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Virtual Variable Sampling Discrete Fourier Transform based Selective Odd-order Harmonic Repetitive Control of DC/AC Converters

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Abstract—This paper proposes a frequency adaptive discrete Fourier transform (DFT) based repetitive control (RC) scheme for DC/AC converters. By generating infinite magnitude on the interested harmonics, the DFT-based RC offers a selective harmonic scheme to eliminate waveform distortion. The traditional DFT-based selective harmonic RC, however, is sensitive to frequency fluctuation since even very small frequency fluctuation leads to a severe magnitude decrease. To address the problem, virtual variable sampling method, which creates an adjustable virtual delay unit to closely approximate a variable sampling delay, is proposed to enable the DFT-based selective harmonic RC to be frequency adaptive. Moreover, a selective odd-order harmonic DFT filter is developed to deal with the dominant odd order harmonic. Because it halves the number of sampling delays in the DFT filter, the system transient response gets nearly 50% improvement. A comprehensive series of experiments of the proposed VVS DFT-based selective odd-order harmonic RC controlled programmable AC power source under frequency variations are presented to verify the effectiveness of the proposed method.

Index Terms—Repetitive control, DC/AC converter, DFT, frequency fluctuation, selective harmonics, virtual variable sampling.

I. INTRODUCTION

DC/AC pulse-width modulated (PWM) power converters are widely used as a significant element in nowadays power conversion applications, e.g. uninterrupted power supply system, renewable energy source converter, programmable AC power source, and other power electronics applications [1]–[9]. For instance, programmable AC power sources provide adjustable sinusoidal output voltages in various bench-top tests, International Electrotechnical Commission (IEC) tests, and power supply tests. Such power supply systems suffer from various disturbances including modeling uncertainty and frequency fluctuation in practical applications. The control method for such systems must be stable and robust to frequency fluctuation and load uncertainties. Although traditional control methods such as proportional-integral (PI) control and state feedback control can stabilize the system, they often cannot ensure satisfactory control accuracy for periodic reference signal tracking and disturbance rejection.

Repetitive control (RC) is an advanced and effective method for periodic signal tracking, which can improve the converter performance with low total harmonic distortion (THD), fast transient response, and low steady state error [10], [11]. To further improve the stability and selectively eliminate harmonic distortions, the DFT-based selective harmonic RC is proposed [12], [13]. The advantages of the DFT-based selective harmonic RC include: it is a selective harmonic RC scheme, which is more effective to compensate harmonic distortion because major waveform distortion mainly concentrates on low odd harmonics; the DFT-based selective harmonic RC has a narrow bandwidth that enables it to deal with distortion on the interested low order harmonics without affecting overall system stability, such as phase margin and gain margin, while traditional RC requires a low-pass filter to maintain the overall system stability. In the DFT-based selective harmonic RC, a selective harmonic filter is created by using the inverse DFT transformation, where the time-consuming filter coefficient calculation is often done offline. Note that the order of the DFT filter is also the sampling number in one reference signal period, and it should be an integer. It is hard to enable the integer-order DFT filter to deal with varying frequency quasi-periodic signals since it is time-consuming to calculate the filter coefficients for a varying integer-order DFT filter in real time or it is impossible to create a fractional-order DFT filter. Since the traditional DFT-based selective harmonic RC is sensitive to the frequency variation, when the signal frequency deviates from the nominal frequency, the performance of the DFT-based selective harmonic RC degrades severely at the fixed sampling rate.

To address the varying-frequency issue, variable sampling/switching frequency techniques were developed to obtain a fixed integer samples per period for wind and aircraft power systems [14], [15]. However, variable sampling methods not only require special variable sampling processing capabilities of the system CPU, but also need to recalculate other digital control parameters to maintain the system stability and robustness [16], [17]. A parallel structure fractional RC scheme was developed [18] in which a correction factor in complex number is inserted to adjust the poles for each parallel RC controller. One major limitation of this approach is that it is difficult to implement the complex correction.

