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Deposited on: 28 March 2017

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How much does a single graduation cohort from further education colleges contribute to an open regional economy?

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Abstract
This paper combines elements of growth accounting and numerical general equilibrium analysis to produce an alternative micro-to-macro modelling approach. This is used to evaluate the macroeconomic impact on the Scottish economy of the human capital generated by a single graduation cohort from Further Education Colleges (FECs). The macroeconomic impact is found to be significant and larger than growth accounting would suggest, due to the associated endogenous investment, employment and competitiveness effects. From a policy perspective this identifies the importance of the conventional teaching role of education institutions and the key function played by FECs in this process.

Keywords: Graduates, Further Education Colleges, Labour Supply, Economic Impact, General Equilibrium
JEL Codes: I23, E17, D58, R13.

Acknowledgments: We acknowledge valuable comments from two anonymous reviewers and the editor and support from the David Hume Institute which provided both funding and access to data as part of its policy dialogue on the role of further education in Scotland. We are also grateful for the input of seminar participants in the School of Education at the University of Glasgow and the European Commission's joint research centre in Seville, as well as conference participants at RSAI-BIS in Cambridge and ERSA in Palermo.
1 Introduction

Further Education Colleges (FECs) are teaching intensive institutions. They typically provide a heterogeneous student body of varying age and ability the opportunity to acquire a wide variety of post-compulsory vocational and academic qualifications. Because of the heterogeneity of both the students and the offered qualifications, the performance of FECs is severely under-researched, as against Higher Educational Institutions (HEIs) and schools. The work of FECs is often justified as contributing to social policy goals, through widening access to educational opportunities and improving income equality. We are not arguing that these aspects of the operation of FECs are unimportant. However, in addition, FECs make an contribution to overall regional economic activity purely through building human capital. In this paper we attempt to quantify this contribution.

Improvements in human capital are often seen as a key means of stimulating national and regional economic growth (EU Commission, 2008; EU Committee of the Regions, 2005; OECD, 2012). However, econometric attempts to identify the macro-economic impact of education are ambivalent, generating a wide range of estimates. Sianesi & Van Reenen (2003) survey over 20 macro growth studies. They conclude that whilst these studies provide valuable qualitative evidence on the link between education and economic output, methodological complications mean that caution is required in using their results to quantify the magnitude of such links.

Growth accounting is an alternative method of capturing the contribution of increased human capital to regional output (Connors and Franklin, 2015). In such an approach increases in human capital can be translated to increases in labour productivity. The present paper extends this method through the incorporation of endogenous changes in prices and factor supply (Stephen and Weale, 2004; Giesecke and Madden, 2006). In particular, a Computable General Equilibrium (CGE) model for Scotland is used to simulate the effect of the increase in human capital generated by a single cohort from existing Further Education Colleges (FECs).
We are restricted to considering the impact of a single year’s FEC output because of data limitations. Detailed information on the output of Scottish FECs is not readily available and the data used in this paper were produced by a specially generated survey.

In this paper we are not concerned with the demand side impacts of the education process. First of all, this is a topic that has already been extensively researched (Florax 1992, Hermannsson et al 2014a). Further, at the time that this research was carried out, the devolved Scottish Government, which funds FEC education in Scotland, operated within a hard budget constraint funded by a fixed, population-based grant from the UK central Government. Therefore any increase in expenditure on further education would be met by reduced expenditure elsewhere, so that any net demand effect would be small. We focus here rather on the neglected supply-side effects of the human capital generated by FECs. For pedagogic reasons the results are presented as the effects of an increase in human capital. However, the most straightforward and intuitive way to interpret the results is as a measure of the change in economic activity that would occur if the education delivered in Scottish FECs in 2011 had been completely ineffective. How much output would subsequently be lost as a result of the lower level of human capital?

2 Evidence of the economic impact of human capital

An extensive international micro-econometric literature documents the private rates of return to additional education in the form of higher earnings. Following Mincer (1974), the standard approach utilises cross-sectional labour market surveys by regressing the log of earnings on the level of education and other controls, such as labour market experience\(^1\). These studies reveal a clear correlation between an individual’s education level and wages (see Psacharopoulos and Patrinos, 2004, for a survey). However, identifying causality is difficult. Interpreted in the spirit of the human capital school, education directly increases human capital, which in turn

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\(^1\) This is typically specified in a quadratic form to capture the increase in wages with experience (learning on the job) and subsequent decline after middle age (see Mincer, 1974, Ch. 4). Ruzik-Sierdzinska et al (2013) provide a summary of recent evidence for several European countries. Age-earnings profiles for the UK are found to be relatively flat. In this analysis the simplifying assumption is adopted that the age-earnings profile is flat over working life and newly qualified workers are given the average wage premium of identical workers.
increases worker productivity and therefore the wage (Becker, 1964; Mincer, 1958; Schultz, 1960). Conversely, the theory of signalling and screening (Arrow, 1973; Spence, 1973; Stiglitz, 1975) suggests that in extremis education does not enhance human capital but simply reveals innate ability. Brown & Sessions (2004) provide an overview to this debate. Additionally, there is a prima facie argument that using the education-earnings relationships obtained through a standard OLS (Mincer) regression will overstate the marginal productivity of human capital as the choice of schooling is endogenous.

Gunderson & Oreopoulos (2010) summarise a range of statistical approaches that have been applied to address this conundrum and the sign and magnitude of the potential biases has been debated for a long time (see, for instance, Griliches, 1977). Examples of alternative methods include: controlling for observed heterogeneity within a longitudinal framework (Blundell et al 2005); controlling for fixed effects using twin samples (Bonjour et al, 2003; McMahon, 2009, Appendix A); and utilising natural experiments (Card, 2001; Dickson, 2013; Krueger & Lindahl, 2001). The resulting consensus is that education affects income per se and is not simply a proxy for unobserved ability (Card, 1999, 2001; Harmon & Walker, 2003).

Similarly, there is an instinctive expectation that wage premia will sharply fall as average educational attainment increases. However, historical observations show wage premia remaining significant and positive over extended time periods, despite increased supply of skilled labour. This outcome is typically attributed to skill-biased technical change, which has simultaneously increased the demand for skilled labour (Goldin & Katz, 2007; Machin, 2004)².

The micro-econometric evidence therefore suggests that education improves an individual’s labour productivity. However, identifying the implied macroeconomic corollary has proven elusive. A significant body of work, reviewed by Sianesi & van Reenen (2003) and Stephens & Weale (2004), has addressed this through regression on cross-country data. However, the range of results is wide. Some authors, such as

² This explanation is consistent with evidence from recent work showing that despite the persistence of stable average wage premia, the variation in individuals' wage premia has increased over time (Green & Zhu, 2010; Walker & Zhu, 2008).
Benhabib & Spiegel (1994) and Barro & Sala-i-Martin (1995), question the very existence of macroeconomic impacts of education, although their findings are contested (Krueger & Lindahl, 2001). At the other extreme, approaches using endogenous growth models suggest implausibly large impacts from education. These are critically discussed in Topel (1999).

