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Investigation of a gas-fuelled water heater based on combined power and heat pump cycles

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Abstract

In this paper, we proposed and demonstrated a novel gas-fuelled hot water system based on combined power and heat pump cycles. It essentially integrates a premixed gas burner, an organic Rankine cycle (ORC) power plant, and an air source heat pump for supplying hot water. An ORC power plant generates mechanical power from the thermal energy produced from the combustion of natural gas in the burner. Subsequently, the generated power directly drives a vapour compression cycle heat pump through a common shaft connecting the expander and the compressor. Cold tap water is headed firstly in the condenser of the heat pump, then in the condenser of the ORC power plant, and finally the flue gas exiting from the burner in a post heater. The flue gas exiting the post heater will be mixed with ambient air to further extract its residual heat in the evaporator of the heat pump. The advantages of the proposed system are threefold. First, the waste heat of the power cycle has been fully recovered. Second, the heat pump operates on a much lower temperature difference, leading to
higher COP. Third, it has no electrical generator or motor, avoiding the transduction losses. A comprehensive analysis has been presented in this paper, and the results show that the proposed system can achieve an overall fuel-to-heat efficiency up to 147% when the cold water is heated from 10 to 65 °C and the ambient air temperature is in the range -5 to 5 °C. The research results demonstrated that the proposed technology has a great potential for hot water applications.

Keywords: Organic Rankine cycle; air source heat pump; water heater; combined cycles
1. Introduction

In order to mitigate the risks of climate change resulting from global warming, it is important to reduce the energy consumption and greenhouse gas emissions. Heating accounts for a large part of the energy consumption in countries with cold winters (e.g., UK), and it usually heavily relies on the burning of fossil fuels such as natural gases and coal. The decarbonisation of the heating sector is therefore pivotal, but it is far more challenging than the power sector due to its nature of decentralised generation and consumption.

There is a wide range of technologies at different stages of development for heating applications, including electrical resistive heaters, gas boilers, heat pumps (HPs), and micro-CHP (combined heat and power) systems. They all have different fuel-to-heat efficiencies that are defined as the ratio of the useful heat production to the energy contained in the consumed fossil fuels [1, 2].

Nowadays, most installed gas boilers are condensing boilers, which have a fuel-to-heat efficiency of around 90% by recovering some heat from their flue gases [2]. Electrical resistive heaters have near 100% efficiency to convert electricity to heat, but their fuel-to-heat efficiency maybe only around 32-40% if electricity is generated from fossil fuels [3]. An electrically driven heat pump (HP) is an ideal alternative technology for domestic heating if low carbon electricity is supplied. However, to achieve any saving of carbon emissions relative to gas boilers, the HP would require a Coefficient of Performance (COP) of over 2.5 with the current electricity supply mix in the UK [4]. In fact, the air source and ground source HPs installed in the UK to
date have $COP$ in the range of 1.2 to 3.3 [5]. Fossil-fuelled power plants have an

efficiency in the range of 34-42%, and the loss in electricity transmission and
distribution is about 6% [6]. Assuming an average $COP$ of 2.5, the overall fuel-to-heat

efficiency of an electrically powered HP would be only around 80-100%.

Gas-driven absorption HPs have been successfully used in large-scale district
heating applications [4]. Domestic scale gas-driven absorption HPs have recently
been introduced to the European market, claiming a fuel-to-heat efficiency around
130-150% [7]. Gas-driven adsorption HPs have also been researched recently, and
they were reported to have a fuel-to-heat efficiency about 130% in laboratory [7]. Air,
water or waste water, soil, geothermal borehole can all be used as heat sources for
heat pumps for different applications. Great efforts had been made to improve the
performance of heat pumps in the past several decades.

For air source heat pump, the operation parameters and control strategy influence
the system performance significantly. Fischer et al. [8] studied the effect of different
control strategies and boundary conditions on the performance of heat pumps and
recommended the trade-off between system complexity and performance. A
self-optimizing control scheme was proposed by Hu et al. to improve the system
performance of an air-source heat pump by using the extremum seeding control
strategy, which is adopted to match the varying ambient temperatures, water outlet
temperatures and discharge pressure [9]. Gupta and Irving [10] developed a model
that can respond to the changing temperatures of the heat source and sink and their
study showed correct response to the changing ambient temperature.
Despite the continuous progresses made in improving the heat pump’s performance, there are still challenges in the practical applications of heat pumps. One is to overcome the low \emph{COP} issue due to the high-temperature lift in winter to reach a desired temperature over 65°C [11]. The poor performance of heat pumps due to high condensation temperature has become a significant obstacle. Furthermore, wide installation of electrically-powered heat pumps will increase peak electricity demand in winter [12, 13], bring challenges to the electrical grid.

The idea of combining power generation cycles and heat pump cycles has been investigated in the past. A concept of ORC driven heat pump system using the same working fluid and sharing a condenser was proposed by Strong [14]. Later on, experimental investigation of this concept was carried out by Demierre et al. [15, 16]. Recently, a hybrid gas engine-driven heat pump system was proposed by Wan et al. [17] and its primary energy ratio, the ratio of the useful energy output to the primary energy input, can achieve up to 1.09. Shang et al. [18] analysed another type of hybrid power-driven heat pump system theoretically. The results showed that the primary energy ratio is 15.8-25.3% higher than that of the conventional gas engine-driven heat pump system by introducing waste heat recovery.

Combined heat and power (CHP) systems for meeting electric and heat demands in cold climate regions have also been widely investigated. Cho et al. [19] performed a conceptual study of a heat pump driven by a CHP system for producing hot water in parallel. It however has a relatively low theoretical \emph{COP} of 3.6-4 because the heat pump operates at large temperature lift. Kang et al. [20] also analysed a parallel
system that uses a gas turbine to drive a heat pump and recovers the gas turbine’s waste heat to produce hot water, but this system again suffers from a lower heat pump COP due to a high condenser temperature. Schimpf and Span [21] numerically analysed a heat pump system that is capable of running in reverse as an ORC by diverting the working fluid between an expansion valve and pump, depending on the flow direction. However, their ORC system does not directly power the heat pump, and the waste heat of the ORC system is used to recharge a geothermal heat source. Liu et al. [22] analysed a system that uses an internal combustion engine to directly drive a heat pump, but the waste heat of the IC engine was used to drive an ORC to generate power instead of heating water. In addition, Siviter [23] proposed to use thermoelectric heat pumps in the condensation process of the Rankine cycle to improve the system performance, and the experimental results showed the potential this concept.

In this paper, we propose a novel natural gas fuelled hot water heater based on combined power and heat pump cycles. The proposed system essentially integrates a premixed gas burner, an ORC power plant, and an air source heat pump for supplying hot water. The thermal energy produced from natural gas in the burner powers an ORC power plant. Subsequently, the generated power directly drives a vapour compression heat pump. Cold tap water is heated by three heat exchangers in series to gradually increase its temperature. It is preheated in the condenser of the heat pump, then heated in the condenser of the ORC power plant, and finally further heated by the flue gas exiting from the burner in a post heater. The flue gas exiting the post heater
will be mixed with fresh ambient air to further extract its residual heat. In response to the changing ambient air temperatures, the proposed system can adjust the mass flow rates of the natural gas to the burner and the fresh air to mix with the flue gas to ensure hot water supply at a temperature of 65 °C.

The proposed water heater has several advantages. First, the waste heat of the power cycle has been fully recovered. Second, such an ORC-HP system design enables the heat pump to work at a much lower condensing temperature than that of a conventional heat pump for supplying hot water at the same temperature, leading to a higher \( \text{COP} \). Third, the mechanical power of expander of the ORC cycle directly drives the compressor of the heat pump, so it eliminates the electrical generator or motor, and thus avoids the transduction losses. Fourth, it can maximise the heat recovery from the flue gases, further improving the system’s efficiency.

