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A Note on the Logic of Worldly Ground

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Abstract: In his 2010 paper ‘Grounding and Truth-Functions’, Fabrice Correia has developed the first and so far only proposal for a logic of ground based on a worldly conception of facts. In this paper, we show that the logic allows the derivation of implausible grounding claims. We then generalize these results and draw some conclusions concerning the structural features of ground and its associated notion of relevance which has so far not received the attention it deserves.

§1. Introduction

Kit Fine is a British philosopher. However, not all philosophers are British. For example, there are also American philosophers. Now, does this latter fact – that there are American philosophers – hold partly because, or in virtue of the fact that Fine is a philosopher? It seems not. Given that Fine is British, it seems that his being a philosopher does not contribute at all to bringing it about, or making it the case, that there are American philosophers. We show that an otherwise attractive-seeming logic for ground, more precisely a logic for what is often called a worldly conception of ground, yields contrary results, and discuss what conclusions should be drawn from this.

§2. Worldly ground

Recent years have witnessed an increasing interest in the notion of ground, where ground is taken to be a kind of non-causal priority among facts. A standard way to motivate this notion is to point to certain uses of the sentential connective ‘because’ or to uses of the phrase ‘in virtue of’ and cognate phrases that are particularly widespread in philosophical discourse.1

Claims to the effect (i) that someone is in a given mental state in virtue of being in a certain physical state, or (ii) that a given ball is red or round because it is red, are typical examples of statements of ground. We shall assume that such statements express that a relation of

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1 This is not the place to give a comprehensive introduction to the debate. For this consider Correia and Schnieder 2012 and Trogdon 2013.
ground obtains between certain facts. For instance, \((ii)\) expresses that the fact that a given ball is red or round is grounded in the fact that it is red.

Theories of ground can be seen as attempts to account for the philosophical use of ‘in virtue of’ and the relation thereby expressed. Intuitions on pertinent uses of ‘because’ or ‘in virtue of’ are therefore not only helpful to motivate the notion of ground, they also provide a crucial, if defeasible, test of adequacy for proposed theories of ground. In particular, if a theory entails an unacceptable ‘in virtue of’-claim, this is a \emph{prima facie} reason to reject it. This, at least, is common practice among participants of the debate, and we will follow it.3

There is a standard distinction between \emph{conceptualist} and \emph{factualist}, or \emph{worldly} conceptions of ground.4 According to the factualist conception, grounding relates relatively coarse-grained ‘worldly’ facts, whereas on the conceptualist view, the grounding relation is in general highly sensitive to how the facts in question are conceptualized.5 While this issue is relevant for a host of topics related to grounding, we will focus on how it affects the \emph{logic} of ground, i.e. the principles and rules that govern the interaction between ground and the logical connectives. Several conceptualists have proposed systems by the lights of which the fact that \(A\) grounds, for instance, the fact that \(A \land A\), the fact that \(A \lor A\), and the fact that \(\neg\neg A\).6 Factualists would deny this.7 For on a worldly conception of facts, the fact that \(A\) is arguably the same as the facts that \(A \land A\), \(A \lor A\) and \(\neg\neg A\) – albeit each time \emph{represented} under a different guise. Given that grounding is irreflexive, factualists thus cannot allow for any grounding relations obtaining in these cases. This does not mean that they cannot hold on to the intuition that, e.g., conjunctive facts are grounded in their conjuncts. This will be the case, however, only if the given conjunctive fact is really different from its conjuncts.

It is not clear that the conceptualist and the factualist conceptions of ground represent mutually exclusive options; they might simply capture different, though related phenomena, both worthy of investigation. To the extent to which they are seen as competing views, however, we want to highlight three points that seem to favour the factualist conception.

\textit{a.} The worldly conception seems more natural given a standard way to \emph{motivate} both the viability and the importance of the notion of ground. For this is usually done by appealing to a picture of \emph{reality} as a \emph{layered structure} (cf. \cite{deRosset:2013}; \cite{Bennett:2011}).