Growth accounting is an alternative approach to estimating the aggregate impact of education (Barro, 1999; Stephens & Weale, 2004). This method simply counts and aggregates inputs in a production function, assigning marginal productivity to those inputs based on the payments they receive. The growth in output is decomposed into the contribution made by changes in all inputs and a residual productivity growth element. The strength of this approach is its transparency. It suggests a clear link from education policy to the macro-economy given a suitably detailed treatment of human capital inputs. However, the method is restricted in that it relies on key simplifying assumptions. We adopt an approach which retains the strengths of aggregating human capital inputs using micro level data, but then applies an extensive structural model to simulate subsequent endogenous adjustments in the economy (Giesecke & Madden 2006, Hermannsson et al 2014b, Hermannsson & Lecca 2016). This identifies the link between the policy lever and the macroeconomic outcome.

3 Analytical model

In a standard growth accounting model, an increase in human capital would be entered as an improvement in labour efficiency: the effective supply of labour would increase. In this section we outline a stylised regional economic model whose outcome can be compared against the standard growth accounting approach. The analytical model presented in this section is expressed in terms of proportional changes and is focused on the long-run impact on aggregate output and factor use. This model is used solely for pedagogic reasons and contains many simplifications not present in the CGE model detailed in Section 5.

In the analytical model all domestically produced goods are sold in an export market and all consumption and investment goods are imported. As we assume that there is

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3 The genesis of this literature is typically attributed to Solow (1956), though for a discussion of precedents, see Griliches (1996).
no change in import prices, this allows us to abstract from the distinction between changes in real and nominal wages. Production is undertaken with no intermediate inputs, so that output is identical to value added (and GDP). The labour force is fixed but employment can vary through changes in the employment rate with accompanying changes in the real wage through a wage curve. Investment and capital stock are endogenous with the cost of capital fixed and determined by interest rates set in extra-regional markets. There is no government expenditure or taxes.\(^4\)

With an increase in labour efficiency of \(\rho(>0)\), the relationship between the proportionate change in employment measured in natural and efficiency units, \(n\) and \(n_E\) respectively, can be expressed as:

\[
(1) \quad n_E = n + \rho
\]

Similarly, the changes in the wage for a unit of labour measured in natural and efficiency units (\(w\) and \(w_E\)) are given as:

\[
(2) \quad w_E = w - \rho
\]

The change in the real (natural) wage, \(w\), is determined by the familiar wage curve relationship (Blanchflower and Oswald, 1994) so that:

\[
(3) \quad w = \beta n \quad \beta \geq 0
\]

where \(\beta\) is the elasticity of the real wage with respect to the employment rate.\(^5\) The change in product demand, \(q\), is negatively related to the change in product price, \(p\):

\[
(4) \quad q = -\eta p \quad \eta \geq 0
\]

---

\(^4\)This assumption is relaxed in the more fully specified CGE model used in Section 5.

\(^5\)The wage curve is usually expressed as a relationship between the real wage and the unemployment rate and this is how it operates in the CGE model outlined in Section 5. However, given that the unemployment rate equals 1 minus the employment rate, the relationship can be reparameterised as one between the real wage and the employment rate, which is more convenient here.
where \(\eta\) is the price elasticity of demand. Recall that in this stylised model, all output is sold in export markets, so that equation (4) implies that the region’s output is a less than perfect substitute for other goods in such markets. This corresponds to the familiar Armington assumption (Armington, 1969).

Given that the economy is in long-run equilibrium, the change in product demand is equal to the corresponding change in supply. The cost minimising change in derived labour demand, measured in efficiency units, is given as

\[
(5) \quad n_E = q - \sigma (w_E - p) \quad \sigma \geq 0
\]

where \(\sigma\) is the elasticity of substitution between capital and labour in production (Heathfield and Wybe, 1987, p. 93). Similarly, given that the change in the cost of capital is assumed to be zero, the derived demand for capital, \(k\), equals:

\[
(6) \quad k = q + \sigma p
\]

Finally, the proportionate change in product price is represented as:

\[
(7) \quad p = \alpha w_E \quad 1 \geq \alpha > 0
\]

where \(\alpha\) is the share of labour inputs in production, given that the change in the price of capital is zero.

In this very simple heuristic model, equations (1) to (7) solve for the seven endogenous variables: \(k, n, n_E, w, w_E, p\) and \(q\). Their values are driven by the exogenous change in the efficiency of labour, \(\rho\), and the elasticity values \(\beta, \eta\) and \(\sigma\), together with the labour share parameter \(\alpha\). To begin, we find expressions for the responsiveness of the use of the two factors of production, labour and capital (\(n\) and \(k\)) to changes in the labour efficiency, \(\rho\).

---

6 Recall that all investment is externally sourced and the interest rate is fixed in extra-regional markets.
The derivatives given in expressions (8) and (9) indicate that the increase in labour efficiency generates endogenous changes in employment and capital use that can be positive or negative. Given the constraints on the parameter values, the denominator in equations (8) and (9) is always positive so that the sign of the numerator in the appropriate equation determines the qualitative effect on the factor use.

Employment change is therefore positive so long as $\eta \alpha + \sigma(1-\alpha) > 1$. There is a stimulus to labour demand coming through the expansion in output and the substitution of labour for capital in production. However, for employment to rise, this stimulus must be greater than the negative impact of the direct increase in labour productivity. The increase in output is driven by a reduction in product price (increased competitiveness): the positive substitution effect is generated by the fall in the price of labour measured in efficiency units.

For capital use the requirement is more straightforward. This increases as long as the stimulus to capital demand from the output effect is greater than the negative impact of the substitution effect. This occurs where the price elasticity of demand for output is greater than the elasticity of substitution in production: that is, where $\eta > \sigma$.

We expect regional economies to be more open than national ones. That is to say, we expect the price elasticity of the product demand function to be relatively high, through the closeness of external markets, and the elasticity in the wage curve to be low, driven in the long run by inter-regional migration. The higher the value of the elasticity of product demand, the more positive is the change in factor use, both for capital and labour, and therefore also the change in output. The impact of a more

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7 In the CGE regional model we often essentially assume an infinitely elastic labour supply in the long run, driven by flow equilibrium migration. However, in the simulations presented in Section 5, for reasons explained there, migration is turned off.
elastic labour supply function reinforces any change in employment driven by increased productivity. That is, if the employment change is positive, it is greater with a more elastic labour supply. However, if employment falls as a result of the efficiency gain, that fall is exacerbated with a lower value for $\beta$. The situation for changes in the capital stock is rather more complex. See Appendix 1.