2. Concept

As shown in Fig. 1, the proposed system integrates an ORC power plant and a vapour compression heat pump. The thermal energy released by the combustion of natural gas is transferred to power the ORC system through the Evaporator-ORC. Cold tap water is preheated in the Condenser-HP, and is then further heated in Condenser-ORC. Finally, it reaches the required supply temperature in the post heater that further recovers heat from the flue gases. After leaving the post heater, the flue gases then mixes with fresh air, entering the Evaporator-HP where their residual heat will be further extracted.
Unlike the conventional CHP systems that drive a generator to produce electricity, the power generated by the ORC is used to directly drive the compressor of a heat pump. The compressor of heat pump and the expander of ORC are directly connected with a common shaft.

The main interests in this paper is to investigate the theoretical limit of performance of the proposed system using available working fluids. A screen of working fluids showed that the best performance was achieved in the simulations when hexane and R134a were used as the working fluid for the ORC system and heat pump, respectively. In fact, some researchers have studied the potential of hexane for ORC power plants [24-26], but it has not been widely used in practical applications so far due to its flammability. On the one hand, as the technologies advance, solutions can be developed to address this challenge. For example, mixing hexane with retardant working fluid can potentially maintain its high efficiency while overcome the concerns of its flammability. On the other hand, flammable working fluids such as propane have already been used for high temperature heat pumps [27-29]. Hence, the results based on the pair of hexane and R134a have been used in this paper to demonstrate the best performance we can achieve in theory using the proposed system.

The Temperature-Entropy diagrams of two thermodynamic cycles are shown in Fig. 2. The green line refers to water, pink for R134a, blue for hexane, red for the flue gas, and nattier blue for the mixture of fresh air and flue gas. Hexane is a dry working fluid and thus only minimal superheat is needed [30, 31].
A programme based on Matlab platform and Refprop database is developed to analyse the thermodynamic performance of the system as shown in Fig. 1. The enthalpies and entropies at different states were calculated by using a database Refprop 9.0. An Aspen Plus model was also used to verify the present Refprop/Matlab model.

The main assumptions used in the modelling are listed as follows.

1. The combustion efficiency is assumed to be 100%.
2. Nitrogen does not take part in the chemical reaction during the combustion process.
3. Both heat and pressure loss in all heat exchangers and pipes are negligible.
4. The combined cycles are operated under steady state conditions.
5. The temperature of flue gas is higher than the acid dew point at the outlet of post heater.
6. The Evaporator-HP is made of anticorrosion materials.
7. There is no power loss at the common shaft between the expander and the compressor.
8. The natural gas is assumed to be pure methane.
9. Assume sufficient excessive air will be supplied to ensure the temperature of combustion products in the evaporator will low enough to prevent the decomposition of the working fluid.
The design target is to provide domestic hot water at 65°C. The proposed system is scalable, and is preferred for relatively large applications. For the convenience of experimental research in the laboratory, the heating power is set as 20 kW in this research. Some other main operating parameters are listed in Table 1.

3.1 Gas burner

The fuel considered in this study is natural gas and it is assumed to be pure methane. The complete reaction combustion of natural gas can be expressed as:

\[ CH_4 + 2(1 + \alpha)(O_2 + 3.76N_2) \rightarrow CO_2 + 2H_2O + 7.52(1 + \alpha)N_2 + 2\alphaO_2 \quad (1) \]

\[ \lambda = 1 + \alpha \quad (2) \]

Here, \( \alpha \) refers to the amount of excess air and \( \lambda \) refers to the excess air ratio [32]. The excess air ratio for natural gas combustion is normally in the ranges of 1.05-1.2 [32-35]. The excess air ratio is set as 1.2 in this study.

For a constant-pressure equilibrium process, the heat released during the combustion process can be defined as:

\[ Q_{comb} = \Delta H \cdot m_{fuel} \quad (3) \]

\[ \Delta H = H_P - H_R \quad (4) \]

Here, \( H_P \) is the enthalpy of the products of combustion that leave the combustion chamber and \( H_R \) is the total enthalpy of the reactants that enter it. The combustion heat of methane is 890.3 kJ/mol. The model of the combustion process of natural gas is validated using the data presented in reference [36]. The predicted combustion temperature for this model is 2332 K when the excess air ratio is 1, slightly different
from 2328 K as predicted in Ref [36]. According to the reaction process as shown in Equation (1), when the excess air ratio is 1.2, the mass fractions of CO$_2$, H$_2$O, N$_2$ and O$_2$ are 6.35%, 5.20%, 75.22% and 13.23% in the product mixture, respectively.

After the combustion process in the burner, additional air will be added to the combustion products to reduce their temperature to the required temperature range 260-280 °C [37] to avoid the decomposition of the refrigerant in Evaporator-ORC.

### 3.2 Pinch-point temperature difference method

The limitation of pinch point temperature difference was considered in the modelling of all heat exchangers during phase change processes to ensure the feasible heat exchanger design in practical applications. Take the evaporation process in the Evaporator-ORC for example, the temperature of flue gas at evaporator inlet ($T_5$), the temperature of hexane at the evaporator inlet ($T_2$), the mass flow rate of flue gas ($m_g$), and the minimum pinch point temperature difference ($\Delta T_{pp}$) are all pre-set. Assuming the bubble point is the pinch point so $T_A = T_B + \Delta T_{pp}$, and the mass flow rate of hexane can be calculated based on the energy conservation as:

$$m_f = \frac{c_{pg} \cdot m_g \cdot (T_5 - T_A)}{h_3 - h_B}$$

Then the energy balance during the process from state 2 to state B can determine the temperature $T_6$ as:

$$T_6 = T_A - \frac{m_f \cdot (T_B - T_f)}{c_{pg} \cdot m_g}$$

where the $c_{pg}$ refers to the specific heat capacity of flue gas.
A procedure was developed to compare the \(T_s-T_2\) and \(T_A-T_B\). If \(T_s-T_2 > (T_A-T_B)\), it means that the assumption of pinch point locating at the bubble point is reasonable. Otherwise, the Pinch Point locates at state 6 and \(T_s=T_2+\Delta T_{pp}\). If \(T_s-T_2 < 0\), \(T_s\) calculated with Equation (6) is lower than the acid dew point of the flue gas, then there is no pinch point limitation in the heat exchange process which limits the amount of heat transfer to the ORC needed to avoid overcooling of the flue gas [38], to ensure \(T_s\) is higher than 120 °C to avoid the acid dew point in the boiler. This pinch point temperature difference (PPTD) method is also adopted in all the other heat exchangers.

3.3 Organic Rankine cycle

The modelling of four main components in the basic ORC is present as follows. The working fluid is pumped from low to high pressure by the pump that consumes mechanical work, and it can be calculated as:

\[
W_{p,\text{ORC}} = m_f \cdot \left(h_3 - h_4\right) / \eta_p
\]

Wherein, \(\eta_p\) refers to the isentropic efficiency of the pump.

The high-pressure liquid enters Evaporator-ORC where it is heated to a saturated vapour at constant pressure. The heat transfer within the Evaporator-ORC can be calculated as:

\[
Q_{eva,\text{ORC}} = m_f \cdot \left(h_3 - h_2\right)
\]
High temperature and pressure vapour of the working fluid expands during the
process 3-4 and the generated power can be calculated as:

$$ W_{\text{ORC}} = m_f \cdot (h_3 - h_4) \cdot \eta_t $$  \hspace{1cm} (10)

Wherein, $\eta_t$ refers to the isentropic efficiency of the turbine.

The vapour enters the Condenser-ORC where it condenses at a constant pressure
to turns into a saturated liquid. The heat transfer to water within the Condenser-ORC
can be expressed as:

$$ Q_{\text{con-ORC}} = m_f \cdot (h_4 - h_1) $$  \hspace{1cm} (11)

$$ Q_{\text{con-ORC}} = m_w \cdot (h_{13} - h_{12}) $$  \hspace{1cm} (12)

Wherein, $m_w$ refers to the mass flow rate of water.

