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PAPER

Vortex instability in turbulent free-space propagation

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Abstract

The spatial structuring of optical fields is integral within many next generation optical metrology and communication techniques. A verifiable physical model of the propagation of these optical fields in a turbulent environment is important for developing effective mitigation techniques for the modal degradation that occurs in a free-space link. We present a method to simulate this modal degradation that agrees with recently reported experimental findings. A 1.5 km free-space link is emulated by decomposing the optical turbulence that accumulates over a long distance link, into many, weakly perturbing steps of 10 m. This simulation shows that the high-order vortex at the centre of the helical phase profiles in modes that carry orbital angular momentum of $|\ell| \geqslant 2\hbar$ are unstable and fracture into many vortices when they propagate over the link. This splitting presents issues for the application of turbulence mitigation techniques. The usefulness of pre-correction, post-correction, and complex field conjugation techniques are discussed.

1. Introduction

The implementation of spatially structured optical fields have resulted in a wide array of scientific and technological advances [1–3]. Real world deployment of many optical metrology and communication systems will involve propagation through atmospheric turbulence [4]. Hence, an understanding of the propagation of spatially structured optical fields in free-space is important, figure 1. The time-dependent and random variations in temperature and pressure of the atmosphere result in a change in the optical density of the air [5]. Current models for free-space transmission of optical fields are based on theories developed for astronomical measurements, which generally assumed an input optical field with a flat wavefront [6, 7]. These theories have been extended for use with spatially structured optical fields [8–10]. However, these theories do not adequately predict the results acquired by free-space ranging experiments in urban environments [11, 12].

In recent years the drive for extra bandwidth in optical communication systems have resulted in many different forms of spatial encoding being demonstrated [2, 4, 13]. One can choose the spatial encoding technique that matches the parameters of the desired free-space system [4]. Common optical lenses have apertures with circular symmetry, hence the circular symmetric Laguerre–Gaussian (LG) field equations are a logical choice. These fields are characterised by orthogonal eigenvalues ℓ and p, corresponding to the azimuthal and radial components respectively. Beams with a transverse amplitude profile of $A(r)\exp(\mathrm{i}\ell\phi)$, such as LG beams, carry an OAM of $\ell\hbar$ per photon, with r and ϕ as the radial and angular coordinates, respectively [14]. A recent demonstration by Krenn *et al* [15], has indicated that spatial intensity modulation of a laser mode is preserved over long distance propagation at distances of up to 143 km. Such demonstrations are inspiring, however raise an important question about the preservation of phase profile of these modes over long distance propagation. The phase structure of spatially structured modes is vital for SDM multiplexing schemes, and systems that require a full state measurement at the receiver such as quantum key distribution. Recent studies have indicated that although the intensity structure of these modes is largely persevered, the phase aberrations on these modes are considerable and result in the break-up of high-order OAM of ℓ, which breaks up into a cluster og ℓ, $\ell = 1$ modes [12]. Such a breakup in high order modes has not commonly been considered in the modelling of line-of-sight point-to-point links. However, vortex splitting has been documented as a concern in the generation of

Figure 1. For any optical system that requires light to propagate through a large body of air such as the connection of two buildings with a communication link within a city, or an environmental monitoring system the temperature and pressure variation across that link result in aberrations to the optical beam resulting in degradation of the wavefront of the light. In systems that rely on the spatial shaping of the wavefront, the mitigation of the aberrations is critical for the operation of the system.

beams that carry OAM, where a high order vortex breaks up in the presence of weak non-cylindrically symmetric aberration that will, upon propagation, break up to give ℓ individual vortices [16–19]. Theoretical studies have indicated propagation through bulk optical atmospheric turbulence can lead to a change in the measured average OAM of an optical field [20, 21]. A direct relationship between change in average OAM and vortex splitting was recently measured across a long distance free-space optical (FSO) link [11]. For the implementation of FSO links it is important to develop experimentally verifiable simulations of the atmospheric aberrations that occur over FSO links.

