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Deposited on: 20 June 2017
Evaluation of analytical models for heat transfer in mine tunnels

Covadonga Loredo¹ David Banks² and Nieves Roqueñí³

¹Universidad de Oviedo, Calle Independencia 13, 33004 Oviedo, Spain
²School of Engineering, James Watt Building (South), University of Glasgow, Glasgow, G12 8QQ, United Kingdom

ABSTRACT

The heat contained in underground flooded mine workings is actively exploited, often via heat pump technology, at a number of locations in Europe and North America. Several different heat exchange configurations may be utilised in this context, but for those (“standing column” and “open loop” arrangements) where mine water reinjection is practised, the rate of heat transfer between mine walls and mine water is a critical process to quantify. The two most commonly-used analytical solutions to this problem have been applied to the same baseline scenario, exploring and comparing their sensitivity, strengths, weaknesses and areas of application. It is found that the Rodríguez-Díaz solution generally predicts heat transfer rates (typically of the order of several tens of W per m of tunnel) greater than the Lauwerier-Pruess-Bodvarsson approach, the difference being due to the fact that the former assumes a radial heat conductive flow geometry in the rock surrounding the mine void, while the latter assumes a less efficient parallel linear flow. The Rodríguez-Díaz solution is more appropriate for approximately cylindrical mine shafts and tunnels. The Lauwerier-Pruess-Bodvarsson approach is more likely to be applicable to tabular mined void geometries. Improvements to the Rodríguez and Díaz model are proposed to enhance its transparency and applicability.

Keywords: heat transfer models, mine geothermal systems, reinjection, open/closed loop

1. Introduction:

1.1. Use of mine water for space heating and cooling

In the recent decades, interest has grown in the potential for the water stored and transmitted in abandoned (or even active) mined systems to be exploited for heating and cooling purposes (Banks et al., 2004, 2003; Bracke and Bussmann, 2015; Hall et al., 2011; Hamm and Bazargan Sabet, 2010; Lund et al., 2011; Peralta Ramos et al., 2015; Preene and Younger, 2014; Raymond and Therrien, 2007; Watzlaf and Ackman, 2006). Indeed several large-scale mine water-based heat pump systems have been constructed around the world: in a European context, the most notable are those at Heerlen, Netherlands (Bazargan et al., 2008; Ferket et al., 2011; Johnston et al., 2008; Verhoeven et al., 2014) and at Mieres, Spain (Jardón et al., 2013), both of which deliver several MW of heating and cooling effect.
The main factors that can influence the success of a geothermal installation in a mined aquifer are: the available sustainable abstraction, the water temperature and its stability, the water hydrochemistry and the available volume of the flooded mined voids and adjacent aquifers (Ordóñez et al., 2012). The presence of a heating / cooling demand in the vicinity of the mine is also critical; if potential users are too far from the mines or if the surface thermal transport system is not efficient enough, costs of pipelines and losses of energy can rapidly negate the energetic and financial viability of the entire system (Banks, 2016; JHI, 2016).

Several different methods can be utilised to extract heat from water contained in abandoned flooded mines (Banks, 2016; Ghomshei, 2007; Hall et al., 2011; Preene and Younger, 2014). These can briefly be summarised as:

(i) Pumping water from an abandoned (or working) mine and passing it through a heat pump or heat exchanger, prior to discharge to a water treatment system, e.g. Caphouse Colliery, near Wakefield, UK (Burnside et al., 2015; Parker, 2011) or surface water recipient (Mieres, Spain). This is referred to as an “open loop” arrangement.

(ii) Pumping water from a mine shaft and passing it through a heat pump or heat exchanger, before reinjecting the thermally spent water back to the mine system at a different location (via another borehole or shaft). This is referred to as an “open loop with reinjection” (e.g. the Shettleston and Lumphinnans schemes in Scotland – (Banks et al., 2009).

(iii) Pumping water from a flooded abandoned mine shaft and passing it through a heat pump or heat exchanger, before reinjecting the thermally spent water back to the shaft at a different depth, as in Markham Colliery, near Bolsover, UK (Athresh et al., 2015). This is known as a “standing column” arrangement and is typically only suitable for relatively modest heat extractions, or for shafts with large depth and diameter or high natural minewater throughflow.

(iv) Submerging a heat exchanger (which may be some form of plate exchanger or simply coils of polythene tubing) in a flooded mine void or treatment pond, and circulating a heat transfer fluid through the heat exchanger, often via a heat pump). Such an arrangement is installed in the shaft of Folldal Mine, Norway (Banks et al., 2004; Peralta Ramos et al., 2015) and in a minewater treatment lagoon at Caphouse Colliery, UK (Banks, 2016; Burnside et al., 2015), and is referred to as a “closed loop” system.
Fig 1 summarizes these different methods. Especially in types (ii) and (iii) where thermally spent water (i.e. cool water in the case of mines providing space heating) is reinjected back into the mine voids, it becomes very important to quantify the subsurface heat exchange - i.e. the rate at which the cool mine water re-acquires heat from the rocks in the walls of the mine voids and passages.

In such cases, the reinjected water (or a portion thereof) will typically gradually flow through a network of mine roadways, drifts and shafts back towards the original point of abstraction. During this passage, it will acquire heat by conductive/convective heat exchange with the walls of the mine voids. If, by the time it returns to the point of abstraction, its temperature has approximately returned to the original temperature of abstraction, the mine water geothermal operation should remain sustainable for a prolonged period. If it has not returned to the original temperature of abstraction, then ‘thermal breakthrough’ of cool water will occur at the abstraction point, the temperature of the abstracted water will fall and the efficiency of heat extraction will decline. The existence of a potential for thermal breakthrough does not necessarily mean that the geothermal scheme will fail immediately, but it may limit the lifetime of the scheme’s thermal and economic viability, and it needs to be fully understood and quantified.