The expression for the responsiveness of the change in output, $q$, to changes in labour efficiency, again derived in Appendix 1, is:

$$
\frac{\partial q}{\partial \rho} = \frac{\alpha \eta (\beta + 1)}{\beta (\sigma (1 - \alpha) + \eta \alpha) + 1} > 0
$$

Given the restrictions imposed on parameter values, expression (10) shows that output always increases as labour efficiency improves. However, it is more useful to benchmark this result against the responsiveness of output$, $q^s$, which would be attributed to the change in labour efficiency in a growth accounting analysis. This is calculated as the share of labour in the production of output, so that:

$$
\frac{\partial q^s}{\partial \rho} = \alpha .
$$

Using equations (10) and (11), both $q$ and $q^s$ are positively related to $\rho$ and

$$
\frac{\partial q}{\partial \rho} > \frac{\partial q^s}{\partial \rho} \text{ iff } \beta (\eta - \sigma) (1 - \alpha) + \eta > 1
$$

The inequality given in expression (12) involves all the parameters of the model and it is clear that actual output change can be more or less than the growth accounting value. If $\eta > \sigma$, so that the elasticity of demand for the product is greater than the elasticity of substitution in production, the inequality is more likely to hold the larger is the value $\beta$ and $\eta$, and the smaller the values of $\alpha$ and $\sigma$. 
It is clear that the conventional micro-to-macro growth accounting procedure can under- or over-estimate the impact of productivity improvements brought about by an increase in human capital and that even in the simplified and stylised model developed in this section, the impact of labour efficiency changes on output is primarily an empirical matter. It depends upon key parameter values underlying important structural and behavioural relationships within the economy. We therefore pursue the analysis further through simulation using a regional Computable General Equilibrium (CGE) model. This model is parameterised on data from a UK region, Scotland, and allows an increase in the detail and scope of the investigation through the model’s more complete specification.

4 Estimating the productivity impact of the 2011 FEC graduation cohort

In order to determine the appropriate labour efficiency shock to apply in the GGE simulations, we first calculate the increase in human capital generated by Scottish Further Education Colleges (FECs) in one year, 2011. Recent UK studies suggest that the graduate wage premium has remained relatively constant. Following Hermannsson et al (2014b) we use this information to motivate the treatment of labour of different skill types in the manner adopted by the standard growth accounting procedure. That is to say, the supply of labour at different skill levels is aggregated into a single stock of human capital, measured in efficiency units. This implies that individuals with different skill levels can be treated simply as having different levels of human capital where the relative wage rates are taken to reflect directly the human capital differentials. Whilst this is a very common procedure, and the bedrock of growth accounting, it is not uncontested. The available evidence is discussed in Section 2. A CGE treatment with a more disaggregated labour market is possible including endogenous skill generation (Giesecke and Meager, 2008; Gieseke et al, 2011). However, the degree of skill disaggregation in the present application means that the data requirements here are too great for such an approach.

Following Acemoglu & Autor (2012), for unskilled workers, $U$, and $k$ types of skilled workers, $S_i$, the human capital stock (labour force) in efficiency units, $N_F$, can be calculated as:

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8 For a detailed discussion of the evidence base see Section 2.2 in Hermannsson et al (2014b).
where $w_U$ is the wage of unskilled workers, $w_i$ is the wage of skill group i and $m_i (= w_i / w_U)$ is the wage premium for skill group i.\(^9\)

Further Education Colleges, F, generate in one academic year, $t$, qualifications, $Q^{F,T}_{i,t}$, at level, $i$, and of type $T$ (vocational or academic). The increase in human capital, $\Delta N^{F}_{E,t}$, generated by the FEC activity in that year is the sum of these qualifications, weighted by the addition to human capital that each student receives through acquiring the qualification, $\Delta m_i^T$. This is shown in equations (14) and (15). The qualifications are represented hierarchically and we assume that achieving an educational qualification raises the recipient one step on that hierarchy. The additional human capital generated when an individual achieves a particular qualification is then the difference between the human capital for that qualification level and the human capital associated with the preceding qualification level.

\[
\Delta m_i^T = m_i^T - m_{i-1}^T.
\]

4.1 2011 Scottish FEC graduation cohort
The information on the number and breakdown of qualifications was collected from all the individual Scottish FECs through a special survey.\(^{10}\) These data are available for only one year but we have no reason to believe that this year is unrepresentative.

\(^9\) Given that we apply this measure in a model in which labour inputs are measured in annual terms, the relative wage measure is for annual wage receipts.

\(^{10}\) This survey was carried out by the David Hume Institute in collaboration with Scotland's Colleges and the Scottish Funding Council, as part of a research initiative on Further Education, the Scottish Labour Market and the Wider Economy. For details see Hermannsson et al (2012).
scale which identifies 5 broad levels.\textsuperscript{11} The UK Office for National Statistics has identified how the educational qualifications used in the Labour Force Survey can be matched to their NVQ equivalents (ONS, 2006, pp. 94-95). The procedure adopted in this paper maps academic and vocational qualifications in Scotland to their equivalent NVQ levels. It is taken from Walker and Zhu (2007a) and is shown in Table 1. This follows ONS conventions closely with some minor modifications (Walker and Zhu, 2007a, pp. 19-21, 55-56).

### Table 1 NVQ levels with the corresponding academic and vocational qualifications.

<table>
<thead>
<tr>
<th>NVQ level</th>
<th>Academic qualification</th>
<th>Vocations qualification</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>&lt;5 GCSE, Standard Grade (General level)</td>
<td>BTEC, SCOTVEC</td>
</tr>
<tr>
<td>2</td>
<td>5+ GCSEs at A-C, Standard Grade (Credit level)</td>
<td>NVQ intermediate level</td>
</tr>
<tr>
<td>3</td>
<td>2+ A-levels/3+Highers</td>
<td>OND, ONC</td>
</tr>
<tr>
<td>4</td>
<td>Undergraduate degree</td>
<td>HNC/HND</td>
</tr>
<tr>
<td>5</td>
<td>PhD, Masters degrees</td>
<td>Non-masters post-graduate qualifications</td>
</tr>
</tbody>
</table>

Source: Walker & Zhu (2007b, Figure 4.1, p. 21).

