Nonlinear properties of the shear dynamo model

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Solar surface features exist on a range of scales. Convection is granulated: granules (∼ 1,500km), mesogranules (5,000 – 10,000km ?), supergranules (∼ 30,000km)
Solar dynamo

- Magnetic field also exists on a range of scales from the granular bright spots to global scales
- Sunspots, flares, prominences, etc.
- 11-year solar cycle evidenced by sunspot activity
Small-scale field: turbulent motions of plasmas amplify magnetic fluctuations via fluctuation dynamo effect

Large-scale field: more complicated, traditionally modeled using mean-field theory

A flow with global net helicity twists and stretches field lines

Large-scale field generated by the \( \alpha \)-effect
Problems with mean-field theory

- Is mean-field theory valid in solar conditions?
- Mean-field theory should only apply when $Rm = UL/\eta$ is small, yet $Rm_{\odot} \gg 1$
- At large $Rms$ increased turbulence causes models to be dominated by small-scale fields
- Poorly correlated EMFs (due to turbulence) lead to a small $\alpha$-effect in large domains relevant to the Sun
Alternatives to mean-field theory

If the mean-field ansatz is not valid under solar conditions then a new mechanism for generating large-scale field is required.

Several proposals have suggested a combination of turbulence and shear to produce large-scale field:

- enhancement of $\alpha$ via greater correlation of small-scale motions by the shear (Courvoisier et al., 2009)
- interaction with a fluctuating $\alpha$-effect (Richardson & Proctor, 2012)
- shear dynamo model (Yousef et al., 2008)
Periodic box MHD simulations performed in a long domain to reduce computing requirements

Forced non-helical motion (no $\alpha$-effect) in the presence of a uniform shear

Large-scale structures in magnetic field can be generated (Yousef et al., 2008)

Structures wander in time and space
Solve the incompressible MHD equations in the presence of a uniform shear flow, \( \mathbf{U} = -Sx\mathbf{\hat{y}} \)

Shear-periodic box subject to a white-noise nonhelical homogeneous isotropic body force, \( \mathbf{f} \)

\[
\frac{d\mathbf{u}}{dt} = u_x S\mathbf{\hat{y}} - \frac{\nabla \rho}{\rho} + \frac{\mathbf{B} \cdot \nabla \mathbf{B}}{4\pi \rho} + \nu \nabla^2 \mathbf{u} + \mathbf{f},
\]

(1)

\[
\frac{d\mathbf{B}}{dt} = -B_x S\mathbf{\hat{y}} + \mathbf{B} \cdot \nabla \mathbf{u} + \eta \nabla^2 \mathbf{B},
\]

(2)

where \( d/dt = \partial_t - Sx\partial_y + \mathbf{u} \cdot \nabla \)

Box dimensions: \( L_x, L_y, L_z \) where \( L_z \gg L_x, L_y \)
Use broadly the same parameter values as the previous work:

- $0.125 \leq S \leq 2$
- $L_x = 1 = L_y, \quad 8 \leq L_z \leq 128$
- Energy injected in a shell centred at $k_f/2\pi = 3$ or, equivalently, $l_f = 1/3$
- Most cases have $\nu = 10^{-2} = \eta$ giving $Rm = Re = u_{rms}/k_f\nu \sim 5$
Kinematic regime

- Growth rate scales linearly with $S$

\[ l_B = \left( \frac{\left\langle (\partial B_y^\perp / \partial z)^2 \right\rangle_z}{\left\langle (B_y^\perp)^2 \right\rangle_z} \right)^{1/2} \]

- Lengthscale, $l_B$, scales as $S^{-1/2}$

- Confirms results of Yousef et al., 2008
Wandering field

$zt$-plots of $B_y$ averaged over $x$ and $y$

$S = 2, L_z = 16$ (normalised by rms value)

$S = 0.5, L_z = 64$ (normalised by rms value)

Large-scale field in $y$-direction wanders in space and time
Two saturated regimes

Saturated state appears to admit two rather different regimes (Teed & Proctor, 2016, 2017)

Clearly seen in different energy equilibration

\[ E_M \ll E_K, \ (S = 2, \ L_z = 16, \ Pm = 1) \]

\[ E_M \sim E_K, \ (S = 0.5, \ L_z = 16, \ Pm = 2) \]
Quenched state (Teed & Proctor, 2016)

\[ l_B \ll l_u, \ (S = 2, \ L_z = 16, \ Pm = 1) \]

Quasi-periodic state (Teed & Proctor, 2017)

\[ l_B \sim l_u, \ (S = 2, \ L_z = 16, \ Pm = 1) \]
Quasi-periodic behaviour

- Two lengthscales: one on the size of the box and another on the intrinsic scale of the kinematic regime
- System moves between periods with $l_B^s \sim L_z$ and $l_B^s \sim l_B^k$
Linear dependence of $I^S_B$ on $I^k_B$

Triangles: values of $I^S_B$ calculated during the periods when $I^S_B \sim L_z$ (box scale).
Squares: values of $I^S_B$ calculated during the periods when $I^S_B \sim I^k_B$ (kinematic scale).
Possible explanation for quasi-cyclic is relaxation oscillations between a ‘mean-vorticity dynamo’ (Elperin, Kleeorin, and Rogachevskii, 2003) and a shear dynamo (for the magnetic field).

Large $z$-dependent shearing flow generated by a vorticity dynamo when field is weak.

Stronger magnetic field suppresses this mechanism $\rightarrow$ weaker vertical shear and operational shear dynamo (Käpylä & Brandenburg, 2009).

Only occurs if the kinetic and magnetic energies are of a similar order (quasi-cyclic state below).
If vorticity dynamo greatly dominates, no large-scale field can be generated by a shear dynamo mechanism. In this case the (weak) magnetic field is generated by a fluctuation dynamo mechanism. Hence lengthscale is reduced to that of the imposed forcing (quenched state below).
Tweaking the model

- Only basic linear shear (dependent on $x$) considered thus far
- Generates large-scale field with some cyclic properties but not solar-like
- Altering shear and/or forcing may promote more cyclic behaviour similar to the solar cycle
- Two main tweaks considered:
  - Changing shear profile; sinusoidal dependence, $z$-dependence
  - Adding a small amount of helicity into the forcing
Tweaking the model - preliminary results

\[
U = (Sx + S_2 \cos(2\pi z/L_z))\hat{y}
\]

\[
S_2 = 0.1
\]

\[
f = f_{nh} + f_h
\]

\[
|f_h| = 0.01
\]

\[
S_2 = 1
\]

\[
|f_h| = 0.05
\]

\[
S_2 = 4
\]

\[
|f_h| = 0.1
\]
Conclusions

- Pure linear shear case shows that the shear dynamo could form the basis for a model of the solar dynamo
- Saturated state admits two regimes: i) quenched state with small-scale field (not solar-like); ii) quasi-periodic state (possibly solar-like)
- Quasi-periodic state displays times of differing field length scale, proportional to the imposed shear rate
- Tweaking the purely linear shear case (z-dependent shear/small amount of helicity) could promote cyclic behaviour in the kinematic phase
- Analysis of further parameter regimes and larger boxes required
- Effects of rotation, compressibility?
Meeting to celebrate Mike Proctor’s retirement!

- Abstract submissions welcome on dynamo theory, MHD, convection, magnetoconvection and other relevant topics.
- Dates: September 11-12, 2017
- Venue: Centre for Mathematics Sciences, Cambridge with conference dinner at King’s College, Cambridge
- Organisers: Rob Teed & Valeria Shumaylova