If the mine water is to be used for space cooling, warm water would be reinjected to the mine, but the same potential issue of thermal breakthrough of warm water at the
point of abstraction still applies and, mathematically, simply becomes the inverse of
the problem described above.

In some schemes, such as that at Heerlen, Netherlands (Bazargan et al., 2008; Ferket et al., 2011; Minewater project, 2011; Verhoeven et al., 2014), the mine system provides the potential for both heating and cooling, depending on the demands of the building mass and the season. In such schemes, the opportunity to manage the mine with some long term balance between net heat extraction and injection becomes possible (i.e. using the mine as a thermal store rather than as a source of heat). This balanced usage of subsurface thermal storage increases the efficiency of the system and minimises the risk of depletion of the thermal reservoir (Minewater project, 2011; Verhoeven et al., 2014), being particularly applicable to buildings and complexes with similar heating and cooling demands over an annual cycle: e.g. hospitals, factories, universities or research institutes with supercomputers (Jardón et al., 2013; Sheldon et al., 2015). Nevertheless, it is still important to understand the rate of heat transfer in mine tunnels and voids and to evaluate the short term risk of thermal breakthrough in periods of intense need for heating or air conditioning (in prolonged hot summer or cold winter spells).

1.2. Heat transfer modelling

The issues of heat transfer in mine voids and thermal breakthrough can be addressed via numerical modelling (Ghoreishi et al., 2012; Renz et al., 2009), which utilises finite element or finite difference solutions of numerical approximations of heat / mass flow equations. Parameter values can be assigned to specific locations in space and time, and complex geometries can be tackled, potentially providing a more realistic representation of many mining scenarios than analytical models. Caution must be taken over numerical dispersion issues, however, and the data requirements are significantly greater than (and often unrealistic, compared with) analytical models (Loredo et al., 2016; McMahon et al., 2001).

Analytical models, on the other hand, use the exact mathematical solutions of flow and heat transport equations. Typically, they can only deal with simple geometries and homogeneous situations (Morgan, 2016). In order to use them, therefore, it is necessary for real complex geologies and geometries to be simplified to approximate to the requirements of analytical scenarios. They are, however, relatively easy to use and can be programmed into spreadsheets. Authors such as Banks (2009, 2015) have argued that analytical models of subsurface heat transport can be used as a rapid and inexpensive ‘first pass’ modelling of geothermal schemes to evaluate their overall feasibility and sustainability. Other authors, such as Ferket et al. (2011) have coupled analytical models of heat transfer in mine voids to discrete ‘pipe network’ hydraulic models, such as EPANET (Rossman, 2000) to construct some impressive and complex modelling tools that have been applied to the Heerlen minewater scheme.

In this paper, we will specifically critically examine and compare the two most commonly-used analytical solutions to heat transfer in mine tunnels filled with flowing groundwater:

1) The Lauwerier-Pruess-Bodvarsson model (Lauwerier, 1955; Pruess and Bodvarsson, 1983)

2) The Rodríguez and Díaz (2009) model
1.3. The Pruess and Bodvarsson (1983) model

Pruess and Bodvarsson (1983) take their solution direction from an old paper by Lauwerier (1955). In evaluating the applicability of this model, it should be borne in mind that the scenario envisaged by Lauwerier (1955) was that a hot fluid (water) was being injected as a horizontal layer into a formation of oil-bearing sand (Fig 2). This is very different, geometrically, from an approximately circular mine tunnel of diameter \(2r_g\) (as envisaged by Rodríguez and Díaz, 2009; Fig 3). In the Lauwerier scenario, the conductive heat transport is essentially 1-dimensional and linear-parallel, away from the hot water layer. In the mine tunnel, the heat flow is radially divergent.

\[ T_s = T_e \text{erfc} \left( \frac{\xi}{2\sqrt{D(\tau - \tilde{\xi})}} \right) U(\tau - \tilde{\xi}) \]  

(1)

Where \(\xi\) is a dimensionless distance in the flow direction, related to the actual distance \(x\) by:

\[ \xi = \frac{x}{2\sqrt{D(\tau - \tilde{\xi})}} \]
\[ \xi = \frac{x_2 \lambda_2}{b^2 \rho_w c_w v_w} \]  

(2)

\[ \nu_w \text{ is the linear water velocity in the injected layer, } \tau \text{ is a dimensionless time, related to real time } t \text{ since injection commenced, by} \]

\[ \tau = \frac{t \lambda_2}{b^2 \rho_1 c_1} \]  

(3)

\[ \rho \text{ refer to density, } c \text{ to specific heat capacity and } \lambda \text{ to thermal conductivity, while the subscripts } w, 1 \text{ and } 2 \text{ refer to the water, the water-saturated sand within the injection layer and the oil-saturated sand outside the injection layer, respectively.} \]

\[ \theta \text{ is the ratio between the volumetric heat capacities of the water-saturated sand and the oil-saturated sand: i.e.} \]

\[ \theta = \frac{\rho_1 c_1}{\rho_2 c_2} \]  

(4)

\[ \text{erfc} \text{ refers to the complementary error function and the function } U \text{ is simply a step function whose value is 0 when the argument is } <0 \text{ and whose value is 1 when the argument is } >0. \]