The numbers of students attending Scottish FECs in 2011 are given in the first data column of Table 2, broken down by the new qualification, if any, that they received in that year. These are separated into academic and vocational NVQ types. 76,152 FEC students completed some form of programme in 2011. Of these, 7,945 received no qualification and 36,136 gained an intermediate qualification that failed to raise his or her status on the NVQ scale. Such qualifications often grant access to, or prepare students for, more advanced courses and are treated as intermediate steps between

\textsuperscript{11} NVQs were developed for England, Wales and Northern Ireland in the 1980s. They form an outcomes-based framework where the NVQs are derived from occupational standards to identify the competencies required to meet each level. A parallel system was implemented in Scotland, the Scottish Vocational Qualifications (SVQ). A discussion of the development of this system is given in Unwin et al (2004, Ch. 3).
NVQ levels.\textsuperscript{12} They are therefore ignored in subsequent parts of this analysis in order to avoid double counting. The remaining 32,071 students received a qualification that represents an interval on the NVQ scale. Over 85\% of the qualifications achieved in Scottish FECs in this year are vocational and just over 50\% are at a vocational NVQ 4 level.\textsuperscript{13}

4.2 Productivity impact of FEC graduates

In order to value the economic benefit of achieving each increment on the NVQ scale, and therefore identify the appropriate $\Delta m_i^T$ values, we use estimates of Scottish wage premia by qualification obtained by Walker and Zhu (2007a, b). These are shown in the second data column in Table 2. Following standard growth accounting procedures, the wage premium associated with a given level of educational qualification is taken to indicate the productivity enhancing effects of education. As is summarised in Section 2 the labour market benefits of qualifications have been frequently estimated using the standard Mincer approach. The analysis of Walker & Zhu (2007a, b) is particularly useful as it estimates wage premia by both academic and vocational NVQ qualification separately for UK regions, including Scotland. This is achieved by pooling ten years of data from the Labour Force Survey, in order to achieve a sufficiently large sample to carry out such detailed analysis.

The findings of Walker and Zhu (2007a, b) are in line with other work in this field and the results are similar for Scotland and Britain as a whole. Their results are reported separately for each gender. In our analysis we average these two figures, implicitly assuming that the gender balance is equal within each increment of the NVQ scale. A drawback of using these estimates is that they are based on old observation and therefore the wage premia could have changed. However, historically wage premia have evolved slowly (see Hermansson et al 2014b for a discussion of this point). Whilst more recent evidence on the wage premia associated with vocational qualifications is not available it is, for example, clear that the wage premia

\textsuperscript{12} Formally qualifications which are between NVQ increments involve the highest level of study (unit) being: Advanced Higher, Higher, Intermediate 2, Intermediate 1 or Access. Also included are: Other Non-Advanced Certificate or equivalent; Other Non-Advanced Diploma or equivalent; National Units alone (formerly National Certificate modules) or any other recognised qualification.

\textsuperscript{13} We assume that each student achieving an NVQ qualification is moving up one NVQ level. The students might already have a qualification at this level, but we expect the probability to be low.
of higher education graduates has held up despite difficult external economic circumstances (Britton et al, 2015).

Following equation (13), workers contribute different amounts of efficiency units of labour to the production process, depending on their skill level. The efficiency level of those workers with no qualification is set to unity and then evidence of the wage premia are used to increase the efficiency units of each worker in accordance with his or her skill level. For example a worker with a level 1 vocational qualification contributes 1.10 efficiency units, someone with level 2 qualification 1.18 and so on.

Table 2 Number of students (FTEs) successfully completing a qualification, the wage premium and efficiency gain per qualification, and the total increase in the human capital for the 2010-11 Scottish FEC cohort of graduates.

<table>
<thead>
<tr>
<th>NVQ Level Vocational</th>
<th>FTEs ( Q_{i,t}^{F,T} )</th>
<th>Wage premium (%)</th>
<th>Efficiency gain ( \Delta m_{i,2011}^{V,P} )</th>
<th>Increased human capital ( \Delta N_{i,2011}^{F,T} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>No qualification</td>
<td>7,945</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intermediate</td>
<td>23,064</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level 1</td>
<td>886</td>
<td>10</td>
<td>0.10</td>
<td>89</td>
</tr>
<tr>
<td>Level 2</td>
<td>3,854</td>
<td>18</td>
<td>0.08</td>
<td>308</td>
</tr>
<tr>
<td>Level 3</td>
<td>5,768</td>
<td>32</td>
<td>0.14</td>
<td>808</td>
</tr>
<tr>
<td>Level 4</td>
<td>16,829</td>
<td>52</td>
<td>0.20</td>
<td>3,366</td>
</tr>
<tr>
<td>Level 5</td>
<td>26</td>
<td>82</td>
<td>0.30</td>
<td>8</td>
</tr>
<tr>
<td><strong>Total NVQ 1-5</strong></td>
<td><strong>27,363</strong></td>
<td></td>
<td></td>
<td><strong>4,578</strong></td>
</tr>
<tr>
<td>Total all qualifications</td>
<td>58,373</td>
<td></td>
<td></td>
<td><strong>4,578</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NVQ Level Academic</th>
<th>FTEs ( Q_{i,t}^{F,T} )</th>
<th>Wage premium (%)</th>
<th>Efficiency gain ( \Delta m_{i,2011}^{V,P} )</th>
<th>Increased human capital ( \Delta N_{i,2011}^{F,T} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>No qualification</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intermediate</td>
<td>13,072</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Level 1</td>
<td>167</td>
<td>18</td>
<td>0.18</td>
<td>29</td>
</tr>
<tr>
<td>Level 2</td>
<td>2,551</td>
<td>30</td>
<td>0.12</td>
<td>306</td>
</tr>
<tr>
<td>Level 3</td>
<td>1,498</td>
<td>46</td>
<td>0.16</td>
<td>240</td>
</tr>
<tr>
<td>Level 4</td>
<td>484</td>
<td>78</td>
<td>0.32</td>
<td>157</td>
</tr>
<tr>
<td>Level 5</td>
<td>8</td>
<td>91</td>
<td>0.13</td>
<td>1</td>
</tr>
<tr>
<td><strong>Total NVQ 1-5</strong></td>
<td><strong>4,708</strong></td>
<td></td>
<td></td>
<td><strong>733</strong></td>
</tr>
<tr>
<td>Total all qualifications</td>
<td>17,780</td>
<td></td>
<td></td>
<td><strong>733</strong></td>
</tr>
</tbody>
</table>

Source: David Hume Institute, Walker & Zhu (2007b, Figure 4.3, Figure 4.5, pp. 12-13), own calculations.
Given these figures it is possible to calculate the efficiency units that FEC graduates bring to the labour market. However, we are initially solely interested in the extent to which the graduates’ efficiency has increased as a result of the FEC course they have completed in the academic year 2010/11. That is, we want to focus on the additional skills provided by the particular course and not the skills already possessed by that student (for example the skills gained at school). Therefore a student completing a level 2 vocational qualification adds 0.08 efficiency units to his or her human capital. This is the difference between the efficiency units associated with a level 2 and a level 1 qualification (1.18 – 1.10 = 0.08). The efficiency gain generated by attaining each type of qualification is shown in the third data column of Table 2.