In this paper a technique is presented for modelling the expected modal degradation of a high-order spatially structured mode that carry orbital angular momentum through simulated point-to-point link atmospheric turbulence similar to that experienced in an urban environment. A numerical modelling techniques is discussed, where the propagation is segmented into 150 short 10 m propagation regions to simulate the effects of micro-scale atmospheric circulation effects. The accumulative effect of all short regions is designed to simulate a 1.5 km free-space channel in an urban environment. Within our modelling we explore the connection between vortex splitting and turbulence strength, indicating vortex splitting ratio that could be used to determine the accumulative turbulence across a point-to-point link. The simulation confirms high-order vortex instability is a concern for point-to-point links shown in recent experimental studies. Potential atmospheric mitigation techniques are compared, indicating single plane post-correction techniques are not appropriate for use in long distance free-space links.

2. Channel mode

Tip-tilt aberrations are generally the most commonly considered atmospheric turbulence effect impacting Gaussian optical modes, however, high-order aberrations can be present within long distance free-space links. To analyse the effects of higher-order optical aberrations in a free-space link one can adopt the Fried parameter r_0, which is a measure of the transverse distance scale over which the refractive index is correlated [5]. To characterise the effect of turbulence on the optical system, the ratio D/r_0 is considered, where D is the aperture of the system. This ratio sets two limiting cases, first when $D/r_0 < 1$: the resolution of the system is limited by its aperture, and second in the case of $D/r_0 > 1$: the atmosphere limits the system's ability to resolve an object [5]. In current models of OAM channel degradation it is assumed that one can take an ensemble average of many turbulent phase screens and represent this as a single phase screen, where this phase screen fits Kolmogorov statistical models of atmospheric turbulence [9]. These models indicated that one would anticipate the received OAM expectation value to be the same as the transmitted mode order [8, 9]. Further theories were extended to incorporate thick atmospheric turbulence effects, and found a similar result as those for the thin phase regime just with a broader range of modal constituents [22]. However, in recent experimental studies it is indicated that these theories do not hold for optical fields with $|\ell| > 1$ propagated over long-distance links, where the measured OAM expectation value at the receiver does not match that of the transmitted mode [11, 12, 20, 21].