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factor in most digital controllers. Non-integer delay length approximated by a finite impulse response (FIR) filter [19–
24] and linear-phase-lag low-pass filter for fractional delay
length compensation [25] were also developed. However, these
frequency adaptive methods cannot be applied to the DFT-
based selective harmonic RC.

This paper develops a virtual variable sampling (VVS) method to enable the DFT-based selective harmonic RC at
a fixed sampling frequency to achieve an accurate, efficient,
and robust RC scheme under frequency fluctuation, where
the virtual variable sampling delay is well approximated by
a VVS unit. Similar to the variable sampling methods by
hardware [14], [15], all the offline-calculated coefficients for
the DFT filter can keep invariant during the system operation.
When frequency varies or fluctuates, the resonant frequency
of the DFT-based selective harmonic RC can be adjusted
accordingly by changing the coefficients of VVS. Since only
several particular odd-order harmonics usually dominate the
distortion in practical applications, it is effective to have a
selective odd-order harmonic RC to exclusively deal with
the signal at the interest harmonic frequencies [26], [27].
Consequently, a VVS DFT-based selective odd-order harmonic
RC scheme is developed. Comparing with traditional DFT-
based selective harmonic RC, the proposed RC scheme has
an only half number of sampling delay which results in half
delay time, so it leads to fast transient response.

The remainder of this paper is organized as follows: Section
II introduces the traditional RC, the DFT-based selective
harmonic RC, and the virtual variable sampling method. The
experimental verification of programmable AC power source
with adjustable frequency is presented in Section III, which is
followed by the conclusions in Section IV.

II. VIRTUAL VARIABLE SAMPLING DFT-BASED SELECTIVE
ODD-ORDER HARMONIC RC

A. Repetitive control

Fig. 1 shows a typical digital control system with plug-in RC
controller [10], where \( R(z) \) is the periodic reference input with
reference frequency \( f_r \) and reference period \( T_r = 1/f_r \), \( G_c(z) \)
is the conventional feedback controller, \( G_p(z) \) is the plant
model, \( D(z) \) is the disturbance, \( Y(z) \) is the system output, and
system sampling frequency is \( f_s \). Plug-in RC controller is a
feedforward controller that consists of RC gain \( K_r \), robustness
filter \( Q(z) \), phase lead filter \( G_f(z) \), and period delay \( z^{-N} \)
in which integral delay length \( N = \lfloor T_r/T_s \rfloor \) is the nearest
integer to the ratio of reference period \( T_r \) to the sampling
period \( T_s = 1/f_s \).

The transfer function of the conventional RC \( G_{rc}(z) \) in Fig.
1 is:

\[
G_{rc}(z) = \frac{U_r(z)}{E(z)} = K_r \frac{z^{-N}Q(z)}{1 - z^{-N}Q(z)}G_f(z)
\]

where \( E(z) = R(z) - Y(z) \) is the system tracking error. The
overall system holds the two stability conditions [11], [22]:
the closed-loop feedback system without RC is stable, and

\[
|Q(z)(1 - K_r G_f(z)G(z))| \leq 1
\]

where \( G(z) \) is the closed-loop transfer function without plug-
in RC, \( G(z) = G_c(z)G_p(z)/(1 + G_c(z)G_p(z)) \). In the fre-
quency domain, RC transfer function in Eq. (1) produces
infinite magnitude gain at all harmonic frequencies of the ref-
ERENCE signal when \( Q(z) = 1 \). Therefore, RC can achieve zero
steady-state error tracking of periodic signals. In practice, a
low-pass filter \( Q(z) \), with the form of \( Q(z) = a_1 z + a_0 + a_1 z^{-1}
\) \((2a_1 + a_0 = 1)\), is added to ensure the system stability. Fig. 2
shows the bode plot of the closed-loop system with a state
feedback (SF) controller before and after a plug-in RC is
added. It is clear that RC generates magnitude peaks at all
harmonic frequencies and the peak magnitudes at high order
harmonics are attenuated by the filter \( Q(z) \) to guarantee the
positive phase margin and gain margin.