If the initial temperature of the reservoir is not 0°C, but some temperature \( T_r \), then

\[ T_s - T_r = \Delta T_1 = (T_e - T_r) \text{erfc} \left[ \frac{\xi}{2\sqrt{\theta(t - \xi)}} \right] U(t - \xi) \]  

(5)

Pruess and Bodvarsson (1983) argued that if the water-injected layer has a porosity of 100%, it simply becomes a water filled fracture or linear void of thickness 2b and that \( \rho_1 c_1 = \rho_w c_w \). Analogous \( \rho_2 c_2 \) simply becomes regarded as the volumetric heat capacity of the host rock. In this case, the error function argument can be rearranged as follows:

\[ \left[ \frac{\xi}{2\sqrt{\theta(t - \xi)}} \right]^2 = \frac{x^2 \lambda_2^2 b^2 \rho_w c_w v_w}{4b^2 \rho_w^2 c_w^2 v_w^2 (v_w - x)} = \frac{x^2 \lambda_2}{4b^2 \rho_w^2 c_w^2 v_w^2 (v_w - x)} \]  

(6)

and

\[ \frac{\xi}{2\sqrt{\theta(t - \xi)}} = \frac{x}{2b} \sqrt{\lambda_2 \rho_w c_w v_w (v_w - x)} = \frac{x}{2b \rho_w c_w v_w} \left( \frac{\lambda_2 \rho c_2}{t - x/v_w} \right) \]  

(7)

Thus, equation (1) simply becomes

\[ T_s - T_r = \Delta T_1 = (T_e - T_r) \text{erfc} \left( \frac{x}{2b \rho_w c_w v_w} \left( \frac{\lambda_2 \rho c_2}{t - x/v_w} \right) \right) U(t - \xi) \]  

(8)

Equation (8) is thus a formula for predicting the temperature \( T_s \) of water in a fracture of aperture 2b at time t and distance x in the scenario where: water injection commences at \( x = 0 \) and \( t = 0 \); the injected water temperature is \( T_e \) and the host rock is initially at temperature \( T_r \).
Pruess and Bodvarsson (1983) then assume that the flow velocity $v_w$ multiplied by the aperture $2b$ is equal to the injection flow rate, $q$ (Kg/s), per unit width ($w$) of the fracture.

$$2b\rho_w v_w = q'w$$

(9)

So:

$$T_s - T_r = \Delta T = (T_e - T_r) \text{erfc} \left( \frac{wx}{qc_w} \sqrt{\frac{\lambda_2 \rho_2 c_2}{t - x/v_w}} \right) U(\tau - \xi)$$

(10)

They further argue that $2wx$ is equal to $S$, the fracture surface area between the point of injection and the point of observation ($x$), which can alternatively be written as $P_x$, where $P$ is the effective perimeter, perpendicular to flow direction, for any generalised flow void. Furthermore, the linear transport time to travel a distance $x$, namely $x/v_w$ is equal to $xAP_w/q$. Thus, for a generalised flow channel:

$$T_s - T_r = \Delta T = (T_e - T_r) \text{erfc} \left( \frac{S}{2qc_w} \sqrt{\frac{\lambda_2 \rho_2 c_2}{t - x/v_w}} \right) U(\tau - \xi)$$

(11)

$$T_s - T_r = \Delta T = (T_e - T_r) \text{erfc} \left( \frac{S}{2qc_w} \sqrt{\frac{\lambda_2 \rho_2 c_2}{t - xAP_w/q}} \right) U(\tau - \xi)$$

(12)

1.4. The Rodríguez and Díaz (2009) model

The solution published by Rodríguez and Díaz (2009) has the advantage that it assumes an approximately radial flow geometry from the outset Fig 3. The approach breaks down the mine gallery, shaft or tunnel, of radius $r_g$, into a series of increments of length $l$. The heat power gain to each increment of tunnel ($\dot{H}_i$) is given by:

$$\dot{H}_i = 2\pi r_g l \dot{h}_i \left( T_r - \frac{T_{ei} - T_{wi}}{2} \right)$$

(13)

The solution that Rodríguez and Diaz eventually derive gives the water exit temperature from the $i^{th}$ increment ($T_{ei}$) as:

$$T_{ei} = \frac{2\pi r_g l U_i T_r + (\rho_w c_w Q - \pi r_g l U_i) T_{wi}}{\rho_w c_w Q - \pi r_g l U_i}$$

(14)

Where $Q$ is the volumetric water flow rate, $\rho_w$ and $c_w$ are the density and specific heat capacity of water, $T_{ei}$ is the entry temperature of the water to the increment and $r_g$ is the tunnel, gallery or shaft radius.
The exit temperature for the $i^{th}$ increment simply becomes the entry temperature for the next increment and the final exit temperature from the tunnel is found as the exit temperature from the last increment.

$$T_{e(i+1)} = T_{si}$$ (15)

The function $U_i$ is defined as:

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{r_g}{\lambda_r} \ln \left( \frac{r_{oi}}{r_g} \right)}$$ (16)

The user of the Rodríguez and Díaz (2009) model should be aware that there is a typographic error in the published version of the above equation, where the logarithmic ($\ln$) term is omitted (Rodríguez, personal communication, 2016).