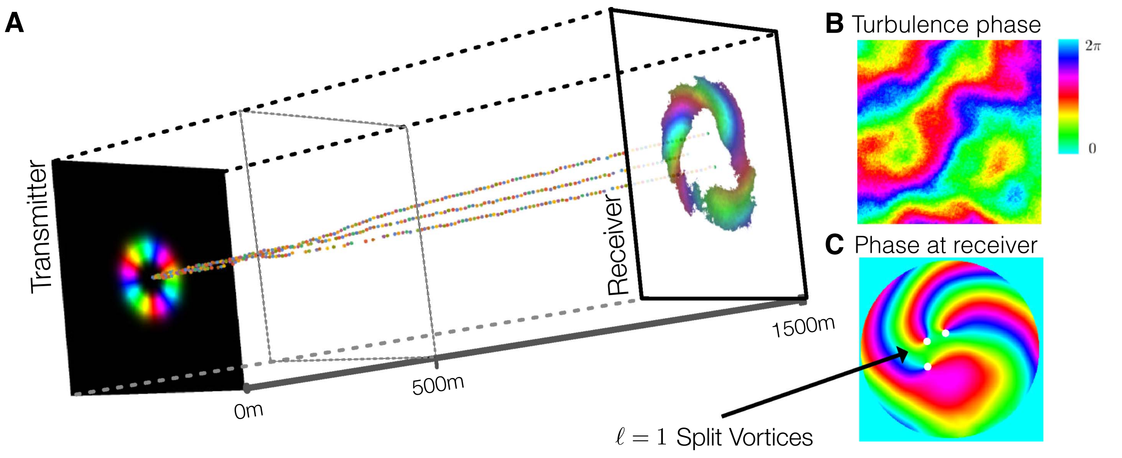

Figure 2. (A) A pure $\ell = 3$ mode is generated at the transmitter, and propagated over a simulated free-space transmission channel of 1.5 km. The channel is segmented in 150 weak turbulence phase screens, each followed 10 m of propagation. The accumulative turbulence over the link is approximately equivalent to a channel with a $D/r_0 = 17$. The vortex position of the 3 split $\ell = 1$ vortices are tracked after each 10 m propagation length and plotted sequentially. A representation of the phase and intensity of the received optical field is overlaid on this plot. (B) A cross sectional image of the accumulative aberration over the 1.5 km link. (C) A cross sectional image of the phase profile of the received optical field, where each of the $\ell = 1$ vortices are highlighted with an overlaid white circle.

The process of taking an ensemble average of many turbulent phase screens holds true for Gaussian modes propagating in turbulence, as the statistical spatial variation of the induced phase distortions is simply accumulated over the free-space channel. However, for spatially structured modes that are perturbed while propagating over a series of turbulent cells the compound effect on the structured mode is more damaging than the effect of simply accumulating the phase distortion over the link. Given a weak perturbation results in the vortices in a high order OAM breaking up in many experimental settings, it would seem logical that aberrations that arise from atmospheric turbulence will result in vortex splitting.

To simulate the long distance propagation, the channel is broken up into 10 m cells corresponding to the micro-scale atmospheric circulation. The propagation within each cell is simulated through plane-wave decomposition. Within each of these cells, a turbulent phase screen, that follows Kolmogorov turbulence theory, is added to the propagating optical field [5]. As such, the aberrations introduced by atmospheric turbulence can be considered as normal random variables, where the ensemble average can be written as $\langle[\phi(\mathbf{r}_1) - \phi(\mathbf{r}_2)]^2\rangle$, where $\phi(\mathbf{r}_1)$ and $\phi(\mathbf{r}_2)$ are two randomly generated phase fluctuations, and is known as the phase structure function [9]. From Kolmogorov statistics it can be shown that this ensemble average must meet the requirement that

$$\langle[\phi(\mathbf{r}_1) - \phi(\mathbf{r}_2)]^2\rangle = 6.88 \left|\frac{\mathbf{r}_1 - \mathbf{r}_2}{r_0}\right|^{5/3}. \tag{1}$$

The turbulence in each cell will be compounded over the length of the channel to yield the turbulence over the full length of the link. In the simulation presented, each phase screen has a $(D/r_0)_{\text{step}} = (D/r_0)_{\text{total}}/150$ that when accumulated over 150 cells yields a channel with a approximate turbulence equivalent to $(D/r_0)_{\text{total}}$, see figure 2.

The thin-phase turbulence regime, is where the accumulative effect of the turbulence can be reduced to a single phase screen that perturbs the optical field similar to the aberrations that occur in an optical lens. This turbulence condition is commonly considered in astronomy, where light from astronomical objects propagates largely unhindered through the vacuum of space and only experience notable aberrations when entering the earths atmosphere. A reciprocal phase distortion can be used to mitigated the turbulence the light encounters. Large aperture telescope systems now commonly use adaptive optical systems that record the phase aberrations through the use of wavefront sensing, and correct the light collected through the use of deformable mirror. Although this technique is very powerful, a study of the appropriateness of it is application is required for optical fields with complexity beyond that of Gaussian optical beams.