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2 We make this assumption purely for ease of expression; nothing in our argument turns on whether ground is strictly a relation between facts. (\textit{For discussion, see Correia and Schnieder 2012a: 10ff.})

3 Cf., e.g., \cite{Correia:2010a:263}.

4 Cf. e.g. \cite{Correia:2010a:256f}; \cite{Correia and Schnieder:2012:14f}.

5 Whether one wants to still call those relata facts or perhaps rather true propositions is of no matter for present purposes. We continue to talk about facts for brevity’s sake. Note that on both conceptions, grounding is hyperintensional, and does not in general allow even logically equivalent sentences to be interchanged salva veritate. Moreover, there are disagreements in both camps as to just how fine- or coarse-grained the relata are to be construed.

6 This holds in \cite{Fine:2012} \emph{Impure Logic of Ground} and also given the principles proposed by \cite{Rosen:2009:117ff.}. Related results can be obtained in \cite{Schnieder:2011}'s \emph{Logie de ’Because’} (2011).

7 This holds at least for \cite{Correia:2010}, on whose system we shall concentrate below. Perhaps not all factualists will give the same verdict on the case. \cite{Audi:2010:700f} takes the relation to be even more coarse-grained.
According to such a picture, the world is not a mere aggregate of facts, but falls into several layers that are connected by various relations of priority. Grounding is then thought of as one such relation. On this kind of view, therefore, grounding emerges as a relation among constituents of the world that exist, and can be individuated, independently of our conceptual (or linguistic) representations of them – a relation that carves reality at its joints. Yet, the conceptualists’ account of ground does not seem to pay this picture its proper due in that it introduces distinctions that are unduly sensitive to our conceptualizations of reality (cf. Correia (2010: 258f)). In other words, conceptualists conflate mere shadows of language with real features of the world.

b. A related worry is that even on the conceptualist view, there should be some non-trivial notion of ‘saying the same thing’ under which the following schema is valid (‘[A]’ abbreviates ‘the fact that A’):

\[
\text{EQUIV}
\begin{align*}
\text{a) If A and B say the same thing and [A] grounds [C], then [B] grounds [C].} \\
\text{b) If C and D say the same thing and [A] grounds [C], then [A] grounds [D].}
\end{align*}
\]

For, surely, not every linguistic difference should make for a difference in ground-theoretic status. But so far, conceptualists have not come up with a viable general criterion to account for that intuition.

c. Finally, conceptualists have not yet shown that the logics they propose are sufficiently well-behaved. For, although the proof-theoretic side of logics of conceptual ground is by now fairly well investigated (cf. esp. Fine 2012), there is still no semantics relative to which we could prove soundness and completeness for the deductive systems. Using an elegant and well-motivated variant of situation semantics (called truth-maker semantics), Fine (2011) establishes soundness and completeness for a set of structural rules for various grounding-operators. As noted in his (2012: 74), however, rules for the interaction between grounding-operators and the truth-functors that are widely accepted among conceptualists turn out to be unsound under this semantics. Strikingly, this is precisely because the semantics cannot discriminate among the semantic values of, e.g. A and A ∨ A.

As we have already seen, the factualist conception takes the idea of grounding as a joint-carving, worldly relation, more seriously. Moreover, in his (2010), Correia has developed a logic of worldly ground that puts this idea on a firm formal footing. Crucially, Correia has offered a semantics with respect to which his logic is provably sound and complete. And while the semantics and some aspects of the logic may seem somewhat contrived, Fine has pointed out that a modified version of his truth-maker semantics allows for a very natural definition of a notion of ground that agrees with Correia’s (cf. Fine (ms. b:11)). Finally, on the Correia/Fine view, a principled account of the notion of saying the same thing relevant to EQUIV is possible; in particular, its logic is that of Angell’s (1977) notion of analytic equivalence.

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The prospects for worldly ground thus look good. It takes the idea of ground as a joint-carving, worldly relation seriously, and appears to have a well-behaved logic in the system proposed by Correia. However, we will show that these apparent advantages notwithstanding, all is not well for proponents of worldly ground.

