee the control outage probability P_s and finite time frequency resource $n = B \cdot T$.

B. Solution

In this subsection, we solve the optimization problem (11) in the following two steps. In the first step, we obtain the optimal power spectral density P_0^* and the optimal time-frequency resource allocation n^* , which are the function of the prediction length K . In the second step, we substitute the optimal P_0^* and optimal n^* into the optimization problem (11), which obtains the relationship between wireless resource consumption E and the prediction length K . Then, we can obtain the optimal prediction length accordingly.

1) *Optimize P_0 and n* : From (11), we have

$$P_0 = (2^C - 1) \frac{N_0}{G}. \quad (12)$$

For simplicity, we consider medium and high SNR regions, i.e., $\gamma \geq 10$ dB. Then, we have $2^C - 1 \approx 2^C$, where $C =$

$\log_2(1 + \gamma)$. The equation (12) can be simplified to

$$P_0 \approx 2^C \frac{N_0}{G}. \quad (13)$$

To minimize the wireless resource consumption in (11), the packet loss probability p_e should be as large as possible, i.e., $p_e = p_s^{\frac{1}{K}}$. Then, we obtain

$$Q^{-1}\left(p_s^{\frac{1}{K}}\right) - \frac{n \log_2(1 + \gamma) - N + \frac{\log_2(n)}{2}}{\sqrt{nV}} = 0, \quad (14)$$

where

$$V = \gamma \frac{2 + \gamma}{(1 + \gamma)^2} (\log_2 e)^2 \approx (\log_2 e)^2. \quad (15)$$

Substituting (13) and (14) into (11), we obtain the following objective function

$$\min_R : E \approx \frac{N_0N}{G} 2^{f(R)}, \quad (16)$$

where

$$f(R) = \frac{R}{N} \left(\sqrt{\frac{N}{R}} Q^{-1}\left(p_s^{\frac{1}{K}}\right) \log_2(e) + N - \frac{1}{2} \log_2\left(\frac{N}{R}\right) \right) - \log_2 R, \quad (17)$$

and N_0N/G is a constant under the fixed value of K . Because 2^x and x have the same monotonicity property, we need to discuss the monotonicity of $f(R)$. Taking the derivative of $f(R)$, we obtain

$$\begin{aligned} \frac{\partial f(R)}{\partial R} &= 1 - \frac{1}{R \ln 2} \\ &+ \frac{1}{N} \left(\frac{1}{2 \ln 2} + \frac{1}{2} Q^{-1}\left(p_s^{\frac{1}{K}}\right) \log_2 e \sqrt{\frac{N}{R}} - \frac{1}{2} \log_2 \frac{N}{R} \right). \end{aligned} \quad (18)$$

In Appendix A, we prove that $\partial f(R)/\partial R > 0$, which indicates that the value of R should be as small as possible. Since $R = N/n$ and $n \leq n_{max}$, the optimal n is n_{max} . Substituting $n^* = n_{max}$ into (6), (12), and (14), we obtain the optimal power spectral density and optimal time-frequency resource allocation as follows:

$$P_0^* = \left(2^{\frac{1}{n^*}} (\sqrt{n^*} Q^{-1}(p_s^{\frac{1}{K}}) \log_2(e) + N - \frac{1}{2} \log_2(n^*)) - 1 \right) \frac{N_0}{G}, \quad (19)$$

$$n^* = n_{max}. \quad (20)$$

2) *Calculate the Optimal K* : Substituting P_0^* and n^* into (11), we obtain the relationship between the wireless resource consumption E and the prediction length K as follows

$$\begin{aligned} \min_K : E &= P_0^* \times n^* \\ &= \left(2^{\frac{1}{n^*}} (\sqrt{n^*} Q^{-1}(p_s^{\frac{1}{K}}) \log_2(e) + N - \frac{1}{2} \log_2(n^*)) - 1 \right) \frac{N_0 n^*}{G}. \end{aligned} \quad (21)$$

Since $N_0 n^*/G = N_0 n_{max}/G$ is a constant, the original optimization problem is equivalent to

$$\min_K : F = \sqrt{n^*} Q^{-1}\left(p_s^{\frac{1}{K}}\right) \log_2(e) + N - \frac{1}{2} \log_2(n^*). \quad (22)$$

Taking the derivative of F , we have

$$F'_K = \sqrt{n^*} \log_2(e) Q^{-1}\left(p_s^{\frac{1}{K}}\right)'_K + L. \quad (23)$$