In modelling the free-space channel as a set of 10 m cells, it is found that an $\ell = 3$ vortex breaks up into 3 $\ell = 1$ vortices that gradually split over the free-space channel. Weak aberrations early in the channel lead to the vortex being split, with subsequent aberrations over the propagation length then further amplifying this modal breakup, figure 3. Therefore, at the receiver the optical field is more naturally represented by ℓ spatially distributed $\ell = 1$ vortices, rather than a single high-order optical field. Spatially distributed $\ell = 1$ vortices can lead to an OAM distribution at the receiver no longer centred on the expected OAM value, $\langle\ell\rangle$, as is shown in

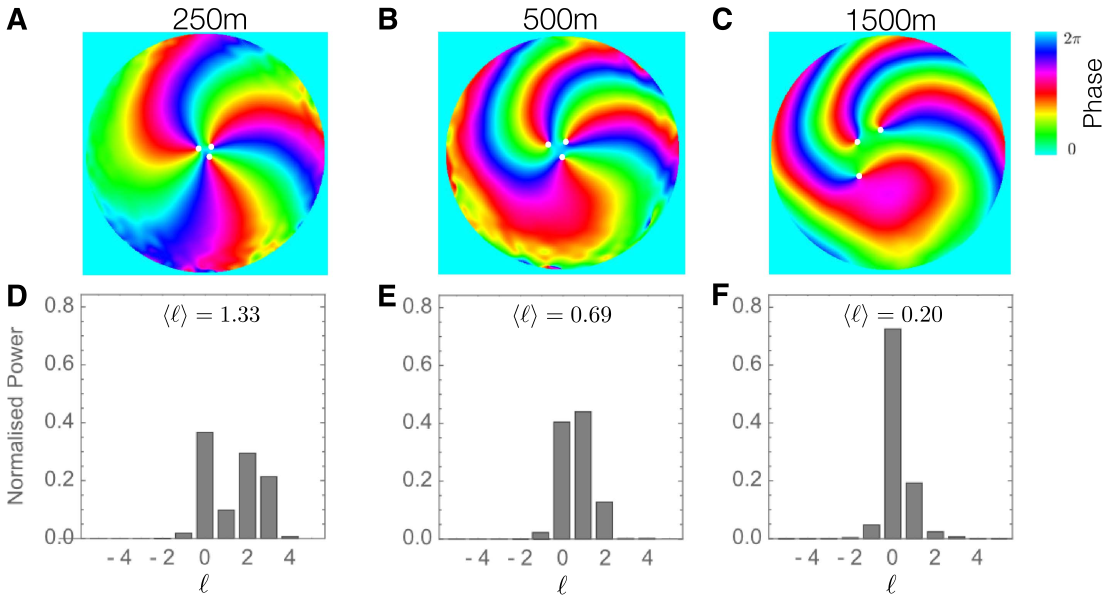

Figure 3. (A)–(C) After propagation through the turbulent optical channel the wavefront is sampled at 250 m, 500 m and 1.5 km respectively. (D)–(F) The inter-modal crosstalk is calculated for each of these sampled wavefronts and the expectation value, $\langle \ell \rangle$, of the received field is determined. In each case $\langle \ell \rangle < 3$.

figures 3(D)–(F). Momentum conservation in an optical fields leads to preservation of the total number vortices under a perturbation.

The degree of vortex splitting can be characterised by a vortex splitting ratio that is defined as $V = \frac{\nabla V_{\langle r \rangle}}{\omega_0}$, where $V_{\langle r \rangle}$ is the average radial distance from the beam origin for the individual vortices and ω_0 is the beam waist of the transmitted mode [11]. After each 10 m of simulated propagation the vortex locations are determined and the average radial distance from the beam origin is calculated to determine the corresponding V value. As the turbulence is randomly varying, the vortex splitting slightly varies. The results are the average of 30 random phase aberrations, similar to the effect of measuring experimentally the vortex locations with a camera with a shutter time of 0.1 s. The error in V was determine by the standard error of the averaged measurements $\frac{\sigma}{\sqrt{n}}$, where σ is the standard deviation and n is the number of samples taken. As the vortex position search is limited to a single pixel in the modelled filed, hence a small offset is applied to V. A range of D/r_0 for both $\ell = 3$ and $\ell = 4$ was determined, figure 4. As the computational time required to generate turbulent phase screen is considerable, pre-determined turbulent phase screens were used. The effective Fresnel number of the simulated link was scaled to yield the presented D/r_0 values. The link is designed to model a 15 mm beam collected by a 150 mm aperture after propagation over a 1500 m link.