![Fig. 2. Bode plot for state feedback (SF) and repetitive controllers (RC) with low-pass filter Q(z)](image)

B. The DFT-based selective odd-order harmonic repetitive
control

As odd-order harmonics usually dominate the waveform
distortion in practice, a DFT-based selective harmonic RC
is added as a feedforward controller to boost loop gains at
selected odd-order harmonic frequencies [12], [13]. Fig. 3
shows the scheme of the DFT-based selective harmonic RC,
in which \( K_r \) is RC gain, \( N_s \) is phase lead step, and \( F_{DFT}(z) \) is
a FIR filter in which \( b(i) \) is the FIR coefficients. The transfer
function of the DFT-based selective harmonic RC is

\[
G_{DFT-RC} = K_r \frac{F_{DFT}(z)}{1 - F_{DFT}(z) \cdot z^{-N_s}}
\]

![Fig. 1. Control system with plug-in RC controller](image)
To attain infinite gains at interested dominant odd-order harmonics, $F_{DFT}(z)$ filter is designed with unity gain and zero phase at interested harmonics. Based on inverse discrete Fourier transform, $F_{DFT}(z)$ filter, with a window length equal to half fundamental period of the reference signal $T_r$, has the transfer function as:

$$F_{DFT}(z) = \frac{4}{N} \sum_{i=0}^{N/2-1} \left( \sum_{h \in N_h} \cos \left( \frac{2\pi}{N} h(i + N_a) \right) \right) z^{-i} \quad (4)$$

where $N$ is one fundamental period delay length, i.e. $N = T_r / T_s$, $N_h$ is the set of selected odd-order harmonic frequencies, and $N_a$ is phase lead step that is similar to the one in the traditional RC [11]. The magnitude analysis of $F_{DFT}(z)$ can be found in the proof in Appendix. As an example, for $N = 200$, $N_a = 3$, and $N_h = 7$, the corresponding magnitude response of $F_{DFT}(z)$ is given in Fig. 4 (a), which only allows the 7th order harmonic to pass through with unit magnitude.

With unity magnitude of $F_{DFT}(z)$ at the selected 7th order harmonics frequency, the magnitude of the DFT-based selective harmonic RC in Eq. (3) reaches infinity at the interested 7th order harmonics as shown in Fig. 4 (b), thus a selective harmonic RC is achieved. The proposed DFT-based selective odd-order harmonic RC has an only half number (i.e. $N/2$) of delay elements compared with the traditional DFT-based selective harmonic RC with a full number (i.e. $N$) of delay elements. The merits of the proposed RC scheme is that it achieves faster response since only half delay units are applied in the controller structure. Because the large gain happens only at selected odd-order harmonic frequencies, the proposed RC scheme does not influence the overall system stability characteristics, such as phase margin and gain margin as shown in Fig. 5.

However, the signal frequency fluctuation will severely degrade the performance of the DFT-based selective harmonic RC. The zoomed inset in Fig. 4 (b) shows that when the signal frequency deviates from the nominal frequency, the magnitude of the DFT-based selective harmonic RC decreases significantly: the magnitude drops from infinity to less than 50 when there is only a 0.1 Hz deviation. It is unfeasible to redesign the DFT filter in Eq. (4) by updating the delay number $N$ in each sampling period to achieve real-time control. Therefore, the traditional DFT-based selective harmonic RC is unable to efficiently deal with frequency fluctuation, which exists in most practical systems.

C. Virtual variable sampling DFT-based selective odd-order harmonic repetitive control

To enable the DFT-based selective odd-order harmonic RC to be frequency adaptive, the VVS method is developed in this section. In the proposed RC scheme, all the coefficients of the DFT filter do not need change when the frequency varies or fluctuates. The infinite magnitude gain of RC at the selective order of harmonics is achieved by changing the coefficients of VVS, which is simple, flexible, low-cost, and easy-to-implement.