The fourth data column in Table 2 gives the total changes in human capital, measured in efficiency units, calculated using equation (14). Adopting this metric, the 2010/11 output from Scottish FECs increased the effective Scottish labour supply by 5,311 efficiency units. This is primarily through the provision of vocational NVQs and particularly at level 4.

4.3 Proportionate impact of the FEC graduation cohort on the human capital stock

Drawing on the Annual Population Survey (APS), it is possible to obtain the number of those between the age of 16 and 64 in Scotland, together with their skill level. This is based on several simplifying assumptions. The APS is accessed via the National Online Manpower Information System (NOMIS) data portal of the Office for National Statistics (ONS). This data source aggregates NVQ4 and NVQ5 qualifications to avoid disclosure. Therefore we abstract from the role of NVQ5 qualifications in the skills base. That is to say, if individuals are recorded as having a skill level of NVQ 4+ we allocate them the efficiency level appropriate for NVQ 4. Furthermore, the APS does not distinguish between academic and vocational qualifications. Therefore we use the average wage premium for a given NVQ level, which implies that within each skill increment, academic and vocational qualifications are assumed to be in equal measure. Finally, those with 'Other qualifications' or where information is not available make up 6% and 5% of the
population respectively. These respondents are treated as though they had no qualification.

Using the wage premia as reported in Walker and Zhu (2007a, b), we calculate the efficiency units of labour contained in the individual level and sum these across the whole working-age population. According to the APS there were 3,378,700 individuals aged 16-64 in Scotland in 2011. This population could supply 4,560,838 efficiency units of labour, which suggest that the average number of efficiency units of labour per working age Scot is 1.35. Using this figure as a denominator we find that our 2011 graduation cohort has increased the amount of available efficiency units of labour by 0.12% (5,311 / 4,560,838 = 0.0012). This is the efficiency shock which is entered into the CGE model.

4.4 Productivity impact of FECs compared to HEIs

It is useful to benchmark the scale and relative importance of FECs in Scottish human capital formation by comparing their output to the corresponding output of the Scottish Higher Education Institutions (HEIs). For this comparison, we focus only on those students that are funded by the Scottish Funding Council, i.e. Scottish domiciled students and students from the European Union outside the UK.¹⁴ For simplicity we assume that only the Scottish students are retained in Scotland following graduation. Information on the number of Scottish domiciled students graduating from Scottish universities is obtained from the Higher Education Statistics Agency (HESA).

<table>
<thead>
<tr>
<th>NVQ level i</th>
<th>FECs Qualifications</th>
<th>Efficiency units</th>
<th>HEIs Qualifications</th>
<th>Efficiency units</th>
<th>Total Qualifications</th>
<th>Efficiency units</th>
</tr>
</thead>
<tbody>
<tr>
<td>NVQ1</td>
<td>1.053</td>
<td>118</td>
<td>0</td>
<td>0</td>
<td>1.053</td>
<td>118</td>
</tr>
<tr>
<td>NVQ2</td>
<td>6.406</td>
<td>615</td>
<td>0</td>
<td>0</td>
<td>6.406</td>
<td>615</td>
</tr>
<tr>
<td>NVQ3</td>
<td>7.266</td>
<td>1,047</td>
<td>0</td>
<td>0</td>
<td>7.266</td>
<td>1,047</td>
</tr>
<tr>
<td>NVQ4</td>
<td>17.313</td>
<td>3,523</td>
<td>21.875</td>
<td>7,109</td>
<td>39.188</td>
<td>10,632</td>
</tr>
<tr>
<td>NVQ 5</td>
<td>34</td>
<td>9</td>
<td>5,135</td>
<td>642</td>
<td>5,169</td>
<td>651</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>32,071</strong></td>
<td><strong>5,311</strong></td>
<td><strong>27,010</strong></td>
<td><strong>7,751</strong></td>
<td><strong>59,081</strong></td>
<td><strong>13,063</strong></td>
</tr>
</tbody>
</table>

¹⁴ We have no information about the domicile of Scottish FEC students but make the assumption here that they are all from Scotland.
To make this calculation, we use exactly the same method for HEIs as we did for FECs. We count the output as qualifications completed in the academic year 2010/11, which for HEIs represents undergraduate, higher and doctoral qualifications. In Table 3 these figures are presented and compared to the FEC data. As far as we are aware, this is the first time that this sort of comparison has been performed for Scottish FECs and HEIs. The Scottish FECs provide a greater absolute number of qualifications over a much wider range of skills than do the HEIs, but the HEIs contribute more additional human capital in terms of efficiency units. Nevertheless, it is still the case that for 2010/11 FECs generate just over 40% of the additional human capital produced in the combined Scottish HEI and FEC sectors.\(^{15}\)

5 CGE model results

To simulate the system-wide impact of this increase in human capital we employ AMOS, a computable general equilibrium (CGE) model of Scotland.\(^{16}\) It is a regional, inter-temporal, general equilibrium variant of the Layard, Nickell and Jackman (1991) model, calibrated to a Scottish Social Accounting Matrix (SAM) for 2006. Essentially this is a much extended version of the simple analytical model presented in Section 3. It has: 3 three domestic transactor groups, namely households, corporations and government; four major components of final demand: consumption, investment, government expenditure and exports; and 25 industrial sectors. Domestic production goes to intermediate demand and to all elements of the final demand categories.

In the variant of the CGE model used in this paper, consumption and investment decisions reflect inter-temporal optimization with perfect foresight (Lecca et al, 2013, 2014). In the period-by-period simulations each period is taken to be a year. This is the period adopted in the econometric work used to parameterise the wage, migration

\(^{15}\) The average cost per efficiency unit of human capital generated in Scottish HEIs is £105,000, 8% higher than in Scottish FECs where the cost is £97,000. This is simply the opportunity cost to Scottish citizens in terms of the foregone public expenditure on other goods and services that could have been otherwise provided or subsidised by the Scottish Government. Furthermore, it only includes the funding of the education institutions themselves and excludes other student-related expenditures, such as maintenance grants.