3. Relative stability of superpositions

Krenn *et al* presented a study showing that the intensity profile of superpositions of OAM modes seem largely unaffected by propagation through turbulent environment of 3 km over the city of Vienna. Modal superpositions were generated and numerically propagated over the simulated 1.5 km channel. It can be seen in figure 5 that after long distance propagation the intensity structure of modal superpositions are fairly well preserved as expected from experimental studies [15]. As the intensity profile of these modal superpositions is encoded at generation, under propagation through mild turbulence the intensity structure is relatively unaffected and the effect on these modes is similar to the aberrations that occur when imaging a distant object.

4. Turbulence mitigation

The mitigation of turbulence is an important challenge facing the deployment of FSO systems. Experimental astronomy has faced this challenge for many years, where many technologies have been developed for correcting the effects of atmospheric turbulence. One example is called guide star, where a Gaussian laser beam is used to replicate a reference star [23]. The light from a reference star collected by the telescope is analysed commonly using a Shack–Heartman wavefront sensor to determine the wavefront aberrations. Once determined an adaptive optical system can be used to correct for wavefront errors that are induced by the turbulent link.

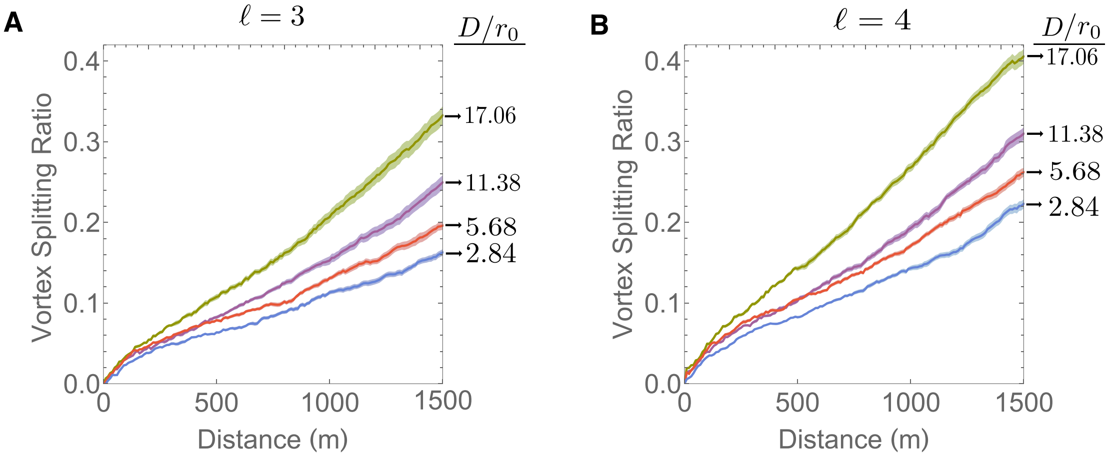

Figure 4. (A), (B) The vortex splitting ratio is determined at 10 m intervals over the full 1500 m simulated link at four different values of D/r_0 for the $\ell = 3$ and $\ell = 4$ respectively. As the presented plot line is the average of 30 random instances of turbulence, the standard error is indicated by the shaded area around each line. These results indicate that this splitting ratio could be used as a method to determine the atmospheric turbulence present in a FSO link.