The heat transfer coefficient $h_i$ is given by combining the Nusselt ($Nu$), Reynolds ($Re$) and Prandtl ($Pr$) numbers of the fluid in the tunnel:

$$h_i = \frac{\lambda_w N u}{2r} = \frac{\lambda_w}{2r} \left[ 0.021 Re^{0.80} Pr^{0.43} \right] = \frac{\lambda_w}{2r} \left[ 0.021 \left( \frac{2 v_w r_g}{\sigma_w} \right)^{0.80} \left( \frac{\rho_w c_w \sigma_w}{\lambda_w} \right)^{0.43} \right]$$ (17)

Where $\omega_w$ is the kinematic viscosity of the water in the tunnel and $r_{oi}$ is defined as the effective thermal radius of the front of heat perturbation from the $i^{th}$ segment of tunnel, it is estimated by Rodríguez and Díaz (2009) from:

$$\frac{r_{oi}}{r_g} = \sqrt{1 + \frac{4h_i}{\rho_c c_r r_g} t}$$ (18)

$\lambda_r, \rho$, and $c_r$ are the thermal conductivity, bulk density and bulk specific heat capacity, respectively, of the rock or sediment hosting the tunnel.

### 2. Comparison of models and sensitivity analysis

#### 2.1. Baseline scenario

These equations can be readily programmed into a spreadsheet and can be tested for a standard scenario, such as that proposed by Rodríguez and Díaz (2009), which assumes an average fluid flow rate of 0.80 m h$^{-1}$ and a calculated Reynolds number $Re$= 680 (laminar flow regime).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flowrate through mine gallery</td>
<td>$q/\rho_w$</td>
<td>10</td>
<td>m$^3$h$^{-1}$</td>
</tr>
<tr>
<td>Water density</td>
<td>$\rho_w$</td>
<td>1000</td>
<td>kg m$^{-3}$</td>
</tr>
<tr>
<td>Water thermal conductivity</td>
<td>$\lambda_w$</td>
<td>0.58</td>
<td>W m$^{-1}$ K$^{-1}$</td>
</tr>
<tr>
<td>Water kinematic viscosity</td>
<td>$\alpha_w$</td>
<td>1.24 x 10$^{-6}$</td>
<td>m$^2$s$^{-1}$</td>
</tr>
<tr>
<td>Mass flow rate</td>
<td>$q$</td>
<td>2.78</td>
<td>kg s$^{-1}$</td>
</tr>
<tr>
<td>Water specific heat capacity</td>
<td>$c_w$</td>
<td>4186</td>
<td>J kg$^{-1}$K$^{-1}$</td>
</tr>
</tbody>
</table>
In Fig 4, Fig 5 and Fig 6 the results of both simulations are compared. For this scenario, thermal breakthrough at the end of the 1000 m gallery occurs at 52.4 days. The Lauwerier-Pruess-Bodvarsson model predicts a temperature at the end of the 1 km gallery equal to 7.94°C after 30 years (Fig 4 and Fig 5). The total heat power gain in the conduit after 30 years is 10.9 kW (10.9 W/m) according to Fig 6. On the other hand the Rodríguez-Díaz model, for the same thermal breakthrough, predicts a temperature at end of the gallery of 12.61°C after 30 years (Fig 4 and Fig 5). The total heat power gain in the conduit after 30 years when using this model (Fig 6) is 65.2 kW (65.2 W/m).

It must be noted that the Lauwerier-Bodvarsson-Pruess model predicts much lower water temperatures and much less heat gain in the mine tunnels than does the Rodríguez-Díaz model. This is largely due to the fact that the Lauwerier-Bodvarsson-Pruess model assumes parallel linear heat conduction to/from the tunnel (which is proportional to \(-\Delta T/\Delta r\), according to Fourier’s Law, where \(r\) is distance from the centre of the tunnel), whereas, the Rodríguez-Díaz model assumes much more efficient radial transport (proportional to \(-\Delta T/\Delta (\ln r)\), according to the line source/sink equation - Banks, 2015 - and hence greater for a given temperature differential).

Fig 4. Comparison of results of Lauwerier-Pruess-Bodvarsson model at 30 years’ simulation time, with original Rodríguez—and Diaz model. Evolution of the water temperature in the mine gallery.
2.2. Sensitivity with respect to flow rate

The variation of the fluid temperature at the exit of the mine tunnel for flow rates in the range 5 to 40 m$^3$ h$^{-1}$ is shown in Fig 7. As would be expected, the greater the flow rate, the lower the emergent temperature, as the heat gained from the tunnel walls is “diluted” in a greater flow. It will be seen that, as the flow rate exceeds 15 m$^3$ h$^{-1}$ the Lauwerier-Pruess-Bodvarsson model begins to asymptotically approach the injection water temperature of 7°C.
If the total heat gain of the water flowing through the tunnel is calculated (Fig 8), a clear difference between the models emerges. The results may at first sight seem perplexing, with greater heat gains for faster flow rates, but lower emergent temperatures. This is simply because the heat gain ($\dot{H}_i$) in any segment of the tunnel equal to the product of the temperature difference between inflow ($T_{ei}$) and outflow ($T_{si}$), the water specific heat capacity ($c_w$) and the flow rate ($q$).

$$\dot{H}_i = q(T_{ei} - T_{si})c_w$$  \hspace{1cm} (19)

or, for the whole tunnel

$$\dot{H} = q(T_{exit} - T_0)c_w$$ \hspace{1cm} (20)

Where $T_{exit}$ is the outflow temperature from the tunnel and $T_0$ is the injection temperature. Thus, as the flow rate increases, the temperature differential tends to decrease (i.e. the outflow temperature decreases) for a given heat gain. With the Rodriguez-Diaz model, the heat gain increases as the flow rate increases, as one would expect, due to the lower average fluid temperature caused by the faster flow rate (i.e. greater temperature difference between rock and fluid). In the Lauwerier-Pruess-Bodvarsson model, it remains almost constant (declining very slowly) as flow rate increases. This is because, for small values of the $erfc$ argument (which is the case in the flow range examined here) in the Lauwerier-Pruess-Bodvarsson model, equation (12) becomes

$$T_s - T_r \approx (T_s - T_i) \times \left(1 - \frac{xP}{2qc_w \sqrt{\frac{\lambda_2 \rho_2 c_2}{t - xA\rho_w/q}}} \right)$$  \hspace{1cm} (21)