In our initial design, the post heater aims to further increase the temperature of
the hot water and the heat transfer can be expressed as:

$$ Q_{\text{post}} = m_w \cdot (h_{16} - h_{13}) $$  \hspace{1cm} (13)

3.4 Vapour compression cycle heat pump

The two-phase refrigerant turns into vapour by absorbing heat from the mixture
of fresh air and flue gas in Evaporator-HP, the heat transfer within the Evaporator-HP
can be calculated as:
Wherein, $m_r$ and $m_{mix}$ refer to the mass flow rate of refrigerant and the mixture gas respectively, $c_{pm}$ refers to the specific heat capacity of gas mixture.

The vapour of refrigerant R134a is then pressurised and circulated by a compressor. The power required is provided by the ORC expander. It can be calculated as:

$$W_{com} = m_r \cdot (h_8 - h_r)$$

(16)

$$W_{com} = W_{ORC}$$

(17)

$W_{com}$ and $W_{ORC}$ refer to the mechanical work consumed by compressor and the mechanical work generated in ORC, respectively.

The high-pressure R134a vapour rejects its heat to the water and turns into liquid. The heat transfer can be calculated as:

$$Q_{con-HP} = m_r \cdot (h_8 - h_b)$$

(18)

$$Q_{con-HP} = m_w \cdot (h_{1_2} - h_{1_1})$$

(19)

The liquid refrigerant R134a exiting the Condenser-HP expands then changes into a low-pressure, low-temperature, two-phase mixture. It can be assumed as an isenthalpic process as the following.

$$h_b = h_{1_0}$$

(20)

### 3.5 Evaluation of the system

Thermal efficiency of the ORC is defined as:
The Coefficient of Performance (COP) of heat pump is defined as:

\[ \text{COP} = \frac{Q_{\text{con-HP}}}{W_{\text{com}}} \]  

(22)

The total heat output of hot water heater can be calculated as:

\[ Q_{\text{total}} = m_w \cdot (h_{t6} - h_{t1}) \]  

(23)

The overall fuel-to-heat efficiency can be defined as:

\[ \eta_{f-h} = \frac{Q_{\text{total}}}{Q_{\text{comb}}} \]  

(24)

### 3.6 Heat transfer area of the heat exchangers

According to the calculated heat capacity, the heat transfer area of the heat exchangers can then be determined [39]. Plate heat exchangers are chosen for the Condenser-ORC, condenser-heat pump, and the post heater owing to its efficient heat transfer and compactness. Hence, all the simulations below are based on plate heat exchangers.

#### 3.6.1 Correlation for working fluid in single-phase state

For the fluid in the sub-cooling zone (state 2 to state B) and in the sup-heating zone (state 8 to state E, and state 4 to state C), heat transfer coefficient \( \alpha_f \) can be calculated according to a semi-empirical equation that is based on Leveque analogy and experimental data [40] as follows.
\[
\text{Re} = \rho \cdot v \cdot d_{eq} / \mu \\
\text{Pr} = c_p \cdot \mu / k \\
1 / f^{0.5} = \cos \beta / (0.18 \cdot \tan \beta + 0.36 \cdot \sin \beta + f_o / \cos \beta)^{0.5} + (1 - \cos \beta) / (3.8 \cdot f_i)^{0.5}
\]

When \( \text{Re} < 2000 \), \( f_o = 64 / \text{Re} \), \( f_i = 579 / \text{Re} + 3.85 \)

When \( \text{Re} \geq 2000 \), \( f_o = (1.8 \cdot \lg \text{Re} - 1.5)^{-2} \), \( f_i = 39 / \text{Re}^{0.289} \)

\[
\text{Nu} = 0.122 \cdot \text{Pr}^{1/3} \cdot (\mu / \mu_{\text{wall}})^{1/6} \cdot (f \cdot \text{Re}^2 \cdot \sin 2\beta)^{0.374}
\]

\[
\alpha = \text{Nu} \cdot k / d_{eq}
\]

\[
d_{eq} = 2b_l / \phi
\]

Here, Re, Nu and Pr refer to the Renold number, Nusselt number and Prandtl number respectively; \( d_{eq} \) refers to the equivalent diameter of working fluid channel; \( b_l \) is corrugation depth and \( \phi \) is the surface enlargement factor of a plate heat exchanger.

Table 2 summarises the main geometric dimensions of the plate heat exchangers.

The correlation mentioned above can be also adopted to calculate the heat transfer coefficient \( (\alpha_w) \) of the cold-water side. Cooper’s pool boiling correlation is adopted for the heat transfer coefficient \( (\alpha_e) \) calculation of the evaporating phase change process [40]:

\[
\alpha_e = 1.5 \times 55 \cdot (p_e / p_{cr})^{(0.12 - 0.2 \cdot \lg R p)} \cdot (-1 \cdot \lg (p_e / p_{cr}))^{-0.55} \cdot q^{0.67} \cdot M^{-0.5}
\]

Here, \( p_e \) and \( p_{cr} \) mean evaporating pressure and the critical pressure of working fluid (MPa), respectively; \( R p \) means the mean asperity height (\( \mu \text{m} \)). According to the supplied plate heat exchanger, the value of \( R p \) is 0.3 in this calculation; \( q \) means the heat flux (W/m\(^2\)), \( M \) means the molar mass of working fluid (kg/kmol).
3.6.2 Correlation for condensation process

The condensation heat transfer coefficient \( \alpha_c \) of all working fluids can be calculated by using the following correlation [41]:

\[
\alpha_c = 0.2092 \cdot \left( \frac{k_i}{d_{eq}} \right) \cdot \text{Re}_l^{0.78} \cdot \text{Pr}_l^{0.33} \cdot \left( \frac{\mu}{\mu_{wall}} \right)^{0.14}
\]  

\( \text{Re}_l = \frac{G \cdot d_{eq}}{\mu} \)  
\( \text{Pr}_l = \frac{c_p \cdot \mu}{k_i} \)  
\( Co = \frac{\rho_s}{\rho_l} \cdot \left( \frac{1}{x-1} \right)^{0.8} \)  
\( Fr_l = \frac{G^2}{(\rho_i^2 \cdot g \cdot d_e)} \)  
\( Bo = \frac{q}{G \cdot i_{fg}} \)  
\( \alpha = \alpha_c \cdot \left( 0.25 \cdot Co^{-0.45} \cdot Fr_l^{0.25} + 75 \cdot Bo^{0.75} \right) \)

Here,

\( C_0 \) – Convection number;

\( B_0 \) – Boiling number;

\( x \) – Quality;

\( G \) – Mass flux of the working fluid (kg/m\(^2\)s);

\( g \) – Acceleration due to gravity (m/s\(^2\));

\( i_{fg} \) – Enthalpy of vaporization (kJ/kg).

Based on the heat transfer coefficient above, the heat transfer area can be obtained as:

\[
A = \frac{Q}{K} / \Delta T_m
\]

\[
1 / K = 1000 / \alpha_{hot-side} + t / \lambda + 1000 / \alpha_{cold-side}
\]
When $\Delta T_i > \Delta T_j$, then

$$\Delta T_m = \frac{\Delta T_i - \Delta T_j}{\ln(\Delta T_i / \Delta T_j)}$$

(41)

Wherein,

$$\Delta T_i = T_{i,in} - T_{i,out}$$

(42)

$$\Delta T_2 = T_{2,out} - T_{2,in}$$

(43)

Here, subscript 1 means the hot fluid, 2 means the cold fluid, subscript “in” means at the inlet, “out” means the outlet.

### 3.6.3 Correlation for Evaporator-HP and post heater

As is well known, the heat transfer coefficient on the gas side is much lower than those on the liquid side. A fin and tube heat exchanger is adopted to increase heat transfer area by utilizing finned heat transfer surfaces. The modelling of the Evaporator-HP is set up based on the fin and tube heat exchanger. The main geometric dimensions of fin and tube heat exchange is shown in Table 3.