§3. The argument

The only properly developed proposal for a logic of worldly ground is that of Correia (2010). This section proves that Correia’s logic entails a close cousin of the above implausible claim of ground (cf. §1).9

Central to Correia’s approach is a notion of factual equivalence (≈), the role of which is to provide a necessary and sufficient condition for two formulae being, under any interpretation, interchangeable in the scope of ground (<). Correia takes the logic of ≈ to be Angell’s logic of analytic equivalence. At first glance, that logic seems well suited to the task. On the one hand, analytic equivalence is narrower even than equivalence in the logic of first-degree entailment. Correia’s idea can thereby allow for pairs like A and A ∨ (A ∧ B) being factually inequivalent, and thus for the former to ground the latter (cf. Correia 2010: 263). On the other hand, the characteristic kinds of pairs over which factualists and conceptualists are divided, like A and A ∨ A come out factually equivalent. It is therefore reasonable to hope that analytic equivalence is sufficiently fine-grained, but not too fine-grained, to play the role of factual equivalence in the factualist’s system.

For our present purposes, the main thing that matters about factual equivalence is that it satisfies the law of distribution of ∨ over ∧:

\[(X) \quad A \lor (B \land C) \approx (A \lor B) \land (A \lor C)\]

We further make use of the following rules for ground:\[10,11\]

\[(\land\text{-Introduction}) \quad A; B; (A \land B) \vdash A; (A \land B) \vdash B / A, B < A \land B\]
\[(\lor\text{-Introduction}) \quad A; (A \lor B) \vdash A / A < A \lor B\]
\[\quad \vdash B / B < A \lor B\]
\[(\text{Factivity}) \quad \Gamma < C / C\]
\[\quad \Gamma, A < C / A\]

9 After we finished the paper, Correia informed us that he has independently discovered the same problem for his logic of factual equivalence, and develops an improvement in an as yet unpublished manuscript called ‘On the Logic of Factual Equivalence’.

10 The rules are written in the format ‘Premise; Premise; … / Conclusion’. The semi-colon is used to separate premises, and the comma to separate the sentences in the left-hand argument place of <, which can be occupied by any finite number of sentences. We have changed Correia’s notation to that employed in Fine’s papers. By ‘!≈’ we denote lack of factual equivalence (i.e. ‘A !≈ B’ abbreviates ‘¬(A ≈ B)’).

11 Note that Correia (2010: 269) in the end replaces the introduction rules by refinements in which the appeal to factual inequivalence is replaced by one to lack of conjunctive-disjunctive containment, i.e. factual equivalence of a premise to some disjunction of conjunctions, one of which contains the conclusion as a conjunct. The change has no real effect on the uses we make of the rules.
The **∧-Introduction** rule captures the intuitive thought that a true conjunction is grounded by its conjuncts, provided it is not factually equivalent to either. (If it is, the irreflexivity of ground prevents it from being grounded by the conjuncts.) The **∨-Introduction** rules capture the intuitive thought that a true disjunction is grounded by any true disjunct, provided it is not factually equivalent with it. The **Factivity** rules encode the compelling principle that only truths are grounded and that only truths are grounds. The **Cut** rule, finally, captures the plausible view that grounding is transitive.

Consider any statements $A$, $B$, and $C$ such that $A$ and $B$ are true, $C$ is either true or false, and the following four assumptions concerning lack of factual equivalence hold:

\[(A1)\]  
\[(A \lor C) \not\approx A\]

\[(A2)\]  
\[(A \lor B) \not\approx B\]

\[(A3)\]  
\[
\left((A \lor B) \land (A \lor C)\right) \not\approx (A \lor B)
\]

\[(A4)\]  
\[
\left((A \lor B) \land (A \lor C)\right) \not\approx (A \lor C)
\]

\[(A1)-(A4)\] are weak assumptions that come out true for most choices of statements $A$, $B$, and $C$.\(^{12}\)

The rules of **∨-Introduction** yield

\(1\)  
\[A < A \lor C\]

\(2\)  
\[B < A \lor B\]

Since $A \lor C$ and $A \lor B$ are true, **∧-Introduction** and two applications of **Cut** yield

\(3\)  
\[A, B < (A \lor B) \land (A \lor C)\]

By (X), it follows that

\(4\)  
\[A, B < A \lor (B \land C)\]

We maintain that (4) has many instances that are implausible, even though the corresponding instances of the assumptions of our derivation are true.