Thus, the power gain in the section is given by
\[
\text{Power} \approx (T_s - T_e)qc_w = (T_s - T_e) \left( \frac{xP}{2qc_w} \frac{\lambda_2 \rho_2 c_2}{\left( t - tA \rho_w \right)^2} \right) qc_w
\]

Thus, the \(qc_w\) term is cancelled out from both sides of the equation and a very modest dependence on \(\left( t - tA \rho_w \right)^{-\frac{1}{2}}\) remains. For high values of \(t\), this dependence is almost negligible.

Fig 8. Analysis of the sensitivity of the heat gain in the mine tunnel (at the end of the simulation) with respect to the fluid flow rate

2.3. Sensitivity with respect to thermal properties of the rock

Fig 9 and Fig 10 examine the sensitivity of the models to the parameterisation of the ground’s thermal properties (Table 2). The thermal conductivity \(\lambda\) is allowed to vary between 1.5 W m\(^{-1}\) K\(^{-1}\) (typical for a shale or claystone - Banks, 2012) up to 3.5 W m\(^{-1}\) K\(^{-1}\) (which might be encountered in a quartz-rich granite or sandstone - Banks 2012).

Fig 9 illustrates that, in the Rodriguez-Diaz model, as one might expect, the emergent fluid temperature is warmer (more heat gained from the tunnel walls) the higher the thermal conductivity is. Interestingly, the effect of thermal conductivity on the Lauwerier-Preuss-Bodvarsson model is much more muted. Fig 10 illustrates that the sensitivity of the tunnel’s heat exchange performance to the volumetric heat capacity is much lower than for the thermal conductivity. The results of both models have been calculated after 5 years’ simulation time.
Table 2. Characteristics of different types of ground considered (http://cte-web.iccl.es/ and http://www.engineeringtoolbox.com/)

<table>
<thead>
<tr>
<th>Ground Type</th>
<th>λ (W m⁻¹ K⁻¹)</th>
<th>c (J kg⁻¹ K⁻¹)</th>
<th>ρ (kg m⁻³)</th>
<th>VHC (MJ m⁻³ K⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand with gravel, wet</td>
<td>2</td>
<td>1045</td>
<td>1950</td>
<td>2.0</td>
</tr>
<tr>
<td>Clay</td>
<td>1.5</td>
<td>2085</td>
<td>1500</td>
<td>3.1</td>
</tr>
<tr>
<td>Shale</td>
<td>2.2</td>
<td>1000</td>
<td>2400</td>
<td>2.4</td>
</tr>
<tr>
<td>Limestone (hard)</td>
<td>1.7</td>
<td>1000</td>
<td>2100</td>
<td>2.1</td>
</tr>
<tr>
<td>Sandstone</td>
<td>3</td>
<td>920</td>
<td>2400</td>
<td>2.2</td>
</tr>
<tr>
<td>Granite</td>
<td>2.8</td>
<td>1000</td>
<td>2600</td>
<td>2.6</td>
</tr>
<tr>
<td>Rodríguez-Díaz baseline scenario</td>
<td>2.78</td>
<td>800</td>
<td>2500</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Fig 9. Analysis of the sensitivity of the fluid temperature emerging from the mine tunnel with respect to the rock thermal conductivity, after a simulation time of 5 years, otherwise parameterized according to the baseline scenario in Table 1.

Fig 10. Analysis of the sensitivity of the fluid temperature emerging from the mine tunnel with respect to the rock volumetric heat capacity, after a simulation time of 5 years, otherwise parameterized according to the baseline scenario in Table 1.

2.4. Sensitivity with respect to tunnel diameter
Fig 11 shows the variation of temperature for the fluid emerging from the mine, for different diameters of simulated mine tunnel, after 5 and 30 years. In the figure, it can be seen that the Lauwerier-Pruess-Bodvarsson model is rather sensitive to this parameter: the calculated temperature increases approximately linearly with respect to the dimension. The Rodriguez-Díaz output, however, increases with tunnel diameter up to c. 8 m, where it reaches a maximum.

It will also be noted from Fig 11 that, as tunnel diameter reaches 10-11 m, after 5 years of simulation, the two models yield similar output temperatures and temperature profiles (Fig 12). This is potentially related to the fact that at such very large diameters, the tunnel curvature becomes rather low and the radial heat flow (assumed by Rodríguez-Díaz) increasingly resembles a linear (parallel) heat flow pattern (as assumed by Lauwerier-Pruess-Bodvarsson). For larger time simulations, due to the quicker thermal exhaustion of the mine tunnel estimated by the Lauwerier-Pruess-Bodvarsson method, this effect is no longer visible at the studied diameters.