To develop the proposed VVS, a virtual variable sampling frequency with sampling period $T_v$ is proposed. In this method, the virtual variable sampling period $T_v$ is created to approximate a virtual unit delay to maintain sampling period $T_r$ as an fixed integral multiple $N_v$ of the reference
fundamental period $T_r$, while the system sampling period $T_s$ keeps a fixed value:

$$T_v = \frac{T_r}{N_v}$$  \hspace{1cm} (5)

When $T_r$ changes or fluctuates, $T_v$ can be easily changed by tuning the parameters of VVS to keep $N_v$ as a constant integer. This is equivalent to the solutions with physical varying sampling frequency [14], [15]. The advantage of the proposed approach over previous approaches is that it is more cost efficient than hardware-based methods. It provides a generic solution to frequency adaptive problems and can be used for all delay-based repetitive control problems.

When the reference period changes from $T_r$ to $\tilde{T}_r$, CS cannot provide fixed integer number of delay periods $N$ for both $T_r$ and $\tilde{T}_r$. By changing sampling period from $T_s$ to $\tilde{T}_s$ accordingly, VS has integer number $N$ delay periods for both $T_r$ and $\tilde{T}_r$. With the same sampling period as that in CS, VVS builds approximated virtual variable delay periods $T_v$ and $\tilde{T}_v$, which is achieved from Fig. 6 (b) with two different sets of parameters, to keep a virtual but fixed number $N_v$ delay periods for both $T_r$ and $\tilde{T}_r$ without changing system sampling period $T_s$.

With virtual variable sampling period $T_v$, the VVS unit delay $z_{v}^{-1}$ is approximated by establishing a $n$th-order Lagrange interpolation polynomial:

$$z_{v}^{-1} = \sum_{i=1}^{n} a_i z^{-i}, \text{ with } a_i = \prod_{j=1,j \neq i}^{n} \frac{T_v - iT_s}{jT_v - iT_s}$$  \hspace{1cm} (6)

Higher order approximation can provide higher accuracy in approximating a variable sampling unit delay. However, higher order approximation costs more computation and a more complicated design. With the tradeoff of approximation accuracy and design complexity, a 3rd-order polynomial is selected in practical applications. Fig. 7 shows the magnitude response of a 3rd-order VVS unit $z_{v}^{-1}$ when the reference signal frequency changes from 59 Hz to 61 Hz, where the VVS is designed at 10 kHz sampling frequency. When reference signal frequency deviates away from 60 Hz with $\delta = (f - 60)$ Hz representing the frequency deviation, VVS unit presents an accurate magnitude at 1 when input signal frequency is below 700 Hz for the system with 10 kHz sampling frequency.

Fig. 6 (a) shows sampling points comparison under a sudden reference period change between constant sampling (CS), variable sampling (VS), and virtual variable sampling (VVS). When the reference period changes from $T_r$ to $\tilde{T}_r$, CS cannot provide fixed integer number of delay periods $N$ for both $T_r$ and $\tilde{T}_r$. By changing sampling period from $T_s$ to $\tilde{T}_s$ accordingly, VS has integer number $N$ delay periods for both $T_r$ and $\tilde{T}_r$. With the same sampling period as that in CS, VVS builds approximated virtual variable delay periods $T_v$ and $\tilde{T}_v$, which is achieved from Fig. 6 (b) with two different sets of parameters, to keep a virtual but fixed number $N_v$ delay periods for both $T_r$ and $\tilde{T}_r$ without changing system sampling period $T_s$.

With the VVS unit delay of $z_v^{-1}$ in Eq. (6), the VVS DFT-based selective odd-order harmonic RC has the transfer function as:

$$F_{DFT}(z_v) = \frac{4}{N_v} \sum_{i=0}^{N_v/2-1} \left( \sum_{h \in N_v} \cos \left( \frac{2\pi}{N_v}h(i + N_a) \right) \right) z_v^{-i}$$  \hspace{1cm} (7)

By applying VVS, the infinite magnitude frequency of the DFT-based selective harmonic RC is able to follow the varying frequency because the virtual sampling frequency can be easily adjusted. Compared with the traditional DFT filter transfer function (4), the delay unit $z_v^{-1}$ is replaced by the VVS unit $z_v^{-1}$. Furthermore, the sampling period of input and output still keeps the system sampling period $T_s$, while all RC analysis in [11] can be conducted with the virtual sampling period $T_v$. Different from the multi-rate sampling [28], aliasing and imaging do not exist in VVS, which eliminate the design of anti-aliasing and anti-imaging filters.