For the fin side (namely the air side), the heat transfer fluid is a mixture of fresh air and flue gases. In order to further recover heat from the flue gases, all the exhaust gases pass through the Evaporator-HP. Fresh air is mixed with flue gases to provide sufficient heat energy needed in the evaporator. The flow rate of fresh air is determined by the required heat in Evaporator-HP and temperature difference between inlet and outlet.

The equivalent heat transfer coefficient of the fin side is expressed as:

$$\alpha_{ef} = \xi \cdot \alpha_o \cdot \eta_s$$

(44)

$\xi$ – The moisture absorption coefficient;
\[ \alpha_o \] – Heat transfer coefficient of air side;

\[ \eta_s \] – Surface effectiveness;

The moisture absorption coefficient \( \xi \) is expressed as follows. If \( d_{am} > d_{we} \),

\[ \xi = 1 + \frac{2460 \cdot (d_{am} - d_{we})}{t_{am} - t_{we}} \], Where, \( t_{am} = \frac{t_{a1} - t_{a2}}{2} \), \( t_{a1}, t_{a2} \) are the mixture temperatures at inlet and outlet. If \( d_{am} < d_{we} \), \( \xi = 1 \). \( d_{am} \) and \( d_{we} \) refer to the mean humidity ratio of the moist air and humidity ratio of saturated air respectively, and \( t_{am} \) and \( t_{we} \) are the mean temperature of the moist air and the wall temperature of tube respectively.

The surface effectiveness \( \eta_s \) can be calculated as:

\[ \eta_s = 1 - \frac{a_f}{a_o} (1 - \eta_f) \] (45)

\( a_f \) – the area of the fin per meter of the tube, \( a_f = \frac{2(S_1 \cdot S_2 - \pi \cdot d_b^2)}{S_f} \);

\( a_b \) – The external surface area of tube per meter tube, \( a_b = \frac{\pi \cdot d_b \cdot (S_f - \delta_f)}{S_f} \);

\( d_b \) – The external diameter of fin collar, \( d_b = d_0 + 2\delta_f \);

\( a_a \) – The total area of the heat exchanger per unit, \( a_a = a_f + a_b \);

\( S_1 \) – Depth of the heat exchanger perpendicular to airflow direction;

\( S_2 \) – Depth of the heat exchanger in airflow direction, \( S_2 = \frac{\sqrt{3} S_1}{2} \);

\( d_0 \) – External diameter of tube;

\( \delta_f \) – Thickness of the fin;

\( S_f \) – Fin spacing;

The fin effectiveness \( \eta_f \) is defined as:
\[ \eta_f = \frac{\tanh(m \cdot h')}{m \cdot h'} \quad (46) \]

where \( m = \sqrt{\frac{2 \cdot \xi \cdot \alpha_f}{\lambda_f \cdot \delta_f}} \).

The equivalent height of the fin \( h': h' = 0.5d_b \cdot (\rho' - 1) \cdot (1 + 0.35 \ln \rho') \)

For the staggered finned tube banks \( \rho' \), \( \rho' = 1.27 \cdot \frac{S_1}{d_b} \cdot \frac{l_1}{l_2} - 0.3 \)

\( l_1 \) – The long side of the hexagon fin;

\( l_2 \) – The short side of the hexagon fin;

In this case, \( l_1 = l_2 \) for the Equilateral hexagon.

The heat transfer coefficient of air side can be expressed as:

\[ \alpha''_o = C \cdot \frac{\lambda_f}{d_{eq}} \cdot \text{Re}_f^n \cdot \left( \frac{S_2}{d_{eq}} \right)^m \quad (47) \]

Here,

\[ A = 0.518 - 0.02315 \left( \frac{S_2}{d_{eq}} \right) + 0.000425 \left( \frac{S_2}{d_{eq}} \right)^2 - 3 \times 10^{-6} \left( \frac{S_2}{d_{eq}} \right)^3; \]

\[ C = A \cdot (1.36 - \frac{0.24 \text{Re}_f}{1000}); \]

\[ n = 0.45 + 0.0066 \cdot \frac{S_2}{d_{eq}}; \]

\[ m = -0.28 + 0.08 \cdot \frac{\text{Re}_f}{1000}; \]

Wherein,

\[ d_{eq} = \frac{2(S_1 - d_o) \cdot (S_f - \delta_f)}{(S_1 - d_o) + (S_f - \delta_f)}; \]

\[ \text{Re}_f = \frac{w_{\max} \cdot d_{eq}}{v_f}; \]

Wherein,
\[ \nu_f - \text{Kinematic viscosity, m}^2/\text{s}; \]

\[ w_{\text{max}} - \text{The wind velocity flowing through the narrowest area,} \]

\[ w_{\text{max}} = \frac{S_f \cdot S_1 \cdot w_f}{(S_1 - d_n) \cdot (S_f - \delta_f)}; \]

Therefore, the equivalent heat transfer coefficient of fin side can be expressed as:

\[ \alpha_e = \alpha'_e \cdot k_1 \cdot k_2 \] (48)

\[ k_1 - \text{Correction coefficient of configuration, } k_1 = 1.1 \text{ for staggered arrangement;} \]

\[ k_2 - \text{Correction coefficient of fin style, } k_2 = 1.2 \text{ for wavy fin;} \]

(1) The heat transfer coefficient of tube side

The tube side heat transfer coefficient \( \alpha_t \) for Evaporator-HP is evaluated from Kandlikar correlation [41, 42].

\[ \frac{\alpha_t}{\alpha_i} = C_1 \cdot (C_0)_{C_i} \cdot (25F \cdot r_f)^{C_i} + C_3 \cdot (B_0)_{C_i} \cdot F_r \] (49)

Wherein,

\[ \alpha_i - \text{The heat transfer coefficient of the liquid refrigerant flowing in the tube,} \]

\[ \alpha_i = 0.023 \left( \frac{g \cdot (1-x) \cdot d_l}{\mu_l} \right)^{0.8} \frac{Pr_l^{0.4} \cdot \lambda_l}{d_l}; \]

\[ C_0 - \text{Convection number, } C_0 = \left( \frac{1-x}{x} \right)^{0.8} \left( \frac{\rho_g}{\rho_l} \right)^{0.5}; \]

\[ B_0 - \text{Boiling number, } B_0 = \frac{q}{g \cdot r}; \]

\[ F_r - \text{Froude number with all flow as liquid, } F_r = \frac{g^2}{9.8 \rho_l^{2} \cdot d_l}; \]

\[ g - \text{Mass flow per unit area per unit time, } \text{kg/(m}^2\cdot\text{s}); \]

\[ \lambda_l - \text{Thermal conductivity of liquid phase, } \text{W/(m} \cdot \text{K)}; \]
\( P_{rl} \) – Prandtl number of liquid phase;

\( \rho_g \) – Density of gas phase, kg/m\(^3\);

\( x \) – Quality;

\( q \) – Heat flux, w/m\(^2\);

\( \rho_l \) – Density of liquid phase, kg/m\(^3\);

\( F_{fl} \) – Determined by working fluid, for R134a, \( F_{fl} = 1.63 \);

if \( C_0 < 0.65 \), \( C_1 = 1.136, C_2 = -0.9, C_3 = 667.2, C_5 = 0.3 \);

if \( C_0 > 0.65 \), \( C_1 = 0.6683, C_2 = -0.2, C_3 = 1058.0, C_5 = 0.3 \);

The overall heat transfer coefficient can be determined from the knowledge of the inside (tube side) and outside (fin side) heat transfer coefficient.

\[
K_{\text{fin-tube}} = \frac{1}{\frac{1}{\alpha_i} \cdot \frac{a_f}{a_i} + \frac{\delta_f}{\lambda} \cdot \frac{a_{of}}{a_m} + r_b + r_o + \frac{1}{\alpha_o \cdot \eta_s}}
\]  

(50)

\( \lambda \) – Thermal conductivity of brass, 383W/m·K;

\( r_0 \) – thermal resistance, 0.001 m\(^2\)K/W;

\( r_b \) – contact resistance of between the fin and the tube, 0.0048;

Based on the heat transfer coefficient above, the heat transfer area of fin side can be obtained as:

\[
A_o = \frac{Q}{K_{\text{fin-tube}}/\Delta T_n} \]  

(51)

\[
A_i = A_o \cdot (\frac{a_o}{\pi d_i}) \]  

(52)

\[
l = A_o / a_o \]  

(53)

\( d_i \) – Internal diameter of tube;
4. Results and discussions

4.1 Comparison between two models

The proposed system was modelled using both ASPEN Plus and the present Refprop/Matlab code, based on the same working condition. In order to compare the simulations of the proposed system using these two models, we selected one case with the evaporating temperature of the heat pump $T_{eva2} = 270.33$ K, the temperature of the gas mixture exiting the Evaporator-HP $T_{15} = 275.55$ K, and an ambient air temperature of $T_0 = 278.15$ K. Tables 4 and 5 show the comparison between the key parameters calculated using these two models.