Fig 12. Comparison of results of Lauwerier-Pruess-Bodvarsson model at 5 and 30 years’ simulation time for a tunnel diameter of 11 m, with Rodríguez-Díaz model (otherwise parameterised according to the baseline scenario in Table 1). Temperature of water at different distances along the mine gallery.
3. Improvements to the Rodríguez and Díaz (2009) model

3.1. Calculation of Thermal Radius

Arguably, one of the main weak points of the Rodríguez and Díaz (2009) model is the derivation of $r_0$ (Equation 18). This is for two reasons:

a) Because the concept of a finite limit to temperature disturbance is a false one, akin to the mythical ‘radius of influence’ of a water well. The line source heat equation (Banks, 2015) relates temperature change ($\Delta T$) at radius $r$ to heat extraction rate $\dot{h}$ in W m$^{-1}$:

$$\Delta T = \frac{\dot{h}}{4\pi \lambda} Ei(z)$$

(23)

Where:

$$Ei(z) = -0.5772 - \ln(z) + \frac{z^2}{(2\cdot2!)} + \frac{z^3}{(3\cdot3!)} - \frac{z^4}{(4\cdot4!)} + ....$$

(24)

$$z = \frac{r^2}{4\alpha_r t}$$

(25)

$$\alpha_r = \frac{\lambda_r}{\rho_r c_r}$$

(26)

Here $\alpha_r$ is the rock thermal diffusivity. It can readily be seen that at any distance for $t > 0$, there will be some finite temperature change.

b) Because Equation (A13) in Rodríguez and Díaz’s (2009) paper makes the dubious assumption that the initial tunnel wall temperature is the average of the fluid entry temperature and the ambient undisturbed rock temperature, with no clear justification.

The current authors have thus decided to take an alternative approach, founded in the logarithmic approximation (sometimes referred to as the Cooper and Jacob, 1946, approximation) of Equation (23), which can be made when $z$ is small ($t$ is large)

$$\Delta T = \frac{\dot{h}}{4\pi \lambda} \left[ \ln\left(\frac{4\lambda_r t}{r^2 \rho_r c_r}\right) - 0.5772 \right]$$

(27)

Thus:

$$\frac{4\lambda_r t}{r^2 \rho_r c_r} = \exp\left(\frac{4\pi \lambda_r \Delta T}{\dot{h}} + 0.5772\right)$$

(28)

Thus $r = \frac{4\lambda_r t}{\rho_r c_r} \frac{1}{\exp\left(\frac{4\pi \lambda_r \Delta T}{\dot{h}} + 0.5772\right)}$ provided $t > \frac{5r^2 \rho_r c_r}{\lambda_r}$

(29)
As in the Rodríguez and Díaz (2009) formulation, $r_0$ is directly related to $\sqrt{\Delta T}$, $r_0$ can be found by setting $\Delta T$ to some arbitrarily low value (say 0.1°C) and finding the radius at which this displacement occurs. Furthermore, at large times, the temperature along the tunnel tends to exhibit a relatively shallow gradient and it seems unnecessary to calculate $r_0$ for every increment. As the Rodríguez and Díaz (2009) solution (Equation 16) depends on $\ln (r_0/r_g)$, it is relatively insensitive to this parameter and a single value of $r_0$ for the entire mine tunnel can be applied, based on the average heat extraction rate $\dot{h}$ from the tunnel.

For the Rodríguez and Díaz (2009) baseline scenario, we can see the effect of these different assumptions and methodologies. Using the original formulation (Equation 18), a value of $r_0$ of 74.7 m is calculated, together with an average heat extraction rate of 65.2 W/m after 30 years.

Using equation (29) the results shown in Table 3 are obtained:

<table>
<thead>
<tr>
<th>Equation (18)</th>
<th>Effective thermal radius $r_0$ after 30 years</th>
<th>Average heat extraction from tunnel during 30 years</th>
<th>Average heat extraction from tunnel after 30 years</th>
<th>Emergent fluid temperature after 30 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation (29) $\Delta T = 0.01°C$</td>
<td>74.7 m</td>
<td>73.4 W m$^{-1}$</td>
<td>65.2 W m$^{-1}$</td>
<td>12.61°C</td>
</tr>
<tr>
<td>Equation (29) $\Delta T = 0.1°C$</td>
<td>54.2 m</td>
<td>78.7 W m$^{-1}$</td>
<td>69.3 W m$^{-1}$</td>
<td>12.96°C</td>
</tr>
<tr>
<td>Equation (29) $\Delta T = 0.5°C$</td>
<td>48.8 m</td>
<td>80.7 W m$^{-1}$</td>
<td>70.8 W m$^{-1}$</td>
<td>13.09°C</td>
</tr>
<tr>
<td>Equation (29) $\Delta T = 1°C$</td>
<td>44.0 m</td>
<td>82.7 W m$^{-1}$</td>
<td>72.3 W m$^{-1}$</td>
<td>13.22°C</td>
</tr>
</tbody>
</table>

It can be seen that the results from the arguably more transparent and simpler approach of Equation (29) yield similar results to the original Rodríguez and Díaz (2009) formulation, and that the result is rather insensitive to the value selected for $\Delta T$, at least up to a value of 0.5°C.

Figures Fig 13–Fig 15 show additional sensitivity analysis, comparing the original Rodríguez and Díaz (2009) formulation with the proposed Cooper-Jacob type approach (equation 29), utilising a $\Delta T$ of 0.01°C.
Fig 13. Analysis of the sensitivity of the fluid temperature emerging from the mine tunnel utilising the original Rodríguez and Díaz (2009) formulation for thermal radius (Equation 18), with the proposed Cooper-Jacob approach (Equation 29) with a $\Delta T$ of 0.01°C, for differing mine tunnel fluid flow rates over a 30 year period.

Fig 14. Analysis of the sensitivity of the fluid temperature within the mine tunnel after 30 years, utilising the original Rodríguez and Díaz (2009) formulation for thermal radius (Equation 18), with the proposed Cooper-Jacob approach (Equation 29) with a $\Delta T$ of 0.01°C, for differing mine tunnel fluid flow rates.
Fig. 15. Analysis of the sensitivity of the fluid temperature emerging from the mine tunnel utilising the original Rodríguez and Díaz (2009) formulation for thermal radius (Equation 18), with the proposed Cooper-Jacob approach (Equation 29) with a ΔT of 0.01°C, for differing rock thermal conductivities over a 30 year period.