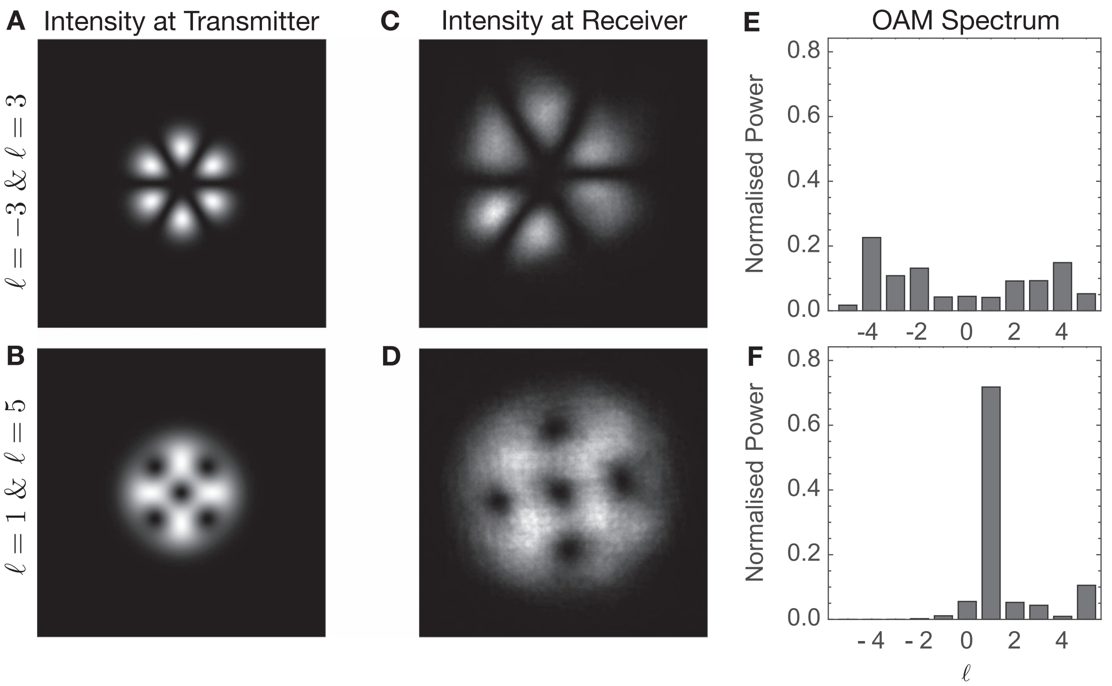

Figure 5. (A) and (B) A modal superposition $\ell = -3$ and $\ell = 3$ and $\ell = 1$ and $\ell = 5$ are generated and propagated over the simulated free-space link respectively. (C) and (D) The intensity profile of the modal superposition after 1.5 km of free-space propagation in a turbulent atmosphere. (E) and (F) The modal decomposition can not always be determined solely by the intensity profile of the optical mode. Hence, to determine the phase aberration introduced by the turbulent channel the modal crosstalk is determined for each of the superpositions.

However, structured optical modes do not behave the same as fundamental Gaussian modes in atmospheric turbulence. For the practical implementation of OAM modes in communication and sensing, it is important to identify an appropriate method for correction. Three techniques are considered: post-correction, where an adaptive optical device is used to correct the received light based on a measurement of the atmospheric turbulence (figure 6(A)); pre-correction, where the optical field is distorted before transmission to match the aberrations over the transmission link using a low bandwidth feedback loop (figure 6(B)); and phase-conjugation, where the received complex field is conjugated and propagated back through the turbulent field prior to encoding of information (figure 6(C)). It is found that none of these techniques fully correct the wavefront aberrations that occur. It is seen that pre-correction offers some advantages over post-correction, which is theorised to be a the result of correcting for defocus over the link. As the beam diverges over the channel,