Since $Q(x)'_x = -\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$, we obtain

$$Q^{-1}(y)'_y = \frac{1}{Q(x)'_x} = -\sqrt{2\pi}e^{\frac{x^2}{2}} = -\sqrt{2\pi}e^{\frac{Q^{-1}(y)^2}{2}}, \quad (24)$$

and

$$Q^{-1}(p_s^{\frac{1}{K}})'_K = \frac{\sqrt{2\pi}p_s^{\frac{1}{K}} \ln p_s}{K^2} e^{\frac{Q^{-1}(p_s^{\frac{1}{K}})^2}{2}}. \quad (25)$$

Substituting (25) into (23), we obtain

$$F'_K = \sqrt{2\pi n^*} \log_2(e) \ln p_s \frac{p_s^{\frac{1}{K}} e^{\frac{Q^{-1}(p_s^{\frac{1}{K}})^2}{2}}}{K^2} + L. \quad (26)$$

In Appendix B, we prove that $F''_{KK} > 0$, which means that F'_K is a monotonically increasing function. We also have $\lim_{K \rightarrow \infty} F'_K = L > 0$. Then, there are two cases to determine the optimal value of K :

- Case 1: When L is large and n_{\max} is small, we have $F'_{K=1} > 0$. Then we have $F'_K > 0$ since $F''_{KK} > 0$. This means that the wireless resource consumption monotonically increases as the value of K grows. Therefore, we obtain the optimal prediction length at $K^* = 1$.
- Case 2: When L is small and n_{\max} is large, we have $F'_{K=1} < 0$. Since $F''_{KK} > 0$ and $\lim_{K \rightarrow \infty} F'_K = L > 0$, there is a stagnation point K_0 , where K_0 satisfies the equation $F'_K = 0$. When $K < K_0$, we have $F'_K < 0$. When $K > K_0$, we have $F'_K > 0$. This means that the wireless resource consumption first decreases as the value of K grows. After passing the stagnation point K_0 , the wireless resource consumption increases as K grows. Therefore, the optimal prediction length is $K^* = K_0$.

IV. SIMULATION RESULTS

In this section, we demonstrate the performance of the proposed co-design. We assume that the power spectral density of AWGN is $N_0 = -174$ dBm/Hz, the wireless gain is $G = -80$ dB, and the SNR is $\gamma \geq 10$. In the packet structure, the length of the head is $H = 32$ bits and the length of each control command is $L = 4, 8, 16$ bits. In PPC, we assume that the control outage probability is $p_s = 10^{-9}$.

In the following, we first provide the simulation results to show the performance of our co-design method. Then, we discuss the reason behind the results theoretically. Note that in our results, the theoretical curves are obtained from (11), (19), and (20). The simulation results are obtained from the developed method in Section III.

A. Results

Fig. 2 provides the relationship between the wireless resource consumption E and the prediction length K , where different time-frequency resource n_{\max} is considered. Here, we assume that the bit length of each control command is $L = 16$ bits. From the figure, we observe that the theoretical curves overlap the simulation curves very well, which verifies our mathematical derivation. In addition, we observe that when $n_{\max} = 25$ Hz·s, the wireless resource consumption E monotonically increases as the value of K grows. However,

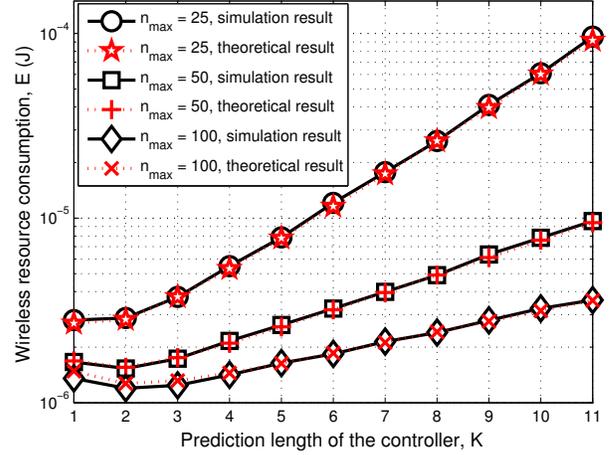


Fig. 2. Wireless resource consumption E versus the prediction length K under different time-frequency resource n_{\max} , where the wireless resource consumption E is the multiplication of transmission time T , frequency bandwidth B , and power spectrum density P_0 of wireless signals. In addition, the prediction length K affects the bit length of each communication packet, i.e., the large value of K is corresponding to large packet and vice versa.