Our first example is very close to the one with which we began this paper. It is a standard view that as far as grounding is concerned, an existential quantification like ‘someone is an American philosopher’ is just like the corresponding (potentially infinite) disjunction. If so, then we may paraphrase the quantification by ‘someone other than Fine is an American philosopher $\lor$ Fine is an American philosopher’. The second disjunct is plausibly grounded-theoretically equated with the conjunction ‘Fine is a philosopher $\land$ Fine is American’. It is easy to check that if we set $A$ to ‘someone other than Fine is an American philosopher’, $B$ to ‘Fine is a philosopher’ and $C$ to ‘Fine is American’, all our premises come out true. So we obtain the conclusion that the fact that someone is an American philosopher is grounded by the facts that someone other than Fine is an American philosopher, and that

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\(^{12}\) Using the refined rules (cp. the previous footnote), we instead require assumptions stating lack of conjunctive-disjunctive containment. These are still true for most choices of $A$, $B$, and $C$. Roughly, as long as our $A$, $B$, and $C$ have independent truth-makers, the assumptions all turn out true.
Fine is a philosopher. In other words, this last fact helps ground the fact that someone is an American philosopher. But this is implausible given an understanding of ground that is mediated by our intuitive understanding of ‘in virtue of’. For, we would not say that the fact that there are American philosophers obtains partially in virtue of some non-American (Fine) being a philosopher.

We note two ways of dramatizing the result. First, we can let C, and thus $B \land C$, be necessarily false. For instance, let A mean that something is a prime number, B that 4 is a number, and C that 4 is prime. It seems highly implausible to hold that the fact that something is a prime number is due in part to the fact that 4 is a number. Second, we may even set C to $\neg B$, obtaining that B helps ground $A \lor (B \land \neg B)$. This seems most counter-intuitive, for it implies that for any truth A and any claim B, the fact that $A \lor (B \land \neg B)$ is due in part to either B or $\neg B$. Consider the fact that snow is white or the number of hairs on my head is both odd and even. Clearly, that this fact obtains is not due even in part to the number of hairs on my head being odd, or due to that number being even, whichever is in fact the case.

We conclude that Correia’s logic of ground has implausible results. While the appeal to analytic equivalence permits us to abstract from mere differences in representation such as may seem to obtain between A and $A \lor A$, it does so at the cost of obliterating real distinctions tracked by our informal understanding of ‘in virtue of’ and cognate phrases.

§4. Variations

It is instructive to note that we can also obtain similar results by appealing to resources that bear no obvious connection to the distributivity principle (X) appealed to before. The basis of these derivations is a structural principle of convexity for ground. It says that if a given truth C is grounded by some collection of facts as well as a subset of the same collection of facts, then it is grounded by any collection of facts that lies between these two, i.e. which is both a subset of the first and a superset of the second collection:

**CONVEXITY:** $\Gamma < C; \Gamma, \Delta, E < C / \Gamma, \Delta < C$

Now if A, B, C are all true, we can (typically) show both that $A < A \lor (B \land C)$ and that $A, B, C < A \lor (B \land C)$. By CONVEXITY, it then follows that $A, B < A \lor (B \land C)$. This is like the above case, only that here we had to assume that C is true.

We are tempted to suggest that typical instances of this grounding claim should be rejected. Roughly, it seems to us that some collection that includes B can only constitute a full ground of $A \lor (B \land C)$ if it also includes C. We admit, though, that intuitions about the appropriate use of ‘in virtue of’ et al on their own lend only limited support to this contention. Consider the fact that Quine is an American philosopher $\lor (Fine is a philosopher$

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13 This slightly reduces the number of statements for which (A1)–(A4) are true. Again, it suffices to assign statements with independent truth-makers to A and B to ensure that the assumptions hold good.