From Figures Fig 13-Fig 15, the following points should be noted:

- The greatest deviation of the Cooper-Jacob (Equation 29) approach is at early times, high flow rates and low thermal conductivities.
- The deviation of the Cooper-Jacob (Equation 29) approach increases with increasing distance along the tunnel.
- The Cooper-Jacob approach with ΔT of 0.01°C tends to result in slightly higher simulated heat gains and emergent fluid temperatures as compared with the original Rodríguez and Díaz (2009) formulation for thermal radius (Equation 18). In Figures Fig 13-Fig 15, the recorded discrepancy after 30 years was for a thermal conductivity of 1.5 W m⁻¹ K⁻¹ at 10 m³ h⁻¹ flow rate was 48.0 kW heat gain as opposed to 41.4 kW (86% of the Cooper-Jacob result).

3.2. Calculation of Nusselt Number

Another potential issue with the original Rodríguez and Díaz (2009) formulation is the use of the Nusselt number, which is a dimensionless number expressing the ratio of convective to conductive heat transfer across a boundary. In Equation (17), Rodríguez and Díaz (2009) cite the Nusselt number (Nu) as a function of the Reynolds (Re) and Prandtl (Pr) numbers:

\[ Nu = 0.021 Re^{0.80} Pr^{0.43} \]  

(30)

For the Rodríguez and Díaz (2009) baseline case, Re = 680, Pr = 9.38 and the calculated Nu from Equation (30) is 10.15.

The exact derivation of this equation is unclear; however, it is similar to the most commonly cited form of the Dittus and Boelter (1985) equation for turbulent flow (Equation 31 - Incropera et al., 2007; Subramanian, 2006, although it should be
noted that the original form was a little more complex than the common cited version - Williams, 2011; Winterton, 1998):

\[ \text{Nu} = 0.023 \text{Re}^{0.8} \text{Pr}^{n} \]  

(31)

Here, \( n \) equals to 0.4 when the fluid is being heated and to 0.3 when is being cooled.

This equation yields a \( \text{Nu} \) of 10.39 for the Rodríguez and Diaz (2009) baseline case. However, the equation is only valid for \( 0.6 < \text{Pr} < 160 \), \( \text{Re} > 10,000 \) and \( L/D > 10 \), where \( D \) is the internal diameter and \( L \) is a characteristic length.

It will be noted, however, that for typical mine tunnel problems, the water flow may be laminar, rather than turbulent - in Fig 7, \( \text{Re} \) is typically less than 2300 (usually regarded as the transition point from laminar to turbulent flow in circular conduits). Thus, the Nusselt number utilised by Rodríguez and Diaz (2009) is probably not applicable to many mining situations. Incropera et al. (2007) state that, for laminar flow, \( \text{Nu} \) becomes independent of \( \text{Re} \) and \( \text{Pr} \), and tends towards a value of 3.66 (for approximately constant surface temperature) to 4.36 (for approximately constant surface heat flux). In an entry region to a conduit, the Nusselt number can be higher, but for a distance into the conduit \( > 0.05 \cdot \text{Re} \cdot \text{Pr}/D \), the steady state values are approached. For the Rodríguez and Diaz (2009) baseline case, this entry region is calculated as 1276 m long. Thus, in the Rodríguez and Diaz (2009) baseline case (Table 1), while the Reynolds Number is too low for truly turbulent flow, the tunnel length is too short for truly laminar flow to have been developed. Thus, one would expect the real Nusselt number to be somewhere between 4 and 10.

For the entry region to laminar flow regimes, Incropera et al. (2007) recommend the use of one of two equations for the average Nusselt Number (\( \overline{\text{Nu}} \)) in the region:

(i) the Hausen equation (32) for \( \text{Pr} > 5 \):

\[ \overline{\text{Nu}} = 3.66 + \frac{0.0688(D/L)\text{Re Pr}}{1 + 0.04[(D/L)\text{Re Pr}]^{1/3}} \]  

(32)

(ii) the Sieder and Tate equation (33) for \( 0.6 < \text{Pr} < 5 \) and \( 0.0044 < \mu_b/\mu_w < 9.75 \):

\[ \overline{\text{Nu}} = 1.86 \left( \frac{\text{Re} \cdot \text{Pr} \cdot D}{L} \right)^{1/3} \left( \frac{\mu_b}{\mu_w} \right)^{0.14} \]  

(33)

Where \( \mu \) is the dynamic viscosity of the fluid and the subscripts \( w \) and \( b \) refer to the fluid adjacent to the wall and the bulk fluid at a quasi-axial position, respectively.

Table 4 illustrates the range of values of \( \text{Nu} \) that could be applied in the Rodríguez and Diaz (2009) model, together with their calculated values for the Baseline Scenario.