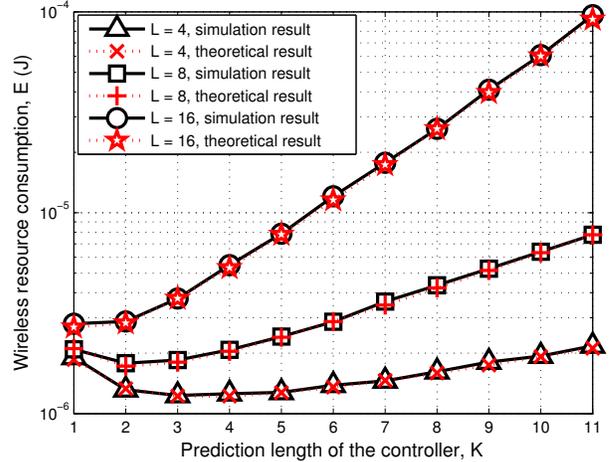


Fig. 3. Wireless resource consumption E versus the prediction length K under different bit lengths of each control command L .

when $n_{\max} = 50$ Hz·s and 100 Hz·s, we observe the U shape in the wireless resource consumption curves E . This trend is reasonable and we will explain the reason in the next subsection.

Fig. 3 demonstrates the relationship between the wireless resource consumption E and the prediction length K , where different bit lengths of each control command L are considered. Here, the time-frequency resource is $n_{\max} = 25$ Hz·s. From the figure, the theoretical curves overlap the simulation curves very well, which verifies our mathematical derivation. In addition, when the bit length of each control command is large, i.e., $L = 16$ bits, the wireless resource consumption monotonically increases as the value of the prediction length K grows. But, when $L = 4$ and 8 bits, we observe the U

shape in the wireless resource consumption curves.

Therefore, from Fig. 2 and Fig. 3, we find that the values of n_{\max} and L affect the relationship between E and K , which further affects the optimal value of K :

- If n_{\max} is small and L is large, we obtain the conventional result that the wireless resource consumption E monotonically increases as the growth of K , i.e., the larger the predication length K is, the larger each communication packet is, and the more wireless resource consumption it needs. Thus, the optimal predication length is $K^* = 1$.
- If n_{\max} is large and L is small, we obtain the unique results that is different from conventional one. The wireless resource consumption E first decreases as the growth of K . After K passes a certain value, E increases as K grows. This means that at the beginning, delivering more bits actually consumes less wireless resource. In other words, designing the control system that minimizes the generated bits is not equivalent to minimizing the wireless resource consumption. The communication-control co-design is expected to choose the optimal value of K to minimize the wireless resource consumption while maintain a certain control outage requirement. For example, in Fig. 3, when $L = 4$ bits, the optimal predication length is $K^* = 3$.

B. Discussion

This subsection discusses the reason behind the observations in Fig. 2 and Fig. 3. From the discussion in Section III, where $E = P_0 n$, we obtain that the minimum E can be obtained when $n = n_{\max}$. Thus, the value of E is actually determined by P_0 , i.e.,

$$E \sim P_0. \quad (27)$$

According to Shannon capacity $C = \log_2(1 + (P_0 B)/(N_0 B))$ the power spectral density P_0 have the same monotonicity as the Shannon capacity, then the value of E can be further determined by C , i.e.,

$$E \sim P_0 \sim C. \quad (28)$$

From (2), (3), and (6), we obtain

$$C \approx R + \sqrt{\frac{V}{n}} Q^{-1}(p_e) = \frac{H + KL}{n} + \sqrt{\frac{V}{n}} Q^{-1}(p_e), \quad (29)$$

where the high order term is ignored. Therefore, the wireless resource consumption E is eventually determined by

$$E \sim \underbrace{\frac{H + KL}{n}}_{\text{The first term}} + \underbrace{\sqrt{\frac{V}{n}} Q^{-1}(p_e)}_{\text{The second term}} \quad (30)$$

In (30), if n is small and L is large, the first term dominates the value of E . Then, E monotonically increases as the value of K grows. In contrast, if n is large and L is small, the second term becomes the dominate component. This is because as K grows, p_e increases and then the second term reduces dramatically. As a result, we can observe the unique results in Fig. 2 and Fig. 3, where the wireless resource consumption

E decreases as K grows. But, when K is large enough, the second term in (30) approaches to 0. Then, the first term becomes the dominate component and we observe the value of E increases as K grows. This is the main reason why we observe the ‘‘U’’ shape in our results.

V. CONCLUSIONS

In this paper, we considered the packetized predictive control in real-time cyber physical systems and proposed a communication-control co-design method to choose the prediction length of the controller so that the wireless resource consumption can be minimized. In particular, we obtained a unique observation that under some situations, the wireless resource consumption reduces as the growth of the traffic volume generated by the control system. Therefore, our result showed the importance of the co-design in real-time cyber physical systems, which is expected to obtain good overall system performance.