\(^{16}\) AMOS is an acronym for A Macro-micro Model Of Scotland.
and investment equations. Real government expenditure is exogenous (although the total tax take is endogenous). This reflects the reality of the funding to the UK devolved authorities at the time. The demand for Scottish exports to the Rest of the UK (RUK) and Rest of the World (ROW) is determined via conventional export demand functions where the price elasticity of demand is set at 2.0. Imports are obtained through an Armington link (Armington, 1969) and therefore relative-price sensitive with trade substitution elasticities of 2.0 (Gibson, 1990). We do not explicitly model financial flows, our assumption being that Scotland is a price-taker in financial markets.

It is assumed that production takes place in competitive industries using multi-level production functions. This means that in each time period all commodity markets are in equilibrium with price equal to the marginal cost of production. Value-added is produced using capital and labour via standard production function formulations so that, in general, factor substitution occurs in response to changes in relative factor-prices. Constant elasticity of substitution (CES) technology is adopted in the production of value added with elasticities of substitution of 0.3 (Harris, 1989). In each industry intermediate purchases are modelled as the demand for a composite commodity with fixed (Leontief) coefficients. These are substitutable for imported commodities via an Armington link, which is sensitive to relative prices. The composite input then combines with value-added (capital and labour) in the production of each sector’s gross output. Cost minimisation drives the industry cost functions and the factor demand functions.
In the simulations reported in this paper, the labour market is characterised by a regional bargaining function in which the bargained real wage is inversely related to the unemployment rate. The bargaining function is parameterised using the regional econometric work reported in Layard et al (1991, 2005). Population is taken to be fixed, implying that the inter-regional migration function is turned off. Detailed discussion of the AMOS model and the underlying structural equations are available in Harrigan et al (1991) for the basic variant and in Lecca et al (2013, 2014) for the inter-temporal extensions. The calibration process implies that the economy is taken to be initially in long-run steady-state equilibrium. This means that if there are no changes in the exogenous variables in the model, the simulated economy would simply reproduce the base values for every period.

As reported in Section 4.3, the direct impact of the 2011 cohort of graduates from FECs in Scotland is to increase labour productivity by 0.12%. It is assumed that the productivity improvement associated with this one cohort of FEC students operates over the 40 years whilst these students are assumed to remain in the labour force. To
simulate the impact of such an economic disturbance we actually run the model for 80 periods where, as stated earlier, each period represents one year. We introduce a 0.12% step increase to labour efficiency in all sectors of the economy in period 1 and maintain this for 40 periods.\textsuperscript{17} The stimulus is then removed for the remaining 40 periods of the simulation. The increase in labour efficiency is the only exogenous change introduced into the model, so that the results should be interpreted as deviations from what would have occurred if labour productivity had remained unchanged. The simulation identifies the supply side impact of one year’s output of Scottish FECs.

Figure 1 shows the evolution of GDP and employment (measured in natural units). These figures are reported as percentage changes measured against their base-year levels. Whilst the simulations are run for 80 periods, for pedagogic reasons we only report the first 60 periods as the economy has essentially returned to its initial equilibrium by that point.\textsuperscript{18} As can be seen, the economy reaches a plateau of higher output and employment quite rapidly, reflecting the forward looking behavioural assumptions of the model. The maximum GDP increase of 0.126% is reached in period 14 and is retained until period 32. However, by period 4, the increase in GDP, at 0.104%, has reached 80% of its maximum value. Employment change is initially negative but becomes positive by period 4. It reaches its maximum level of 0.012% in period 13 and retains this until period 33.\textsuperscript{19} Once the FEC cohort leaves the labour force in period 41, there remains a short period where a legacy effect occurs, including a sharp stimulus to employment in period 41. In that period, labour efficiency returns to its base-year value but the physical capital stock is higher than its original level. This generates a temporary small increase in employment, whose size

\textsuperscript{17} The impact of acquired skills on labour efficiency might vary with work experience, with the Mincer equation only picking up the average impact over the individual’s whole working life. Whilst this would affect the evolution of the resulting economic effects, it would be of less importance for their maximum impact or the discounted values.

\textsuperscript{18} In order to operationalize the forward-looking model we need to impose long-run equilibrium condition in the final period (period 80). However, as is clear from Figure 1, the model is effectively in long-run equilibrium by period 60.

\textsuperscript{19} The relatively lower effects on economic activity in the short run are driven by the assumption that we introduce an unannounced efficiency shock into an economy taken to be initially in long-run equilibrium. Therefore the economy takes some time to adjust to this unanticipated supply-side shock. The medium term impacts are a better measure of the continuing impact of one year’s output from a stable FE system.
is reduced over time as disinvestment gradually brings the capital stock back to its original absolute level and sectoral configuration.

Table 4 shows the impacts on a wider range of key economic variables for periods 1, 30 and 40 in. Again these are given as percentage changes from their base-year values. Period 1 is the short run, where the capital stock is fixed both in aggregate and across industrial sectors. Period 30 represents a year in which the labour efficiency impacts have reached a stable, maximum level and period 40 is just prior to the efficiency improvement’s being withdrawn.

Table 4 Impact on key economic variables in periods 1, 30 and 40 of a temporary 0.12% increase in labour productivity lasting for 40 periods (% changes from base).

<table>
<thead>
<tr>
<th>Variable</th>
<th>1</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>0.068</td>
<td>0.126</td>
<td>0.111</td>
</tr>
<tr>
<td>Consumer Price Index</td>
<td>-0.021</td>
<td>-0.059</td>
<td>-0.059</td>
</tr>
<tr>
<td>Unemployment Rate</td>
<td>0.231</td>
<td>-0.189</td>
<td>-0.036</td>
</tr>
<tr>
<td>Total Employment</td>
<td>-0.015</td>
<td>0.012</td>
<td>0.002</td>
</tr>
<tr>
<td>Nominal Wage</td>
<td>-0.048</td>
<td>-0.038</td>
<td>-0.055</td>
</tr>
<tr>
<td>Real Wage</td>
<td>-0.026</td>
<td>0.021</td>
<td>0.004</td>
</tr>
<tr>
<td>Replacement Cost of Capital</td>
<td>-0.020</td>
<td>-0.053</td>
<td>-0.053</td>
</tr>
<tr>
<td>Households Consumption</td>
<td>0.019</td>
<td>0.052</td>
<td>0.051</td>
</tr>
<tr>
<td>Investment</td>
<td>0.212</td>
<td>0.113</td>
<td>0.008</td>
</tr>
<tr>
<td>Capital Stock</td>
<td>0.000</td>
<td>0.115</td>
<td>0.089</td>
</tr>
<tr>
<td>Export to RUK</td>
<td>0.030</td>
<td>0.096</td>
<td>0.094</td>
</tr>
<tr>
<td>Export to ROW</td>
<td>0.037</td>
<td>0.088</td>
<td>0.087</td>
</tr>
</tbody>
</table>