**Table 4.** Various values of Nusselt Number applied to Rodríguez and Diaz (2009) baseline case. \( \text{Re} = 680, \text{Pr} = 9.38 \). In the Sieder and Tate equation (33), the ratio of viscosities is assumed to be 1.
<table>
<thead>
<tr>
<th>Source</th>
<th>Condition</th>
<th>Nu</th>
<th>Heat gain after 30 years</th>
<th>Emergent fluid temperature after 30 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dittus and Boelter - Eqn. 31</td>
<td>Turbulent flow</td>
<td>10.39</td>
<td>65.3 kW</td>
<td>12.62</td>
</tr>
<tr>
<td>Rodríguez and Díaz - Eqn. 30</td>
<td>Turbulent flow</td>
<td>10.15</td>
<td>65.2 kW</td>
<td>12.61°C</td>
</tr>
<tr>
<td>Sieder and Tate Eqn. 33</td>
<td>Entry region to laminar flow. (Pr&lt;5)</td>
<td>5.48</td>
<td>59.7 kW</td>
<td>12.14°C</td>
</tr>
<tr>
<td>Hausen Eqn. 32</td>
<td>Entry region to laminar flow. (Pr&gt;5)</td>
<td>4.93</td>
<td>58.3 kW</td>
<td>12.01°C</td>
</tr>
<tr>
<td>Laminar flow</td>
<td>Constant surface heat flux</td>
<td>4.36</td>
<td>56.5 kW</td>
<td>11.86°C</td>
</tr>
<tr>
<td>Laminar flow</td>
<td>Constant surface temperature</td>
<td>3.66</td>
<td>53.6 kW</td>
<td>11.61°C</td>
</tr>
</tbody>
</table>

For the Rodríguez and Díaz (2009) Baseline scenario, arguably the most appropriate equation is the Hausen equation (32), where the tunnel is effectively the Entry Region to a laminar flow regime. With respect to the Hausen equation, the Nusselt formulation used by Rodríguez and Díaz (2009) overestimates the heat transfer (in the baseline case) by 12% and the emergent temperature (relative to an injection baseline of 7°C) also by 12%.

4. Discussion and conclusions

Abandoned flooded mine voids can be utilised efficiently both as thermal stores and as subsurface heat exchangers, providing an environmentally friendly source of heating and cooling. Thus, mine water, traditionally regarded as an environmental liability, can be converted into an economic and environmental asset.

To assess the performance of mine tunnels and roadways as heat exchangers, some form of modelling is usually needed. While detailed numerical modelling work may be required prior to implementation, analytical modelling at an early stage of feasibility study can provide insight into the sustainability of mine water abstraction - heat exchange - reinjection schemes. Two different, commonly used analytical models of heat exchange in a mine tunnel have been examined and compared.

The Lauwerier-Pruess-Bodvarsson model (Pruess and Bodvarsson, 1983) yields very conservative (i.e. low) values of mine tunnel heat exchange capacity. This is primarily because the model was developed for planar or tabular flow horizons (i.e. aquifer layers or fractures), rather than quasi-circular mine tunnels. In other words, the original Lauwerier-Pruess-Bodvarsson model assumed 1 dimensional parallel heat conduction away from the water-bearing horizon or fracture, whereas, in real circular tunnels, heat is conducted away more effectively in a radial and divergent manner.

The Rodríguez and Díaz (2009) model results in more realistic heat exchange capacities for mine tunnels and assumes radial and divergent heat conduction from (or to) the tunnel. The modelled heat exchange capacity increases as fluid flow and rock thermal conductivity increase, and decreases slowly with increasing time (as the heat in the rocks surrounding the tunnel is slowly depleted), as common sense would
predict. The Rodríguez and Díaz (2009) model does, however, suffer from at least three flaws:

Firstly, the published version of the model (Rodríguez and Díaz, 2009) contains a typographical error, where a logarithmic term is omitted from Equation 16.

Secondly, the theoretical basis for calculating the effective radius of the thermal front around the tunnel is, arguably, a little contrived. In this paper, a fresh approach is adopted, namely, the well-known Cooper-Jacob logarithmic approximation of the line source heat function. This approach arguably has a more transparent theoretical basis, but suffers from the modest drawbacks that (i) it requires an iterative process to solve, and (ii) it requires a subjective assumption to be made regarding the temperature differential ($\Delta T$ in Equation 29) that defines the effective edge of the thermal front. The two approaches have been compared and it is found that, using a $\Delta T$ of down to 0.01°C, the original Rodríguez and Díaz (2009) approach slightly overestimates the radius of the thermal front and thus underestimates the overall heat exchange compared with the Cooper-Jacob approximation. However, the differences are modest, especially for low-to-moderate flow rates and long times, and the original Rodríguez and Díaz (2009) approach is conservative in nature.

Thirdly, and more importantly, the calculation of the Nusselt number in the Rodríguez and Díaz (2009) approach assumes turbulent flow conditions. In reality, in large diameter tunnels, there is a strong probability that the calculated Reynolds Number will fall within the laminar flow regime. This means that the Rodríguez and Díaz (2009) model will tend to overestimate the heat transfer if flow is not truly turbulent. This can easily be modified in a spreadsheet environment, however, and Equation (17) can be modified to use a value of $Nu$ appropriate to turbulent flow, laminar flow or the Entry Region to a non-fully developed laminar flow regime. In the case of Rodríguez and Díaz (2009)'s Baseline Scenario (Table 1), the use of the turbulent $Nu$ overestimated heat exchange in the tunnel by 12% as compared to the use of a $Nu$ more appropriate for a laminar flow Entry Region condition.

In conclusion the Rodríguez and Díaz (2009) model is more appropriate for simulating heat exchange in conduit-like mine tunnels and roadways than the Lauwerier-Pruess-Bodvarsson (Pruess and Bodvarsson, 1983) approach. Care should be taken, however, to use a Nusselt number in Equation (17) which is appropriate to the flow regime within the tunnel. It is arguable that the Pruess and Bodvarsson (1983) approach would be more applicable to planar or tabular mined void geometries.

**Acknowledgements**

This research was supported by the EU Research Fund for Coal and Steel (RFCS) LoCAL Project RFCR-CT-2014-00001.

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