APPENDIX

A. The Proof of the inequality $\partial f(R)/\partial R > 0$

Because $\gamma \geq 10$ dB, we have $R = \log_2(1 + \gamma) \geq 3.46$ and

$$1 - \frac{1}{R \ln 2} \geq 1 - \frac{1}{3.46 \times \ln 2} = 0.583 > 0. \quad (31)$$

Thus we can obtain $\partial f(R)/\partial R > 0$ as long as

$$\frac{1}{2} Q^{-1}(p_s^{\frac{1}{K}}) \log_2 e \sqrt{\frac{N}{R}} - \frac{1}{2} \log_2 \frac{N}{R} > 0. \quad (32)$$

Next, we will prove the validation of (32). Let

$$h(x) = \frac{\log_2 e}{2} x - \frac{1}{2} \log_2 x^2, \quad (33)$$

and take the derivative of $h(x)$, we obtain

$$\frac{\partial h(x)}{\partial x} = \frac{\log_2 e}{2} - \frac{1}{\ln 2 \times x}. \quad (34)$$

For $x > 0$ in Equ (34), $h(x)$ is an U-shaped function and the stagnation point is $x = 2/(\ln 2 \cdot \log_2 e)$. Thus, the minimum value of $h(x)$ is $h(2/(\ln 2 \cdot \log_2 e)) = 0.4427 > 0$. We obtain

$$\log_2 e \frac{1}{2} \sqrt{\frac{N}{R}} - \frac{1}{2} \log_2 \frac{N}{R} > 0. \quad (35)$$

According to [17][18], we adopt $K \leq 11$. Since the inverse Q-function is a monotonic decreasing function, we have

$$Q^{-1}(p_s^{\frac{1}{K}}) \geq Q^{-1}(p_s^{\frac{1}{11}}) = 1.0279 > 1. \quad (36)$$

Then, we have

$$\begin{aligned} & Q^{-1}(p_s^{\frac{1}{K}}) \log_2 e \frac{1}{2} \sqrt{\frac{N}{R}} - \frac{1}{2} \log_2 \frac{N}{R} \\ & > \log_2 e \frac{1}{2} \sqrt{\frac{N}{R}} - \frac{1}{2} \log_2 \frac{N}{R} \geq 0.4427 > 0. \end{aligned} \quad (37)$$

Substituting (31) and (37) into (18), we obtain the following result

$$\frac{\partial f(R)}{\partial R} > 0. \quad (38)$$

B. The Proof of the inequality $F''_{KK} > 0$

Since $\sqrt{2\pi n^*} \log_2(e) \ln p_s < 0$ in (26), $F''_{KK} > 0$ is equivalent to

$$\left(\frac{p_s^{\frac{1}{K}} e^{\frac{Q^{-1}(p_s^{\frac{1}{K}})^2}{2}}}{K^2} \right)'_K \geq 0. \quad (39)$$

Since $\ln(x)$ and x have the same monotonicity, we can construct the following function

$$\begin{aligned} g(K) &= \ln\left(\frac{p_s^{\frac{1}{K}} e^{\frac{Q^{-1}(p_s^{\frac{1}{K}})^2}{2}}}{K^2} \right) \\ &= \frac{\ln(p_s)}{K} + \frac{1}{2} Q^{-1}(p_s^{\frac{1}{K}})^2 - 2 \ln(K). \end{aligned} \quad (40)$$

Taking the derivative of $g(K)$, we obtain

$$\begin{aligned} g'_K &= Q^{-1}(p_s^{\frac{1}{K}}) \frac{\sqrt{2\pi} p_s^{\frac{1}{K}} \ln(p_s)}{K^2} e^{\frac{Q^{-1}(p_s^{\frac{1}{K}})^2}{2}} \\ &\quad - \frac{\ln(p_s)}{K^2} - \frac{2}{K}. \end{aligned} \quad (41)$$

For $K = 1, 2, \dots, 10$, we can use exhaustive search to verify $g'_K < 0$. For $K \geq 11$, we have

$$Q^{-1}(p_s^{\frac{1}{K}}) \frac{\sqrt{2\pi} p_s^{\frac{1}{K}} \ln(p_s)}{K^2} e^{\frac{Q^{-1}(p_s^{\frac{1}{K}})^2}{2}} < 0, \quad (42)$$

Then we obtain

$$g'_K \leq -\frac{\ln(p_s)}{K^2} - \frac{2}{K} < 0. \quad (43)$$

Therefore, we obtain that $g(K)$ is a monotonically decreasing function and $F''_{KK} > 0$.

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