III. EXPERIMENTAL VERIFICATION: SINGLE-PHASE PROGRAMMABLE AC POWER SUPPLY

A. System Modeling and State Feedback Controller

Fig. 8 shows a DFT-based selective odd-order harmonic RC controlled single-phase programmable AC power source of DC/AC inverter, where $E_n$ is the DC bus voltage; $L$ and $C$ form an LC filter. The performance of AC source are tested.
under linear load $R$ and nonlinear rectifier load with $C_r$, $L_r$, and $R_r$. The output voltage $V_{out}$ and inductor current $I_L$ are two states for state feedback controller; $V_{in}$ is the input PWM voltage.

\[ v(k+1) = \begin{pmatrix} \varphi_{11} & \varphi_{12} \\ \varphi_{21} & \varphi_{22} \end{pmatrix} v(k) + g_1 u(k) \]

where \( \varphi_{11} = 1 - T_s/(RC) + T_s^2/(2RC^2) - T_s^2/(2LC) \), \( \varphi_{12} = T_s/C - T_s^2/(2RC^2) \), \( \varphi_{21} = -(T_s/L + T_s^2/(2RLC)) \), and \( g_1 = E_n T_s^2/(2LC) \).

The state feedback controller has the form as:

\[ u = -k_1 v(k) - k_2 i(k) + g v_{ref}(k) \]

where \( k_1, k_2 \) and \( g \) are state feedback controller parameters, \( v_{ref} \) is the reference sinusoidal voltage. With state feedback controller (9), the transfer function can be rewritten as:

\[ G(z) = \frac{m_1 z + m_2}{2 z^2 + p_1 z + p_2} \]

where \( p_1 = -(\varphi_{22} - g_2 k_2) - (\varphi_{11} - g_1 k_1) \), \( p_2 = (\varphi_{11} - g_1 k_1)(\varphi_{22} - g_2 k_2) - (\varphi_{12} - g_1 k_2)(\varphi_{21} - g_2 k_1) \), \( m_1 = g_1 k_1 \), \( m_2 = g_2 k - g_1 k_1(\varphi_{22} - g_2 k_2) \) [11].

### TABLE I

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC voltage, $E_n$</td>
<td>250 V</td>
<td>Inductor, $L$</td>
<td>3 mH</td>
</tr>
<tr>
<td>Capacitor, $C_r$</td>
<td>10 $\mu$F</td>
<td>PWM frequency</td>
<td>10 kHz</td>
</tr>
<tr>
<td>Sampling frequency, $f_s$</td>
<td>10 kHz</td>
<td>Linear load, $R$</td>
<td>200 $\Omega$</td>
</tr>
<tr>
<td>Rectifier capacitor $C_r$</td>
<td>60 $\mu$F</td>
<td>Rectifier inductor $L_r$</td>
<td>3 mH</td>
</tr>
<tr>
<td>Rectifier resistance $R_r$</td>
<td>200 $\Omega$</td>
<td></td>
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</tr>
</tbody>
</table>

With parameters in Table I and state feedback controller \( k_1 = 0.4 \), \( k_2 = 7 \), and \( g = 1.4 \), the closed-loop transfer function for 60 Hz single-phase 110V DC/AC inverter is derived as:

\[ G(z) = \frac{0.592 z + 0.012}{z^2 - 0.81 z} \]

The feedback control system is stable with poles \( p_1 = 0 \), \( p_2 = 0.81 \) in the unit cycle. With only feedback controller, a severe phase lag and high waveform distortion are observed in the output waveform under linear load and rectifier load conditions shown in Fig. 10. The major distortion components appear on first 10 harmonic frequencies.

Fig. 9. Experiment setups

Fig. 9 shows the experimental testbed, in which the state feedback controller and plug-in VVS DFT-based selective odd-order harmonic RC are designed in Matlab Simulink and implemented by dSPACE DC1103 to control the H-bridge IGBT converter. The output voltage and current waveform are recorded via sampling circuit by Control Desktop for real-time control and performance analysis.