Table 4 summarises the calculated the temperatures, heat transfer, work transfer within the key components of the proposed system, as well as the difference between the two models. An Aspen plus model is used as the benchmark. The calculated temperature using these two models are all very close to each other, and the differences are all less than 1%. It can also be seen that there is only very small difference (namely <4.8%) in the calculated heat transfer at each heat changers between the predictions by the two models. This can be attributed to the different control strategies and the different PPTD approaches used for calculating the heat
transfer. For example, the heat absorbed by water in the Condenser-HP ($Q_{\text{cond-HP}}$) predicted by Aspen plus model is 1.5% higher than that of the present model, while the heat rejected by the Condenser-ORC calculated by the Aspen plus model is 4.8% lower than that of the present model. For the Aspen Plus model, the water temperature $T_{16}$ is controlled directly by adjusting the mass flow rate of the fuel, while for the present Refprop/Matlab model, both the temperature of flue gas $T_e$ and the mass flow rate of natural gas can be adjusted.

Table 5 shows the calculated performance indicators of this case study using these two models. The $COP$ of the heat pump subsystem calculated by the Aspen model is around 5.7% higher than that predicted by the present model, while the thermal efficiencies predicted by two models are almost the same. The total heat power output predicted by the present model is 1.4% higher than the Aspen plus model. The predicted overall fuel-to-heat energy efficiencies are almost the same.

Nevertheless, the results shown in Table 4 and 5 demonstrate a good agreement between the results calculated using these two models, offering us the confidence to use the present model to further analyse the performance and characteristics of the proposed system. In this study, Refprop/Matlab is adopted in the following analysis since the modelling of each component and the control strategy sometimes are not available for the Aspen Plus model, which is a disadvantage for the designing and system optimization. However, the Refprop/Matlab model can overcome this disadvantage. This is the reason why Refprop/Matlab model was adopted.
4.2 Comparison of system performance using different ORC working fluids

The performance of thermodynamic cycles strongly depend on the properties of working fluids. Ideally, the working fluids should meet a list of criteria such as stability, non-fouling, non-corrosiveness, non-toxicity and non-flammability. However, not all the desired criteria could be satisfied in the present ORC design. Dreschler et al. [43] proposed working fluid selection based on thermal efficiency as the key criterion.

A comparison of system performance between R245fa, R123 and hexane are conducted and the result are shown in Table 6. The system with different working fluids are assumed to operate under the same working conditions. The evaporation pressure and condensation temperature of ORC are set to be 3000 kPa and 61 °C, respectively.

There are three parameters to evaluate the system performance, thermal efficiency for ORC, COP for heat pump cycle, and the overall fuel-to-heat efficiency for the whole system. It can be seen in Table 6 that the maximum thermal efficiency of ORC can be obtained by using hexane and that the maximum COP can be obtained by using R245fa. The maximum fuel-to-heat efficiency was achieved using hexane when the ambient temperature is in the range from -5 °C to 5 °C. In order to explore theoretical limit of performance of this system, hexane is used as the ORC working fluid in the following. As explained early on, although the hexane is flammable, mixing hexane and retardant working fluid is a possible way to maintain its high efficiency but overcome the issues of its flammability.
4.3 Effects of the ambient air temperature and the condensation temperature of the ORC

Using the obtained model that has been verified by Aspen Plus to some extent, a set of comprehensive simulations has then conducted to optimise the proposed system in response to the changing ambient temperatures. The design target is to heat cold tap water from 10 to 65 °C and deliver 20 kW heating power when the ambient air temperature varies in the range from -5 to 5 °C. The condensation temperature of ORC $T_{\text{con1}}$ is an important parameter that strongly affects the water temperature $T_{13}$, the exiting temperature of flue gas $T_6$, and then the final hot water temperature $T_{14}$. Therefore, the effects of the ambient temperature $T_0$ and the condensation temperature $T_{\text{con1}}$ on the performance of the whole system are studied in detail. The evaporation pressure within the Evaporator-ORC is set at 3000 kPa since the ORC can obtain relatively high thermal efficiency when it is operated closed to the critical pressure.

In practical applications, mass flow rates of pumps or compressors are usually used to control the operation of thermodynamic systems. There are three fluids exchanging energy with the system, including natural gas, fresh air, and water. The mass flow rate of water is 0.087 kg/s and both the temperature of cold tap water and the target temperature are set as 10 and 65 °C, respectively. So only the mass flow rates of natural gas and fresh air are variable. In this system, the mass flow rate of natural gas is controlled to match the system with the changing ambient air temperature.
As shown in Fig. 3, the mass flow rate of natural gas decreases as ambient air temperature increases from -5 to 5 °C for any given condensation temperature within the Condenser-ORC. More natural gas is required to compensate the reduction of heat extracted from the ambient air as its temperature decreases. For a given ambient temperature, the mass flow rate of the required natural gas firstly decreases and then increases as the condensation temperature the ORC system increases from 55 to 62 °C, leading to a minimum value when the ORC condensation temperature is around 61 °C.

Fig. 4 presents the variation of mass flow rate of fresh air as ambient temperature varies. It is determined by the heat required in the Evaporator-HP and the residual heat carried by the flue gases. We can control the mass flow rate of the natural gas burnt in the burner to control the temperature of the mixture of fresh air and flue gases before they enter the Evaporator-HP. For the combined ORC-HP system, the thermal energy leaving it has two parts: the heat output carried away by the hot water, which is kept constant as 20 kW; and the thermal energy carried away by the gas mixture exiting the Evaporator-HP. The energy input also includes two parts: the heat produced by the combustion of natural gas and the heat extracted from fresh air flowing in the system. According to the first law of thermodynamic, all the energy entering the system should equate to the energy leaving it. Hence, the variation trend of mass flow rate of fresh air in Fig. 4 is opposite to that of the mass flow rate of natural gas as shown in Fig. 3.

In general, for a given condensation temperature within the Condenser-ORC, as
the ambient air temperature increases, the mass flow rate of air increases so more heat can be extracted from it (see Fig. 4), and less natural gas is needed (see Fig. 3). For a given ambient temperature, as the condensation temperature of ORC increases from 55 to 62 °C, the required air flow rate firstly increases and then decreases.

Figures 5 and 6 present the effects of the ambient air temperature and the ORC condensation temperature on the heat capacities of heat exchangers, including the Condenser-HP, Condenser-ORC, and the post heater. In Fig. 5, heat capacities of both the Condenser-HP and the post heater increase as the ambient air temperature increases, while that of the Condenser-ORC decreases, when the total heat output is kept as 20 kW. As the temperature of ambient air increases, the evaporating temperature of the heat pump and the exiting temperature $T_{15}$ rises. On the one hand, more heat energy can be extracted from the mixture of fresh air and flue gases via the Evaporator-HP. Consequently, the water temperature $T_{12}$ increases. On the other hand, less natural gas is needed in the burner, and consequently less power output is generated by the ORC system for a given condensation temperature in the Condenser-ORC. As a result, the heat capacity of Condenser-ORC decreases.