14 Modulo the relevant conditions of factual inequivalence; this caveat will be left implicit in what follows.
∧ Fine is British). Does this fact obtain wholly in virtue of the facts that Quine is an American philosopher and that Fine is a philosopher? Or is it only the fact that Quine is an American on its own, which fully grounds the disjunctive fact? Intuition does not seem to decide the case.

However, in the presence of some additional plausible assumptions, CONVEXITY also implies the results previously found to be unacceptable. Firstly, according to a very popular view, grounding is an internal relation in the sense that if some facts \( \Gamma \) ground another \( C \), they do so in every world in which all of \( \Gamma \) and \( C \) obtain.\(^{15}\)

**INTERNALITY:** \( \Gamma \prec C \rightarrow \Box((\Lambda \Gamma \land C) \rightarrow \Gamma \prec C) \)

Now assume that grounding is necessarily internal and that \( A \) and \( B \) are true. Assume further that \( A \land B \land C \) is contingently false, so \( C \) is contingently false and compossible with \( A \) and \( B \). Then in any world in which \( A, B \) and \( C \) are all true, we have \( A, B \prec A \lor (B \land C) \). But since grounding is internal in every world, and all of \( A, B, \) and \( A \lor (B \land C) \) obtain in the actual world, it follows that in the actual world \( A, B \prec A \lor (B \land C) \). So every instance of this schema which is derivable by the above method, and in which \( A, B, \) and \( C \) are compossible, can also be obtained from CONVEXITY and INTERNALITY.

Secondly, it is plausible that we should also recognize a non-factive counterpart of the notion of ground.\(^{16}\) If so, it seems natural to think that a structural principle like CONVEXITY should hold for non-factive ground (\( \prec \)) if it holds for factive ground:

**CONVEXITY*:** \( \Gamma \prec C; \Gamma, \Delta, E \prec C / \Gamma, \Delta \prec C \)

Given the obvious connecting principle that if \( \Gamma \prec C \) and \( \Lambda \Gamma \land C \), then \( \Gamma \prec C \), we can then derive all the above unwelcome results.

Since Correia’s logic for ground does not have modal operators, INTERNALITY is not derivable in it. Still, it is a very plausible seeming claim, and one that Correia has elsewhere committed himself to. CONVEXITY provably holds in Correia’s logic.\(^{17}\) As for CONVEXITY*, Correia has no operator for non-factive ground in his system, but it is clear what would correspond semantically to such an operator. If we were to introduce \( \prec \) accordingly, CONVEXITY* would be derivable.\(^{18}\) We conclude that the most plausible way of avoiding the

\(^{15}\) Correia (2005: 61) and (2014a: 88), Bennett (2011: 32f), Audi (2012: 697) and deRosset (2013: 20) all accept principles that clearly entail INTERNALITY. Also Fine (2012: 76) accepts it. Leuenberger (2014) presents counterexamples to INTERNALITY.

\(^{16}\) See e.g., Fine (2012: 48f) and Correia (2014b: 36).

\(^{17}\) Proof sketch: Say that \( C \) is disjunctively contained in \( A \) (\( A \geq^d C \)) if \( A \) is factually equivalent to some disjunction of which \( C \) is a disjunct. Application of Correia’s Reduction Theorem (2010: 19) to the premises and conclusion of CONVEXITY reveals that it suffices to show that if the conjunction of the sentences \( \Gamma \) and the conjunction of the sentences in \( \Gamma, \Delta, \) and \( E \) is disjunctively contained in \( C \), then so is the conjunction of the sentences in \( \Gamma \) and \( \Delta \). This can be shown essentially by applications of the distributivity of \( \lor \) over \( \land \) and \( \land \) over \( \lor \). We should note that Correia voices some dissatisfaction with the Reduction Theorem. However, against the background of Fine’s truthmaker semantics, it seems perfectly well motivated.

\(^{18}\) The proof is a proper part of the proof of CONVEXITY sketched above.
results of §3 also involves a rejection of the principles of convexity.\(^\text{19}\) As we see it, their problematic character also points to an important fact about the notion of \textit{relevance} that guides central intuitions invoked by theorists of ground. We elaborate on this issue in the next section.