The labour efficiency shock is modelled as if every employee can subsequently produce 0.12% more output (other things being equal). The corollary is that with the existing choice of production techniques, 0.12% less employment is required for every unit of output. This produces a short-run (period 1) increase in GDP of 0.068% together with downward pressure on prices. Exports to both the rest of the UK (RUK) and the rest of the world (ROW) rise but this is accompanied by a fall in employment of 0.015%. However, these changes trigger further adjustments in the economy. Increased labour productivity stimulates the return to capital which, in turn, leads to an increase in investment. This expansion in capacity produces further price
reductions and a subsequent additional stimulus to exports. Subsequently positive employment impacts also produce a further stimulus to GDP through increases in household consumption. The results for period 30 therefore show a rise in GDP, capital stock and employment of 0.126%, 0.115% and 0.012% respectively. The competitiveness of the economy has improved with a larger fall in the cpi, and exports to the RUK and ROW are now higher by 0.096% and 0.088%. The unemployment rate has fallen, with a subsequent rise in the real wage of 0.021%.

In period 40 the increases in GDP, 0.111%, and employment, 0.002%, are lower than at their peak. A comparison of these results with those for period 30 reveals that the export and household consumption figures are very similar. However, in period 40 there is a markedly lower value for the increase in capital stock, at 0.089%, and a particularly low increase in investment at 0.008%. Essentially, at this point gross investment is not covering depreciation, so that the capital stock is moving back towards its base year level. This reflects the behaviour of forward looking agents adjusting to the future removal of the productivity stimulus. However, the fact that the capital stock in period 40 is greater than in the base year generates a continuing supply-side benefit to the economy in terms of increased competitiveness after period 40. This is reflected in higher GDP and employment levels. As the capital stock adjusts back to its original level this positive supply-side effect unwinds.

It is of interest to compare the change in GDP identified in Table 4 and Figure 1 with the change that would have been predicted using standard growth accounting methods. First, the growth accounting approach would have calculated the increase in GDP as just operating over the 40 periods during which the skill-enhanced cohort remained in the labour market. There would be no identified legacy effects. Second, the annual GDP impact would be the proportionate increase in labour productivity times the share of labour in Scottish GDP (as revealed in the Scottish Input-Output tables). This equals $0.12 \times 0.62 = 0.074$. In each of the 39 periods immediately after period 1, the simulation results shown in Figure 1 are greater than this figure. Once the maximum GDP change plateau is reached, that is once the full capital adjustments have been made, the simulation model generates a GDP increase 70% higher than that predicted using the growth accounting approach.
The reasons for this divergence are straightforward. As is apparent from the second column of results reported in Table 4, at its maximum the increase in labour efficiency generates an endogenous increase in employment and capital stock of 0.012% and 0.115% respectively. Their contribution to increased GDP would not be captured using conventional growth accounting. Further, this increase in capital stock continues to have a positive effect on GDP and employment after the direct efficiency increase has been withdrawn.

It is also important to recognise that the CGE model incorporates the fall in the price of human capital as a whole as its supply increases. By period 30, the real wage rises by 0.02% but this is less than the increase in human capital (0.12%). Using equation (2) we can see that the price of labour measured in efficiency units has fallen by 0.10%. This has clear distributional implications, especially for those workers who have not increased their skills and whose overall wage is reduced as a consequence. However, in practice the role of FECs is often to provide training for those at the lower end of the skills spectrum. Therefore the skills provided by the FECs can be seen as offsetting some of the competitive disadvantage incurred by non-graduate workers as HE participation has increased. It is apparent however, that those workers that are not investing in human capital are even more disadvantaged as the average skill level of the labour supply increases.

An alternative way of measuring the importance of the additional human capital produced by Scottish FECs is to calculate the present value of the stream of annual increments to Scottish GDP generate by this one cohort. Using the HM Treasury discount rate (HM Treasury, 2003), this produces a value of just under £2.3 billion. The cost to the Scottish Government, through the Scottish Funding Council, of covering the tuition costs for this cohort of Scottish FEC students was £515 million, less than a quarter of the discounted sum of the increase in GDP. From the CGE simulation results it is also possible to calculate the increased tax take in Scotland that the expansion in economic activity would generate.\(^{20}\) The present value of the increase in indirect and employment tax receipts is £764 million.

\(^{20}\) Of course, although the Scottish Government decides on the level of Further and Higher Education spending in Scotland, at present it receives a very small share of its income in locally raised taxes. However, this will change with the implementation of the Scotland Act (2012) and the adjustments to
5.1 Regional Openness

We have labelled the region as open, with finance capital perfectly mobile and exports taking a large share of output. However, there is no migration in the model, which has two distinct aspects. First, all FEC students taught in Scotland are assumed to remain in Scotland for their working life. Second, there is no endogenous migration generated by any subsequent change in economic activity. It is useful to discuss the implications of openness for the estimated impacts.

In the simulation reported in Figure 1 and Table 4 firms face an infinitely elastic supply of finance. However, an indication of the implication of a more restricted capital supply can be identified in the results for period 1 shown in Table 4. In this period, capital stocks are fixed. Note that the GDP increase is just over half the value in period 30 where the capital stock is fully adjusted, and period-1 employment actually falls. Similarly, for export elasticities, extensive work using the AMOS model suggests that with a supply-side improvement, increasing the trade elasticities increases the GDP and employment impact, as exports and import substitution respond to increased competitiveness.

For labour mobility we begin with the assumption that all those qualified from Scottish FECs are retained, essentially because we have no information about the mobility of FEC students. In a similar analysis for HE graduates in Scotland, evidence suggests that up to 20% of graduates out-migrate and the economy-wide implications are investigated through sensitivity analysis (Hermansson et al., 2014). In that analysis the qualitative impacts do not change but their magnitude is proportionately reduced. We would expect a similar result for FE graduates but for the rates of outmigration to be much lower than those for HEIs, given the nature of the students and qualifications.

We also assume that there is no endogenous migration generated by the change in activity following the productivity shock. This is because such migration would

Scotland’s devolved fiscal position resulting from the recommendations of the Smith Commission (2014).
endogenously change the characteristics of the labour supply in a way in which we presently cannot model.\textsuperscript{21} However, we know that the impact on Scottish GDP and employment would be increased if migration were allowed. The real wage change would be closer to zero in period 30, generating lower unit costs and thereby increasing regional competitiveness.

6. Conclusions
There are a wide range of potential benefits that can accrue from the education provided by Higher and Further Education Institutions. Many of these benefits are both difficult to measure and value. We therefore do not attempt anything approaching a cost benefit analysis here.\textsuperscript{22} Nevertheless, a fundamental contribution of educational institutions is the increased skills that they generate and improvement in human capital has been identified as an important element in regional and national development strategies. In this paper we are particularly interested in quantifying the role of FECs in human capital formation in Scotland and the subsequent impact on the Scottish economy.