In Fig. 6, the ambient temperature is kept constant at 5 °C. The heat capacity of the post heater firstly decreases as the ORC condensation temperature increases, and then stays constant when it is above 60 °C. This is due to a fixed temperature $T_6$. The calculated heat capacities of the two condensers both increase as the ORC condensation temperature increases. This can be attributed to that the temperature of flue gases is set as above 120 °C to prevent corrosion in the Evaporator-ORC.
Figure 7 presents the share of the heat supply to the water through those three heat exchangers under various operational conditions. Interestingly, the results clearly show that the heat supply is dominated by the two condensers (i.e., Condenser-HP and Condenser-ORC), while the heat supplied through the post heater is marginal. For a given ORC condensation temperature, the variation of ambient air temperature has little effect on the distribution of heat supply. It can also be seen that, for a given ambient air temperature, the ORC condensation temperature can strongly affect the distribution of heat supply. For example, when $T_0=5 \, ^\circ\text{C}$, the heat supplied by the post heater decreases from 13.61% to 1.42% when the ORC temperature increases from 55 to 61 °C. From the analysis above, it can be inferred that the post heater can only supply very limited amount of heat to the water, especially when ORC condensation temperature is high. Hence, it might be beneficial to remove it from the system to improve the overall cost-effectiveness.

Figures 8, 9 and 10 show the calculated heat transfer area of the Condenser-HP, Condenser-ORC, and the post heater, respectively. According to Equation (39), the heat transfer area of each heat exchanger depends on the heat capacity, the heat transfer coefficient and LMTD.

In Fig. 8, for a given ORC condensation temperature, the heat transfer area of Condenser-HP decreases as the ambient temperature increases. As mentioned in the modelling section, the heat transfer area of the Condenser-HP is divided into two parts, the single phase part from State 8 to State E (see in Fig. 2) and the condensation process (two phase) part from State E to State 9. The variation of ambient temperature
directly affects the condensation temperature of the heat pump. As the ambient
temperature increases, the heat transfer coefficient of both single phase and
condensation process increase due to a higher condensation temperature in the heat
pump. The LMTD of condensation process part decreases while that of the single
phase part increases. The combined effects of these factors will lead to a decrease in
the heat transfer area of single phase part and an increase in condensation process part.
As the reduction of heat transfer area in the single phase part is larger, the total heat
transfer area of Condenser-HP presents a downward trend.

Figure 9 shows that, for a given ORC condensation temperature, the heat transfer
area of Condenser-ORC decreases as the ambient temperature increases. Moreover,
there exists a minimum heat transfer area of the Condenser-ORC, which first
decreases and then increases when the ORC condensation temperature increases. The
method used for the heat transfer area calculation of the Condenser-HP is also applied
to calculate the heat transfer area of Condenser-ORC. As mentioned above, a higher
ambient temperature will lead to a higher condensation temperature of the heat pump,
thus a higher water temperature $T_{12}$ and a higher heat transfer coefficient. The LMTD
of single-phase part decreases, while that of the condensation process part increases.
Similarly, the heat transfer area of Condenser-ORC increases with the ambient
temperature.

In the post heater, the heat transfer area is calculated by using the correlation for
the single-phase since no phase change happens during the heat transfer process. It is
clearly shown in Fig. 10 that the heat transfer area of the post heater decreases with
the increasing ambient temperature and condensation temperature. As the ambient
temperature increases, the inlet temperatures of flue gases $T_6$ increases while the
water temperature $T_{13}$ decreases. Although the heat capacity of the post heater
increases (see in Fig.5), the heat transfer area of the post heater decreases as both the
LMTD and heat transfer coefficient increase. Moreover, as the heat transfer
coefficient without phase change is much smaller than that with phase change, the
heat transfer area of the post heater with smaller heat capacity is even higher than that
of the two condensers.

A fin-and-tube heat exchanger is adopted for the Evaporator-HP because the heat
transfer area at the air side is much larger than other heat exchangers. In Fig. 11, the
left graph shows the calculated heat transfer area at fin side as a function of ORC
condensation temperature and ambient air temperature. In general, it increases as the
ambient air temperature increases for all tested ORC condensation temperatures. For a
given ambient temperature, the fin area firstly increases and then decreases as the
ORC condensation temperature increases from 55 to 62 °C. The right graph in Fig. 11
shows the calculated heat transfer areas at tube side as a function of ORC
condensation temperature and ambient air temperature. The variation of the required
heat transfer area of tube side is the similar to that of fin side since the ratio between
them remains constant.

The effect of the ORC condensation temperature on the thermal efficiency of
ORC is shown in Fig. 12. The results show that the thermal efficiency of the ORC
system decreases as its condensation increases. Ideally, a low condensation
temperature is preferred to obtain high thermal efficiency. However, trade-off is needed to achieve the maximum overall fuel-to-heat efficiency in the proposed system.

The $COP$ of a heat pump strongly depends on the temperature lift between its evaporator and condenser. The exiting temperature $T_{15}$ of the mixture should be lower than the ambient air temperature in order to extract heat energy from fresh air. Therefore, the evaporation temperature in the Evaporator-HP needs to be set below the ambient air temperature. Fig. 13 shows the calculated $COP$ as a function of ambient air temperature for different ORC condensation temperatures. The results show that heat pump can reach a $COP$ up to 5.56.

A fuel-to-heat efficiency is introduced in this research to evaluate the performance of different heating technologies. Fig. 14 shows that the calculated fuel-to-heat efficiency as a function of ambient air temperature and the ORC condensation temperature. In general, it increases as the ambient air temperature increases this is because the HP has a higher $COP$ at higher ambient air temperature as shown in Fig. 13. For a given ambient temperature, it firstly increases and then decreases when the ORC condensation temperature increases. It reaches its maximum value at the condensation temperature of 61 °C. Among all the operation conditions considered, the fuel-to-heat efficiency reaches its maximum value of 1.47 when the ORC condensation temperature and the ambient temperature is 61 °C and 5 °C, respectively.
4.3 Optimised model

4.3.1 Optimised working condition

Through the analysis above, the integrated system can then be optimised to achieve the maximum fuel-to-heat energy efficiency. One case with an ambient air temperature of 5 °C is used as example to demonstrate an optimised model. The optimum ORC condensation temperature was calculated as 61 °C as shown in Fig. 14. The properties and key parameters of the optimised model are summarised in Tables 7 and 8.

Table 7 lists the properties of different state points of the combined cycles as shown in Fig. 1. Table 8 shows the heat supply of the three heat exchangers. The water is heated in Condenser-HP, Condenser-ORC, and post heater in series, and the heat supply by these three heat exchangers are 9.42 kW, 10.29 kW, and 0.28 kW, respectively. The natural gas flow rate is 0.000244 kg/s, and heat produced in the burner is 13.66 kW. The Evaporator-ORC transfers 11.79 kW heat to the ORC system, and it produces 1.81 kW shaft power. The HP extract 7.61 kW heat from the gas mixture via the evaporator, and delivers 9.42 kW heat to the water via its condenser at a temperature of 36.3 °C. The original purpose of using the post heater is to recover heat from the flue gas. It is clearly shown that the heat supply by the post heater accounts for only 1.41% of the total heat energy.

4.3.2 Effect of post heater

From the above analysis we can learn that the post heater plays limited role,
especially for a relative high ORC condensation temperature. Therefore, the system performance of the model without post heater was investigated in this part and the results are compared with that of the model with post heater.