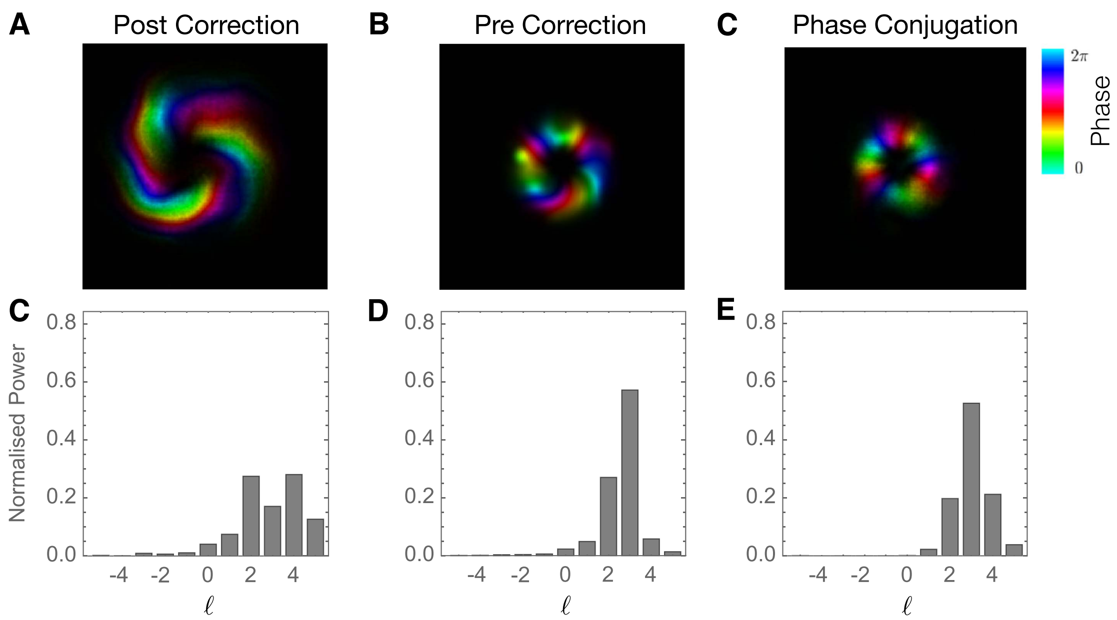

Figure 6. (A) An $\ell = 3$ mode is propagated over the simulated free-space channel. Using a computational equivalent to guide star, the accumulative optical turbulence is determined and a corrective phase screen is applied. A composite image of the phase and intensity is presented at the far-field of the corrective phase screen. (B) The determined accumulative turbulence is instead applied to the $\ell = 3$ mode prior to propagation over the free-space channel. (C) After receiving the perturbed optical mode, the complex conjugate of the optical field is calculated and the field propagated through the turbulent channel in reverse. (D)–(F) A modal spectrum of the optical field after attempted turbulence mitigation. A perfect mitigation would result in all of the optical power in the $\ell = 3$ channel.

the turbulence effect increases with an increase in effective beam diameter. This increase in beam diameter limits the effectiveness of post-correction, as the effective turbulence strength has changed from transmitter to receiver.

Phase conjugation would generally allow for the full correction of the optical field, in the situation where all the light that is transmitted is received. Unfortunately given the limited optical aperture of the simulated system, phase conjugation will not fully reverse the optical aberrations as information is lost in transmission. This indicates that full recovery of the optical field will be challenging when a OAM communication system is implemented as it is difficult to fully capture the optical field after greater long distances. At high light levels, techniques such as optical multiple input multiple output signal processing could be used to fully correct the field [24, 25]. However, at the single photon level the challenge is much greater, it is expected that a combination of technologies will be required to resolve this challenge.

5. Conclusion

In conclusion, an overview of the a segmented approach to modelling turbulence is discussed. It is seen that high-order vortices, where $\ell > 1$, breakup into $\ell, \ell = 1$ vortices during propagation in a relatively weak turbulence regime. This modelling technique will allow for characterisation of channels with variance in turbulence conditions along the length of the link. Three forms of atmospheric turbulence correction have been tested: post-corrections, pre-correction, and phase conjugation. Our simulated results indicate that pre-corrections and phase conjugation outperform post-correction as these limit the divergence of the propagating optical field. Further, it is important that for near-perfect correction one would need to collect the entire optical field at the receiver or have a more advanced correction technique. This experimental evidence inspired modelling method presented in this paper will allow for the development and testing of future methods to mitigate atmospheric turbulence.

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