With the sampling period $T_s$, the discrete-time state space of the single-phase inverter system in Fig. 8 can be written as:

\[ G(z) = \frac{m_1 z + m_2}{2 z^2 + p_1 z + p_2} \]

where \( p_1 = -(\varphi_{22} - g_2 k_2) - (\varphi_{11} - g_1 k_1) \), \( p_2 = (\varphi_{11} - g_1 k_1)(\varphi_{22} - g_2 k_2) - (\varphi_{12} - g_1 k_2)(\varphi_{21} - g_2 k_1) \), \( m_1 = g_1 k_1 \), \( m_2 = g_2 k - g_1 k_1(\varphi_{22} - g_2 k_2) \) [11].

B. VVS DFT-based selective odd-order harmonic RC design

With the reference signal frequency $f_r = 60$ Hz ($T_r = 1/60$s) and sampling frequency $f_s = 10$ kHz ($T_s = 0.1\mu$s), the delay length for the traditional DFT-based selective harmonic RC is the closest sampling integer $N = \lceil T_r/T_s \rceil = 166.67 = 167$. From the form of VVS unit delay in Eq. (6), \( z_s^{-1} \) is approximated by \( z^{-1} \), \( z^{-2} \), and \( z^{-3} \). The adjustable VVS period is between $T_s$ and $3T_s$, which makes the virtual variable frequency varies between $f_s/3$ and $f_s$. Delay length $N_v$ in the experiment is selected as 80 where the virtual sampling period is around $2T_s$, thus VVS can have large frequency adjustment range around the reference frequency. The VVS unit delay is obtained from Eq. (6) as:

\[ z_s^{-1} = 0.038 z^{-1} + 0.993 z^{-2} + 0.045 z^{-3} \]

Since the distortion appears mainly on 3rd, 5th, 7th, and 9th order harmonics shown in Fig. 10, the selected harmonic set $N_h$ in Eq. (7) is set as $N_h = 1, 3, 5, 7, 9$; The phase lead step $N_a$ is designed to cancel the DSP computational delay and phase delay of plant $G_p(z)$. The design of RC gain $K_r$ and phase lead step $N_a$ can be found in [11], [29], which is based on the accurate system modeling. In practice, the theoretical value does not make the system work at the best state because of system uncertainty and DSP computation delay [30], [31]. By experimental tests, the RC gain $K_r$ is set as $K_r = 1$ to achieve a fast response to a load or frequency change; the phase lead step $N_a$ is selected as $N_a = 3$, which provides phase lead compensation for model phase lag and computational time delay. The coefficients of the DFT filter $F_{DFT}(z)$ are calculated offline. When reference frequency changes, the VVS unit delay is updated based on Eq. (6).
C. Experimental results and comparison

Programmable AC power sources provide frequency and amplitude adjustable AC voltage for different applications. Under various conditions, a high-performance programmable AC power source is required to generate the very clean sinusoidal output voltage. The system performances are compared between the traditional DFT-based selective harmonic RC and VVS DFT-based selective odd-order harmonic RC in terms of transient response, steady-state performance, load switch, and frequency fluctuation under different kinds of loads at variable frequencies.

1) Transient response: Figs. 11 and 12 show the transient responses and tracking errors after applying the traditional DFT-based selective harmonic RC and VVS DFT-based selective odd-order harmonic RC with 60 Hz reference frequency under rectifier load, respectively. When the system operates under state feedback control, the peak tracking error is about 50 V due to the phase lag and magnitude error. After applying the DFT-based selective harmonic RC, the tracking error converges in 0.06 s. Because the proposed DFT-based selective odd-order harmonics RC is an only half reference period delay compared with the traditional DFT-RC with one period delay, the transient speed gets about 50% improvement, which is shown as 0.025 s in Fig. 11.

2) Steady-state performance: Figs. 13 and 14 show the steady-state performance of the DFT-based selective harmonic RC and VVS DFT-based selective odd-order harmonic RC under linear load and rectifier load for 59 Hz, 60 Hz, and 61 Hz applications.