As shown in Table 9, for both models, the thermal efficiency of the ORC subsystem and the COP of the heat pump subsystem are more or less the same, while the fuel-to-heat efficiency of system without a post heater is a bit smaller than that of the system with a post heater. After removing the post heater, the temperature of exhaust gas $T_{17}$ becomes higher, which will require less fresh air since the evaporation temperature in Evaporator-HP is fixed. As a result, less energy can be exacted from the ambient air thus lead to a slightly lower fuel-to-heat efficiency.

5. Conclusion

This paper proposes a novel natural gas fuelled water heater that essentially integrates an ORC power plant with a heat pump, with waste heat recovery. Heat energy produced from combustion of natural gas in a burner powers an ORC power plant. The generated power directly drives a vapour compression cycle heat pump. Cold tap water is heated by several heat exchangers in series to gradually increase its temperature. The flue gas exiting is mixed with ambient air to further extract its residual heat in the evaporator of the heat pump.

The simulation results demonstrated that the proposed system has great potential to improve the efficiency of domestic hot water applications. The advantages of the proposed system include:
(1) Unlike using a heat pump to directly heat cold tap water to the required
temperature, the proposed system raises the water temperature using several heat
exchangers in series, so the condensation temperature of the heat pump can be
significantly reduced, leading to a much higher \textit{COP}.

(2) The waste heat of the ORC power cycle and flues have been recovered, so the
overall fuel-to-heat energy utilisation efficiency of natural gas can be maximised.

(3) The proposed system can adjust the heating loads between the
Condenser-ORC and Condenser-HP when the ambient air temperature varies.

(4) The mechanical power of expander of the ORC cycle directly drives the
compressor of the heat pump, so it eliminates the electrical generator or motor, and
thus avoids the transduction losses

(5) For all the tested conditions, the research results show that there is an
optimum ORC condensation temperature of 61 °C to deliver hot water at 65 °C when
the ambient air temperature is 5 °C, leading to a maximum fuel-to-heat efficiency of
147%.

(6) It was also found that the post heater could only provide a very small amount
of heat supply. From the thermodynamic point of view, a post heater could slightly
improve the fuel-to-heat efficiency. However, from viewpoint of cost-effectiveness, it
may be removed from the proposed system so the whole system and control strategy
can be largely simplified.

In a summary, this research shows that the proposed natural gas fuelled water
heater can potentially achieve a much higher fuel-to-heat energy efficiency than other
gas fuelled heating technologies. It has great potential to make a contribution to the carbon reduction of domestic heating sector.

Acknowledgment

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Reference


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<td>$B_o$</td>
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<tr>
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<tr>
<td>$C_o$</td>
<td>Convection number</td>
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<td>Specific heat at constant pressure</td>
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<td>Coefficient of performance</td>
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<td>$d_0$</td>
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<td>Mean humidity ratio of wet air</td>
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<td>$d_i$</td>
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<td>$F_{fl}$</td>
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<td>Efficiency</td>
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<td>ORC</td>
<td>Organic Rankine cycle</td>
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<td>Coefficient of performance</td>
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<td>Pinch point temperature difference</td>
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<td>Logarithmic mean temperature difference</td>
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**Nomenclature**
Fig. 1 Schematic diagram of the proposed natural gas fuelled water heater system

Fig. 2 Temperature-Entropy diagrams of both the ORC (hexane) and heat pump cycle (R134a)
Fig. 3 The required natural gas for delivering 20 kW at 65 °C hot water supply to match different ambient temperatures and ORC condensation temperatures.

Fig. 4 The required fresh air to be mixed with the flue gas before entering the inlet of Evaporator-HP to match the different ambient temperature.
Fig. 5 Heat capacities of the three heat exchangers of the system for a given ORC condensation temperature of 61 °C.

Fig. 6 Heat capacities of the three heat exchangers of the system for a given ambient air temperature of 5 °C.
Fig. 7 Share of heat supply by the three heat exchangers under different optional conditions: (a) $T_{\text{cond-ORC}}=55 \, ^\circ\text{C}$ and $T_0=-5 \, ^\circ\text{C}$; (b) $T_{\text{cond-ORC}}=55 \, ^\circ\text{C}$ and $T_0=5 \, ^\circ\text{C}$; (c) $T_{\text{cond-ORC}}=61 \, ^\circ\text{C}$ and $T_0=-5 \, ^\circ\text{C}$; (d) $T_{\text{cond-ORC}}=61 \, ^\circ\text{C}$ and $T_0=5 \, ^\circ\text{C}$;
Fig. 8 The required heat transfer area of the Condenser-HP

Fig. 9 The required heat transfer area of the Condenser-ORC

Fig. 10 The required heat transfer area of Post heater
Fig. 11 The required heat transfer area of both the fin side and tube side of the Evaporator-HP

Fig. 12 The effect of ORC condensation temperature on thermal efficiency of ORC
Fig. 13: The effect of ambient temperature on the COP of the heat pump.

Fig. 14: The fuel-to-heat efficiency against ambient temperature and ORC condensation temperature.
Table 1 Key operating parameters of the combined cycles

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atmosphere pressure, $P_0$ [kPa]</td>
<td>101</td>
</tr>
<tr>
<td>Efficiency of pump, $\eta_1$</td>
<td>0.9</td>
</tr>
<tr>
<td>Efficiency of turbine, $\eta_2$</td>
<td>0.7</td>
</tr>
<tr>
<td>Efficiency of compressor, $\eta_3$</td>
<td>0.7</td>
</tr>
<tr>
<td>Efficiency of combustion, $\eta_4$</td>
<td>1</td>
</tr>
<tr>
<td>Temperature of return water, $T_{11}$ [°C]</td>
<td>10</td>
</tr>
<tr>
<td>PPTD in evaporator of ORC, [°C]</td>
<td>30</td>
</tr>
<tr>
<td>PPTD in Post heater, [°C]</td>
<td>5</td>
</tr>
<tr>
<td>PPTD in condenser of heat pump, [°C]</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 2 Key geometric dimensions of plate heat exchangers [42]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chevron angle $\beta$ [degree]</td>
<td>60</td>
</tr>
<tr>
<td>Surface enlargement factor, $\phi [/]$</td>
<td>1.117</td>
</tr>
<tr>
<td>Plate width, $L_w$ [m]</td>
<td>0.119</td>
</tr>
<tr>
<td>Plate thickness, $t$ [m]</td>
<td>0.0003</td>
</tr>
<tr>
<td>Mean asperity height, $R_p$ [μm]</td>
<td>0.3</td>
</tr>
<tr>
<td>Corrugation depth, $b_1$ [m]</td>
<td>0.00224</td>
</tr>
<tr>
<td>Equivalent diameter of liquid side, $d_{eq}$ [m]</td>
<td>0.004</td>
</tr>
<tr>
<td>Equivalent diameter of gas side, $d_{eqg}$ [m]</td>
<td>0.009</td>
</tr>
<tr>
<td>Coefficient of thermal conductivity, $\lambda$ [kW/(m·K)]</td>
<td>0.00163</td>
</tr>
</tbody>
</table>
Table 3 Key geometric dimensions of fin and tube heat exchangers

<table>
<thead>
<tr>
<th>Parameters</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tube outside diameter, $d_0$ [m]</td>
<td>0.00952</td>
</tr>
<tr>
<td>Depth of the heat exchanger perpendicular to airflow direction, $S_1$ [m]</td>
<td>0.0254</td>
</tr>
<tr>
<td>Depth of the heat exchanger in airflow direction, $S_2$ [m]</td>
<td>0.022</td>
</tr>
<tr>
<td>Fin spacing, $S_f$ [m]</td>
<td>0.0002</td>
</tr>
<tr>
<td>Thickness of the fin, $\delta_f$ [m]</td>
<td>0.000014</td>
</tr>
<tr>
<td>Face velocity, $W_f$ [m/s]</td>
<td>3</td>
</tr>
<tr>
<td>Fouling resistance of tube side, $r_0$ [m²K/kW]</td>
<td>2.5</td>
</tr>
<tr>
<td>Fouling resistance of fin side, $r_1$ [m²K/kW]</td>
<td>1</td>
</tr>
<tr>
<td>Thermal conductivity of copper, $\lambda_c$ [kW/(m·K)]</td>
<td>0.383</td>
</tr>
<tr>
<td>Thermal conductivity of aluminium, $\lambda_{al}$ [kW/(m·K)]</td>
<td>0.22</td>
</tr>
<tr>
<td>Mean humidity ratio of wet air, $d_{am}$ [g/kg]</td>
<td>5.1</td>
</tr>
</tbody>
</table>