\section*{§5. Relevance}

It is commonly held that one of the distinctive features of the notion of ground is that it imposes a constraint of \textit{relevance} on the grounds of a given fact; it is this feature of the notion which renders it \textit{non-monotonic}, i.e. such that adding arbitrary facts to a ground of a fact does not in general yield another ground of that fact.\(^\text{20}\) In addition, the notion of \textit{full} ground imposes a constraint of \textit{sufficiency}: if some facts are to constitute a full ground of some further fact, then their obtaining must in some appropriate understanding be sufficient for the grounded fact’s obtaining. Metaphorically speaking, these two constraints put bounds on the set of a fact’s full grounds both from below and from above. It is bound “from below” by the sufficiency constraint, which forces us to put enough into a given collection of facts that they are jointly sufficient for the fact to be grounded. It is bound “from above” by the relevance constraint, which prevents us from enlarging the collection of facts that is to be the ground in arbitrary ways.\(^\text{21}\)

It is plausible to suppose that the two constraints, on some suitable understanding, are jointly sufficient: if a collection of facts \(\Gamma\) is, in the appropriate senses, both sufficient and relevant for the obtaining of fact \(C\), then \(\Gamma < C\). At any rate, it seems very unclear what constraints might be satisfied in all typical examples for ground that cannot plausibly be subsumed under either the heading of sufficiency or that of relevance. But then it may seem as though ground \textit{should} satisfy Convexity. For suppose that (i) \(\Gamma < C\) and (ii) \(\Gamma, \Delta, E < C\). Then by (i) \(\Gamma, \Delta\) is sufficient for \(C\). Moreover, by (ii), a superset of \(\Gamma, \Delta\) is relevant for \(C\), so \(\Gamma, \Delta\) must also be relevant. What our discussion shows is that this very tempting line of reasoning must be flawed somehow.

We would like to suggest that the flaw consists in an overly simplistic picture of relevance. We assumed, in effect, that if a given collection is relevant in the pertinent sense to a given fact, so is any subset of that collection. This follows immediately if the notion of relevance is understood \textit{distributively}, so that the relevance of a given collection consists simply in the relevance of each member. We are inclined to hold that a notion of relevance that can do justice to our intuitive understanding of ‘in virtue of’ must be understood \textit{collectively}, so that some facts can jointly be relevant to another without each member on its

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\(^\text{19}\) There is an intriguing connection between Convexity and (X) in Fine’s truthmaker semantics for worldly ground (cf. Fine, ms b). In that semantics, both turn crucially on a condition of convexity that Fine imposes on ground-theoretic content: that if something is both a part of a truthmaker of a proposition and has a truthmaker of the same proposition as part, then that object is itself a truthmaker of the proposition. It would be very interesting to investigate the logics of factual equivalence and ground obtained by dropping the condition of convexity on Finean propositions.


\(^\text{21}\) We owe the picture to Fine (ms a).
own, or indeed arbitrary subcollections, being thus relevant. In particular, we suggest that A, B, C are jointly relevant, in the appropriate sense, to A ∨ (B ∧ C), but A, B is not. The reason is that B is relevant only in combination with either C or a full ground of C. As long as A is neither, the collection A, B is not suitably relevant for A ∨ (B ∧ C).

§6. Conclusion

We have shown that an initially plausible logic attempting to take the intuition of grounding as a worldly relation seriously entails unacceptable consequences. How can an adherent of a worldly conception of ground react? Perhaps the most natural option is modifying the logic. Our derivations relied on either the distributivity principle (X) for factual equivalence or the CONVEXITY-principles. One thus might consider giving up these principles. However, since this would involve giving up on Angell’s system as a guide for interchangeability in the scope of <, factualists would lose their respective principled account – one of the advantages Correia’s logic seemed to offer. It is also unclear whether such a modified notion of factual equivalence allows for a well-behaved logical system – a second advantage the notion seemed to enjoy compared to conceptualist views. The question whether the notion of worldly ground has an attractive logic, and how it compares to conceptualist rivals, is still open.22

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