![Fig. 11. Transient response of the traditional DFT-based selective harmonic RC under rectifier load under nominal reference frequency](image)

![Fig. 12. Transient response of VVS DFT-based selective odd-order harmonic RC under rectifier load under nominal reference frequency](image)

![Fig. 13. Steady-state performance of the traditional DFT-based selective harmonic RC under linear load and rectifier load for 59 Hz, 60 Hz, and 61 Hz applications](image)

![Fig. 14. Steady-state performance of VVS DFT-based odd-order harmonic RC under linear load and rectifier load for 59 Hz, 60 Hz, and 61 Hz applications](image)
1 Hz fluctuation especially under rectifier load. Fig. 14 shows that, under VVS DFT-based selective odd-order harmonic RC, the steady-state outputs stay unchanged sinusoidal waveform under different kinds of loads when the frequency is under fluctuation. This is because the VVS is able to adaptively adjust the virtual sampling period according to varying signal period.

### TABLE II

<table>
<thead>
<tr>
<th></th>
<th>The DFT-based selective harmonic RC</th>
<th>VVS DFT-based selective odd-order harmonic RC</th>
</tr>
</thead>
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<tr>
<td></td>
<td>RMS error (V)</td>
<td>THD</td>
</tr>
<tr>
<td>59 Hz</td>
<td></td>
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<tr>
<td>LL</td>
<td>4.69</td>
<td>1.49%</td>
</tr>
<tr>
<td>RL</td>
<td>8.86</td>
<td>7.16%</td>
</tr>
<tr>
<td>60 Hz</td>
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<td></td>
</tr>
<tr>
<td>LL</td>
<td>1.81</td>
<td>1.19%</td>
</tr>
<tr>
<td>RL</td>
<td>2.11</td>
<td>1.51%</td>
</tr>
<tr>
<td>61 Hz</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LL</td>
<td>3.16</td>
<td>1.27%</td>
</tr>
<tr>
<td>RL</td>
<td>8.56</td>
<td>5.53%</td>
</tr>
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</table>

The tracking accuracy of PWM inverters is normally evaluated in two important aspects: (1) THD concerning how sinusoidal the output is (in terms of power quality); (2) tracking error in root mean square (RMS) concerning the absolute control accuracy. Table II shows steady-state performance comparison for variable input frequency (59 Hz, 60 Hz, and 61 Hz) between the DFT-based selective harmonic RC and VVS DFT-based selective odd-order harmonic RC with linear load (LL) and rectifier load (RL). When reference input frequency deviates away from 60 Hz, the advantage of VVS DFT-based selective odd-order harmonic RC becomes significant. For 59 Hz and 61 Hz reference inputs, VVS DFT-based selective odd-order harmonic RC has lower tracking error (RMS errors are less than 2 V) and THD (THDs are about 1%). For the traditional DFT-based selective harmonic RC, serious performance degradation occurs with 59 Hz and 61 Hz input signals. Compared with the traditional DFT-based selective harmonic RC, VVS DFT-based selective odd-order harmonic RC under LL shows 5% (59 Hz) and 84% (61 Hz) improvement in terms of THD and 39% (59 Hz) and 78% (61 Hz) improvement in terms of RMS tracking error. Similarly, VVS DFT-based selective odd-order harmonic RC under RL shows 29% (59 Hz) and 83% (61 Hz) improvement in terms of THD and 43% (59 Hz) and 77% (61 Hz) improvement in terms of RMS tracking error.

3) Load switch: Figs. 15 and 16 show the transient response of VVS DFT-based selective odd-order harmonic RC under sudden load switches from no load to linear load and from no load to rectifier load, respectively. The figures show that the output voltage recovers from sudden step load change in 1 cycle (0.017 s) when the linear load is switched on, and recovers within about 2 to 3 cycles (about 0.04 s) when rectifier load is switched on. This demonstrates that proposed VVS DFT-based selective odd-order harmonic RC is robust to load changes and has a quick transient response when the load changes.