Table 4 Parameters of the combined cycle

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Heat pump cycle</th>
<th>ORC cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ASPEN Plus</td>
<td>Refprop/ Matlab</td>
</tr>
<tr>
<td>$T_{12}$, K</td>
<td>307.05</td>
<td>306.48</td>
</tr>
<tr>
<td>$T_{con2}$, K</td>
<td>305.75</td>
<td>307.98</td>
</tr>
<tr>
<td>$T_{eva2}$, K</td>
<td>270.33</td>
<td>270.33</td>
</tr>
<tr>
<td>$T_{15}$, K</td>
<td>274.55</td>
<td>274.55</td>
</tr>
<tr>
<td>$Q_{con-HP}$, kW</td>
<td>10.01</td>
<td>9.86</td>
</tr>
<tr>
<td>$Q_{eva-HP}$, kW</td>
<td>8.18</td>
<td>7.94</td>
</tr>
<tr>
<td>$m_{f2}$, kg/s</td>
<td>0.054</td>
<td>0.054</td>
</tr>
<tr>
<td>$W_{com}$, kW</td>
<td>1.84</td>
<td>1.92</td>
</tr>
<tr>
<td>$Q_{post}$, kW</td>
<td>1.16</td>
<td>1.13</td>
</tr>
</tbody>
</table>
### Table 5 Evaluation of System performance

<table>
<thead>
<tr>
<th></th>
<th>ASPEN Plus</th>
<th>Refprop/Matlab</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combusted energy of fuel</td>
<td>15.6</td>
<td>15.6</td>
<td>0</td>
</tr>
<tr>
<td>COP of heat pump</td>
<td>5.45</td>
<td>5.14</td>
<td>5.7%</td>
</tr>
<tr>
<td>ORC thermal efficiency, %</td>
<td>14.92</td>
<td>14.96</td>
<td>0.3%</td>
</tr>
<tr>
<td>Overall fuel-to-heat efficiency</td>
<td>136.0</td>
<td>137.2</td>
<td>0.9%</td>
</tr>
<tr>
<td>Total heat absorbed by water, kW</td>
<td>21.07</td>
<td>21.36</td>
<td>1.4%</td>
</tr>
</tbody>
</table>

### Table 6 Comparison of system performance using different ORC working fluids

<table>
<thead>
<tr>
<th>Working fluid</th>
<th>Fuel-to-heat efficiency</th>
<th>Thermal efficiency</th>
<th>COP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hexane</td>
<td>R245fa</td>
<td>R123</td>
</tr>
<tr>
<td>$T_0=-5$</td>
<td>1.343</td>
<td>1.208</td>
<td>1.254</td>
</tr>
<tr>
<td>$T_0=-3$</td>
<td>1.366</td>
<td>1.224</td>
<td>1.271</td>
</tr>
<tr>
<td>$T_0=-1$</td>
<td>1.389</td>
<td>1.240</td>
<td>1.288</td>
</tr>
<tr>
<td>$T_0=1$</td>
<td>1.414</td>
<td>1.257</td>
<td>1.307</td>
</tr>
<tr>
<td>$T_0=3$</td>
<td>1.441</td>
<td>1.275</td>
<td>1.327</td>
</tr>
<tr>
<td>$T_0=5$</td>
<td>1.471</td>
<td>1.296</td>
<td>1.349</td>
</tr>
</tbody>
</table>
Table 7 Parameters of different states at optimal condition

<table>
<thead>
<tr>
<th>state</th>
<th>T(°C)</th>
<th>P(kPa)</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>61</td>
<td>79</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>62.2</td>
<td>3000</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>233.8</td>
<td>3000</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>139.0</td>
<td>79</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1809</td>
<td>101</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>120</td>
<td>101</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>0.5</td>
<td>298</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>51.7</td>
<td>958</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>37.8</td>
<td>958</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>0.5</td>
<td>298</td>
<td>0.26</td>
</tr>
<tr>
<td>11</td>
<td>10</td>
<td>101</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>36.3</td>
<td>101</td>
<td>0</td>
</tr>
<tr>
<td>13</td>
<td>65.1</td>
<td>101</td>
<td>1</td>
</tr>
<tr>
<td>14</td>
<td>5.1</td>
<td>101</td>
<td>1</td>
</tr>
<tr>
<td>15</td>
<td>3.5</td>
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<td>1</td>
</tr>
<tr>
<td>16</td>
<td>65.9</td>
<td>101</td>
<td>1</td>
</tr>
<tr>
<td>17</td>
<td>71.7</td>
<td>101</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 8 Heat transfer rate and power transfer of the key components

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat capacity of Condenser-ORC, kW</td>
<td>10.29</td>
</tr>
<tr>
<td>Heat capacity of Condenser-HP, kW</td>
<td>9.42</td>
</tr>
<tr>
<td>Heat capacity of the Post Heater, kW</td>
<td>0.28</td>
</tr>
<tr>
<td>Mass flow rate of natural gas, kg/s</td>
<td>0.000244</td>
</tr>
<tr>
<td>Heat produced by the burner, kW</td>
<td>13.66</td>
</tr>
<tr>
<td>Heat capacity of Evaporator-ORC, kW</td>
<td>11.79</td>
</tr>
<tr>
<td>Power generated in Expander, kW</td>
<td>1.81</td>
</tr>
<tr>
<td>Mechanical work consumed by pump, kW</td>
<td>0.108</td>
</tr>
<tr>
<td>Mechanical work consumed by compressor, kW</td>
<td>1.81</td>
</tr>
<tr>
<td>Heat capacity of Evaporator-heat pump, kW</td>
<td>7.61</td>
</tr>
<tr>
<td>Thermal efficiency of ORC</td>
<td>15.34%</td>
</tr>
<tr>
<td>COP</td>
<td>5.21</td>
</tr>
<tr>
<td>Fuel-to-heat efficiency of the combined cycle</td>
<td>147.1%</td>
</tr>
</tbody>
</table>
Table 9 Comparison of system performance with and without post heater

<table>
<thead>
<tr>
<th>Ambient temperature</th>
<th>Fuel-to-heat efficiency</th>
<th>Thermal efficiency of ORC</th>
<th>COP</th>
<th>Fuel-to-heat efficiency</th>
<th>Thermal efficiency of ORC</th>
<th>COP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_0=-5$</td>
<td>1.343</td>
<td>4.256</td>
<td>1.323</td>
<td></td>
<td></td>
<td>4.256</td>
</tr>
<tr>
<td>$T_0=-3$</td>
<td>1.366</td>
<td>4.421</td>
<td>1.345</td>
<td></td>
<td></td>
<td>4.421</td>
</tr>
<tr>
<td>$T_0=-1$</td>
<td>1.389</td>
<td>0.153</td>
<td>4.597</td>
<td>1.369</td>
<td>0.153</td>
<td>4.597</td>
</tr>
<tr>
<td>$T_0=1$</td>
<td>1.414</td>
<td>4.786</td>
<td>1.394</td>
<td></td>
<td></td>
<td>4.786</td>
</tr>
<tr>
<td>$T_0=3$</td>
<td>1.441</td>
<td>4.989</td>
<td>1.421</td>
<td></td>
<td></td>
<td>4.989</td>
</tr>
<tr>
<td>$T_0=5$</td>
<td>1.471</td>
<td>5.206</td>
<td>1.450</td>
<td></td>
<td></td>
<td>5.206</td>
</tr>
</tbody>
</table>