There are numerous studies that link an individual’s human capital to his or her wage income. However, the relationship between education and macro-economic performance is less straightforward to determine and empirical results are much more varied. The key contributions of this paper is to use a micro-macro Computable General Equilibrium method which builds on, but goes beyond, the conventional growth accounting approach.

The empirical results have important policy relevance. FECs are known to play a valuable role in improving accessibility and equity in post-school education in Scotland (Sutton, 2012). However, their overall contribution to human capital formation has been difficult to measure and therefore under-appreciated. This paper shows that in Scotland, FECs are an important source of additional human capital. The total impact on Scottish output of the qualifications received from FECs is around 70\% of the impact of HEIs, with a lower cost per unit of improvement.

\textsuperscript{21} Essentially the size of the efficiency change would vary endogenously with the extent of migration.

\textsuperscript{22} There is a literature that identifies various non-monetary benefits of education to its recipient, as well as the wider monetary and non-monetary impacts of education on society as a whole (see, for example, McMahon, 2004, 2009 and Hermannsson et al, 2016).
Further, the paper quantifies the subsequent impact on the economic activity of the region. In this particular case, the standard growth accounting method would underestimate the GDP impact by 70%. The CGE simulation results suggest that one year’s output from Scottish FECs generates a 0.126% increase in GDP over a number of decades, equivalent to a present value of just under £2.3 billion.

The primary aim of the analysis in this paper is to provide an indication of the regional economic impact of the human capital generated by FECs. As far as we are aware, this is the first attempt to provide such information for Scotland. This is the first step in such an analysis, which has required a number of simplifying assumptions to be adopted. Data was only available for a single cohort, but future work would benefit from a consistent and systematic time series of data for the qualifications generated in the Scottish FEC sector as a whole with more detailed information concerning the gender balance, mobility and domicile status of its students. Furthermore, it would be desirable to consider the interaction of initial qualification levels and the subsequent impact of learning by doing on labour productivity and also a greater disaggregation of the work force. This would allow a more nuanced treatment of the impact of different types of qualifications and how qualifications interact with experience to vary the wage premium of an individual over time. Furthermore, it would be useful to extend the model by endogenising the wage premia associated with particular qualifications. This would make it possible to model changes in wage premia driven by anticipated future changes in labour supply and/or labour demand. Overall, however, we don't expect such refinements to change the qualitative findings of this paper, but they could inform more precisely the timing of impacts and their sensitivity to potential changes elsewhere in the economy.
Appendix 1: Derivation of the expressions for q and n

Substituting equation (2) in the text into (3) in the text gives:

(A1.1) \( w_E = \beta n - \rho \)

Substituting equation (5) in the text into (7) in the text produces:

(A1.2) \( q - n_E = \sigma w_k (1 - \alpha) \)

Substituting equations (4) and (7) in the text into (A1.2) to eliminate q and rearranging produces:

(A1.3) \( n_E = -w_E \left( \sigma(1 - \alpha) + \eta \alpha \right) \)

Substituting equation (1) in the text into (A1.3) and rearranging gives:

(A1.4) \( w_E = -\frac{n + \rho}{\sigma(1 - \alpha) + \eta \alpha} \)

Substituting (A1.4) into (A1.1) and rearranging produces the expression for n:

(A1.5) \( n = \frac{\rho(\sigma(1 - \alpha) + \eta \alpha - 1)}{\beta(\sigma(1 - \alpha) + \eta \alpha) + 1} \)

Combining equations (2), (4) and (7) in the text gives:

(A1.6) \( q = -\eta \alpha (w - \rho) \)

Substituting equation (3) in the text into (A1.6) then produces:

(A1.7) \( q = \eta \alpha (\rho - \beta n) \)
Then substituting equation (A1.5) into (A1.7) and rearranging produces an expression for the value of \( q \).

\[(A1.8) \quad q = \alpha \rho \frac{\eta(\beta + 1)}{\beta(\sigma(1 - \alpha) + \eta \alpha) + 1}\]

Substituting equation (4) into equation (6) in the text to eliminate \( q \) gives:

\[(A1.9) \quad k = (\sigma - \eta)p\]

Substituting equation (7) in the text and equation (A1.9) produces:

\[(A1.10) \quad k = \alpha(\sigma - \eta)w_E\]

Then substituting equation (A1.4) into equation (A1.10) gives an expression for \( k \).

\[(A1.11) \quad k = \frac{\alpha \rho (1 + \beta)(\eta - \sigma)}{\beta(\sigma(1 - \alpha) + \eta \alpha) + 1}\]

Differentiating expressions (8), (9) and (10) with respect to \( \eta \) and \( \beta \) gives:

\[(A1.12) \quad \frac{\partial^2 n}{\partial \rho \partial \eta} = \frac{\alpha(1 + \beta)}{(\beta(\sigma(1 - \alpha) + \eta \alpha) + 1)^2} > 0\]

\[(A1.13) \quad \frac{\partial^2 n}{\partial \rho \partial \beta} = -\frac{(\sigma(1 - \alpha) + \eta \alpha - 1)(\sigma(1 - \alpha))}{(\beta(\sigma(1 - \alpha) + \eta \alpha) + 1)^2} < 0 \quad \text{iff} \quad \sigma(1 - \alpha) + \eta \alpha > 1\]

\[(A1.14) \quad \frac{\partial^2 k}{\partial \rho \partial \eta} = \frac{\alpha(1 + \beta)(\beta \sigma + 1)}{(\beta(\sigma(1 - \alpha) + \eta \alpha) + 1)^2} > 0\]

\[(A1.15) \quad \frac{\partial^2 k}{\partial \rho \partial \beta} = \frac{\alpha(\eta - \sigma)(1 - \sigma(1 - \alpha) - \eta \alpha)}{(\beta(\sigma(1 - \alpha) + \eta \alpha) + 1)^2}\]
\[ \frac{\partial^2 q}{\partial \rho \partial \eta} = \frac{\alpha(\beta + l)(\beta \sigma(1 - \alpha) + 1)}{(\beta(\sigma(1 - \alpha) + \eta \alpha) + l)^2} > 0 \]

\[ \frac{\partial^2 q}{\partial \rho \partial \beta} = \frac{\alpha \eta(l - \sigma(1 - \alpha) + \eta \alpha)}{(\beta(\sigma(1 - \alpha) + \eta \alpha) + l)^2} < 0 \quad \text{iff} \quad \sigma(1 - \alpha) + \eta \alpha > 1 \]
References