4) Frequency fluctuation: Figs. 17 and 18 show the transient response comparison of the traditional DFT-based selective harmonic RC and VVS DFT-based selective odd-order harmonic RC when the reference frequency fluctuates. When the reference frequency changes from 60 Hz to 61 Hz, VVS DFT-based selective odd-order harmonic RC achieves very low tracking error and the peak-to-peak value about 10 V after a certain transient process. On the contrary, the peak-to-peak tracking error of the traditional DFT-based selective harmonic RC is about 30 V after the transient process. Therefore, the proposed VVS DFT-based selective odd-order harmonic RC shows obvious frequency adaptability with the reference frequency changing.

### IV. Conclusion

In this paper, a simple, flexible, generic, and easy-to-implement virtual variable sampling method is proposed to address the performance degradation for variable frequency applications of the traditional DFT-based selective harmonic RC. With fixed system sampling period, a virtual variable sampling period is achieved by building an approximated virtual unit delay. Based on this, the DFT-based selective
delay based controller design such as classic repetitive control variable sampling also provides a flexible solution for other when the frequency is under fluctuation. The proposed virtual the traditional DFT-based selective harmonic RC especially tracking performance and faster transient performance than VVS DFT-based selective odd-order harmonic RC offers better faster transient response. Experimental results also show that DFT-based selective harmonic RC, and then leads to up 2 times number of delay elements comparing with the traditional DFT-based selective harmonic RC, and then leads to up 2 times faster transient response. Experimental results also show that VVS DFT-based selective odd-order harmonic RC offers better tracking performance and faster transient performance than the traditional DFT-based selective harmonic RC especially when the frequency is under fluctuation. The proposed virtual variable sampling also provides a flexible solution for other delay based controller design such as classic repetitive control [32].

**APPENDIX**

Proof of DFT filter performance:

For a particular harmonic \( h \), the DFT filter without concern-

\[
F_{\text{DFT}}(z) = \frac{4}{N} \sum_{i=0}^{N/2-1} \left[ \cos \left( \frac{2\pi}{N} hi \right) \right] z^{-i}
\]

(13)

where \( j \) is imaginary unit, \( \omega \) is signal angular frequency and \( \omega T_s = 2\pi k/N \) (\( k \in \mathbb{N} \) is the input signal harmonic order).

\[
F_{\text{DFT}}(z) = \frac{4}{N} \sum_{i=0}^{N/2-1} \left[ \cos \left( \frac{2\pi}{N} hi \right) \right] z^{-i}
\]

\[
= \frac{1}{2} \left[ \cos \left( \frac{2\pi}{N} (h+k) i \right) + \cos \left( \frac{2\pi}{N} (h-k) i \right) \right] - \frac{1}{2} \left[ \sin \left( \frac{2\pi}{N} (h+k) i \right) - \sin \left( \frac{2\pi}{N} (h-k) i \right) \right]
\]

(14)

When \( h \) and \( k \) are both odd number, i.e. both \( h+k \) and \( h-k \) are even number, then

\[
\sum_{i=0}^{N/2-1} \left[ \cos \left( \frac{2\pi}{N} (h+k) i \right) \right] = 0;
\]

(15)

\[
\sum_{i=0}^{N/2-1} \left[ \sin \left( \frac{2\pi}{N} (h+k) i \right) \right] = 0;
\]

(16)

\[
\sum_{i=0}^{N/2-1} \left[ \sin \left( \frac{2\pi}{N} (h-k) i \right) \right] = 0.
\]

(17)

Thus, the DFT-based filter has unity magnitude and zero phase at the desired harmonics, \( k = h \), and zero magnitude and zero phase at the other harmonics, \( k \neq h \):

\[
F_{\text{DFT}}(z) = \frac{2}{N} \sum_{i=0}^{N/2-1} \left[ \cos \left( \frac{2\pi}{N} (h-k) i \right) \right]
\]

\[
= \begin{cases} 
1, & \text{when } k = h \\
0, & \text{when } k \neq h
\end{cases}
\]

(18)

**REFERENCES**


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